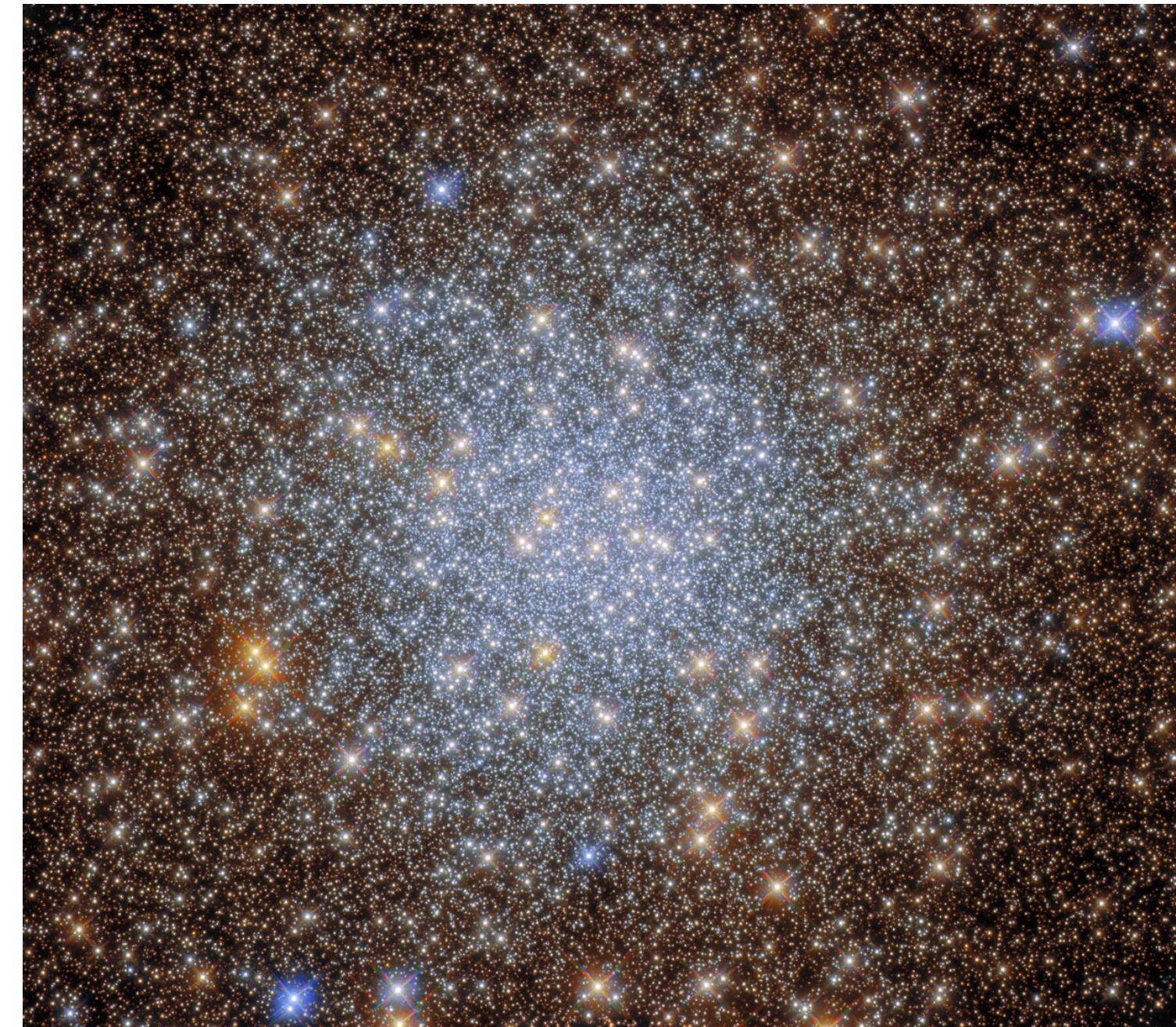
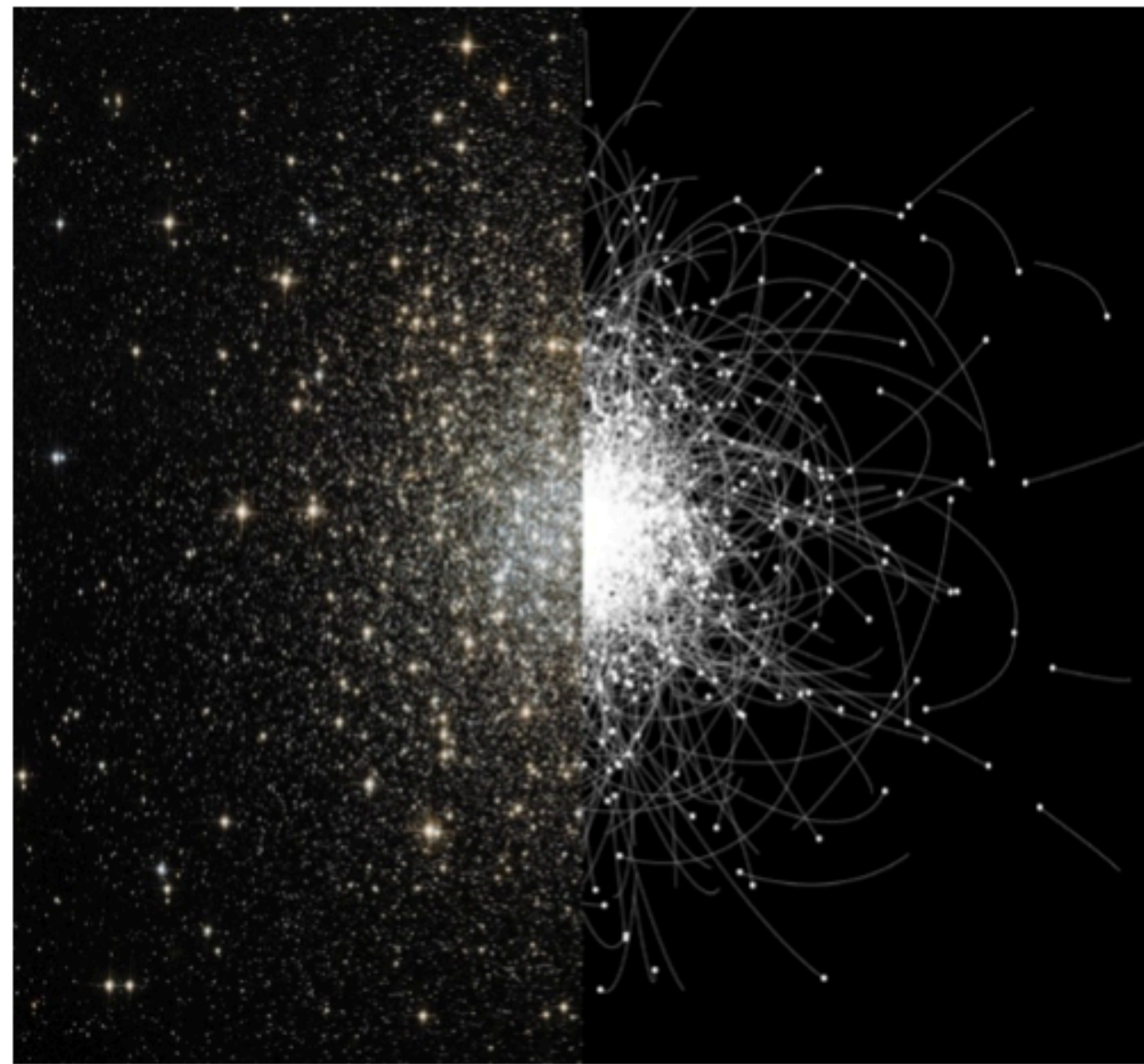


Star Cluster Dynamics and Evolution



Geoplanet Doctoral School Lecture Course (Spring 2024)

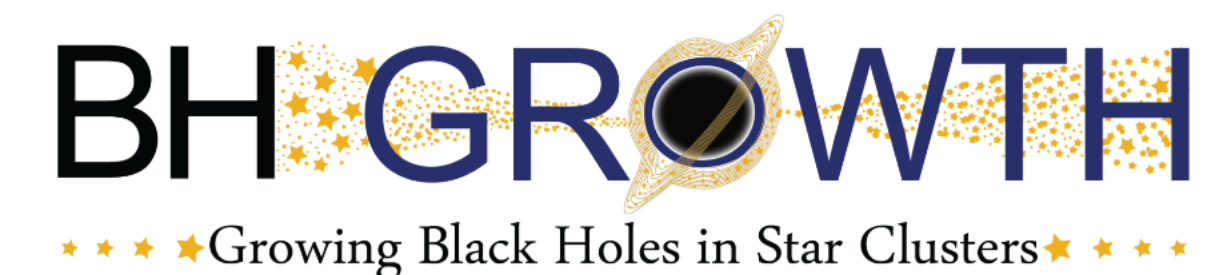
Mirek Giersz & Abbas Askar

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Warsaw, Poland

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askar@camk.edu.pl



Course outline

- Lecture 1: Introduction to star clusters and stellar dynamics
- Lectures 2 - 4: Collisionless and collisional stellar dynamics
- Lecture 4-5 : Direct N -body and Monte Carlo method for evolving star clusters
- Lecture 6: Thermodynamics of stars clusters and simple star cluster models
- Lecture 7: Internal physical processes in star cluster evolution
- Lecture 8: External physical processes in star cluster evolution
- Lectures 9-10: Observations and astrophysical importance of star clusters
- Lecture 10-11: Black holes in star clusters and formation of gravitational wave sources
- Lecture 12: Summary lecture with key takeaways + instructions on assignment

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What happens to stellar-mass black holes in clusters?

- Black hole formation, retention and growth within stellar clusters
- Role black holes play in cluster evolution
- Formation of gravitational wave sources in globular clusters

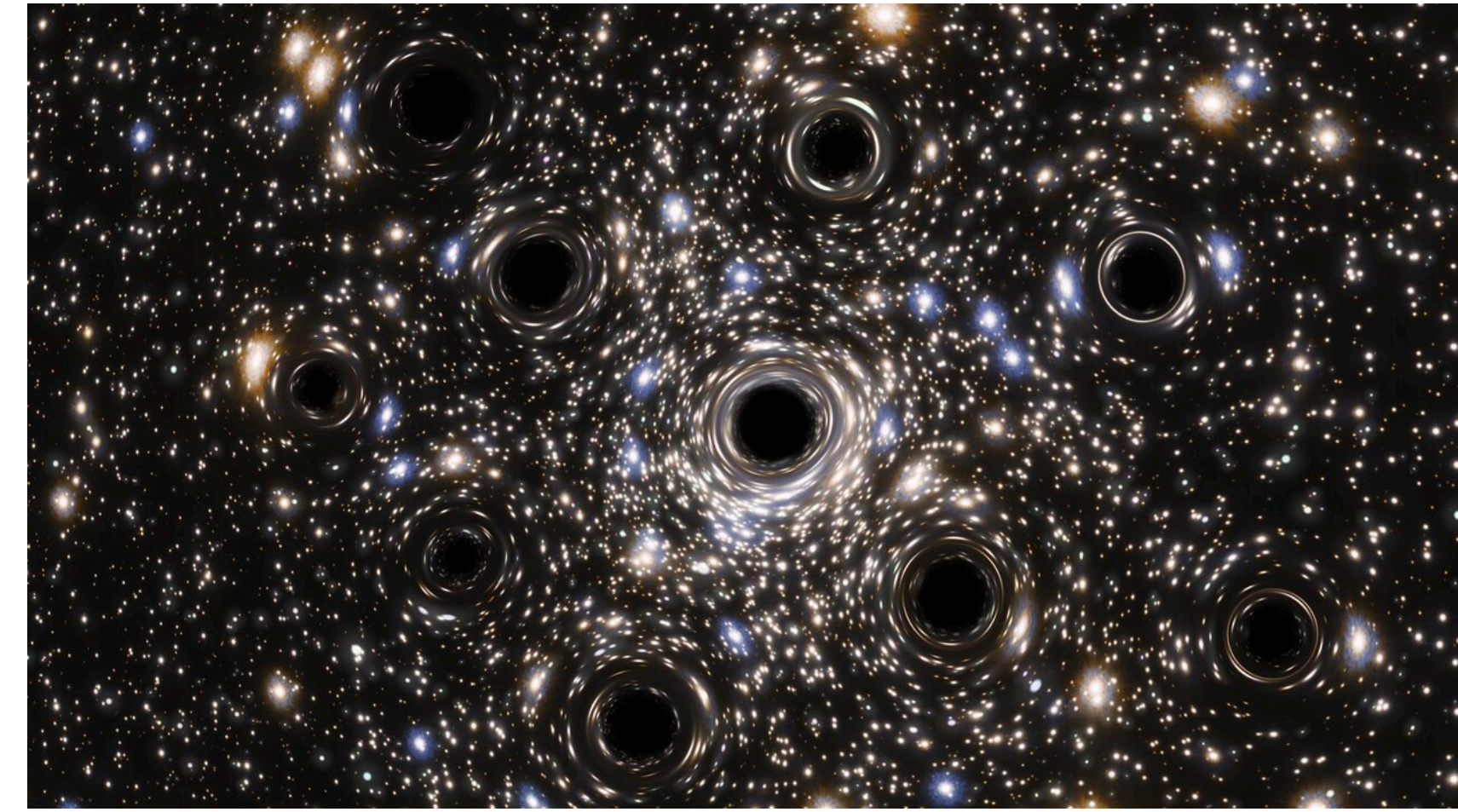
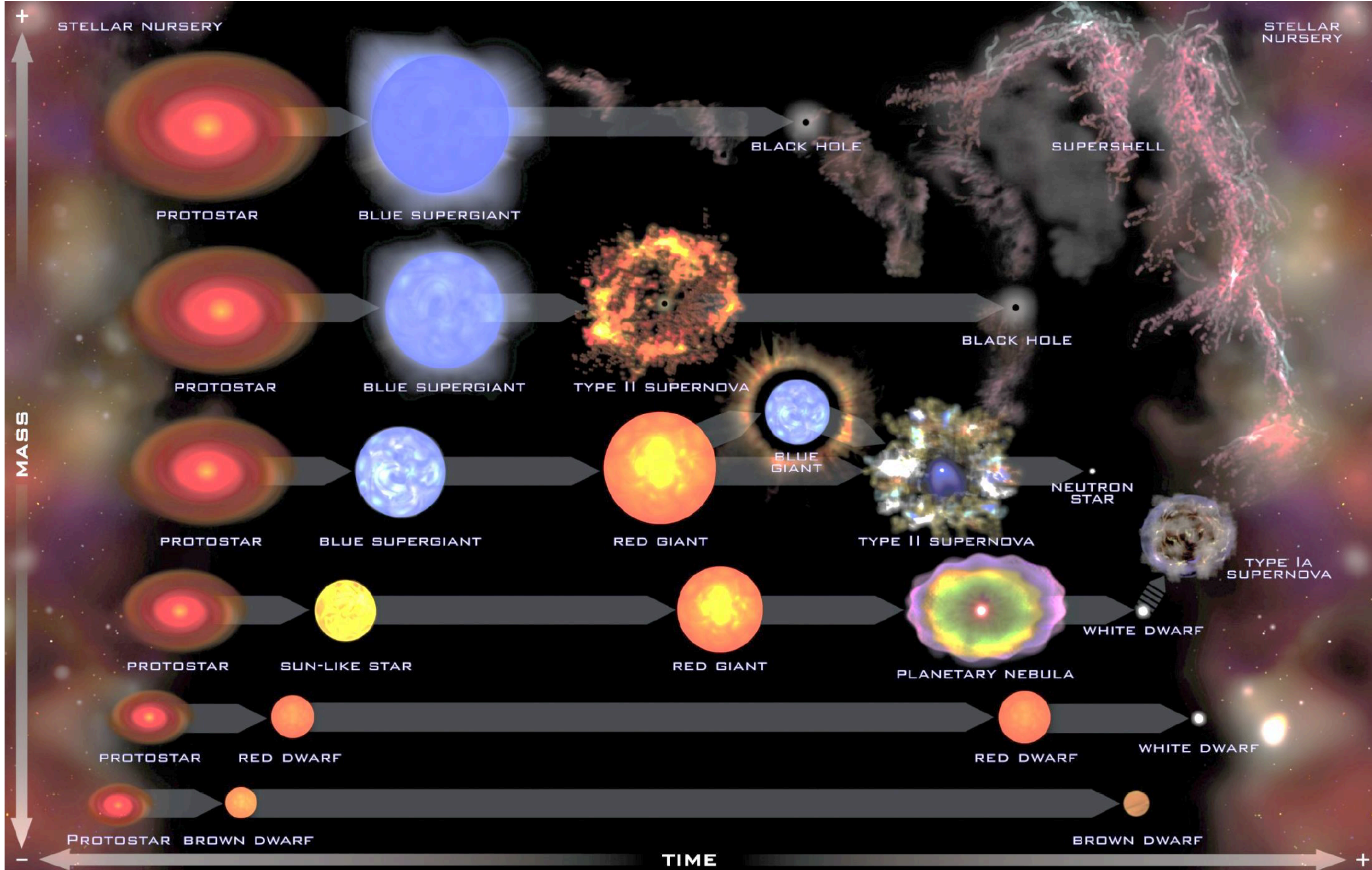


