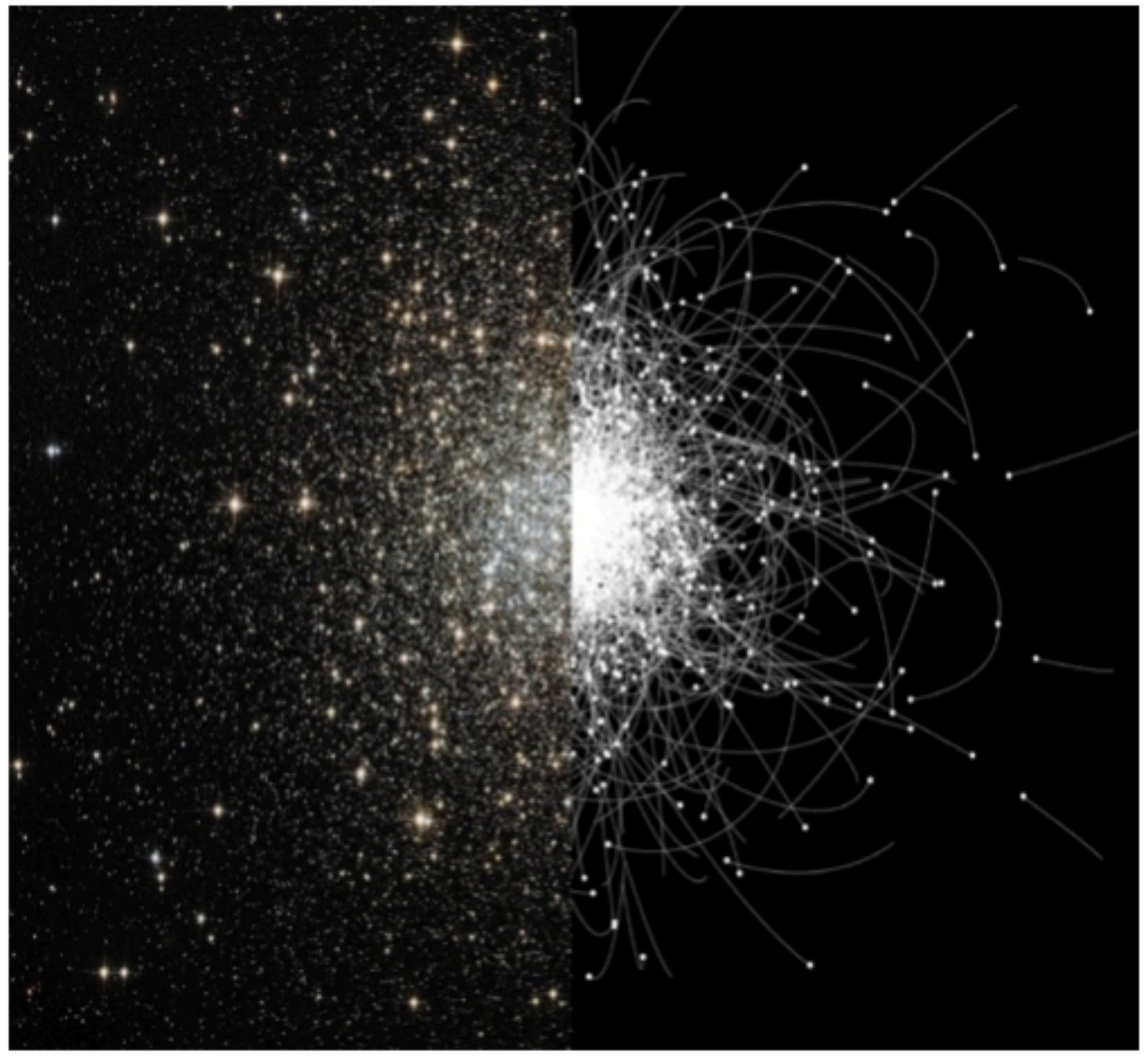
Star Cluster Dynamics and Evolution



Geoplanet Doctoral School Lecture Course (Spring 2024)

