Star Cluster Dynamics and Evolution





Geoplanet Doctoral School Lecture Course (Spring 2024) MOCCA

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*** Growing Black Holes in Star Clusters* * * *





Long-term survival of black holes in globular clusters





Black hole dynamics in globular clusters

- Black holes segregate to the center of the cluster \rightarrow interact with each other and surrounding stars
- Chaotic binary-single and binary-binary interactions involving black holes
- Formation of binary black holes through exchange encounters

Shrinking of binary black holes through interactions \rightarrow can merge due to gravitational wave radiation within a Hubble time (more in next part of the lecture)





3-body encounters in stellar clusters

- For a bound binary system: $E_{int} = -\frac{G m_1 m_2}{2 a} = -E_b$
- Binary shrinking/hardening:
- Single star extracts internal energy of the binary (stars final kinetic energy is higher than initial) - Flyby
- Binary softening or lonization:
- Single star transfers kinetic to the binary
- Exchange interaction:
 - Single star can replace one of the binary companions

Heggie/Hill's (1975) Law: In 3-body encounters hard binaries become harder and soft binaries tend to become softer



Credit: Michela Mapelli Lecture notes on Collisional Dynamics



Black hole dynamics in globular clusters

- Dynamical interactions also eject tight binary black holes out of the cluster due to dynamical recoil (scattering kick)
- Black hole population in clusters depletes with time \rightarrow depletion time depends on cluster initial properties
 - Black hole subsystem heats surrounding stars (Mackey et al. 2007;2008, Breen & Heggie 2013)
 - Initially dense clusters \rightarrow more interactions \rightarrow faster depletion of black holes
 - Less dense clusters \rightarrow fewer interactions \rightarrow slower depletion of black holes
- Initially dense clusters \rightarrow have shorter initial 2-body relaxation times \rightarrow dynamically older \rightarrow important for producing binary black holes (next part of the lecture)





Credit: Breen & Heggie (2013)

Binary Hardening in Fly-by Encounters						
		II		IV		
sn			BH subsystem	Restored balanced		
ore Radi	BH subsystem formation	Balanced		/		
Ŭ		evolution Core bounce	ļ	Second core		
		in BH subsystem	n 	bounce		

MOCCA simulations of 3 GC models: Mass evolution

N = 700,000, Initial binary fraction (IBF) = 10%, Z = 0.05 Z_{\odot} , BHs kicks scaled by mass (Belczynski et al. 2002)

 $0.08 \text{ M}_{\odot} \leq M_{ZAMS} \leq 100 \text{ M}_{\odot}$ (Kroupa 2001 IMF) 450000 Belczynski et al. 2002 (for BH masses) Number of BH progenitors ~ 1900



Simulating the Evolution of Globular Clusters - Abbas Askar Geoplanet Doctoral School - PhD Lecture Course



MOCCA simulations of 3 GC models: Core radius evolution

N = 700,000, Initial binary fraction (IBF) = 10%, Z = 0.05 Z_{\odot} , BHs kicks scaled by mass (Belczynski et al. 2002)

 $0.08 \text{ M}_{\odot} \leq M_{ZAMS} \leq 100 \text{ M}_{\odot}$ (Kroupa 2001 IMF) Belczynski et al. 2002 (for BH masses) Number of BH progenitors ~ 1900



Early Core contraction can be seen in some models

Models from MOCCA Survey Database I (Askar et al. 2017)

Core Radius [pc]

Core expansion driven by black holes

Evolution of black holes in globular clusters

N = 700,000, Initial binary fraction (IBF) = 10%, Z = 0.05 Z_{\odot} , BHs kicks scaled by mass (Belczynski et al. 2002)

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- Dynamically young clusters with low initial densities will retain more black holes up to a Hubble time
- Also see: Morscher (2015), Wang et al. (2016), Kremer et al. (2018; 2020), Arca Sedda et al. (2018), Askar et al. (2018), Askar et al. (2019), Weatherford et al. (2018;2020)