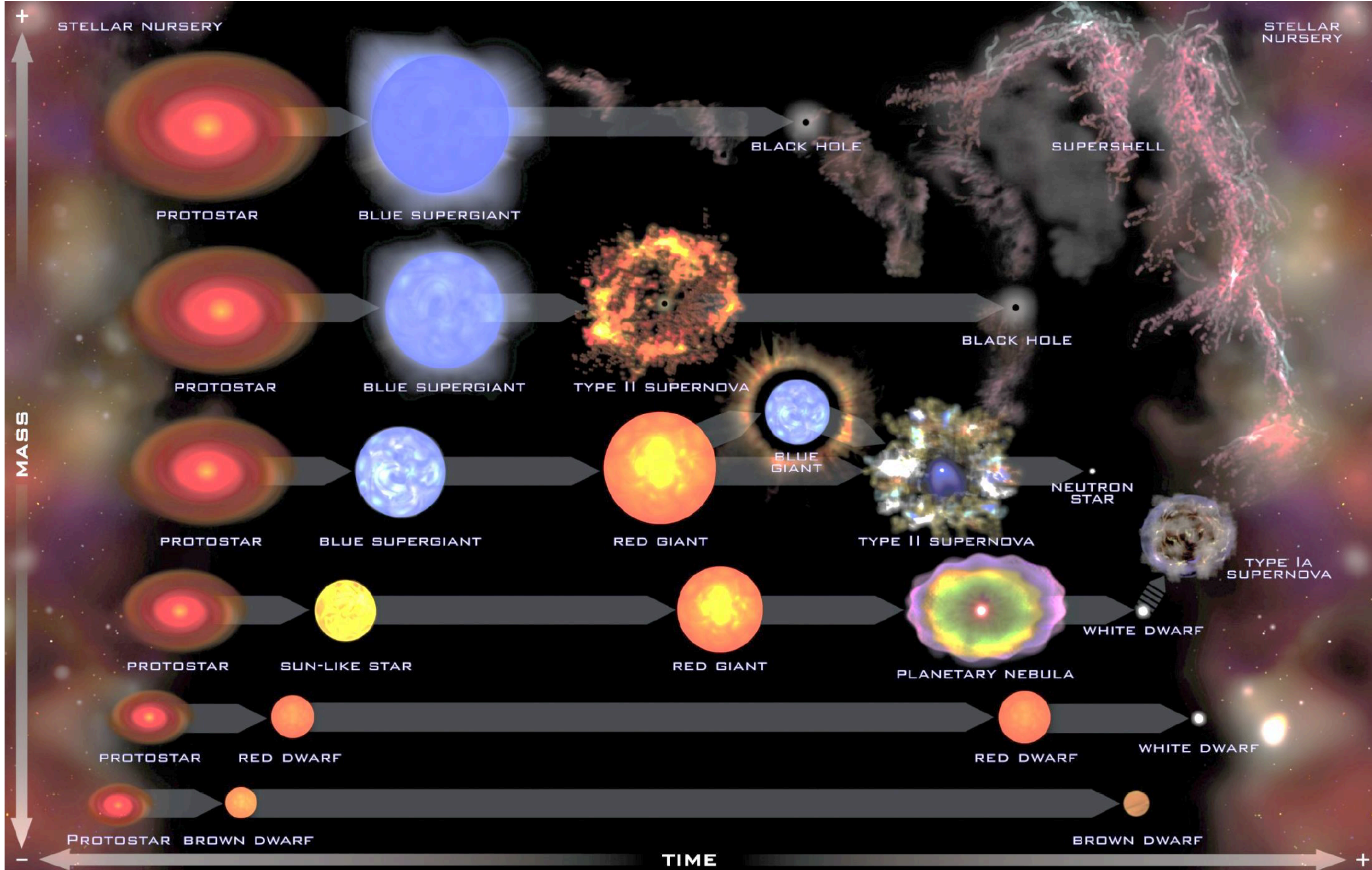
Image Credit: ESA/Hubble,
N. Bartmann

Stellar evolution in a nutshell



Credit: Thomas Tauris

Stellar evolution in a nutshell: Black hole formation



BH progenitors

$$M_{\text{ZAMS}} \gtrsim 18 - 20 M_{\odot}$$

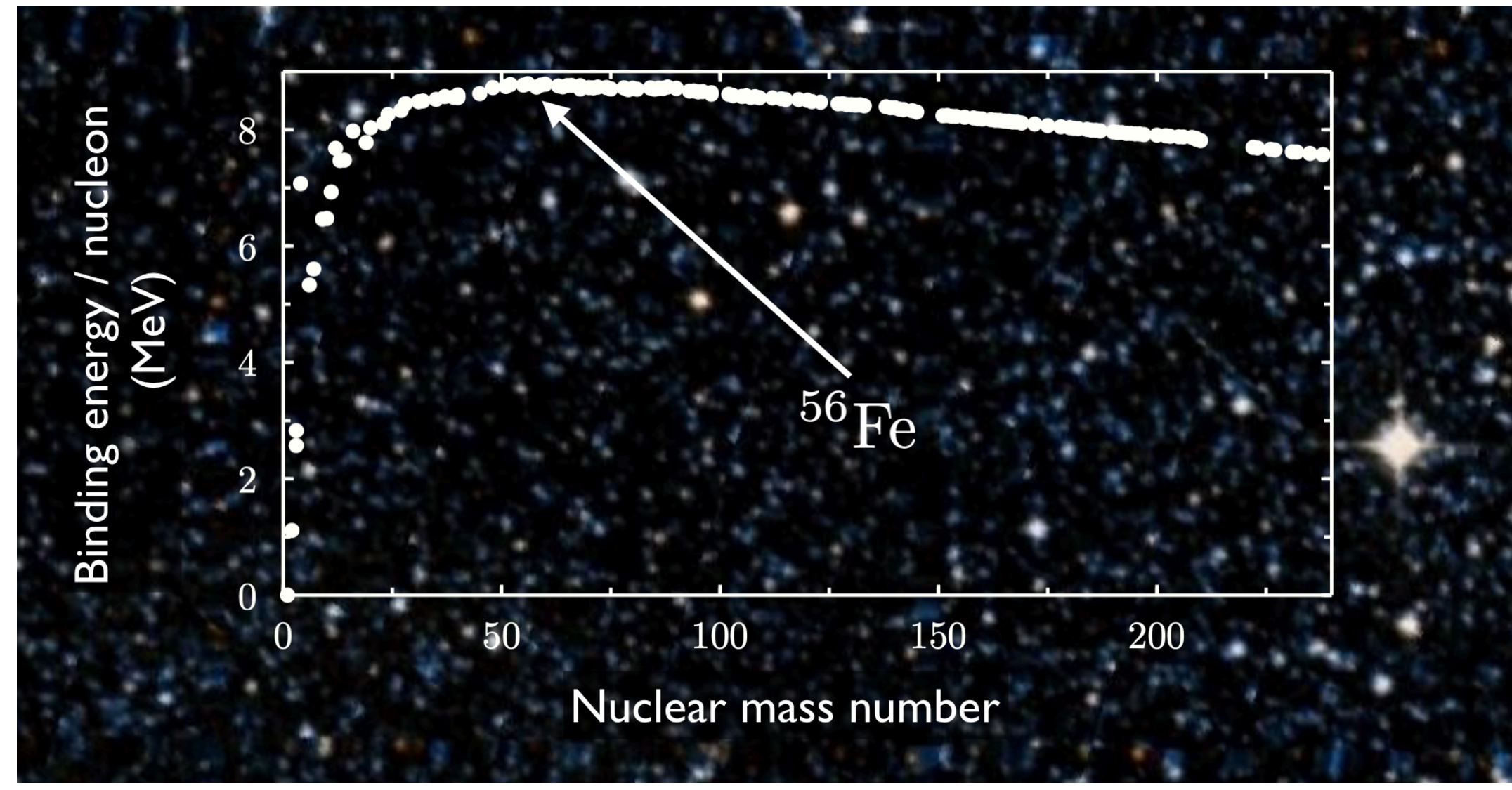
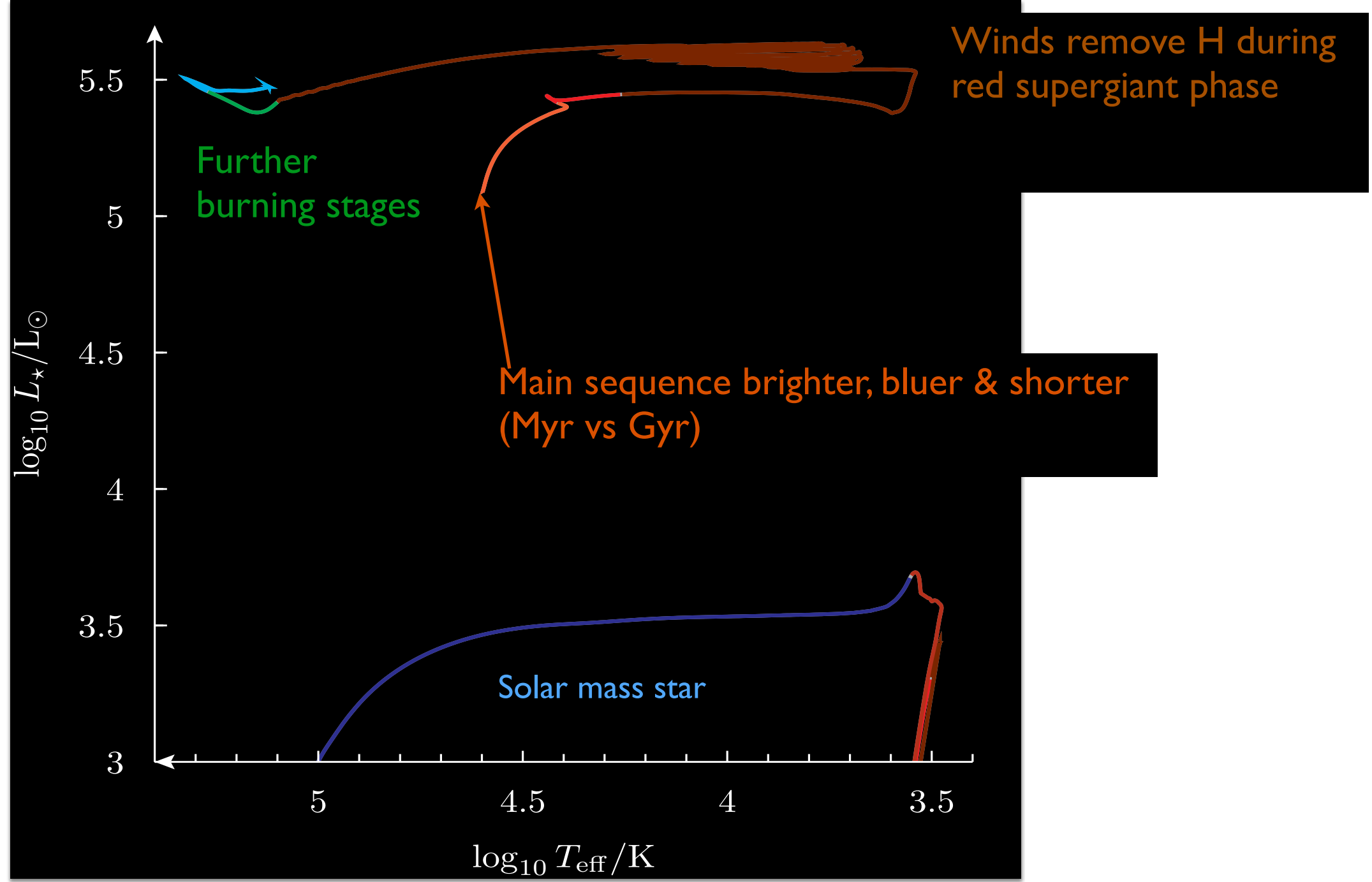
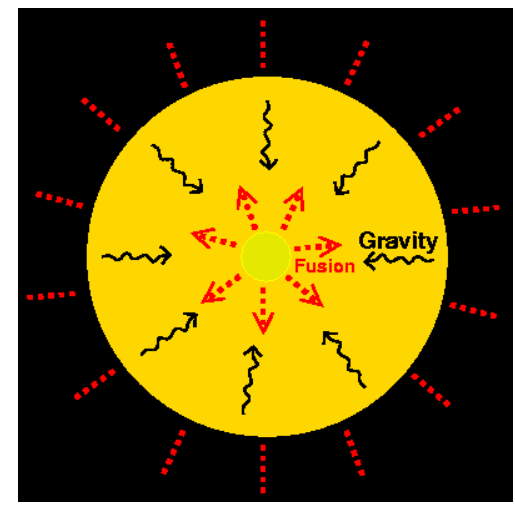
Evolution time
(~ few Myr to 30 Myr)

$$\tau_{\text{nuclear}} \propto M^{-2.5}$$

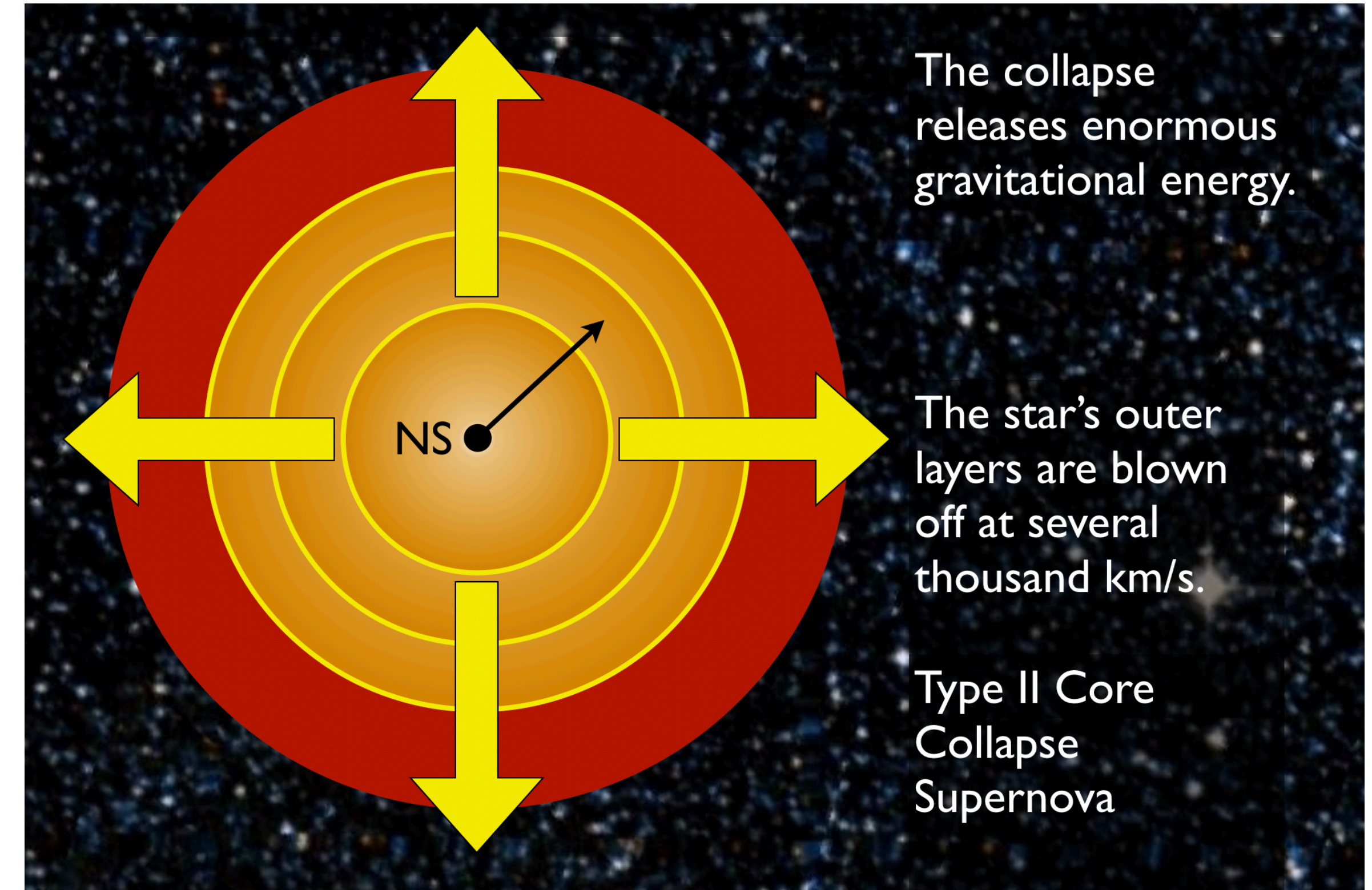
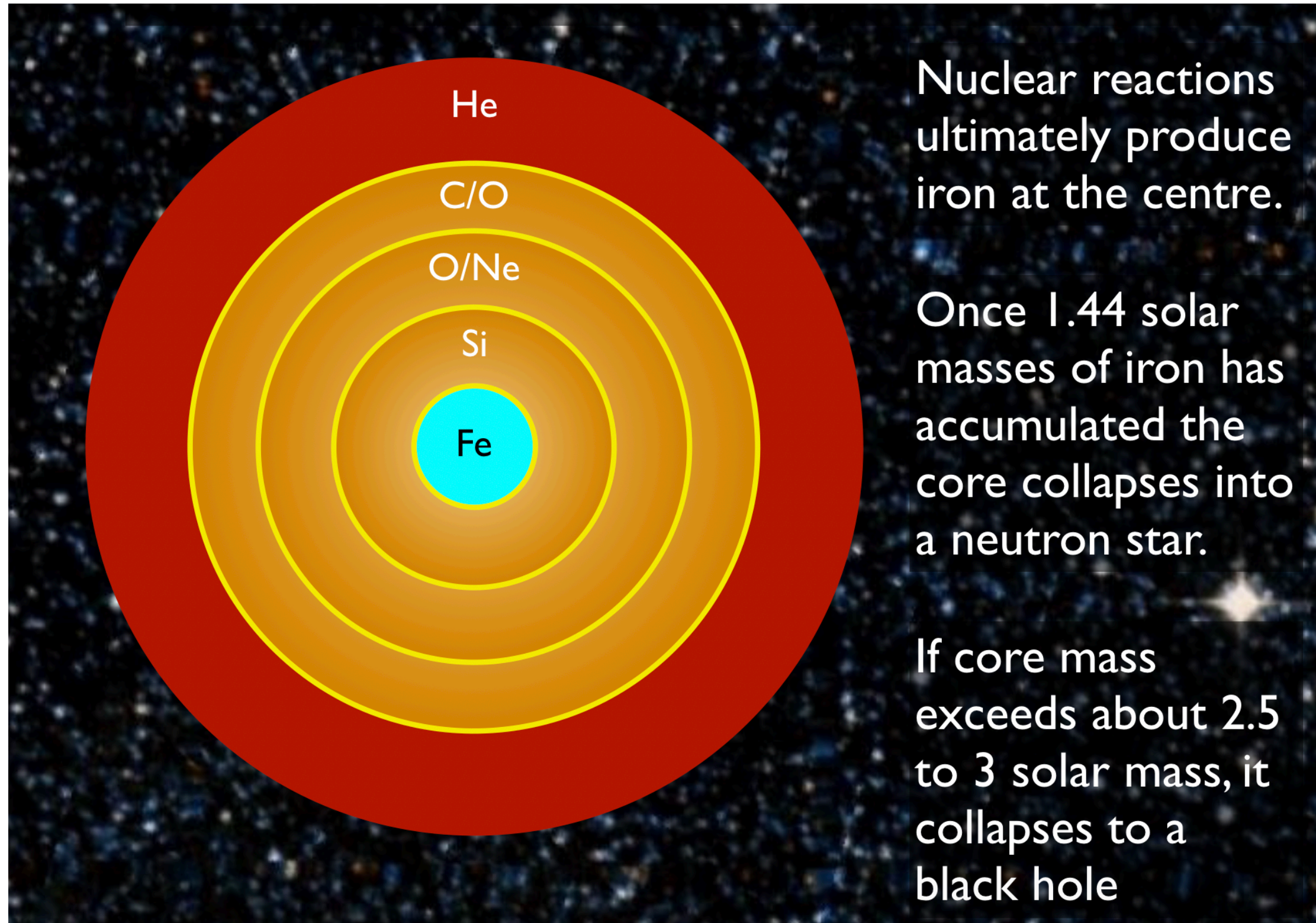
Credit: Thomas Tauris

Black hole formation: supernova models and remnant masses

- Massive stars go through a series of burning phases, fusing the ashes of previous burning phases until a core of iron is built up in its center.
- Energy is released when smaller nuclei fuse
- Neutrons and protons are more bound in larger nuclei:
- Iron has maximum binding energy: no more energy released by fusion → stellar core starts collapsing because pressure drops



Death of a massive star: core collapse supernova

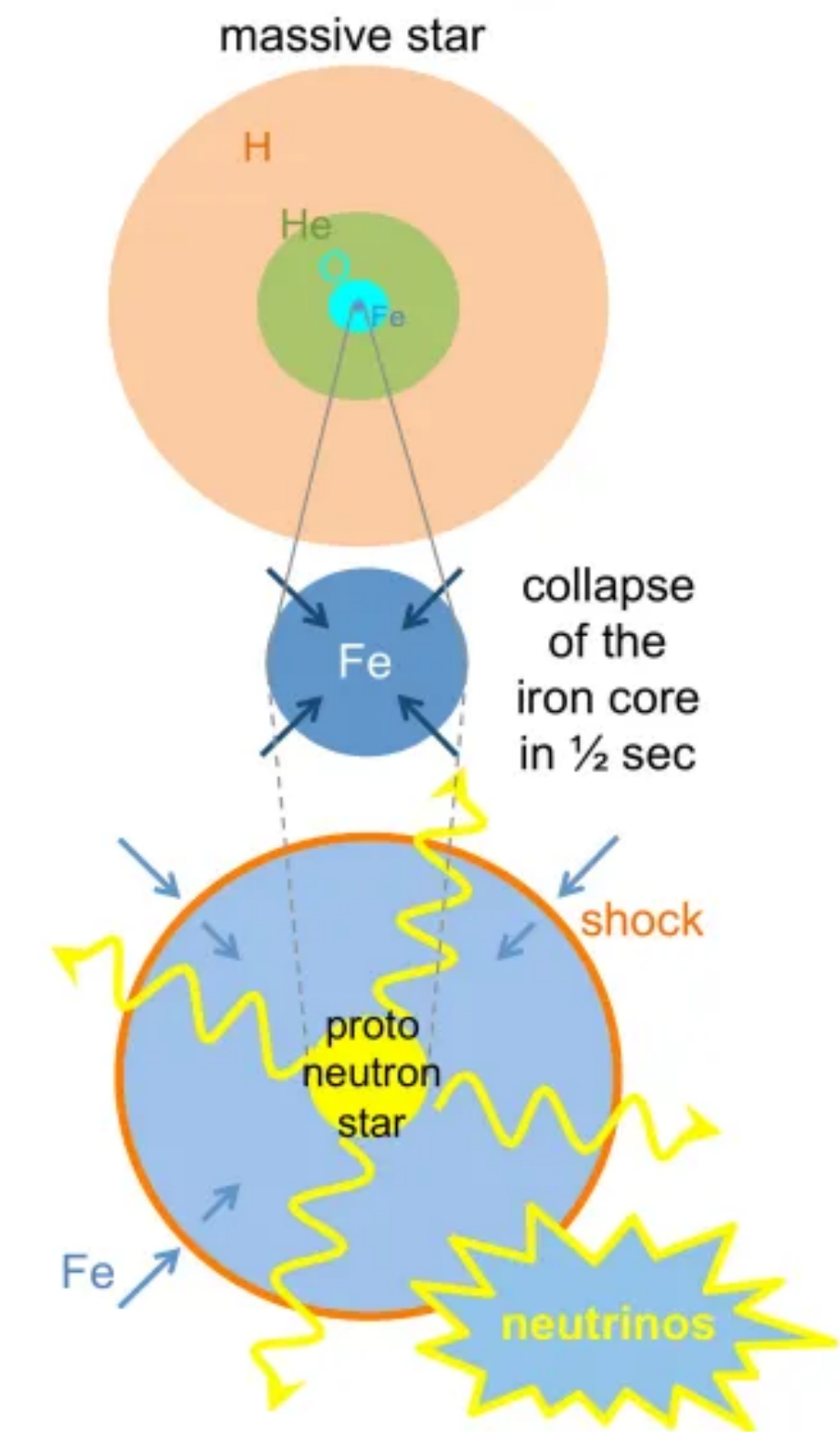


The collapse of a stellar core releases up to 10^{53} erg of gravitational potential energy

How this energy is converted into explosion energy is still an active area of research!