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CAMK PAN

National Science Centre Poland

MOCCA

BH GROWTH
Growing Black Holes in Star Clusters

Course format and outline

- Lecture 1: Introduction to star clusters and stellar dynamics
- Lectures 2 - 4: Collisionless and collisional stellar dynamics
- Lecture 5 : Direct *N*-body and Monte Carlo method for evolving star clusters
- Lecture 6: Thermodynamics of stars clusters and simple star cluster models
- Lecture 7: Simulating realistic star clusters: important physical processes
- Lecture 8: Role of binaries in star clusters and energy generation: dynamics and heating
- Lectures 8-9: Observations and astrophysical importance of star clusters
- Lecture 10-11: Black holes in star clusters and formation of gravitational wave sources
- Lecture 12: Summary lecture with key takeaways + instructions on assignment

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- Lecture 7: Internal physical processes in star cluster evolution
- Lecture 8: External physical processes in star cluster evolution
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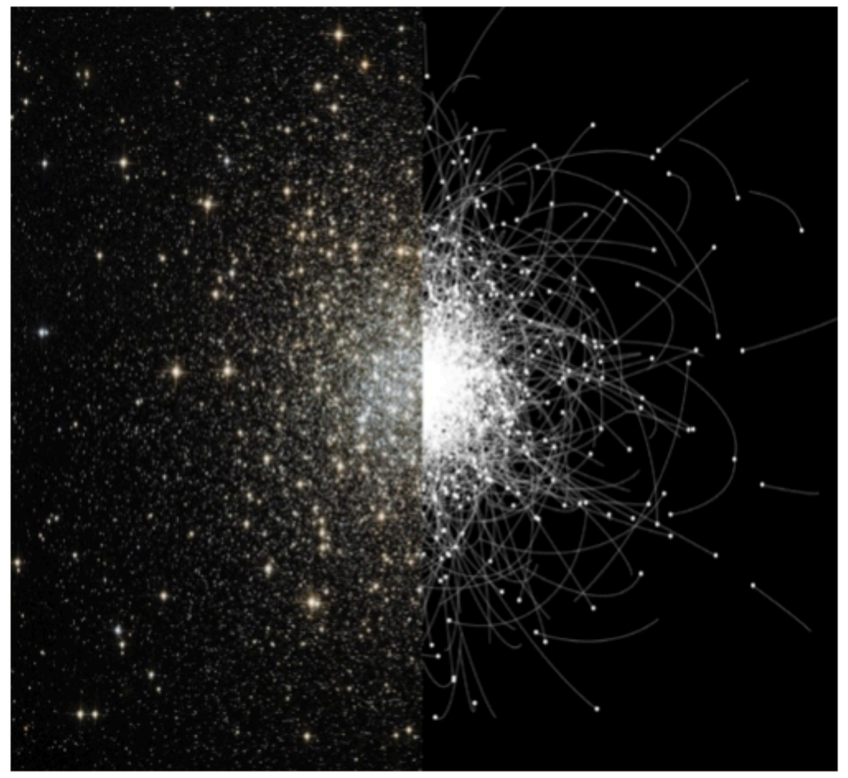
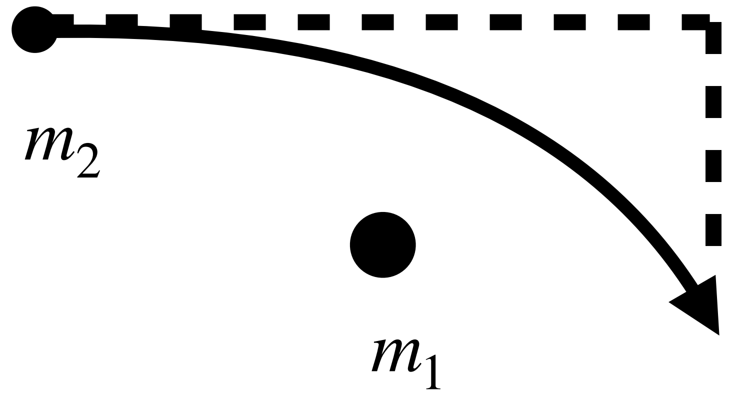
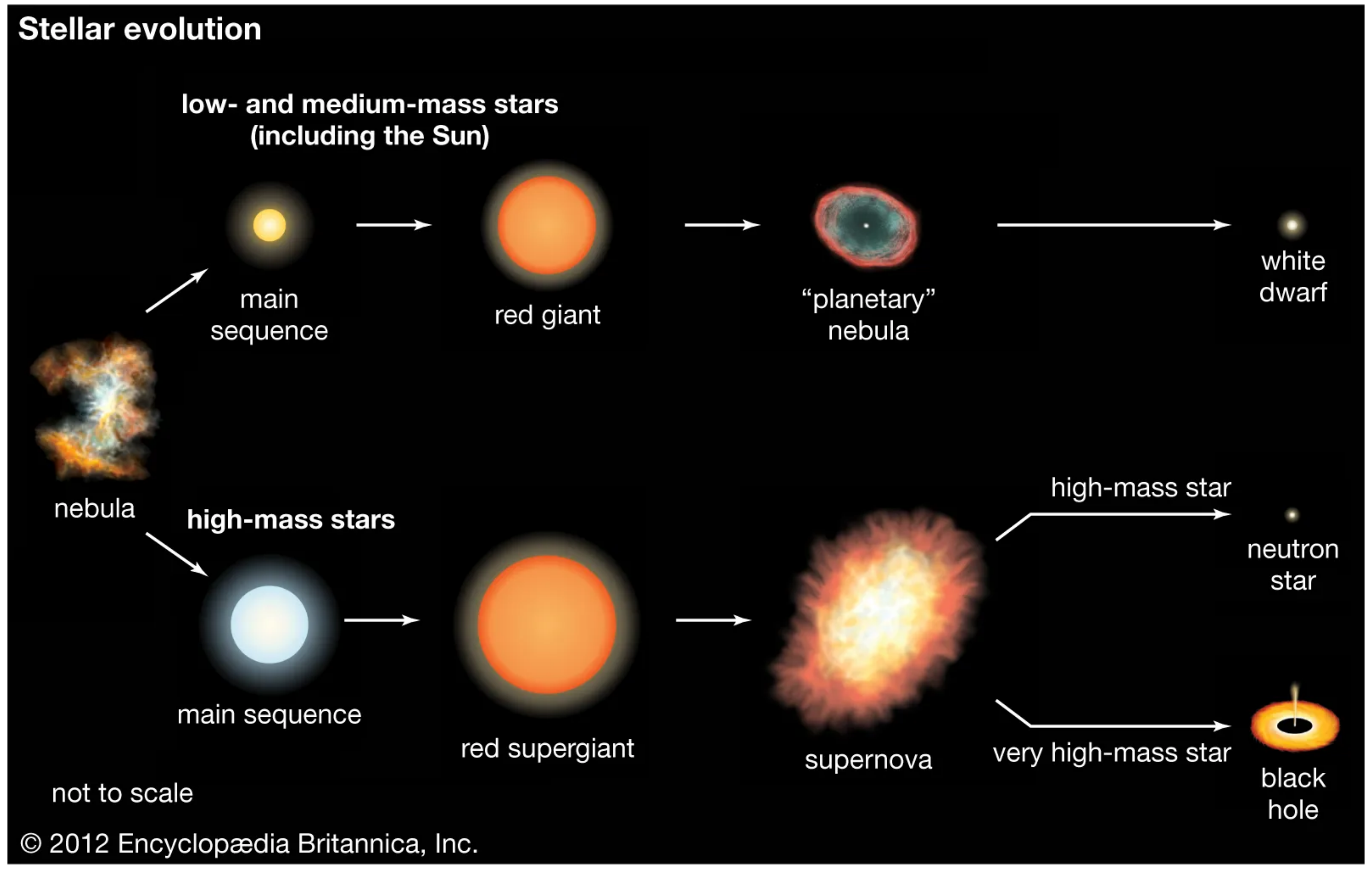
Internal physical processes that drive cluster evolution