Evolution of black holes in globular clusters

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Black holes and observable properties of globular clusters

N = 700,000 Initial binary fraction (IBF) = 10% $Z = 0.05 Z_{\odot}, W_0 = 6, r_h = 2.4 \text{ pc}, r_t = 120 \text{ pc}$ $0.08 \text{ M}_{\odot} \leq \text{M}_{ZAMS} \leq 100 \text{ M}_{\odot}$ (Kroupa 2001 IMF)

MOCCA Simulations of Globular Clusters

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Dragon Simulations: Black holes in globular clusters

Different populations of objects at 12 Gyrs – Wang et al. 2016 (NBODY6++GPU) : Initial half-mass radius 7 pc \rightarrow long half-mass relaxation time \rightarrow BH Natal Kicks (Belczynski et al. 2002)

Observations of luminous stars simulated with COCOA (Askar et al. 2018)

brightness

DRAGON 2 (D2-R7-IMF01)

Hydra GPU Cluster, Garching. Germany

dissolve faster than models which kick out most BHs (high natal kicks; no fallback)

MOCCA Simulations of Tidally-Filling Globular Cluster Models (Giersz et al. 2019)

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Star cluster models in which we retain a larger number of BHs (reduced natal kicks; fallback)

Black Hole Subsystem (BHS) survives up to cluster dissolution time

• BHS evolution has a strong influence on the final stages of cluster dissolution

dissolve faster than models which kick out most BHs (high natal kicks; no fallback)

• Dissolution also in direct N-body simulations

Star cluster models in which we retain a larger number of BHs (reduced natal kicks; fallback)

Disruption time depends on the number of objects and initial concentration

• Large N clusters that are more concentrated take a longer time to dissolve

- For tidally-filling star cluster, the BHS evolves very slowly due to smaller overall relaxation time
- % of the cluster mass

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For tidally-underfilling models. The evolution of the BHS is much faster and BHS properties change quickly Cluster dissolution stars when the cluster mass is about 20% of its initial mass and the BHS is about a few

Giersz et al. 2019

- BHs segregate and form a subsystem at about 1 Gyr, but mass segregation is going on up to about 3 Gyr
- Balanced evolution up to 7 Gyr \rightarrow loss of equilibrium
- BHS seems to decouple from the rest of the clusters and dissolves.

- The BHS evolves according to Breen & Heggie (2013) theory - balanced evolution with the whole cluster
- The BHs in BHS are slowly "kicked out" on the half-mass relaxation time scale.

- Cluster dissolution for a tidally-filling model is connected with the decoupling of the BHS from the rest of the cluster and the loss of dynamical equilibrium by other objects → luminous star become hot
- BHS stops collapsing further at when cluster mass fraction is equal to about 0.2 - slow increase of the BHS Lagrangian radii → The BHS starts is disrupting itself

- Cluster is constantly losing mass due to tidally stripping →
 escape velocity becomes smaller as mass decreases →
 more objects can be removed from the cluster due to BHS
- Lower velocity dispersion in the core results in higher interaction rate and a larger escape rate
- Cluster loses its dynamical equilibrium and dissolves

Key Points

- Initial black hole retention in globular clusters depends on the natal kicks that they receive (uncertain)
 - Black hole properties depend on the evolution of its progenitor star (winds, exact supernoval mechanism and their dependence on initial parameters)
- Black holes segregate to the center of the cluster where they can dynamically interact with other black holes and stars
 - Long term survival of black holes in globular clusters is determined by the initial properties of the stellar cluster
 - Initially dense clusters \rightarrow more interactions \rightarrow faster depletion of black holes
 - Less dense clusters \rightarrow fewer interactions \rightarrow slower depletion of black holes
- Black holes retained in stellar clusters heat surrounding stars and influence the observable properties of the globular cluster, e.g. core radius, half-light radius and central surface brightness
- Black holes can lead to the dissolution of tidally filling clusters

Gravitational wave detections of binary black holes

- ~83 merging binary black holes detected by LIGO-Virgo-KAGRA (01, 02, 03a, 03b)
 - Observed merger rate: ~ 18 44 Gpc⁻³ yr⁻¹(Abbott et al. 2020)
- 2 merging neutrons stars and 5 BH-NS merger
- Key Question:
 - What is the astrophysical origin of these gravitational wave sources?
- Possible Answers:
 - Isolated binary evolution
 - Dynamical formation in dense stellar environments
 - Mergers in field triples, accretion discs of active galactic nuclei
 - Primordial black holes?