Black hole formation: supernova models and remnant masses

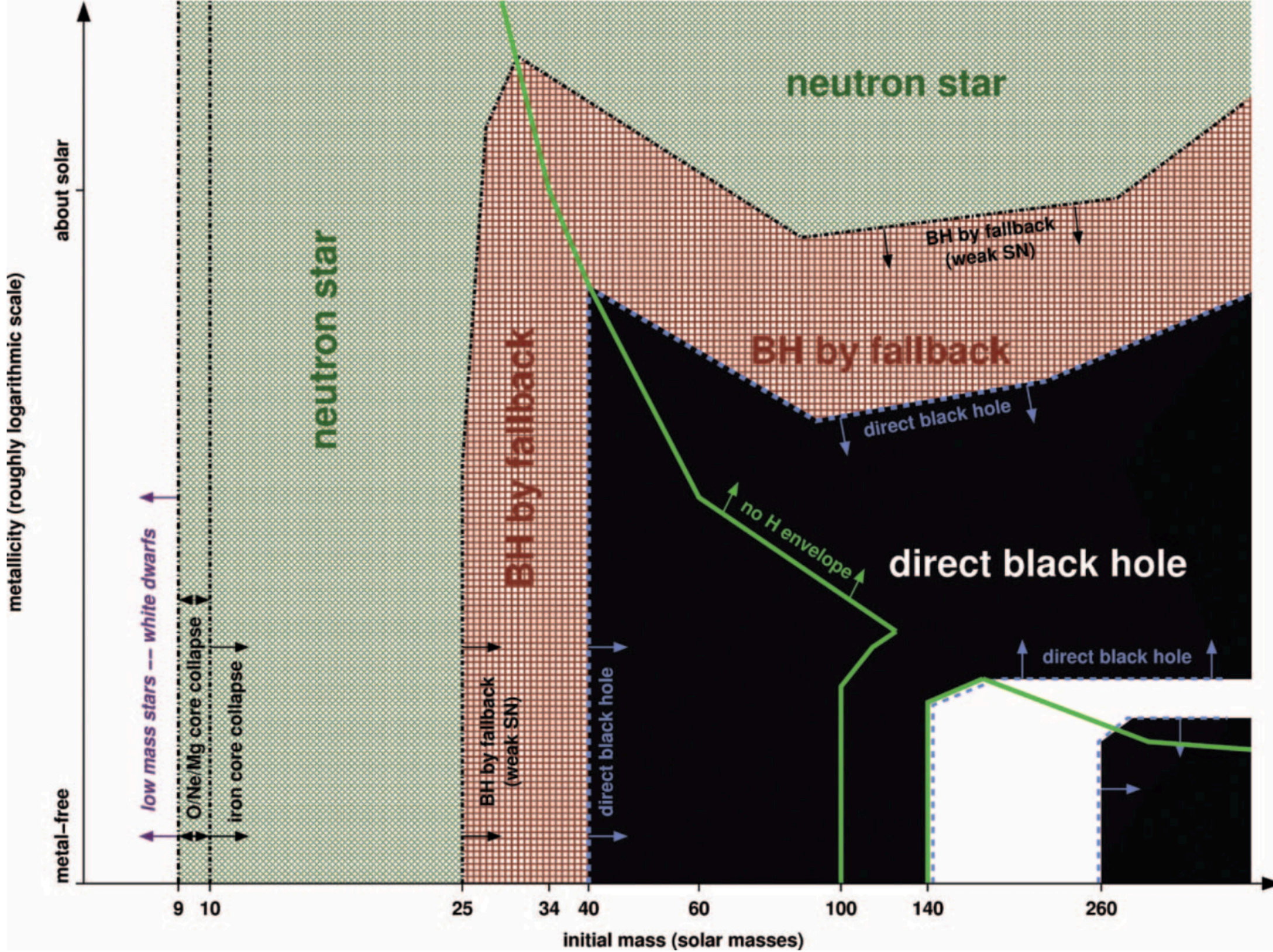
- Convective engine: Potential energy is converted into thermal energy (mostly thermal energy of neutrinos) and core bounces driving shocks
- Shocks must reverse collapse of outer layers
- Star cannot explode if the binding energy of the envelope is larger than the SN energy
- Depends on energy released in the supernova, the mass and radius of the envelope of the evolved star
- Supernova outcome depends on the 'fallback' of the outer layers
- How much material falls back on to the collapsing core after the supernova



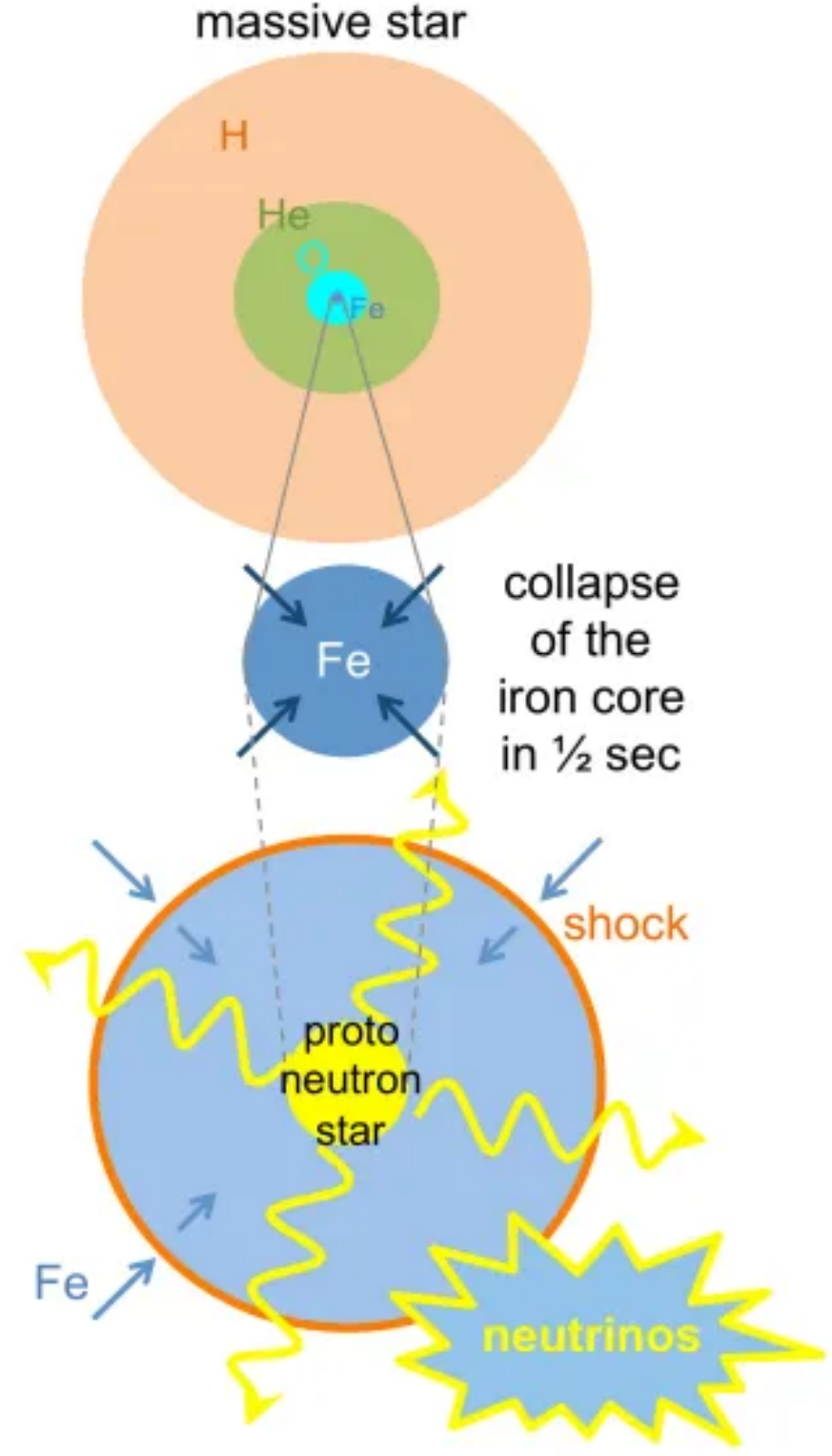
Foglizzo et al. 2015

Black hole formation: supernova models and remnant masses

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- Depends on energy released in the supernova, the mass and radius of the envelope of the evolved star
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- Extremely uncertain! Depends on:
 - explosion energy
 - progenitor's mass and metallicity

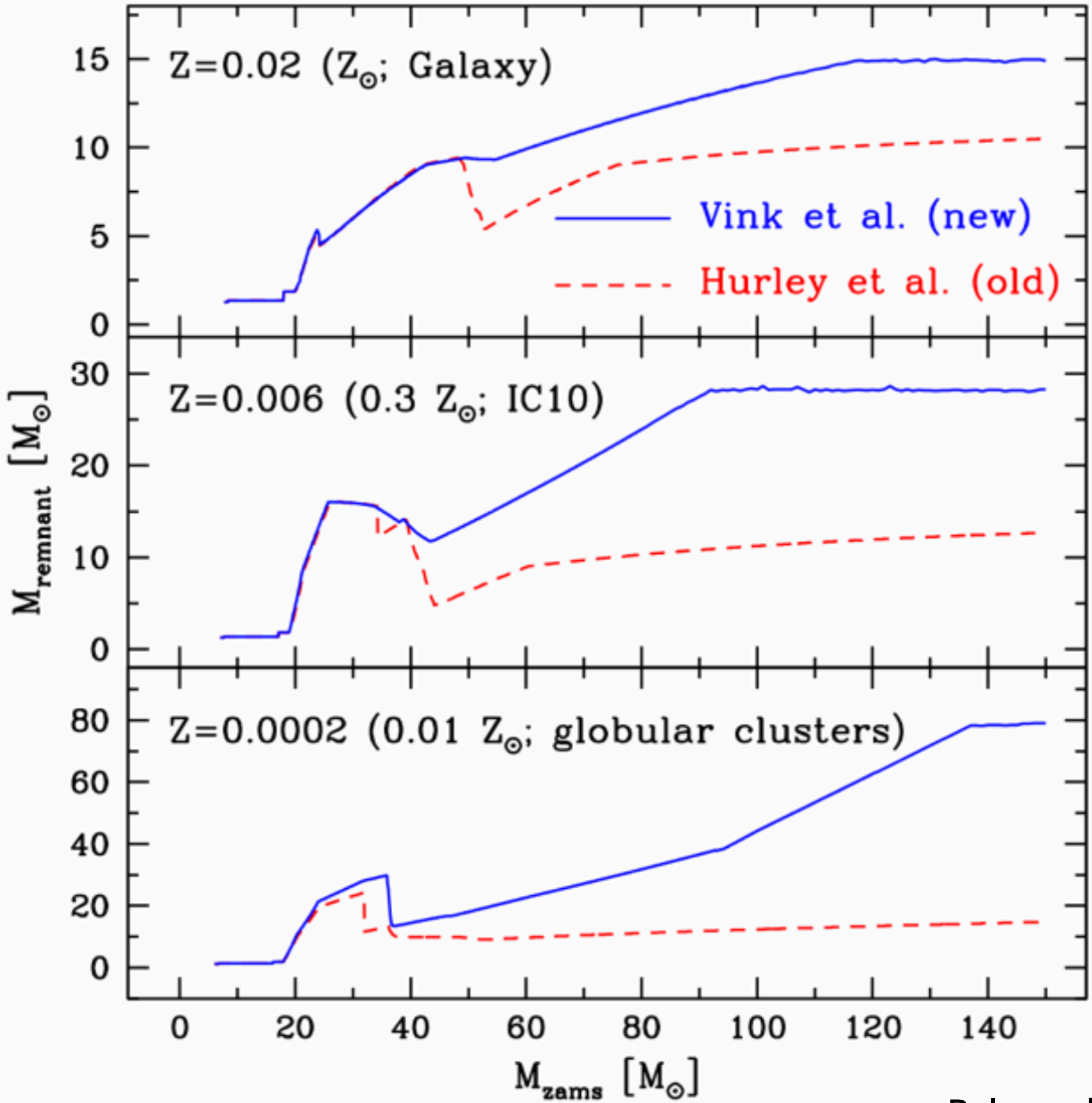


Foglizzo et al. 2015

From Heger et al. 2003 (also see Belczynski et al. 2002)

Dependence of black hole mass on the evolution of its progenitor

- Final black hole depends on initial mass, metallicity, winds, supernova model (Belczynski et al. 2016, Spera & Mapelli 2017, Giacobbo et al. 2018)

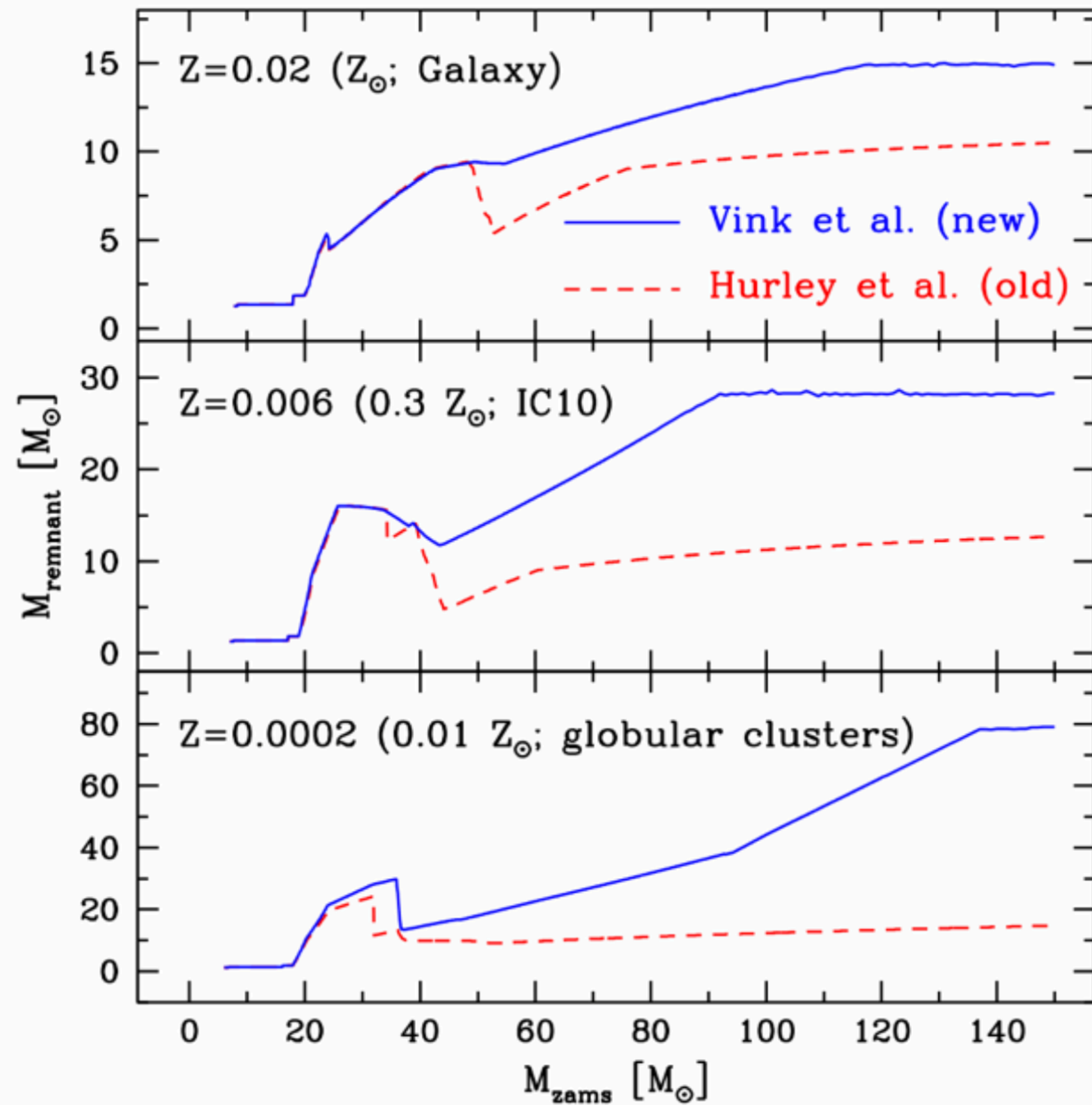


Impact of stellar winds on ZAMS mass vs BH mass

Belczynski et al. 2010

Black hole formation and pair instability supernova

- Final black hole depends on initial mass, metallicity, winds, supernova model (Belczynski et al. 2016, Spera & Mapelli 2017, Giacobbo et al. 2018)



Belczynski et al. 2010

- If a star is very massive and hot it can produce γ -ray photons in its core
- γ -ray photons scattering with atomic nuclei results in pair-pair production: $\gamma \rightarrow e^{-} + e^{+}$
- Missing pressure from γ -ray photons results in collapse of the core during oxygen burning
- High temperatures in the core can ignite remaining material resulting in an explosion that can leave no remnant
- Or strong mass loss due to pulsations (pulsation pair instability supernova)