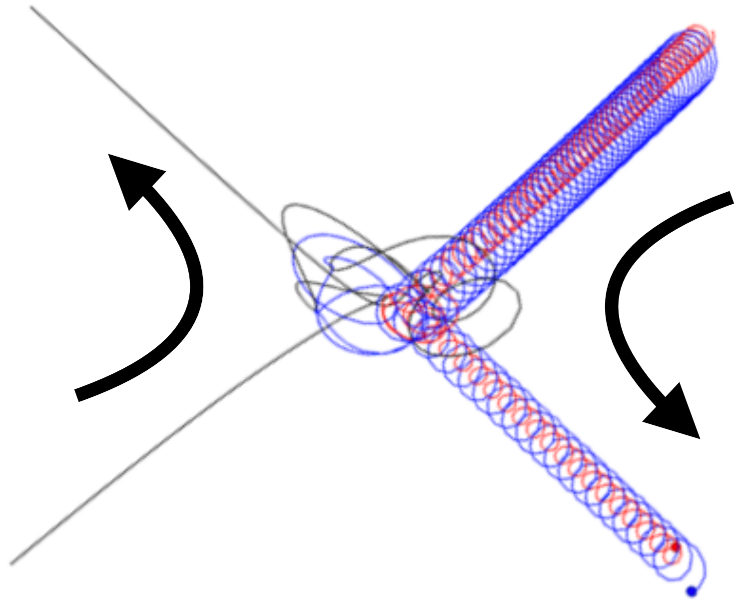
- **Gravitational Dynamics**
  - Mutual gravitational interactions among stars significantly modify their initial orbits: 2-body relaxation