Image Credit: LIGO-Virgo-KAGRA/Northwestern Univ./Aaron Geller

Gravitational Wave Sources: Binary Black Holes

Black holes segregate to the center of the cluster → interact with each other and surrounding stars

- Chaotic binary-single and binary-binary interactions involving black holes
 - Formation of binary black holes through exchange encounters
 - Mergers can occur during these interactions (Samsing 2018, Samsing, Askar, Giersz 2018, Rodriguez et al. 2018 a,b)
- Hardening of binary black holes through interactions \rightarrow binary becomes 'useful' \rightarrow can merge due to gravitational wave radiation within a Hubble time

$$\tau_{\rm gr} \simeq 10^{10} yr \left(\frac{a_{\rm bin}}{3.3R_{\odot}}\right)^4 \frac{1}{(m_1 + m_2)m_1m_2} \cdot \left(1 - e^2\right)^{7/2}$$
 (Pet

Gravitational Wave Sources: Binary Black Holes

- Dynamical interactions also eject tight binary black holes out of the cluster due to dynamical recoil (scattering kick)
 - Can merge due to gravitational wave emission outside the cluster
- Black hole population in clusters depletes with time \rightarrow depletion time depends on cluster initial properties
 - Black holes heat surrounding stars (Mackey et al. 2007;2008, Breen & Heggie 2013)
 - Initially dense clusters \rightarrow more interactions \rightarrow faster depletion of black holes
 - Less dense clusters \rightarrow fewer interactions \rightarrow slower depletion of black holes
- Initially dense clusters that are dynamically older produce more 'useful' binary black holes

Credit: Breen & Heggie (2013)

Evolution of black holes in globular clusters

N = 700,000, Initial binary fraction (IBF) = 10%, Z = 0.05 Z_{\odot} , BHs kicks scaled by mass (Belczynski et al. 2002)

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Escaping binary black holes and their properties

N = 700,000, Initial binary fraction (IBF) = 10%, Z = 0.05 Z_{\odot} , BHs kicks scaled by mass (Belczynski et al. 2002)

Models from MOCCA Survey Database I (Askar et al. 2017)

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Dynamical formation of a binary black hole

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• 2 single black holes form in the cluster from the evolution of massive stars

- Both end up in 2 different binaries following numerous dynamical interactions
- Form a binary after a binary-binary exchange interaction and are ejected from the cluster
- Will merge outside the cluster after 208 Myr since the beginning of cluster evolution
- An example of a dynamically formed BBH from Askar et al. (2017) Based on interaction diagrams first presented in Rodriguez et al. (2016)

- $t_{\rm merg} = t_{\rm esc} + t_{\rm GW}$ Peters (1964)
- $t_{\rm merg} = 145 + 63$
- $t_{\rm merg} = 208 \,\,{\rm Myr}$

Producing binary black holes in globular clusters

- Simulated 2000 GC models with different initial parameters as part of the MOCCA-Survey Database I (Askar et al. 2017)
 - Black hole natal kicks computed according to the mass fallback prescription given by Belczynski et al. (2002) \rightarrow **1007 GC models**
 - 9000 Escaping BBHs (Total: 17,121) • Systematically search for merging binary black holes that escape of 8000 BBHs Inside GC Models (Total: 3,435) 7000 merge inside the cluster Merging BBHs 6000 17,121 'useful' BBHs escaped the cluster 5000 4000 3,435 BBHs merged inside the cluster within a Hubble time Number 3000 2000 Most mergers inside the cluster occur within the first 500 Myr of 1000 cluster evolution 2000 0 4000 6000 8000 10000 12000 Escapers can contribute to binary mergers at later times Merger Time [Myrs]