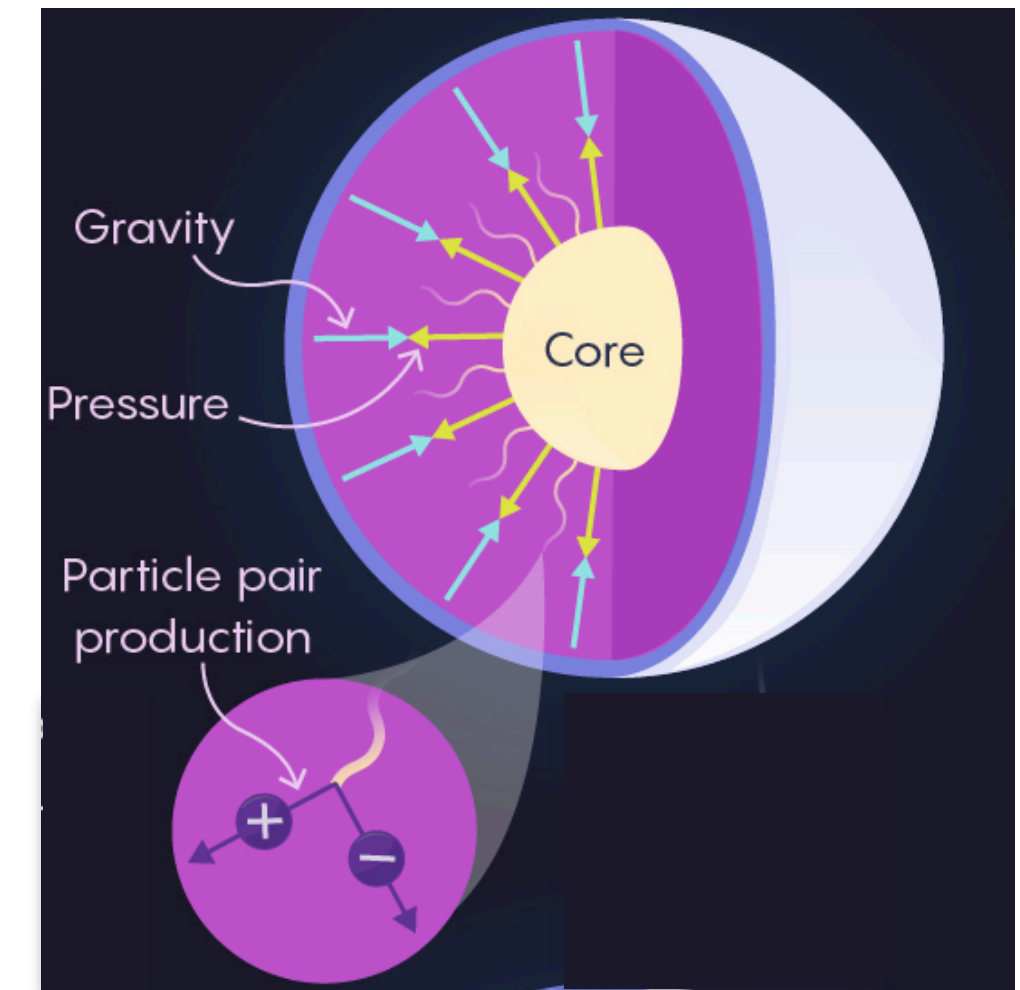
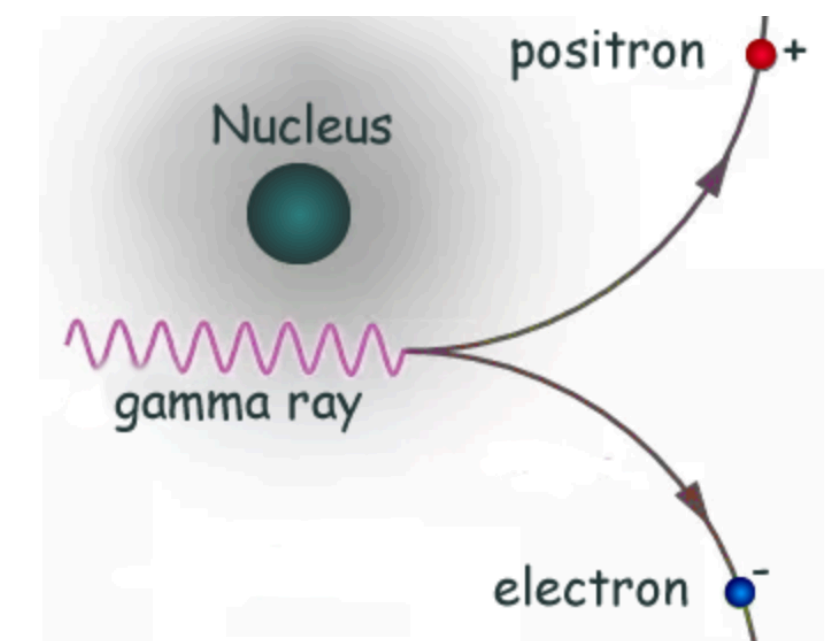
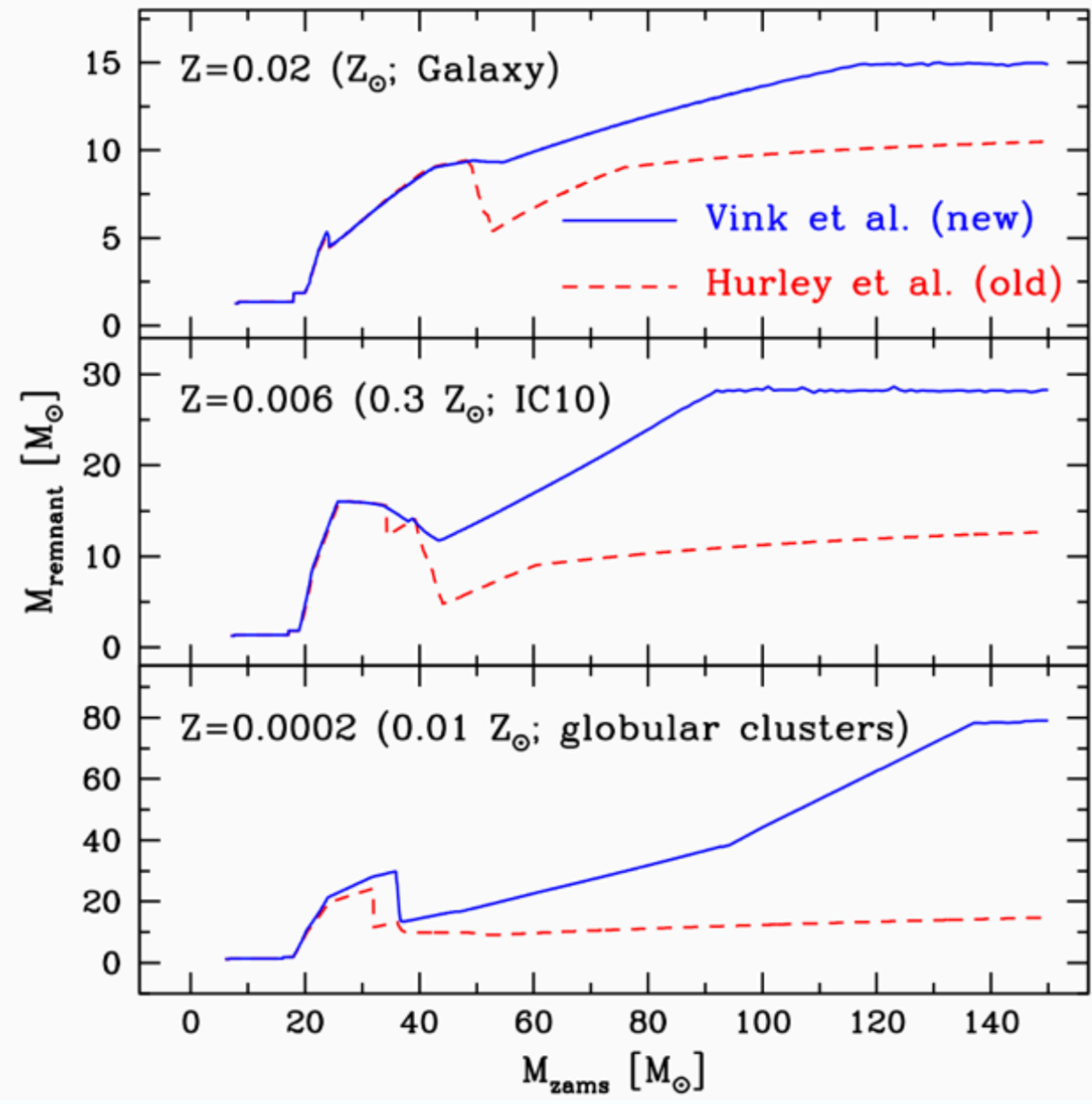


Image Credit: Lucy Reading-Ikkanda/Quanta Magazine

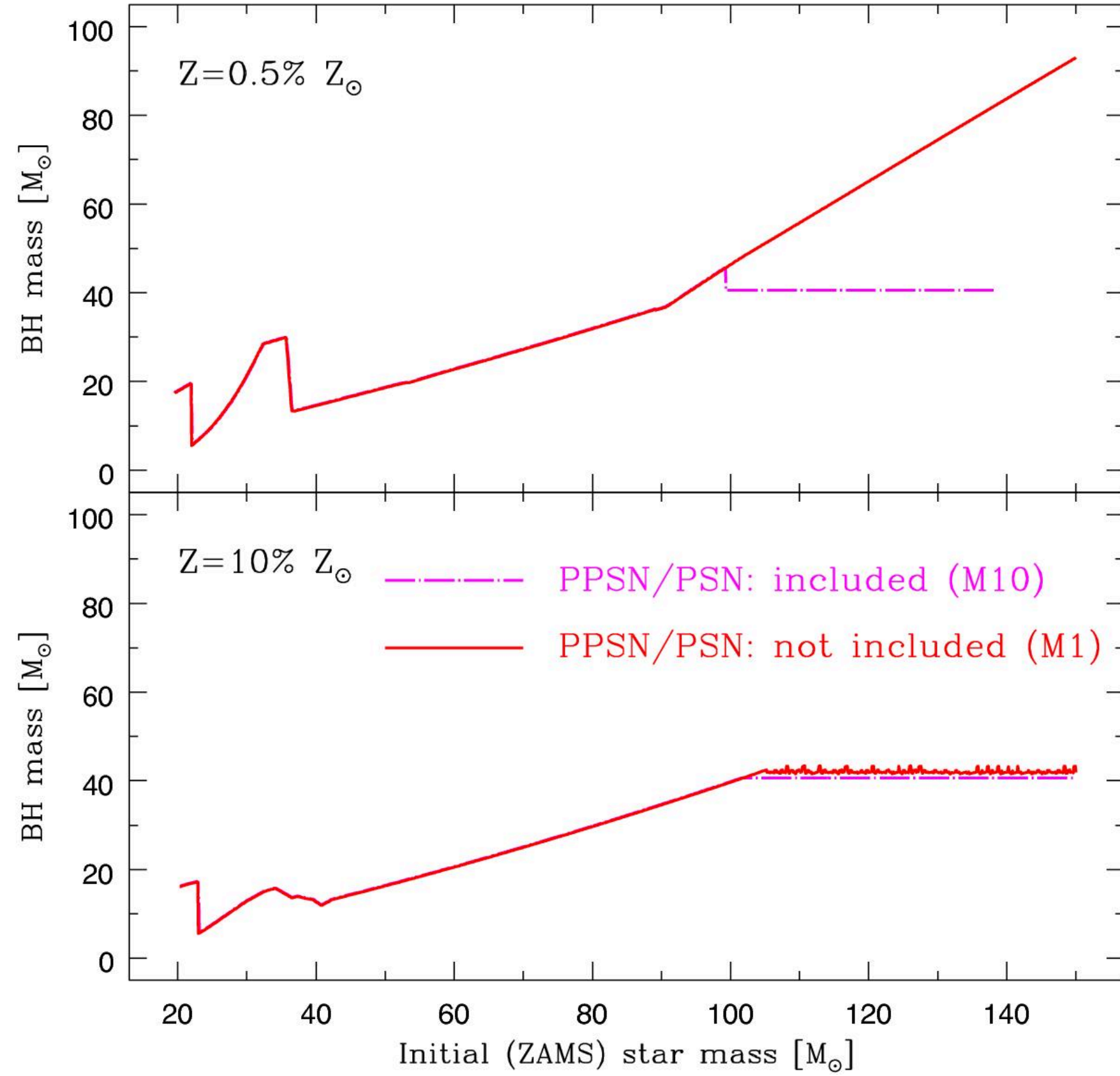


Dependence of black hole mass on the evolution of its progenitor

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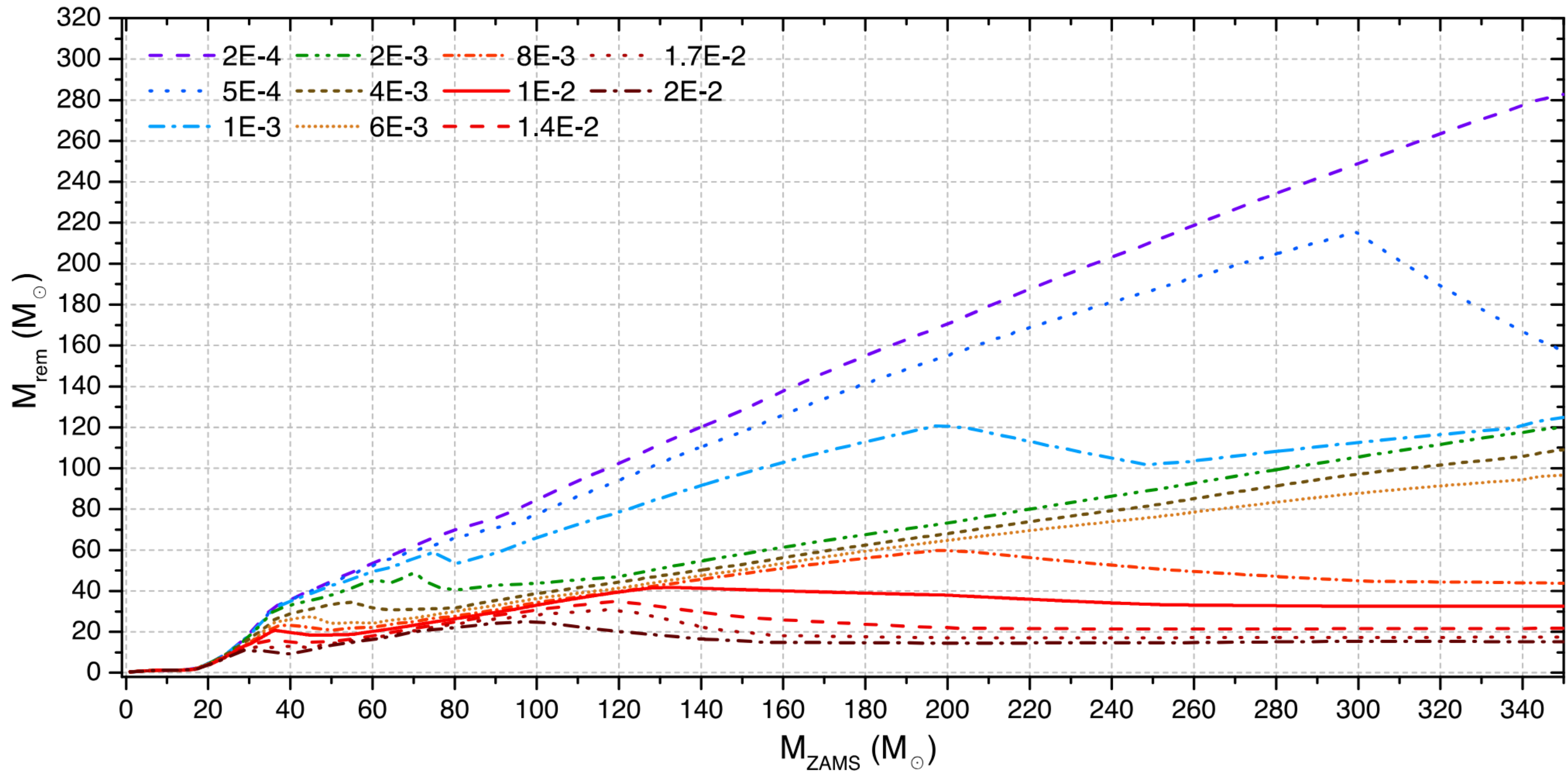
Belczynski et al. 2010



Belczynski et al. 2016

Dependence of black hole mass on the evolution of its progenitor

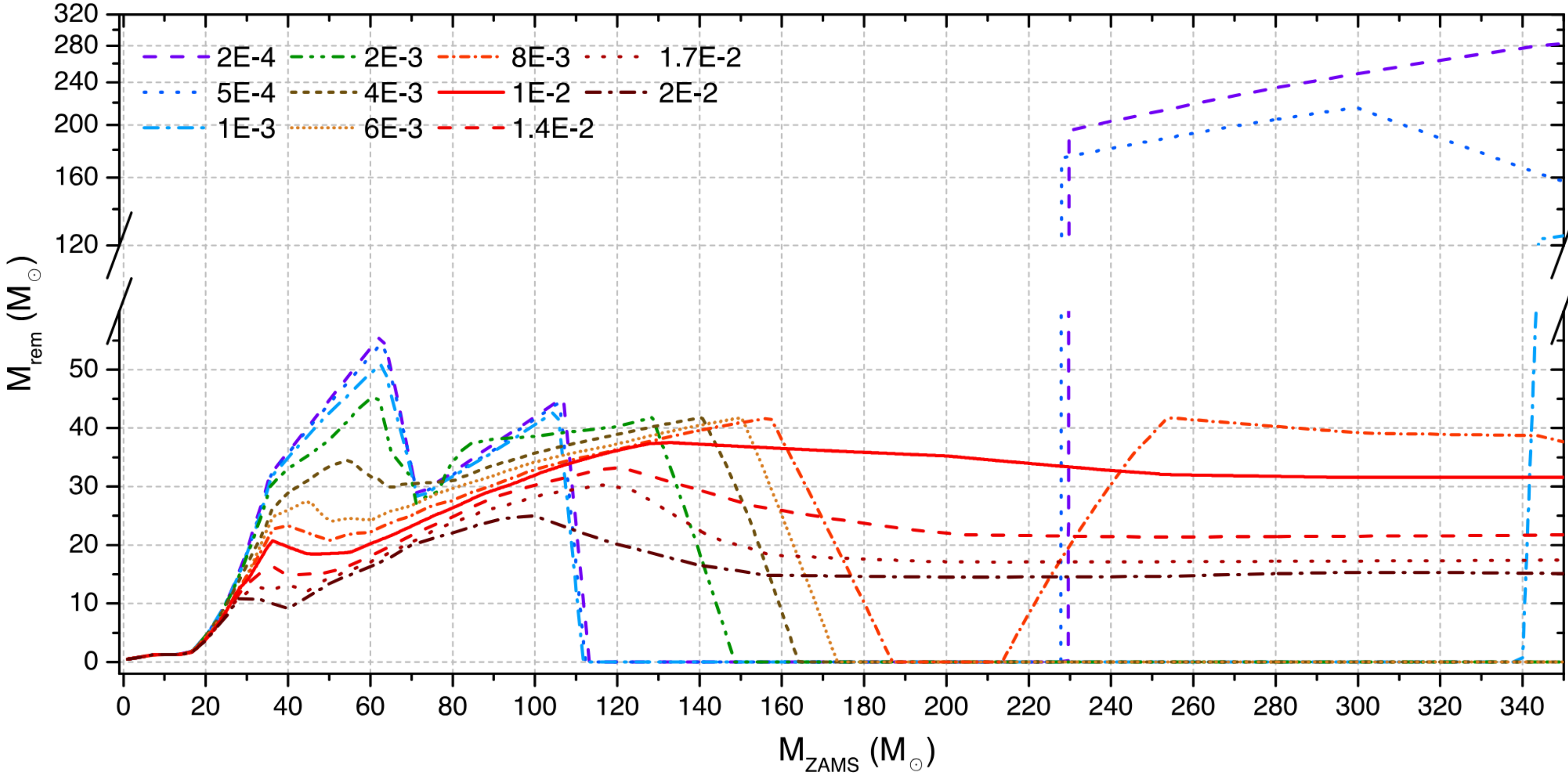
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Spera & Mapelli (2017) - without pair and pulsation pair instability supernova

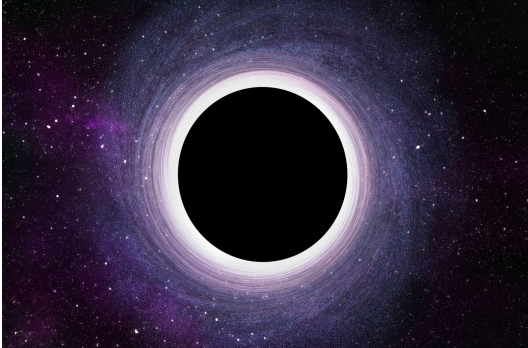
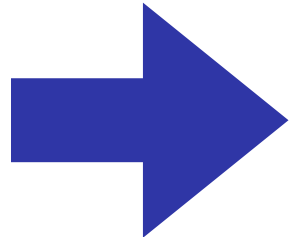
Dependence of black hole mass on the evolution of its progenitor

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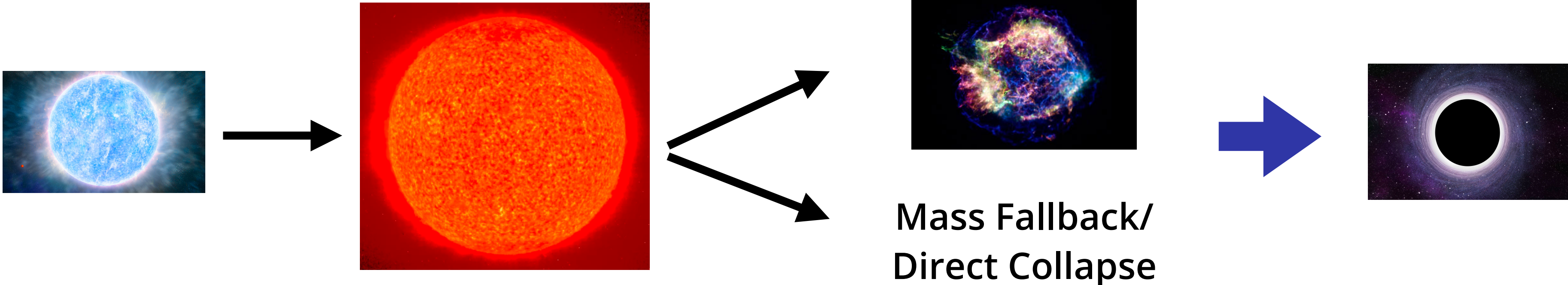
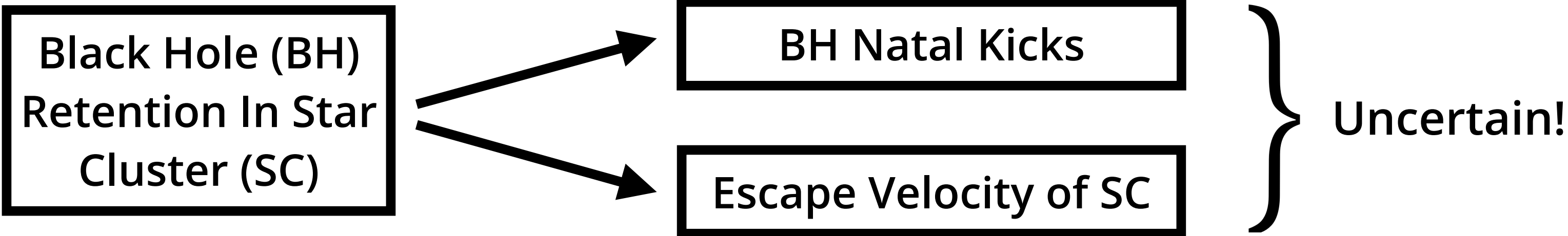
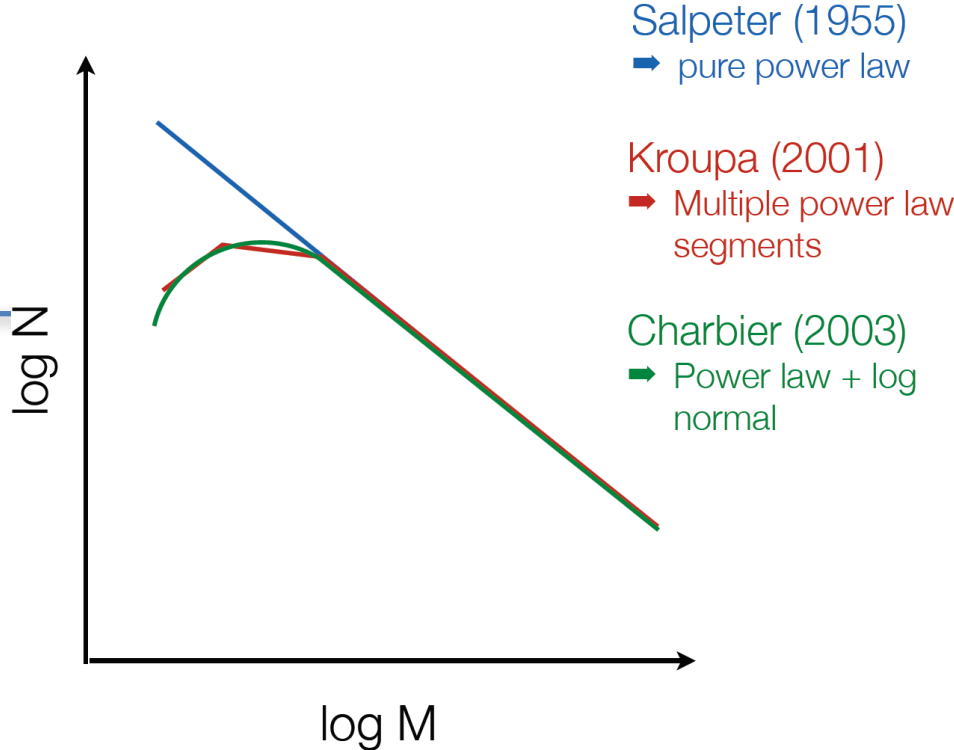
Spera & Mapelli (2017) - with pair and pulsation pair instability supernova