- **Stellar and binary evolution**

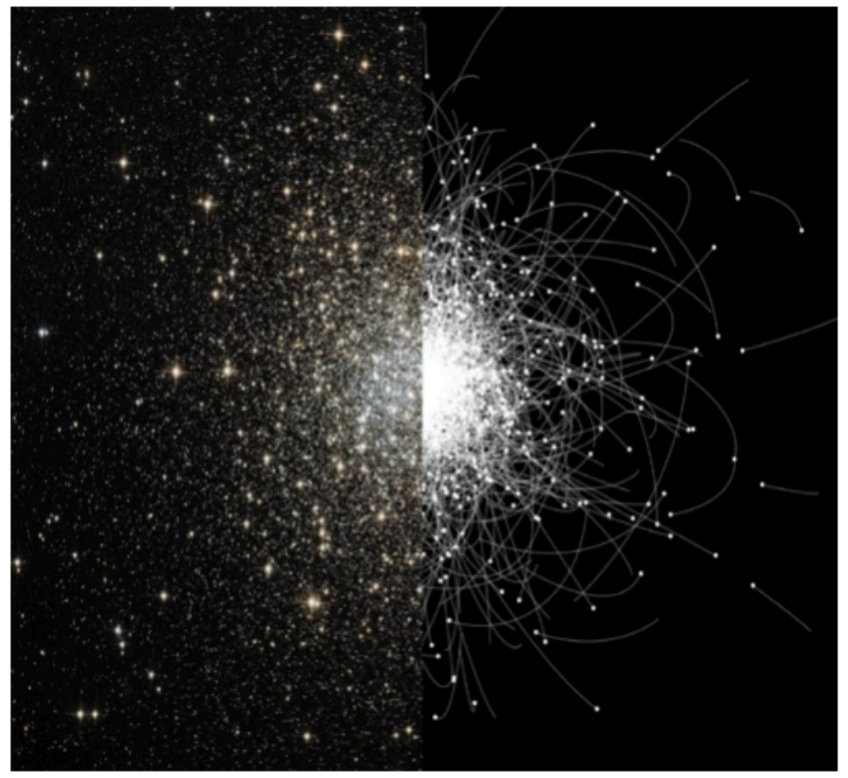


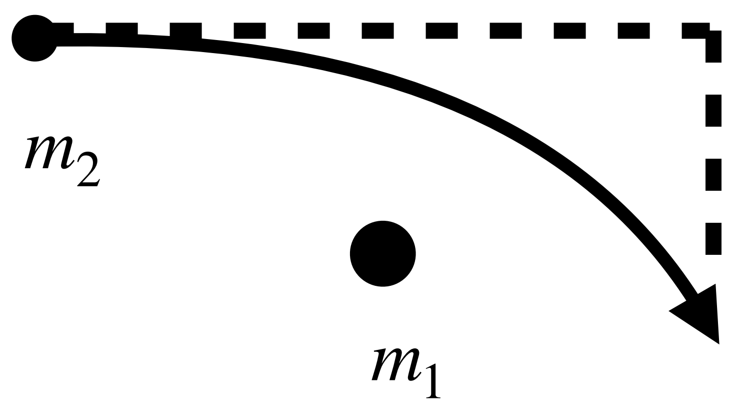


- **Role of binaries: dynamics and cluster heating**

Internal physical processes that drive cluster evolution

- **Gravitational Dynamics**
  - Mutual gravitational interactions among stars significantly modify their initial orbits: 2-body relaxation





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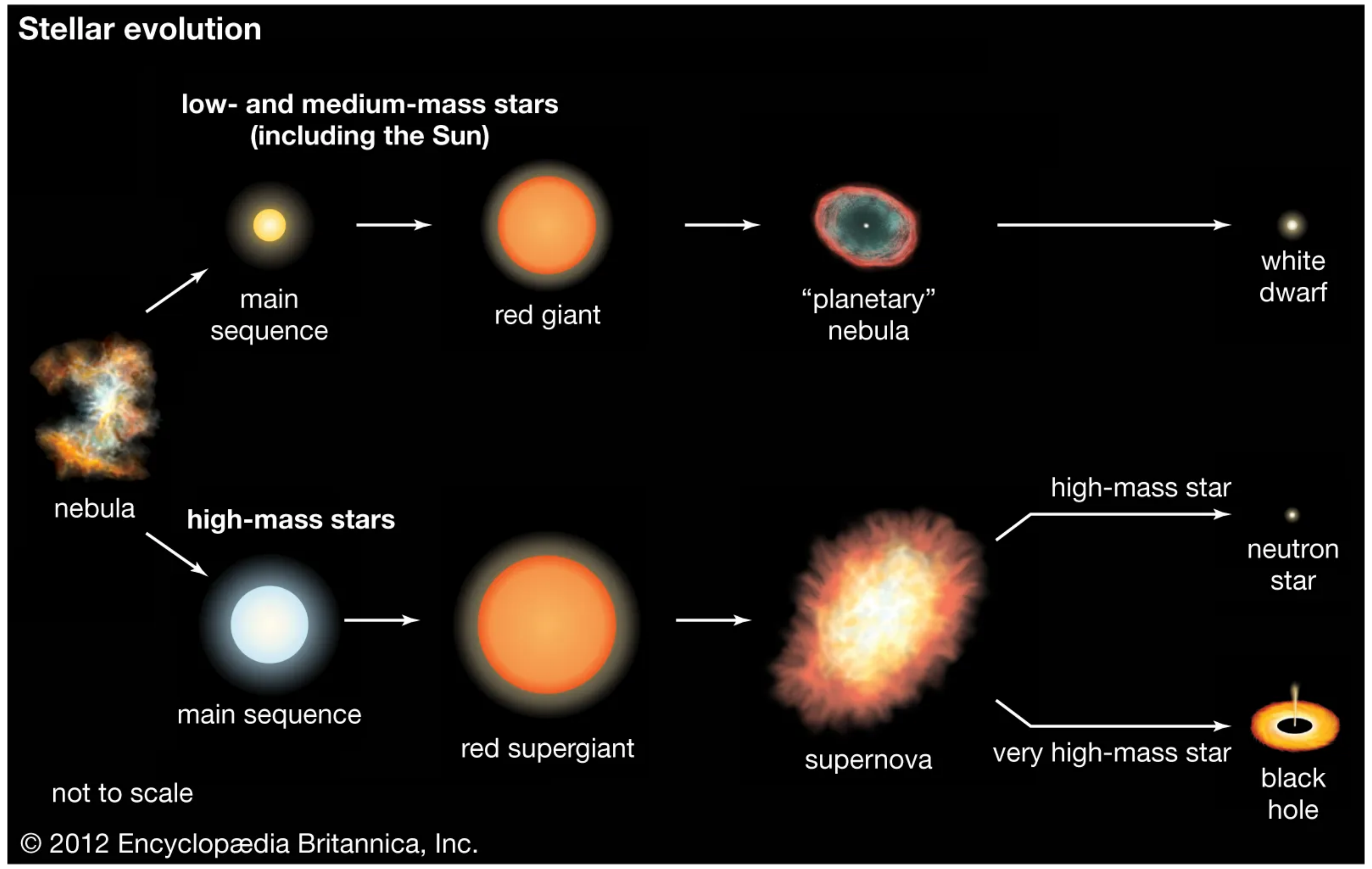