Models from MOCCA Survey Database I (Askar et al. 2017)

Merger rates for binary black holes from globular clusters

- Estimated local merger rate density as done for isolated field BHs (Bulik, Belczynski & Rudak 2004).
 - GC star formation rate as a function of redshift (Katz & Ricotti 2013) - Peak in GC Formation at about redshift (z) of 3
- Local merger rate density of BBHs originating from GCs: 5.5 Gpc⁻³ yr⁻¹ (Askar et al. 2017)
- Consistent with independently calculated rates by Rodriguez et al. (2016), Park et al. (2017)
- Rodriguez & Loeb (2018) \rightarrow 15 Gpc⁻³ yr⁻¹

Differential rate density per unit chirp mass Updated Fig. 4 from Askar et al. (2017) Credit: M. Szkudlarek

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- Rodriguez & Loeb (2018) \rightarrow 15 Gpc⁻³ yr⁻¹
- Open question: How much star formation took place in globular clusters?
- Currently $\sim 0.1 1\%$ of galaxy stellar mass is in globular clusters (Harris et al. 2014)
- May have been $\geq 10\%$ at z > 3 (Muratov & Gnedin 2010)

Rodriguez & Loeb (2018)

Binary black hole production and cluster properties

Number of merging binary black holes as a function of initial cluster mass (M_0), average density (ρ_h) and binary fraction ($f_{h,0}$) (Hong, Vesperini, Askar et al. 2018)

Hong, Vesperini, Askar et al. (2018) Hong, Askar et al. (2020)

Eccentric binary black holes mergers in clusters

- Non-negligible probability of experiencing a very close passage during a resonant encounter
- Significant orbital energy and angular momentum are carried away from the system by gravitational wave radiation \rightarrow can result in rapid, highly-eccentric black hole mergers (e > 0.1)
- Rate of such capture mergers: 0.5 2 Gpc⁻³ yr⁻¹ see Samsing (2018), Samsing, Askar, Giersz (2018), Rodriguez et al. (2018) a,b)
- Very rarely single black holes may also capture each other and merge (Samsing et al. 2020)
- Hierarchical three-body mergers (Samsing & Ilan 2018, Veske et al. 2020)
- For eccentric mergers during binary-binary interactions, see Zevin et al. (2018)
- See also contribution from triple systems (Antonini, Toonen & Hamers 2017)

Credit: Samsing et al. (2020)

GW190521 and LVK observations of black holes in the upper-mass gap

- Pair and pulsational pair instability supernovae prevent formation of black holes with masses in the range: ~ 50^{+20}_{-10} – 120 M_{\odot} → upper mass gap of black holes
- LVK Observations of massive stellar-mass black holes:

LVK Merger Event	Primary Mass $[M_{\odot}]$	Secondary Mass $[M_{\odot}]$	Effective Spin $\chi_{\rm eff}$	Luminosity Distance (Gpc)	Redshift (z)
GW190521_030229	$95.3^{+28.7}_{-18.9}$	69 ^{+22.7} -23.1	$0.03^{+0.32}_{-0.39}$	$6.1^{+4.9}_{-3.1}$	$0.64^{+0.28}_{-0.28}$
GW190403_051519	$88^{+28.2}_{-32.9}$	$22.1^{+23.8}_{-9.0}$	$0.70^{+0.15}_{-0.27}$	$8.00^{+5.99}_{-3.99}$	$1.14^{+0.64}_{-0.49}$
GW190426_190642	$106.9^{+41.6}_{-25.2}$	$76.6^{+26.2}_{-33.6}$	$0.19^{+0.43}_{-0.40}$	$4.35^{+3.35}_{-2.15}$	$0.70^{+0.41}_{-0.30}$
GW200220_061928	87^{+40}_{-23}	61^{+26}_{-25}	$0.06^{+0.40}_{-0.38}$	$6.1^{+4.9}_{-3.1}$	$1.14^{+0.64}_{-0.49}$