Black hole formation and retention in globular clusters

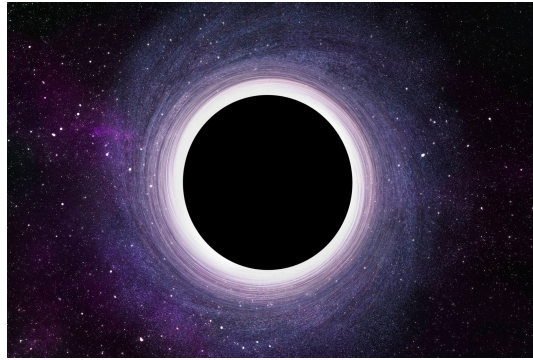
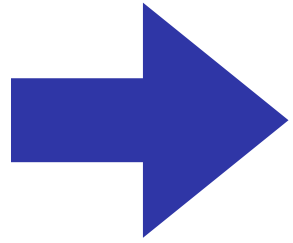


- Evolution Time: 4 - 30 Myrs
- ~ 2 black holes for every 1000 stars (typical IMF, $\alpha = 2.3$)

$M_{ZAMS} \gtrsim 18 - 20 M_{\odot}$

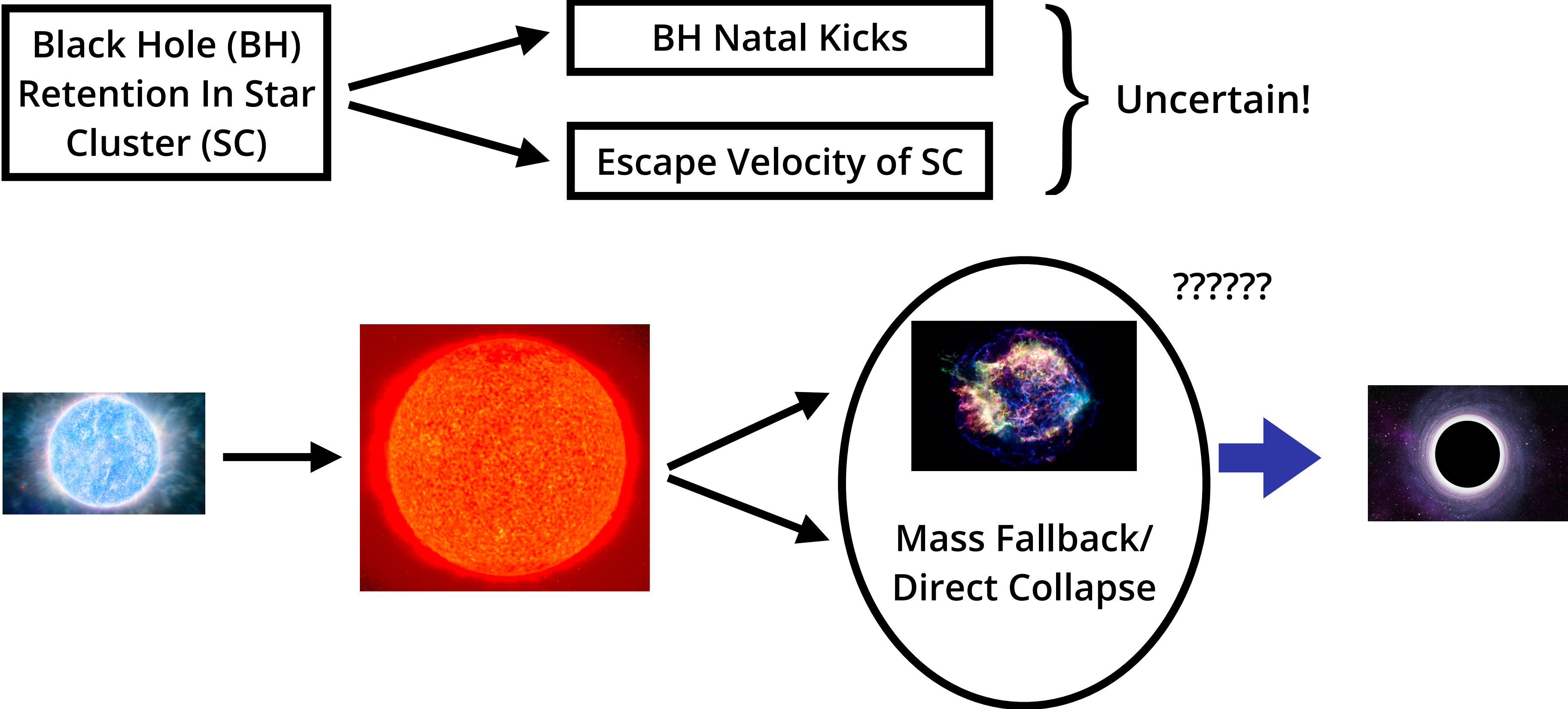


Black hole formation and retention in globular clusters

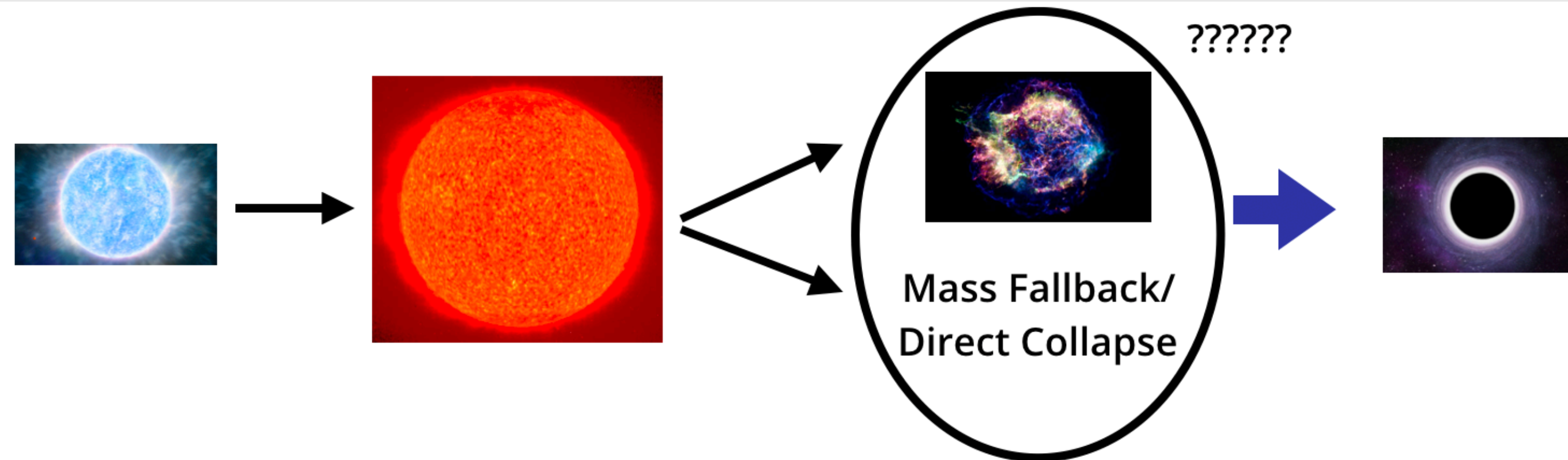


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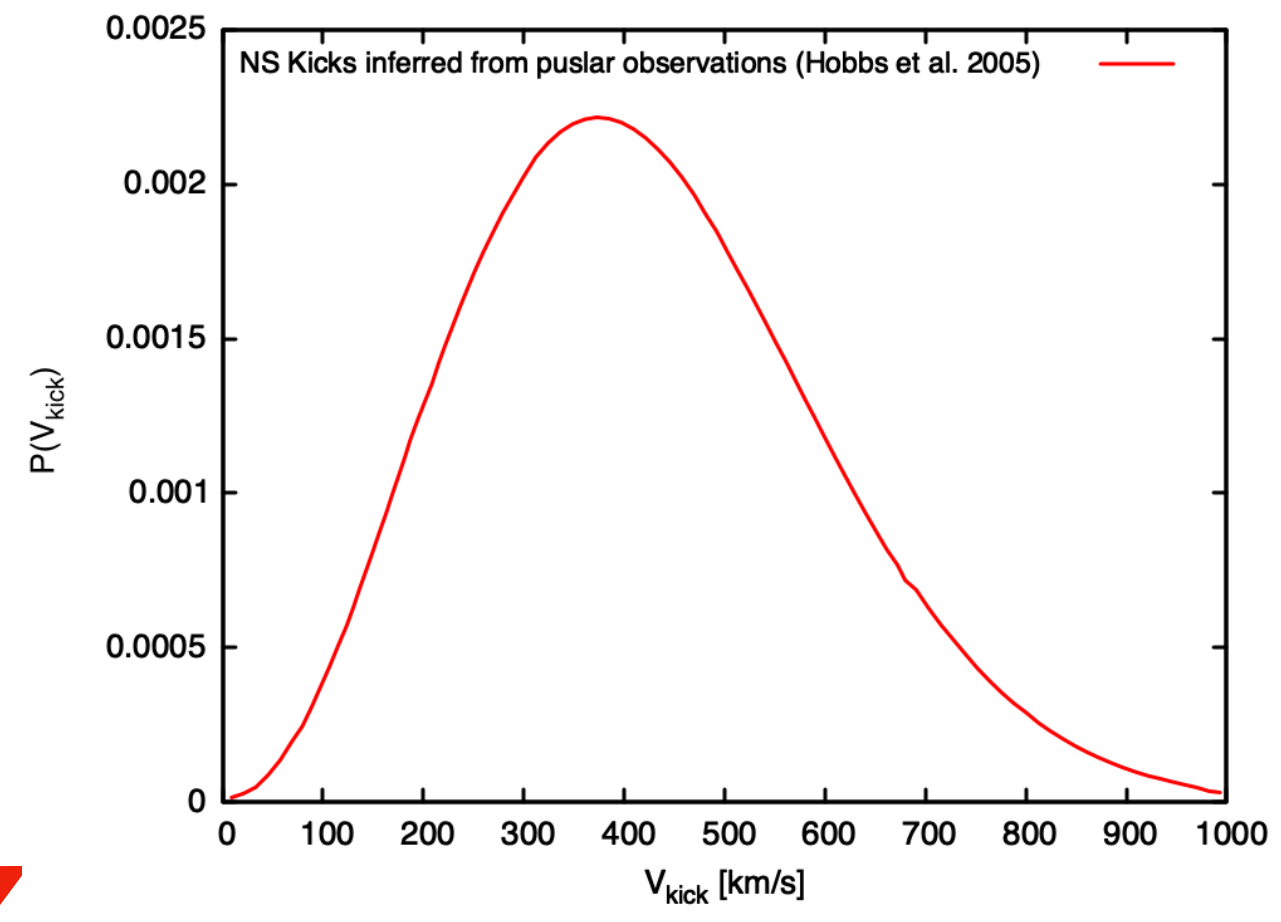
Black hole formation and retention in globular clusters



- **Core-collapse SN** → asymmetric mass ejection and/or neutrino emission → Natal kicks as high as for neutron stars? → 200 – 500 km/s up to 1000 km/s (see page 165 in course textbook)
(Repetto et al. 2012; Janka 2013, 2017, Observations: Mirabel et al. 2002, Hobbs et al. 2005)
 - **Mass Fallback (Failed SN)/Direct Collapse** → Low natal kicks/no kicks? (Fryer 1999, Heger et al. 2002, Belczynski et al. 2002, 2010, Fryer et al. 2012, Mandel 2016, Amaro-Seoane & Chen 2015 **Observations:** Reynolds et al. 2015, Adams et al. 2016, Allan et al. 2020)
- **Momentum conservation and/or Fallback** → kicks scaled down linearly with BH Mass → Final BH Mass depends on initial mass, metallicity, winds, supernova model (Belczynski et al. 2016, Spera & Mapelli 2017, Giacobbo et al. 2018)

Dependence of black hole retention in globular clusters on natal kicks

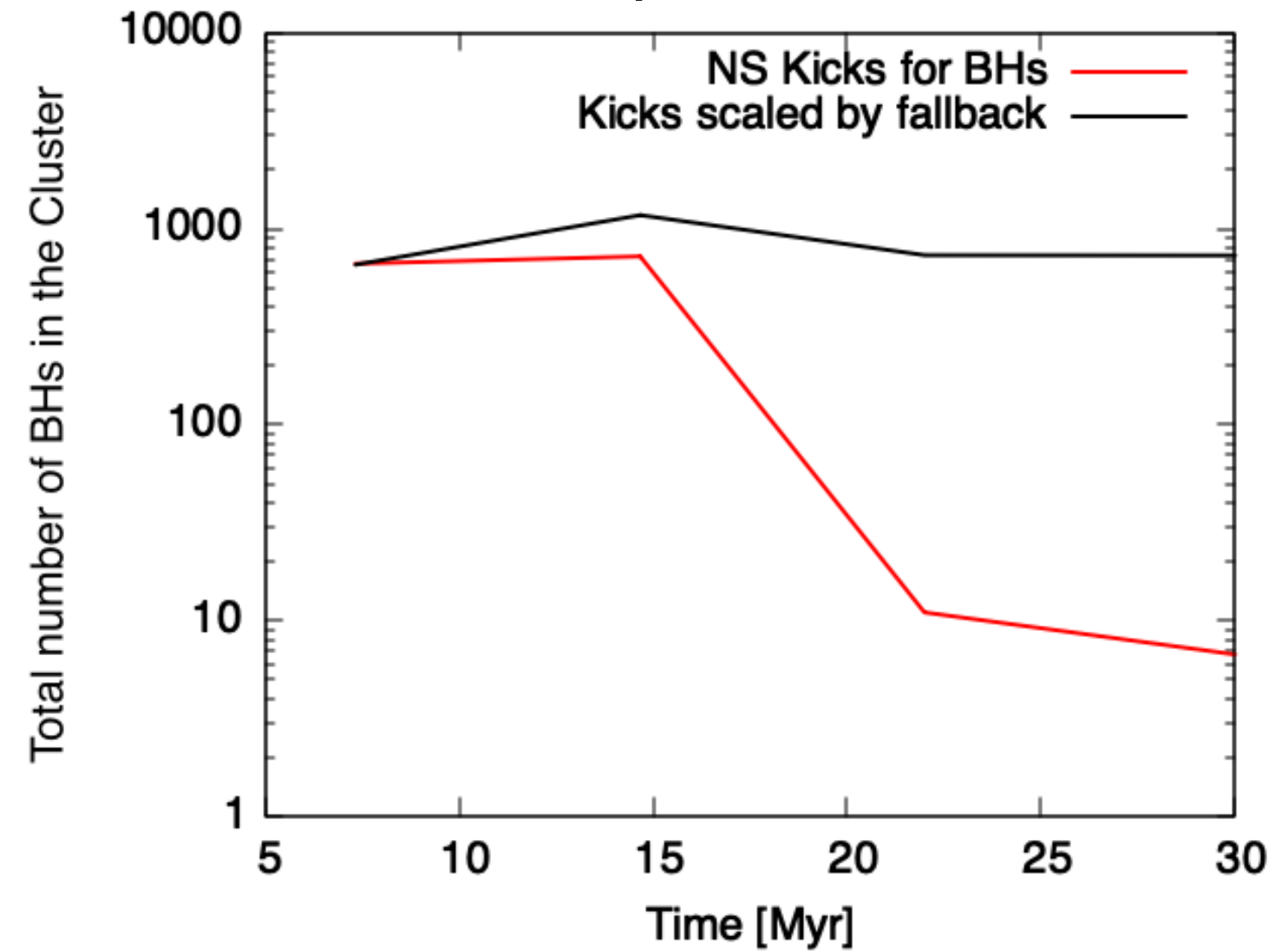
$N = 700,000$
 Initial binary fraction (IBF) = 10%
 $Z = 0.05 Z_{\odot}$
 $r_h = 4.8 \text{ pc}$, $r_t = 120 \text{ pc}$, $V_{\text{esc}} = 33 \text{ km/s}$
 $0.08 M_{\odot} \leq M_{\text{ZAMS}} \leq 100 M_{\odot}$ (Kroupa 2001 IMF)
 Belczynski et al. 2002 (for BH masses)
 Number of BH progenitors ~ 1900



Taken from Fig. 3 in Abbott et al. 2017

- 2 cases ($N = 700,000$):
1. **Neutron star kicks (Hobbs et al. 2005) for black holes**
 2. Black hole kicks scaled by fallback (Belczynski et al. 2002) and momentum conservation

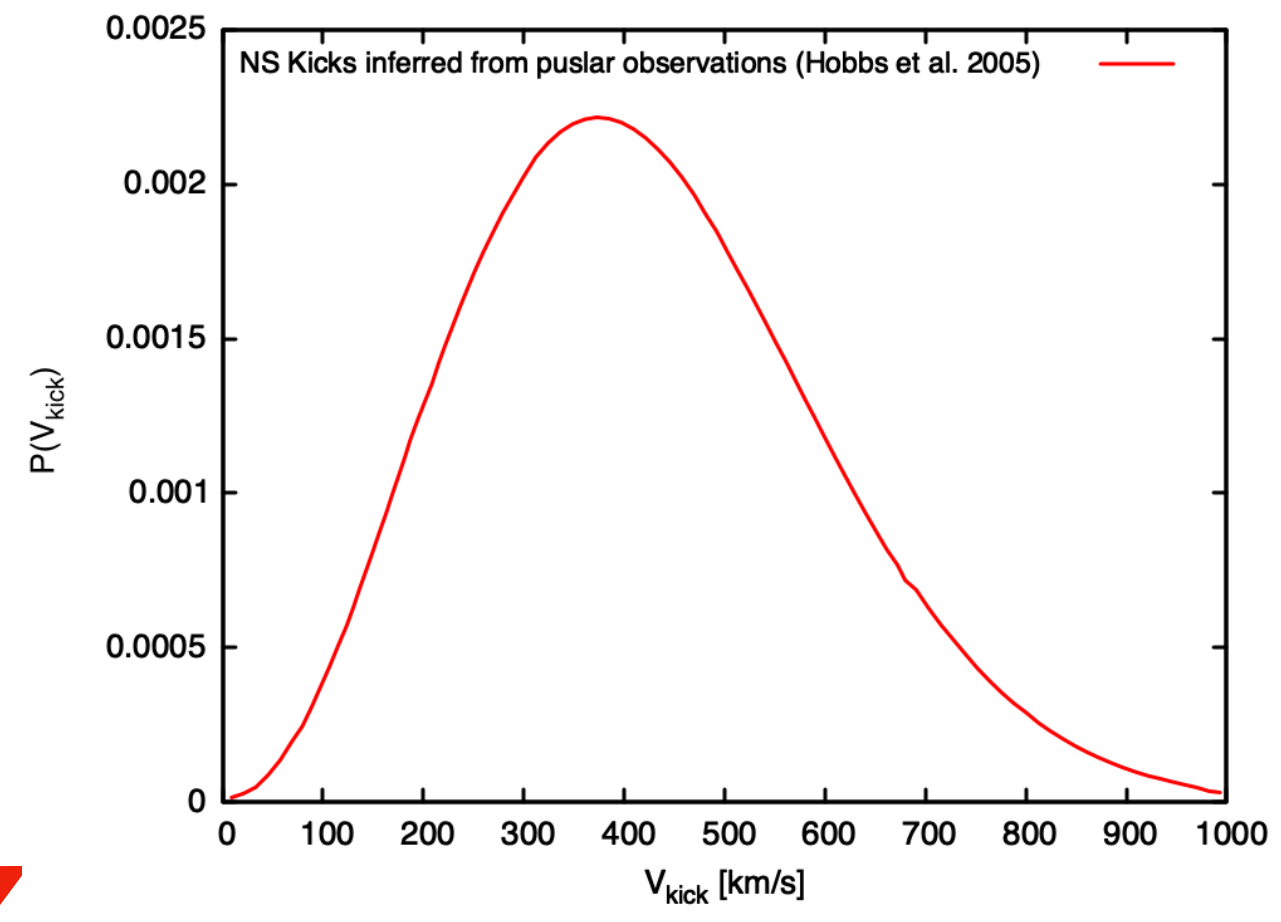
Retention fraction after supernovae explosions