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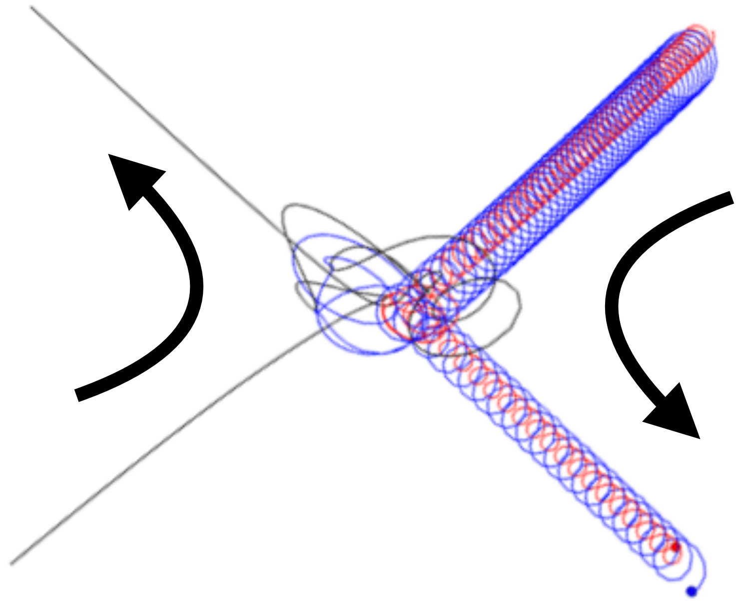