Image Credit: Lucy Reading-Ikkanda/Quanta Magazine

Data from GWTC-2.1 and 3 (Abbot et al. 2020; 2021) https://www.gw-openscience.org/

- Two sBHs (1G) merge due to gravitational wave (GW) emission and form a more massive BH (2G)
- In a dense stellar cluster, this merged BH (2G) can pair up and merge with another BH (1G or 2G)
- Most straightforward way for growing BHs and one of the proposed formation channels for GW events like GW190521

Rodriguez et al. (2019; 2020), Arca Sedda et al. (2020; 2021), Fragione et al. (2020) Kremer et al. (2020), Samsing & Hotokezaka (2020), di Carlo et al. (2020), Dall'Amico et al. (2021), Mapelli et al. (2021), Banerjee (2022)

Repeated BH could lead to the runaway growth of an IMBH $\sim 10^2 - 10^4 M_{\odot}$

Miller & Hamilton (2002); Mouri & Taniguchi (2002); Portegies Zwart & McMillan (2002)

Gerosa & Berti (2017)

Image credit: LIGO/Caltech/MIT/R. Hurt (IPAC)

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- Most straightforward way for growing BHs
- Problem: Can be difficult to retain a merged BH in a dense environment due to GW recoil kicks

(e.g., Merritt et al. 2004; Holley-Bockelmann et al. 2008)

- If GW recoil kick magnitude is larger than the escape speed of the cluster then merged BH will escape
- Magnitude of GW depends on mass ratio of merging BH and the magnitude and orientation of their spins

 Magnitude of GW recoil kick depends on mass ratio of merging BHs and the magnitude and orientation of their spins

- If sBH birth spins are low then 2G BHs can potentially be retained in environments like globular clusters
 - receive large recoil kicks \rightarrow harder to retain 3G and 4G BHs
- 2020; Fragione et al. 2022)

Assuming isotropic spin directions GW recoil kicks calculated using van Meter (2010)

Morawski et al. (2018)

• 2G BHs are likely to have to have large spins values (close to 0.7) \rightarrow 2G+1G and 2G+2G merger products will

• Better chances for retaining merged BHs in NSCs due to higher escape velocities (Gerosa & Berti 2019; Antonini et al.

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Birth spins of BHs are highly uncertain!

Hierarchical mergers of black holes in stellar clusters: birth spins?

- Highly uncertain:
- Depends on the efficiency of angular momentum transport during the evolution of the progenitor
- Very efficient angular momentum transport from core to envelope \rightarrow very low birth spins for black holes (Fuller & Ma 2019)

orbit misalignment

Fig.1 from Rodriguez et al. 2016

Hierarchical mergers of black holes in stellar clusters: birth spins?

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Fig.1 from Rodriguez et al. 2016

- LVK observations strongly suggest low spin magnitudes and isotropic distribution of spin-orbit misalignment angle
- Consistent with dynamical formation

orbit misalignment

Hierarchical mergers of black holes in stellar clusters

- For 1G birth spins close to $0 \rightarrow 60\%$ of second generation BHs will be retained in the cluster
- For 1G birth spins of $0.5 \rightarrow 3\%$ of second generation BHs will be retained in the cluster

Results from MOCCA simulations

Mergers in the mass gap

N = 2.5 million stars (1.4 × 10⁶ M $_{\odot}$) between 0.08 M $_{\odot} \le$ M_{ZAMS} \le 150 M $_{\odot}$

Z = 0.05 Z $_{\odot}$ (1 model with Z = 0.01 Z $_{\odot}$)