Models from MOCCA Survey Database I (Askar et al. 2017)

Dependence of black hole retention in globular clusters on natal kicks

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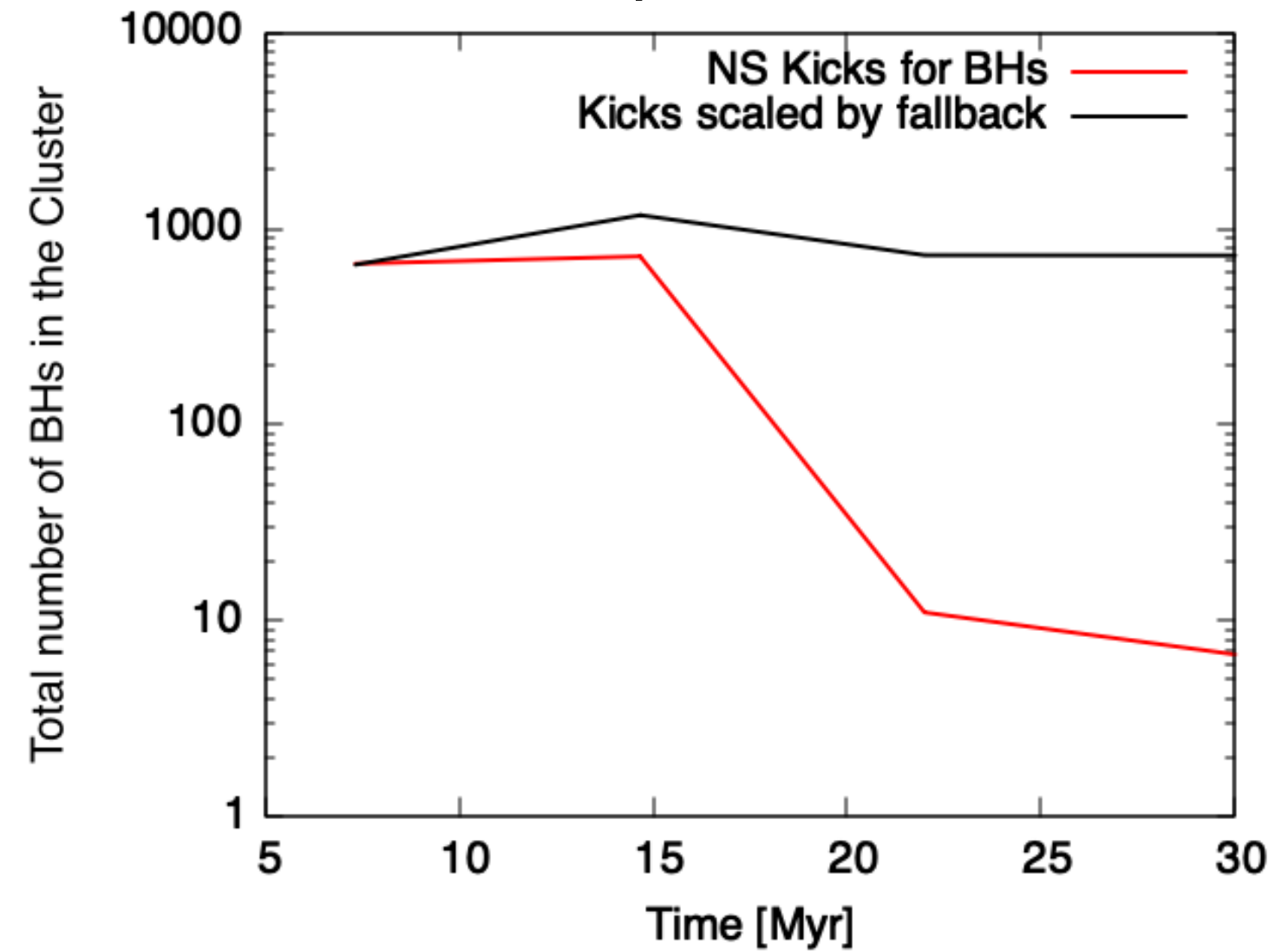
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1. **Neutron star kicks (Hobbs et al. 2005) for black holes**
 2. Black hole kicks scaled by fallback (Belczynski et al. 2002) and momentum conservation

Retention fractions of black holes in globular clusters can be as high as 55% or lower than 1% depending on how you compute natal kicks!

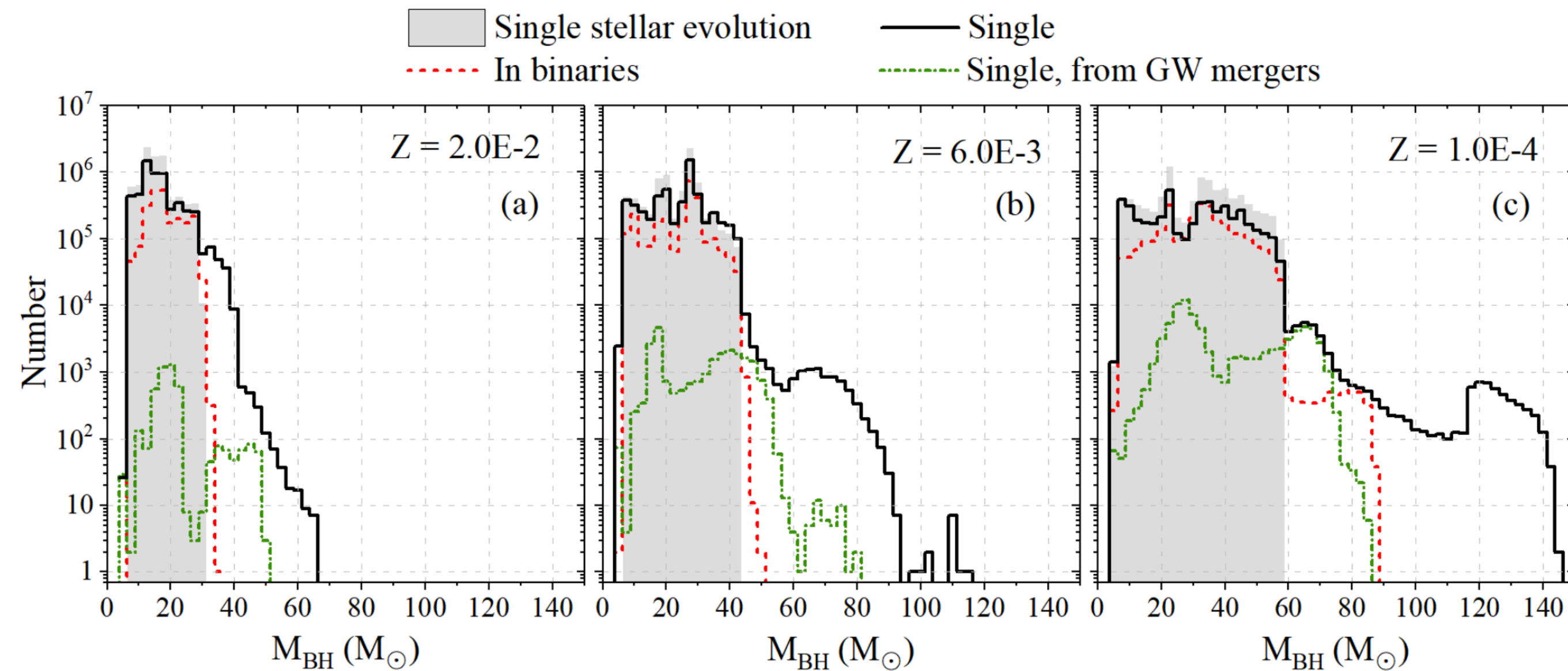
Models from MOCCA Survey Database I (Askar et al. 2017)

Retention fraction after supernovae explosions



Retention fraction and mass function of stellar-mass BHs

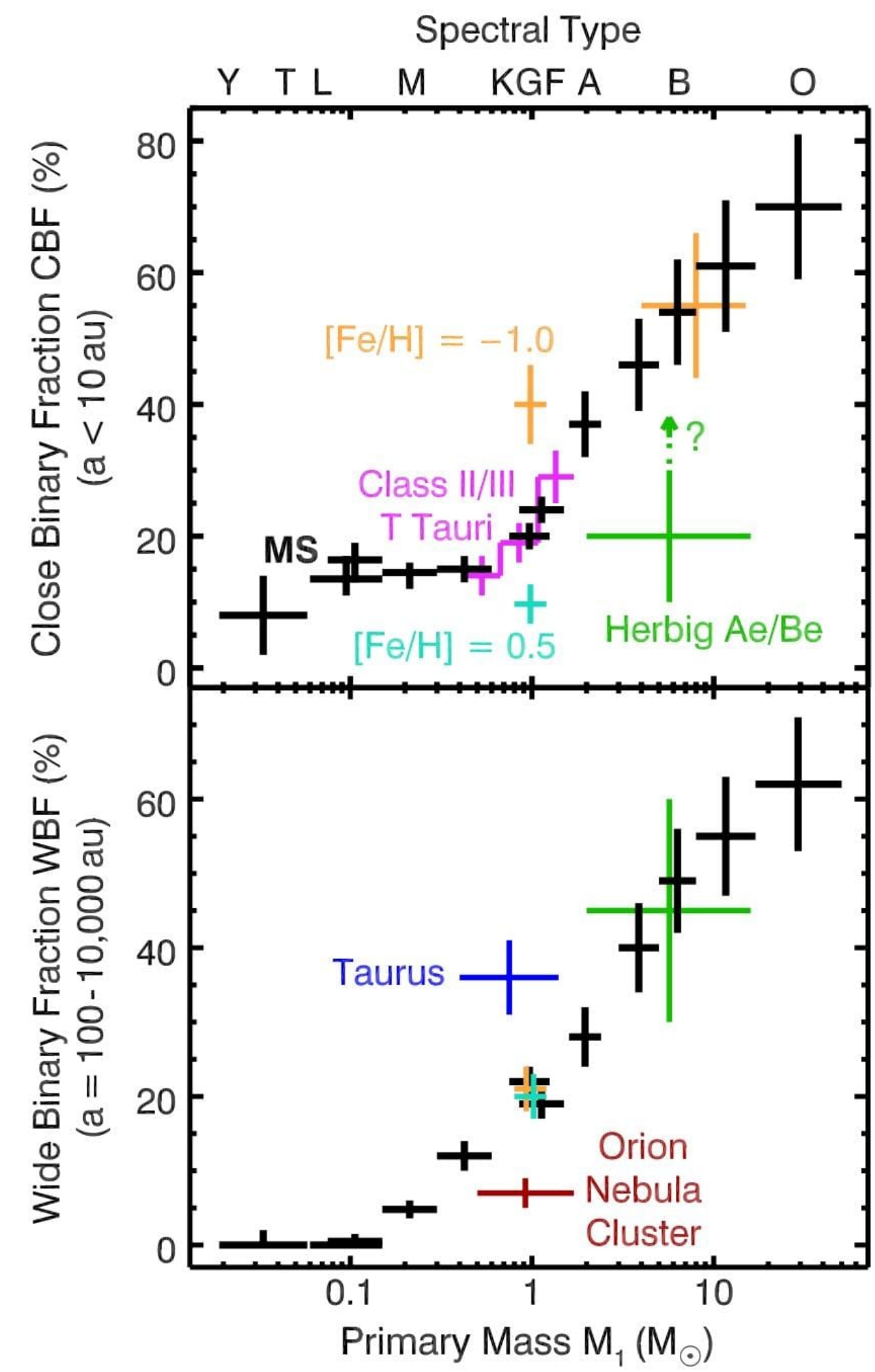
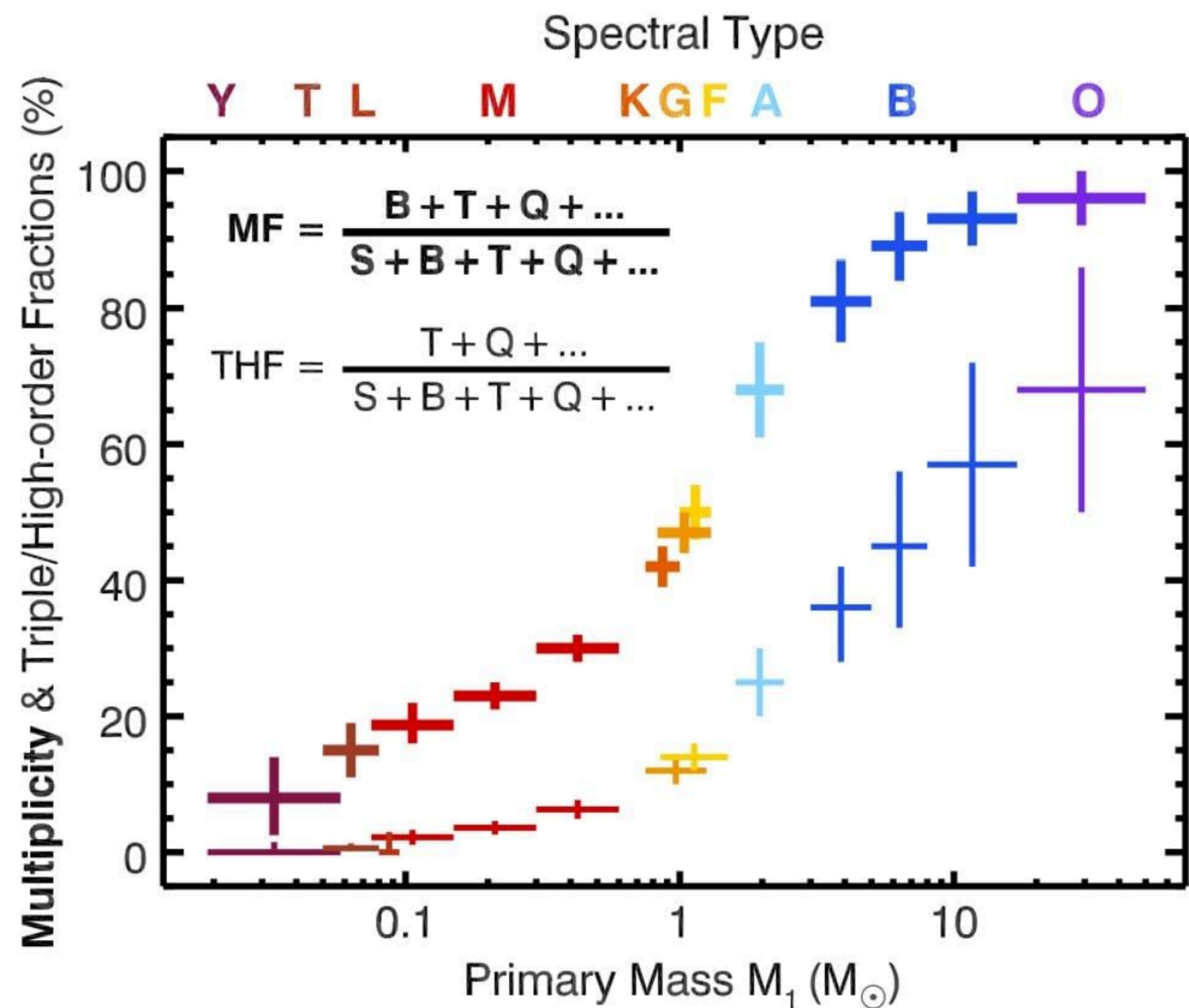
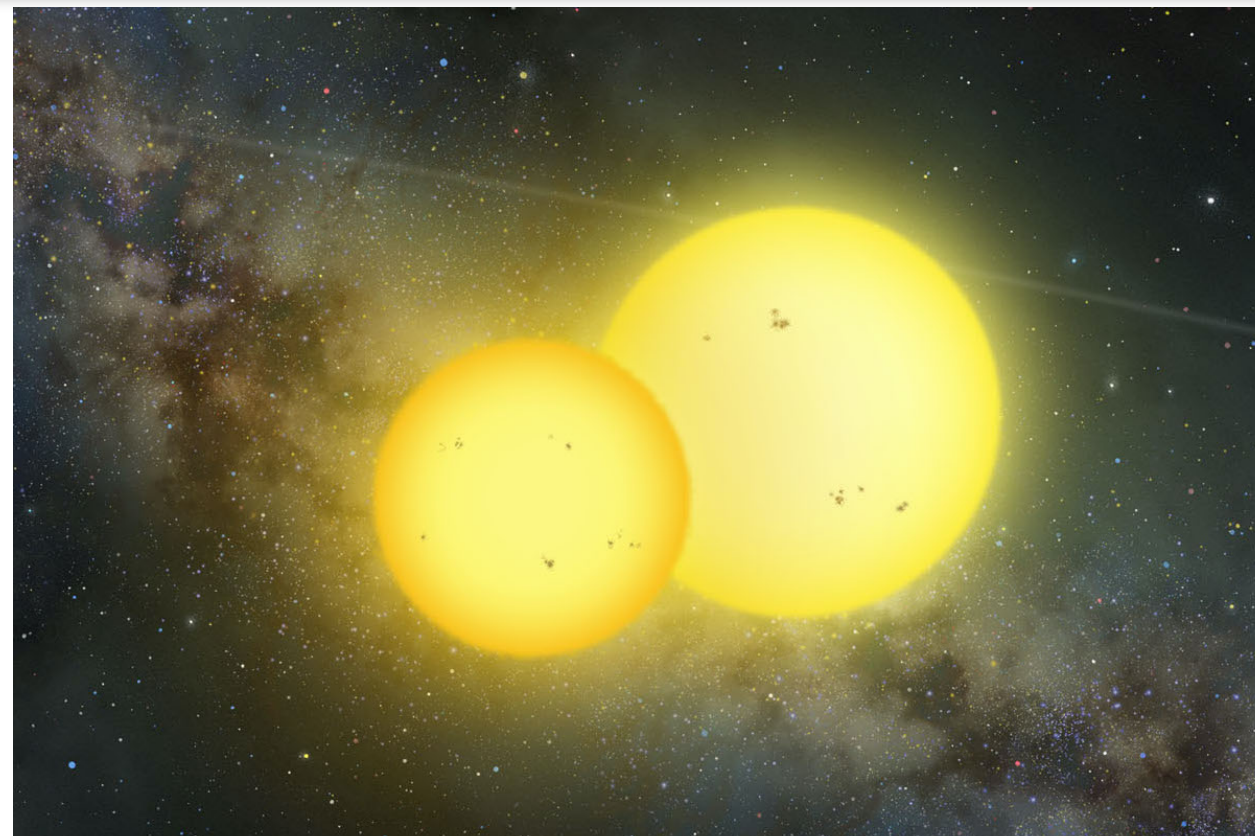
- Retention fraction (number of retained black holes/number of all BH progenitors) of stellar-mass BHs depends on natal kicks that they receive
- **Highly uncertain**
- Mass function of retained black holes depends on the evolution and properties of the progenitors
- Initial ZAMS masses and metallicities
 - Mass loss due to stellar winds
 - Supernova mechanism → fallback or no fallback
- **Highly uncertain**
- Binarity of massive stars can also affect BH mass function



Spera et al. 2018

Recall from lecture 7: Binary evolution processes → mass transfer, common envelope evolution