 R_h = 0.8 pc (ρ_0 = 4 \times 10⁶ M $_{\odot}$ pc⁻³) and 2 pc (ρ_0 = 2.5 \times 10⁵ M $_{\odot}$ pc⁻³) Initial binary fraction 5% and 25%

Updated treatment for stellar winds, natal kicks and remnant masses (Kamlah et al. 2022)

Birth Spins of Black Holes = 0.1 (Fuller & Ma 2019)

GW Recoil Kicks Included

BHs in the mass gap are mostly 1G+2G Mergers Few 2G+2G mergers

Maximum black hole mass from stellar evolution depends on metallicity and prescriptions for progenitor evolution \rightarrow up to 45 M $_{\odot}$

(Belczynski et al. 2016 ; Banerjee et al. 2020)

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Binary black holes in open/young stellar clusters

- Higher fraction of star formation takes place in open and young clusters compared to globular clusters
- Formation not limited to a given cosmic epoch
- More efficient at producing binary black holes at higher metallicities compared to isolated binary evolution
 - 90% of the mergers take place outside the cluster (Di Carlo et al. 2020)
 - Inclusion of post-Newtonian terms could lead to more in cluster mergers (Banerjee 2017; 2020)
- Produce more low mass ratio mergers
- Local merger rates of binary black holes originating in young stellar stellar: $50 100 \text{ Gpc}^{-3} \text{ yr}^{-1}$ (Di Carlo et al. 2020)

Di Carlo et al. (2020)

Binary black hole in nuclear stellar clusters

- Nuclear star clusters (NSCs) are extremely dense ($\sim 10^6 - 10^7 \text{ M}_{\odot} \text{ pc}^{-3}$) and massive star clusters ($\sim 10^6 - 10^9 \text{ M}_{\odot}$) occupying the nucleus of most galaxies.
- Formation Scenarios:
 - Globular cluster infall and mergers (Tremaine et al. 1975)
 - In-situ star formation (Loose et al. 1982)
- Most host supermassive black holes
- Tens of thousands of stellar mass black holes (Hailey et al. 2018)
- Presence of a supermassive BH can pump up eccentricity of stellar mass BBHs via Kozai - Lidov mechanism
- Local Merger rate: 3 8 Gpc⁻³ yr⁻¹ (Arca Sedda 2020) & 2 - 25 Gpc⁻³ yr⁻¹ (Rodriguez & Antonini 2018)

Credit: Lucy Reading-

Arca Sedda (2020)

Dense stellar environments and binary black holes

	Open Clusters & Young Massive Clusters		Globular Clusters	Nuclear Star Cluste
		<image/>		NGC205
Mass	$100-\lesssim 10^4~{ m M}_\odot$	$10^4 - 10^5 {\rm ~M}_{\odot}$	$10^4 - 10^6 {\rm ~M}_{\odot}$	$10^5 - 10^8 { m M}_{\odot}$
Radius	~ 1 – few pc	~ 1 – 10 pc	~ 10 – 30 pc	~ 1 – 5 pc
Central Density	$\lesssim 10^3 {\rm ~M}_{\odot} {\rm ~pc}^{-3}$	$\gtrsim 10^3 {\rm M}_{\odot} {\rm pc}^{-3}$	$\gtrsim 10^4 - 10^5 \mathrm{M}_{\odot} \mathrm{pc}^{-3}$	$10^5 - 10^7 \ \mathrm{M_{\odot} \ pc^{-3}}$
Ages	~ 1 Myr to few Gyr	a few to $\lesssim 100 \text{ Myr}$	≳ 8 – 13 Gyr	Age Spread
Local Merger Rates for Binary Black Holes	~ 50 - 100	Gpc ⁻³ yr ⁻¹	$\sim 5 - 15 \text{ Gpc}^{-3} \text{ yr}^{-1}$	~ 1 – 10 Gpc ⁻³ yr ⁻

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40 pc

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Radius	~ 1 – few pc	~ 1 – 10 pc	~ 10 – 30 pc	~ 1 – 5 pc
Central Density	$\lesssim 10^3 {\rm ~M}_{\odot} {\rm ~pc}^{-3}$	$\gtrsim 10^3 {\rm M}_{\odot} {\rm pc}^{-3}$	$\gtrsim 10^4 - 10^5 \mathrm{M}_{\odot} \mathrm{pc}^{-3}$	$10^5 - 10^7 \ \mathrm{M_{\odot} \ pc^{-3}}$
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40 pc

Fast runaway: Stellar collisions resulting in IMBH formation

- BH progenitors can quickly (≤ 10 Myr) segregate to the center in clusters with high initial central densities ($\rho_0 \gtrsim 10^6 \ {\rm M}_{\odot} \ {\rm pc}^{-3}$)
- These massive stars can undergo runaway collisions leadings to the formation of a very massive star (VMS; $10^2 - 10^3 M_{\odot}$)
- VMS could potentially directly collapse into a massive BH seed (more likely to happen if metallicity is low ($Z \leq 10^{-3}$ or 5% Z_{\odot}) since stellar winds are weaker)
- This pathway could lead to the formation of an IMBH ($\sim 10^2 - 10^4 M_{\odot}$) in the densest star clusters Portegies Zwart et al. (1999; 2004); Gürkan et al. 2004, Giersz et al (2015); Mapelli (2016), Reinoso et al. (2018); Alister Seguel et al. (2020); Di Carlo et al. (2021)

Fraction of clusters producing an IMBH ~ 0.10 - 0.40 Models from MOCCA Survey Database I (Askar et al. 2017)

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Initial central of 1200 models from MOCCA Survey Database I (Askar et al. 2017, Hong, Askar et al. 2020, Askar et al. 2023):

Initial Stellar Cluster Density [M_opc⁻³]

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- These massive stars can undergo runaway collisions leadings to the formation of a very massive star (VMS; $10^2 - 10^3 M_{\odot}$)
- VMS could potentially directly collapse into a massive BH seed (more likely to happen if metallicity is low ($Z \leq 10^{-3}$ or 5% Z_{\odot}) since stellar winds are quenched)
- This pathway could lead to the formation of an IMBH $(\sim 10^2 - 10^4 M_{\odot})$ in the densest star clusters Portegies Zwart et al. (1999; 2004); Gürkan et al. 2004, Giersz et al (2015); Mapelli (2016), Reinoso et al. (2018); Alister Seguel et al. (2020): Di Carlo et al. (2021)

Other pathways for growing black holes in dense star clusters

(A) Repeated or hierarchical mergers of stellar-mass BHs

(B) Fast runaway: Stellar collisions resulting in IMBH formation

(C) Slow runaway: Gradual growth of a stellar-mass BH

(D) Binary mergers leading to IMBH formation

Massive stars evolve into black holes

massive star

(E) Gas accretion by stellar-mass BHs

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Possible pathways for growing black hole mass in star clusters

(C) Tidal disruption/collision of stars with black holes

Direct collisions between stars leading to black hole growth

(D) Binary evolution mergers/accretion

Merger of massive stars in binary evolution leading to black hole formation

Merger of a star with a black hole or mass transfer in a binary system leading to the growth of a black hole

(E) Gas accretion by black holes

Accretion of intercluster gas by stellar-mass black holes may result in their growth

Askar et al. 2023

Key ideas

- Dense clusters can efficiently form 'useful' binary black holes through dynamical interactions:
 - Major channel: Exchange during binary-single encounters
 - Binary black holes can be hardened and made 'useful' due to encounters
- Maximum local merger rate contribution from globular and nuclear stellar clusters is $\sim 25 \text{ Gpc}^{-3} \text{ yr}^{-1}$
- If all stars originate in young stellar clusters then their contribution to the local merger rate could be about $\sim 50 - 100 \text{ Gpc}^{-3} \text{ yr}^{-1}$
- Dynamics within dense star clusters can lead to black hole growth \rightarrow formation of IMBH (more on this from Mirek)