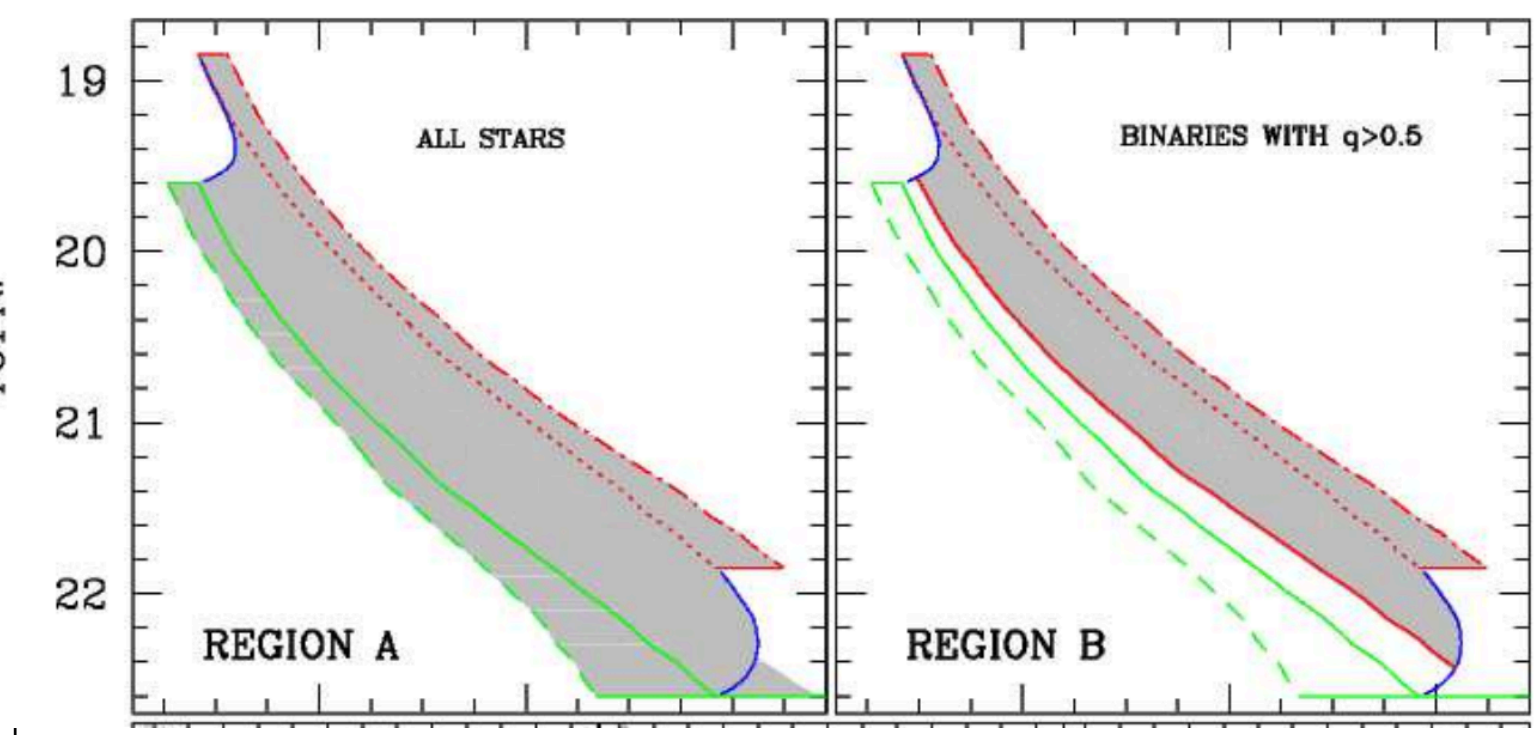
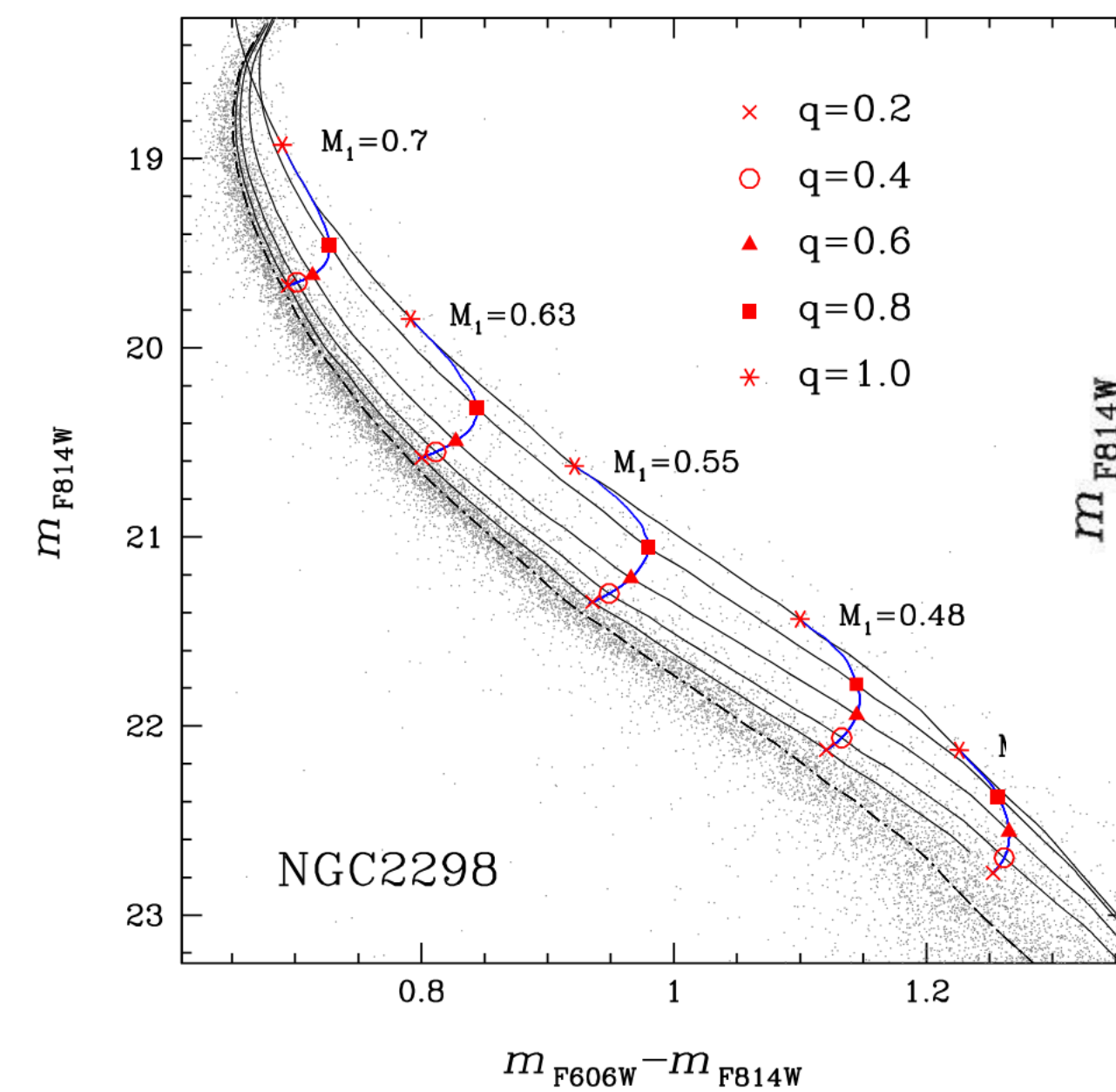
Multiplicity of massive stars



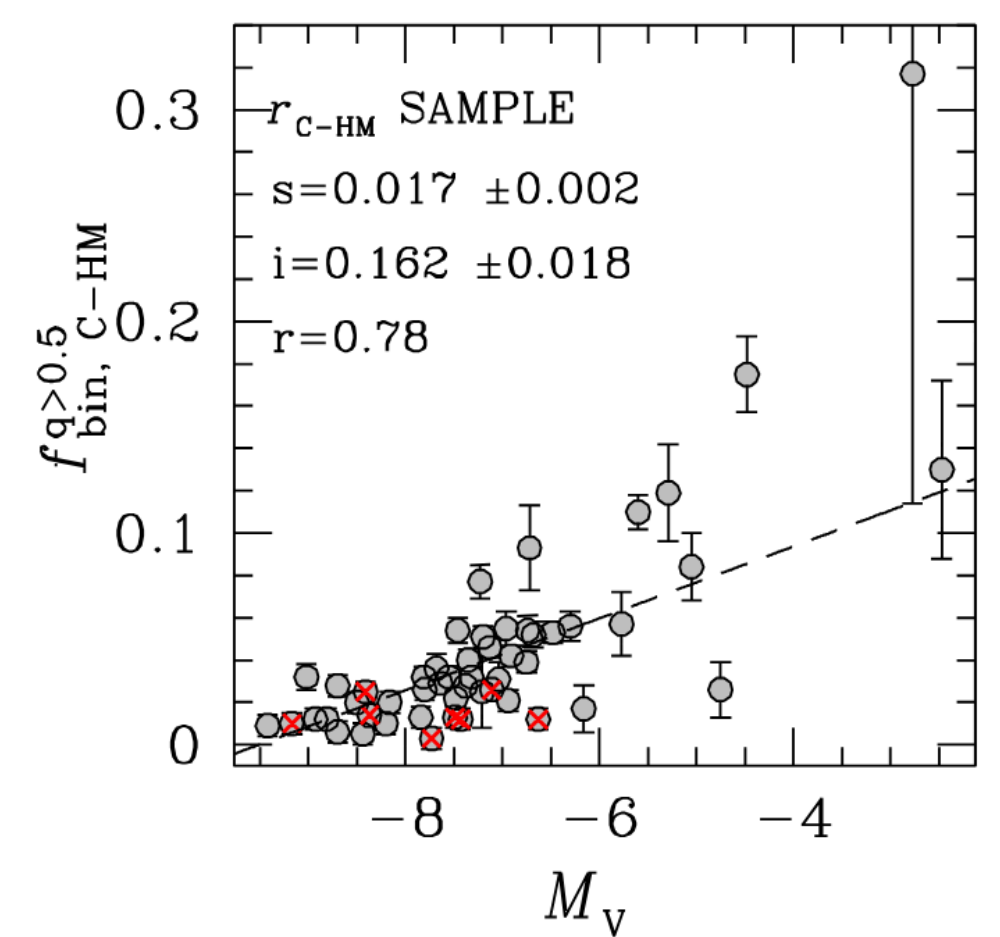
Offner et al. 2023: <https://arxiv.org/abs/2203.10066>

Observed binary fraction in globular clusters

- Observed binary fraction ($\frac{N_{bin}}{N_{single} + N_{bin}}$) of most Galactic globular clusters ranges from a few to up to ~20%
- Few clusters like Arp 2 have a high present-day binary fraction of 70%
- Spectroscopic identification of binaries in NGC 3201 (Giesers et al. 2019)
 - binary fraction in the core of 6.75%
- Birth binary fraction could be significantly larger than present-day binary fraction (Kroupa 1995; Leigh et al. 2015; Belloni et al. 2018)



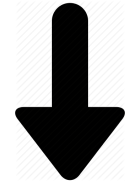
From Milone et al (2011)



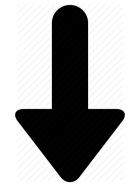
What happens to retained black holes in star clusters?

Old Theoretical Picture

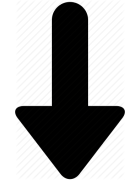
Black holes segregate to the center (due to dynamical friction)



Form a subsystem that behaves like an isolated cluster



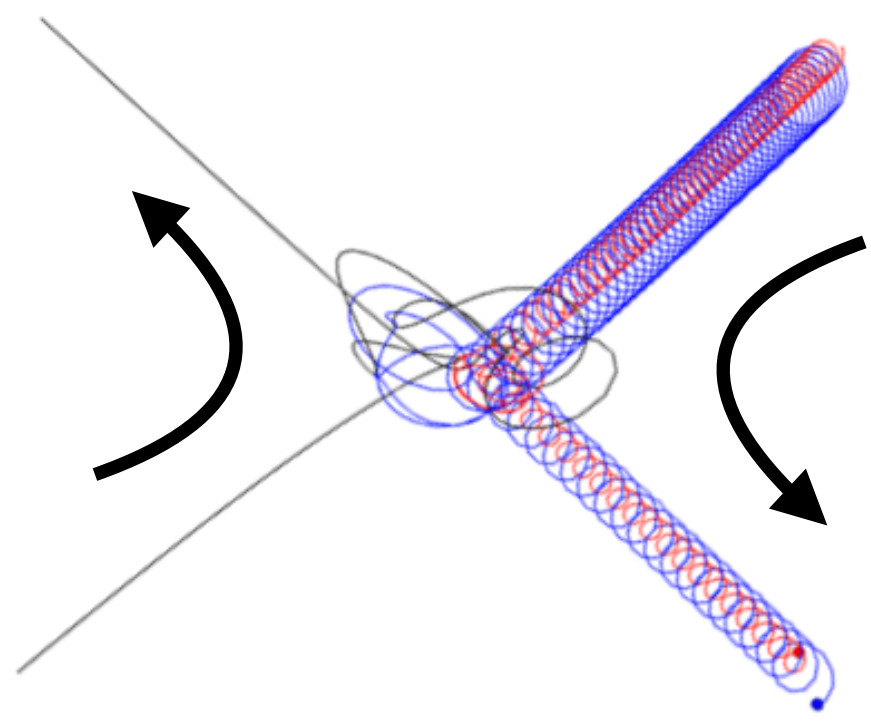
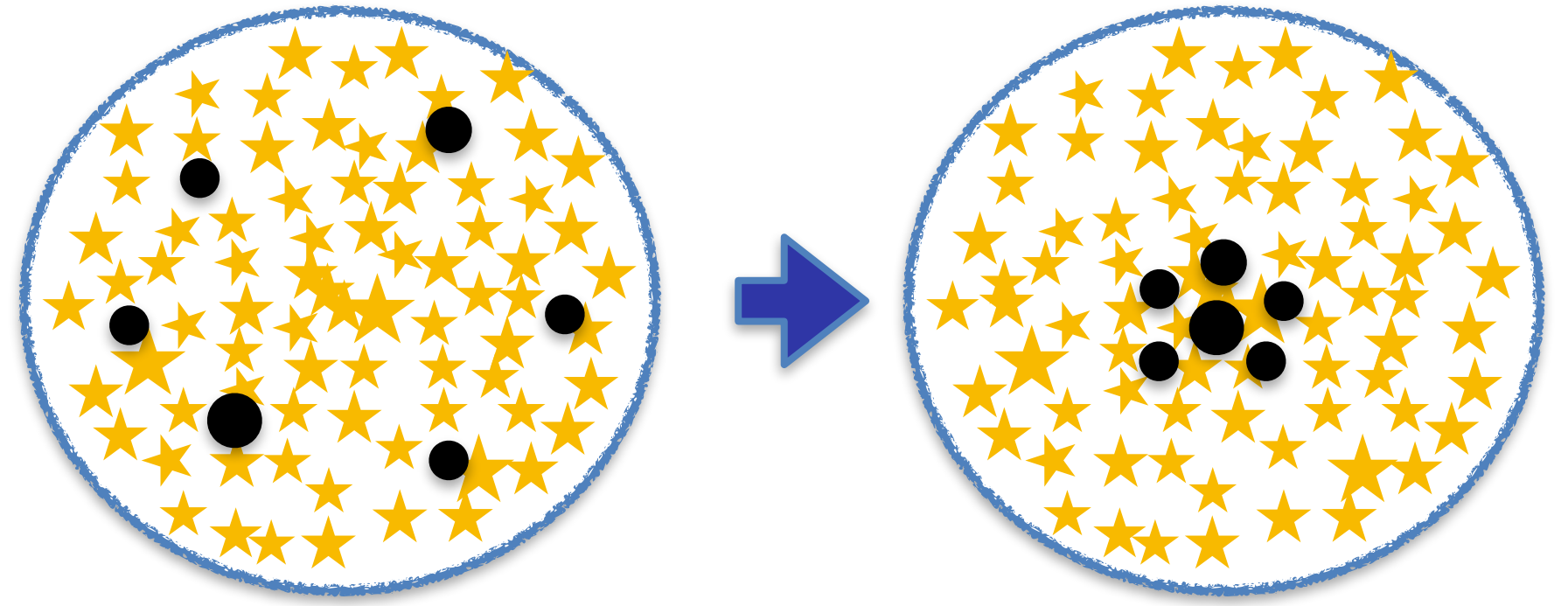
Black holes strongly interact with each other and escape very quickly (~1 Gyr)



1 or 2 black holes(at best) may survive in a globular cluster up to a Hubble time (Kulkarni, Hut, McMillan 1993; Sigurdsson & Hernquist 1993).

$$t_{df} \sim \frac{m}{M} t_{rlx}$$

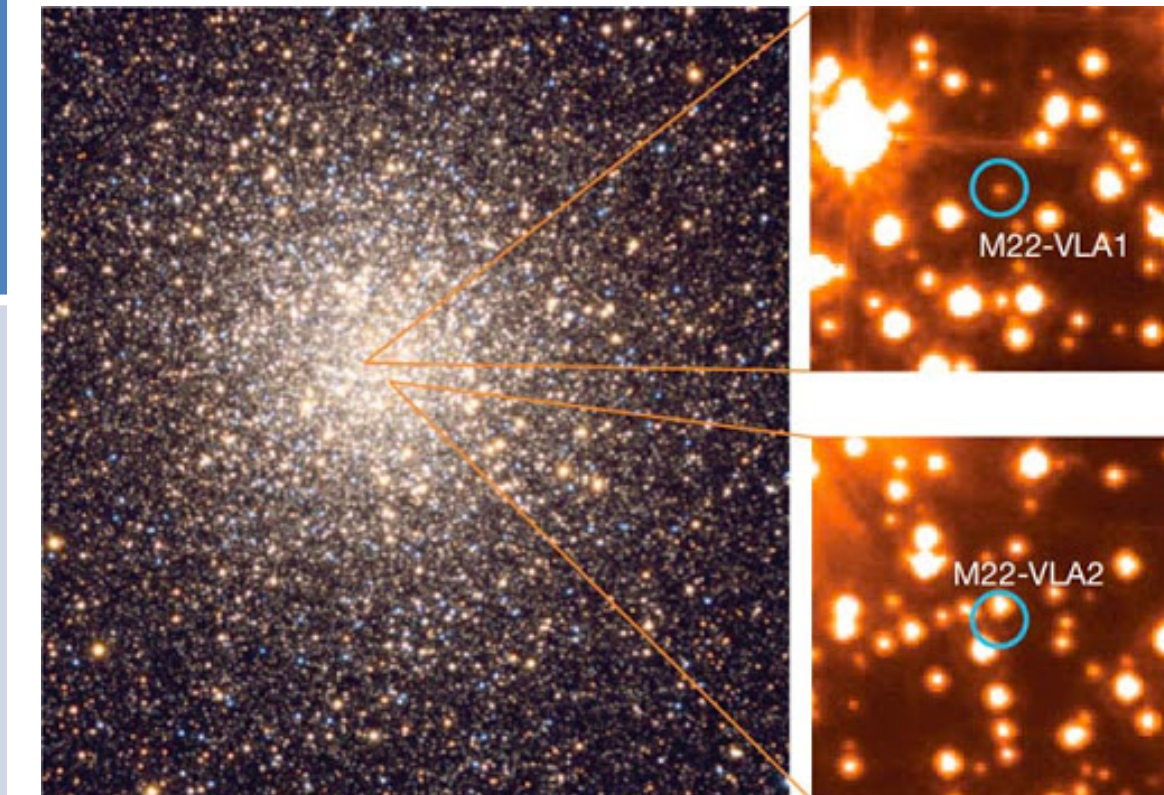
Mass Segregation



Black holes should not be observed in old globular clusters!

Observations of stellar-mass black holes in globular clusters

Type of Black Holes (BHs)	Observational Method	Observations
Accreting BHs in Binary Systems	X-ray/Radio Observations	<ul style="list-style-type: none"> • 2 candidates in M22 (Strader et al. 2012) • 1 candidate in M62 (Chomiuk et al. 2013) • Ultracompact BH-WD binary in 47Tuc (Bahramian et al. 2017) • BH-Red Straggler binary in M10 (Shishkovsky et al. 2018) • ULXs observed in a GC in the elliptical galaxy NGC 4472 (Maccarone et al. 2007)
Detached BHs in Binaries with a Luminous Companion	Radial Velocity Measurement	<ul style="list-style-type: none"> • 3 detected using MUSE in NGC 3201 (Giesers et al. 2018; 2019) $M \sin i = 7.68 \pm 0.50 M_{\odot}, 4.40 \pm 2.8 M_{\odot}$ and $4.531 \pm 0.21 M_{\odot}$
Isolated BHs	Gravitational Microlensing	<ul style="list-style-type: none"> • VVV Survey: BH candidate in NGC 6553? (Minniti et al. 2015) - Uncertain



2 BH candidates in M22 (Strader et al. 2012)



NGC 3201

Long-term survival of black holes in globular clusters

Segregating black holes form a subsystem
↓
Black hole subsystem provides energy (conduction) to support the rest of the cluster
↓
Evolution of black hole subsystem is governed by the overall relaxation time of the cluster
↓
Black holes do not escape quickly if a cluster has a long half-mass relaxation time

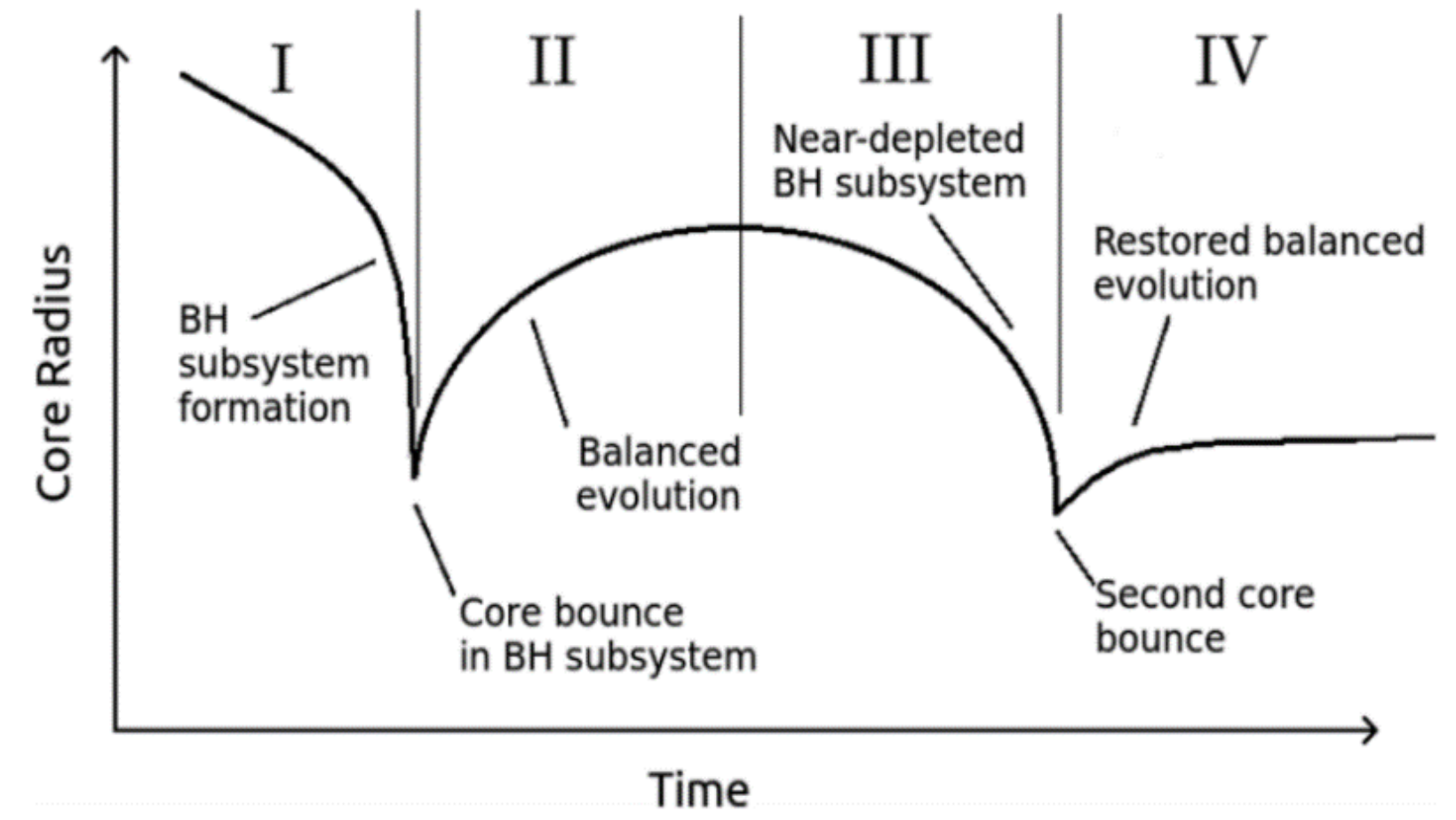
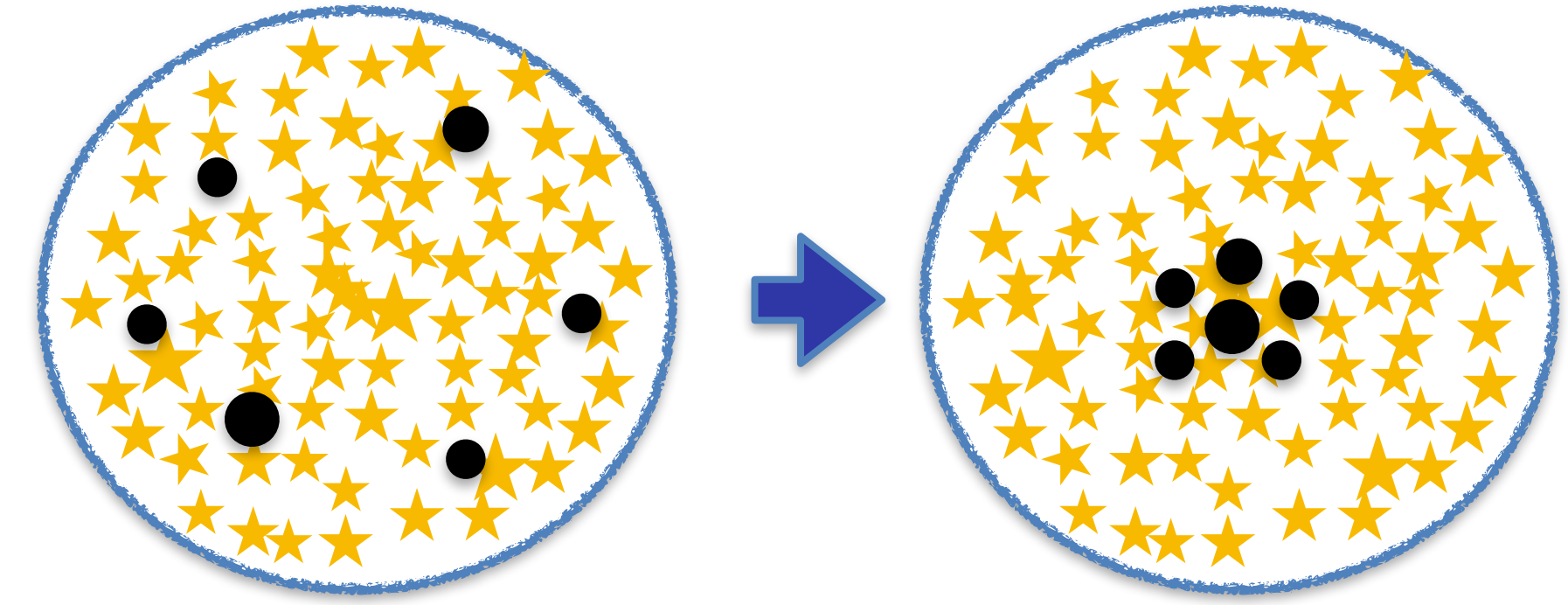
Black holes sit inside a deep potential well

Isolated clusters expand on relaxation timescales

Black holes drive expansion of the whole cluster

Some globular clusters could still contain many black holes!

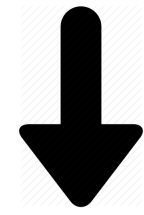
Mass Segregation



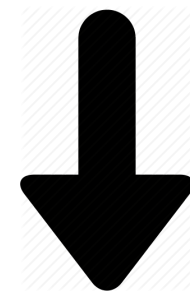
Credit: Breen & Heggie (2013)

Black hole dynamics in globular clusters

- Black holes segregate to the center of the cluster → interact with each other and surrounding stars

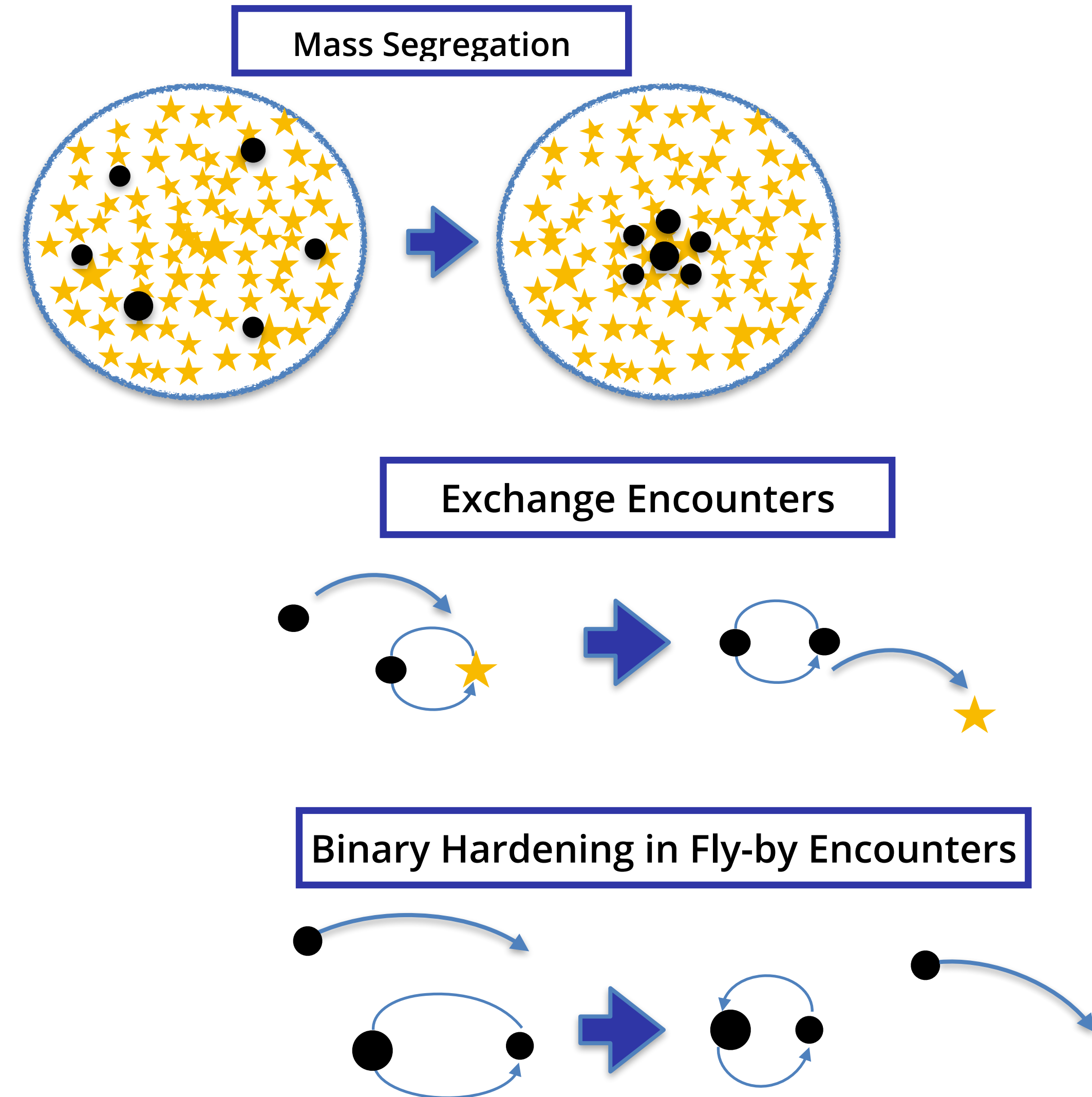


- Chaotic binary-single and binary-binary interactions involving black holes
- Formation of binary black holes through exchange encounters



- Shrinking of binary black holes through interactions → can merge due to gravitational wave radiation within a Hubble time (more in next part of the lecture)

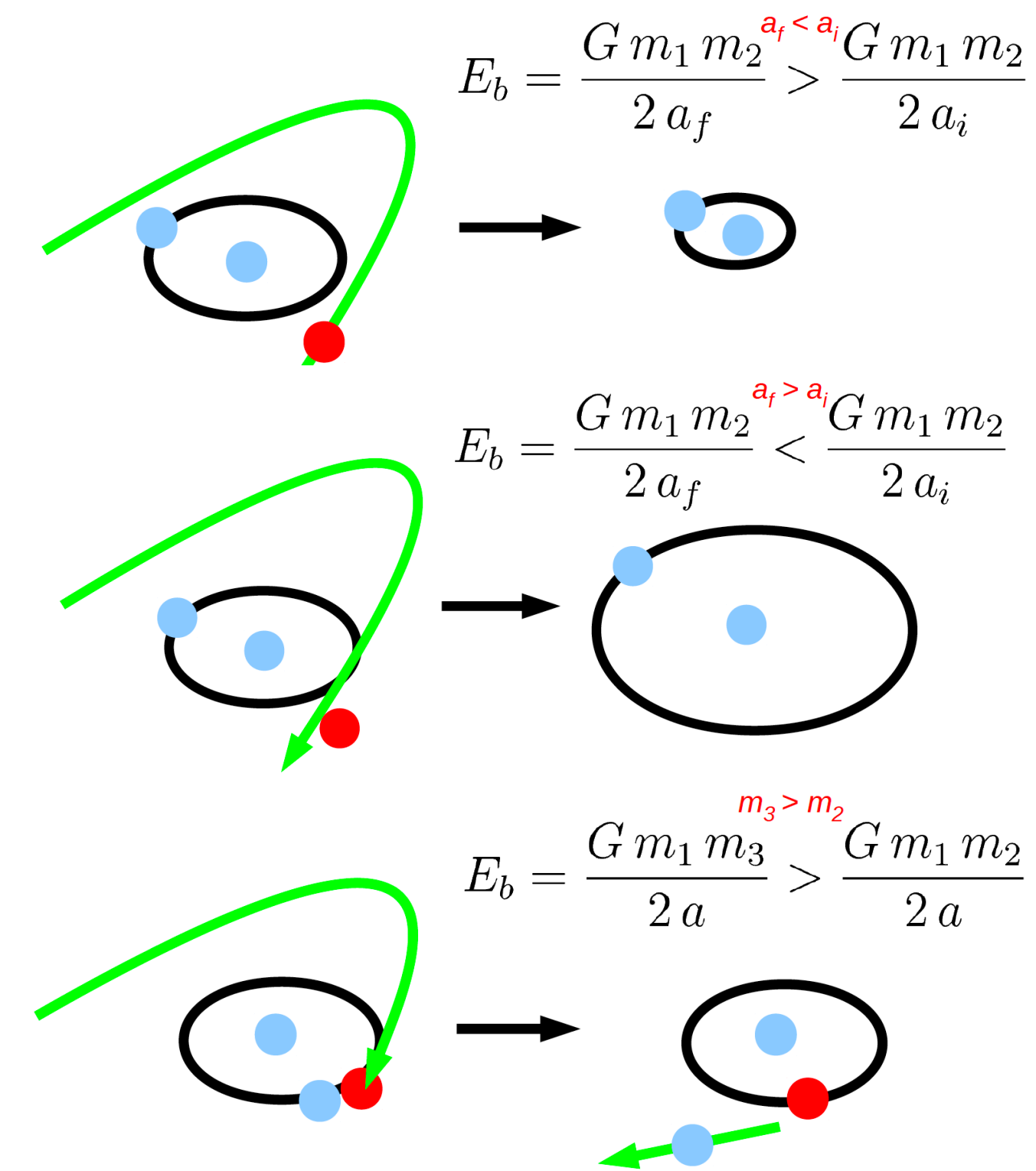
$$\tau_{\text{gr}} \simeq 10^{10} \text{yr} \left(\frac{a_{\text{bin}}}{3.3R_{\odot}} \right)^4 \frac{1}{(m_1 + m_2)m_1m_2} \cdot (1 - e^2)^{7/2} \quad (\text{Peters 1964})$$



3-body encounters in stellar clusters

- For a bound binary system: $E_{int} = -\frac{G m_1 m_2}{2 a} = -E_b$
- Binary shrinking/hardening:
 - Single star extracts internal energy of the binary (stars final kinetic energy is higher than initial) - Flyby
- Binary softening or Ionization:
 - Single star transfers kinetic to the binary
- Exchange interaction:
 - Single star can replace one of the binary companions

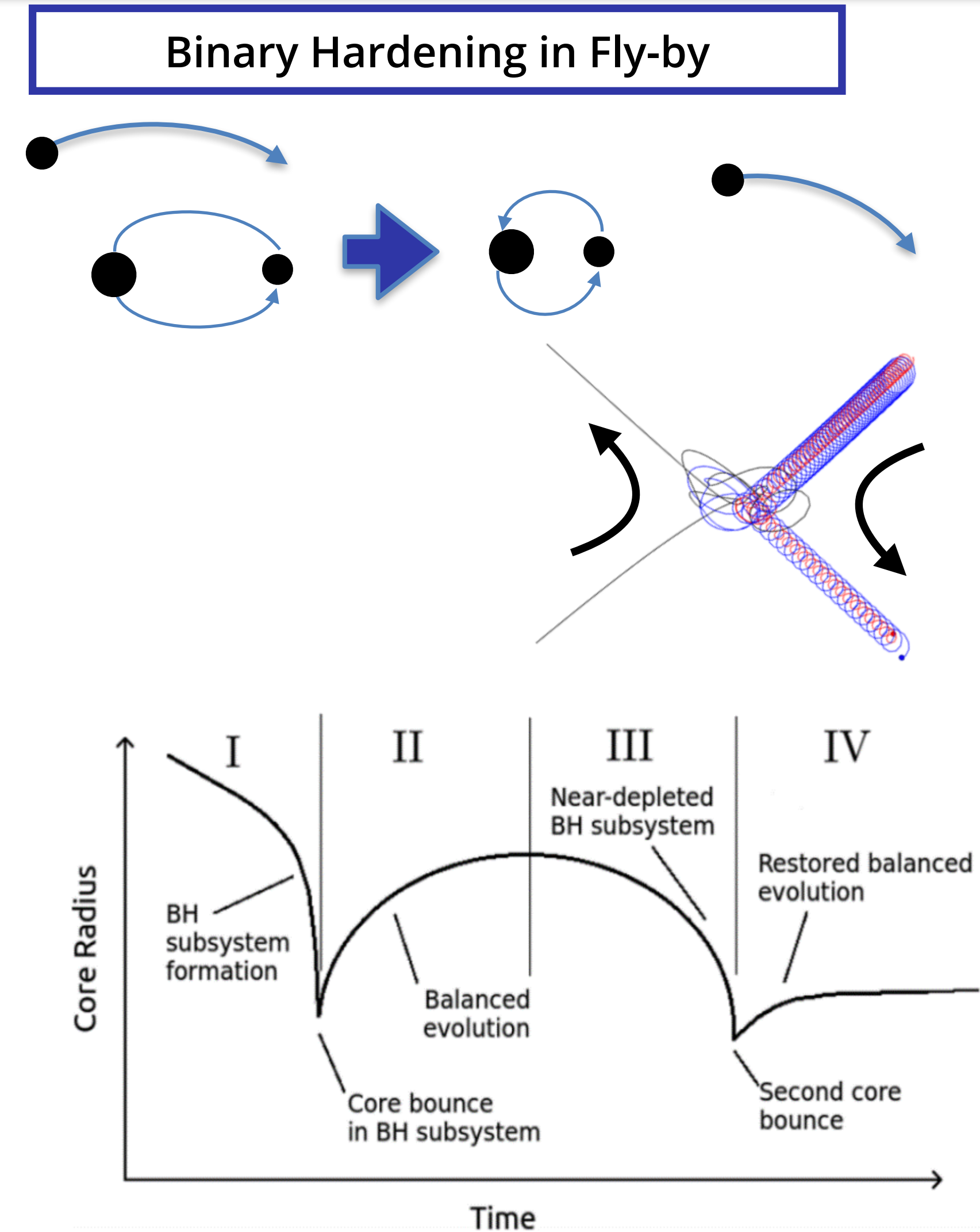
Heggie/Hill's (1975) Law: In 3-body encounters hard binaries become harder and soft binaries tend to become softer



Credit: Michela Mapelli
Lecture notes on Collisional Dynamics

Black hole dynamics in globular clusters

- Dynamical interactions also eject tight binary black holes out of the cluster due to dynamical recoil (scattering kick)
- Black hole population in clusters depletes with time → depletion time depends on cluster initial properties
- Black holes heat surrounding stars (Mackey et al. 2007;2008, Breen & Heggie 2013)
- Initially dense clusters → more interactions → faster depletion of black holes
- Less dense clusters → fewer interactions → slower depletion of black holes
- Initially dense clusters → have shorter initial 2-body relaxation times → dynamically older → important for producing binary black holes (next part of the lecture)



Credit: Breen & Heggie (2013)