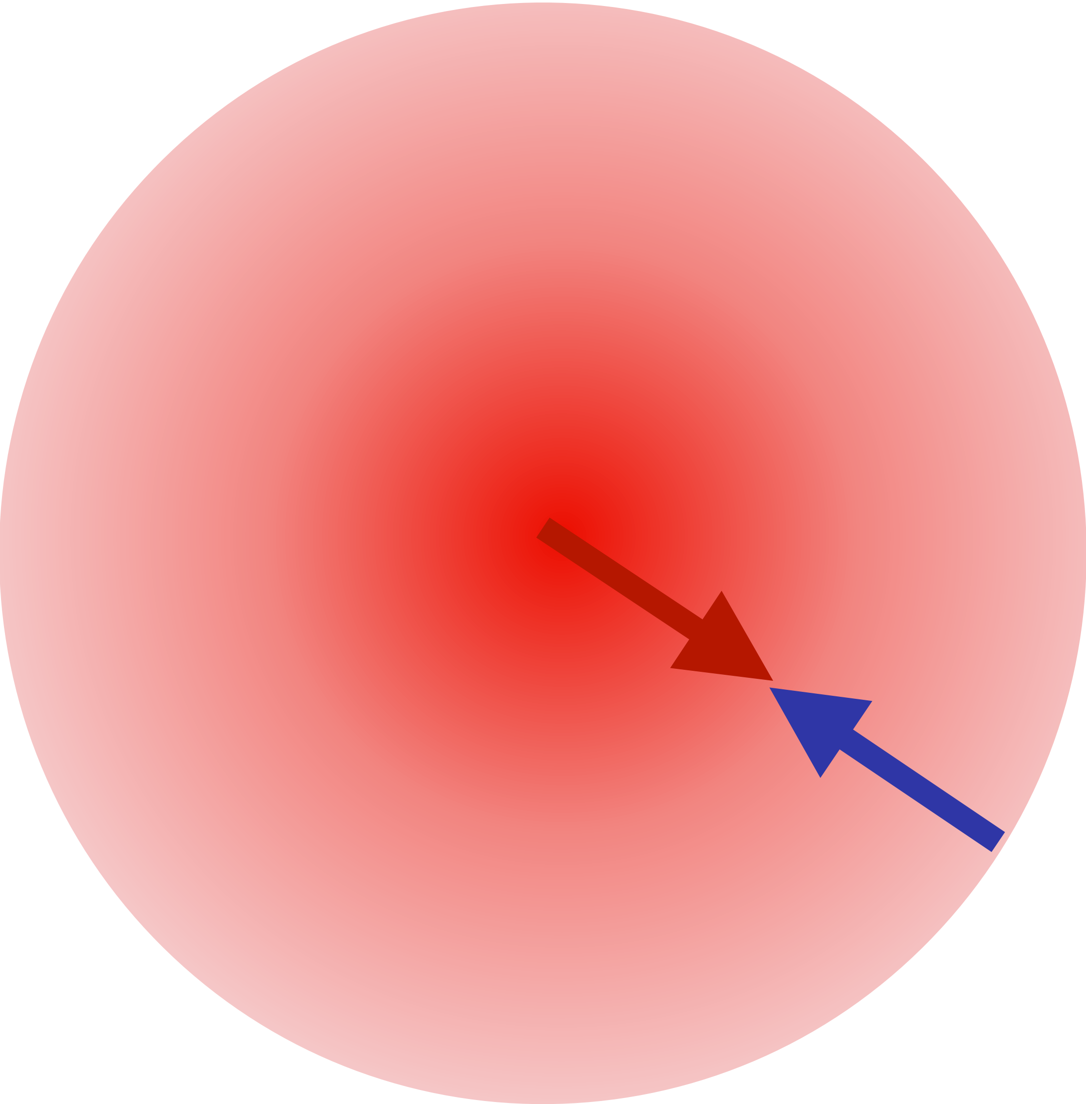
Reminder: How do star clusters form?

- Stars are born in large clouds of molecular gas
- Molecular clouds are composed of hundreds of solar masses of material → stars (almost) always form in groups or clusters

Pillars of Creation within the Eagle Nebula
Credit: *JWST*

Reminder: Formation of stellar clusters

- Dense molecular clouds → cold 10° to 50° K (-263° C)



Gas Pressure = **Gravitational Attraction**

$$R_J \sim \sqrt{\frac{T}{\rho}}$$

Temperature

Density

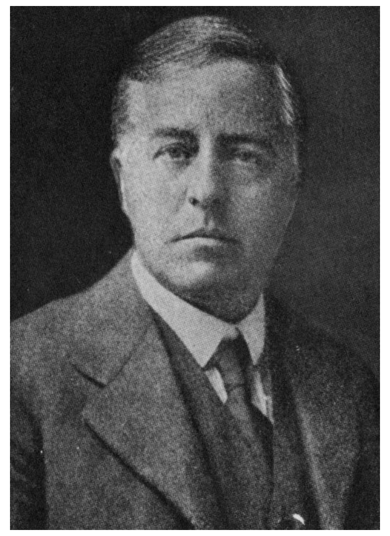
Radius where thermal expansion energy = gravitational potential energy

$$M_J = \rho R_J^3 \sim \sqrt{\frac{T^3}{\rho}}$$

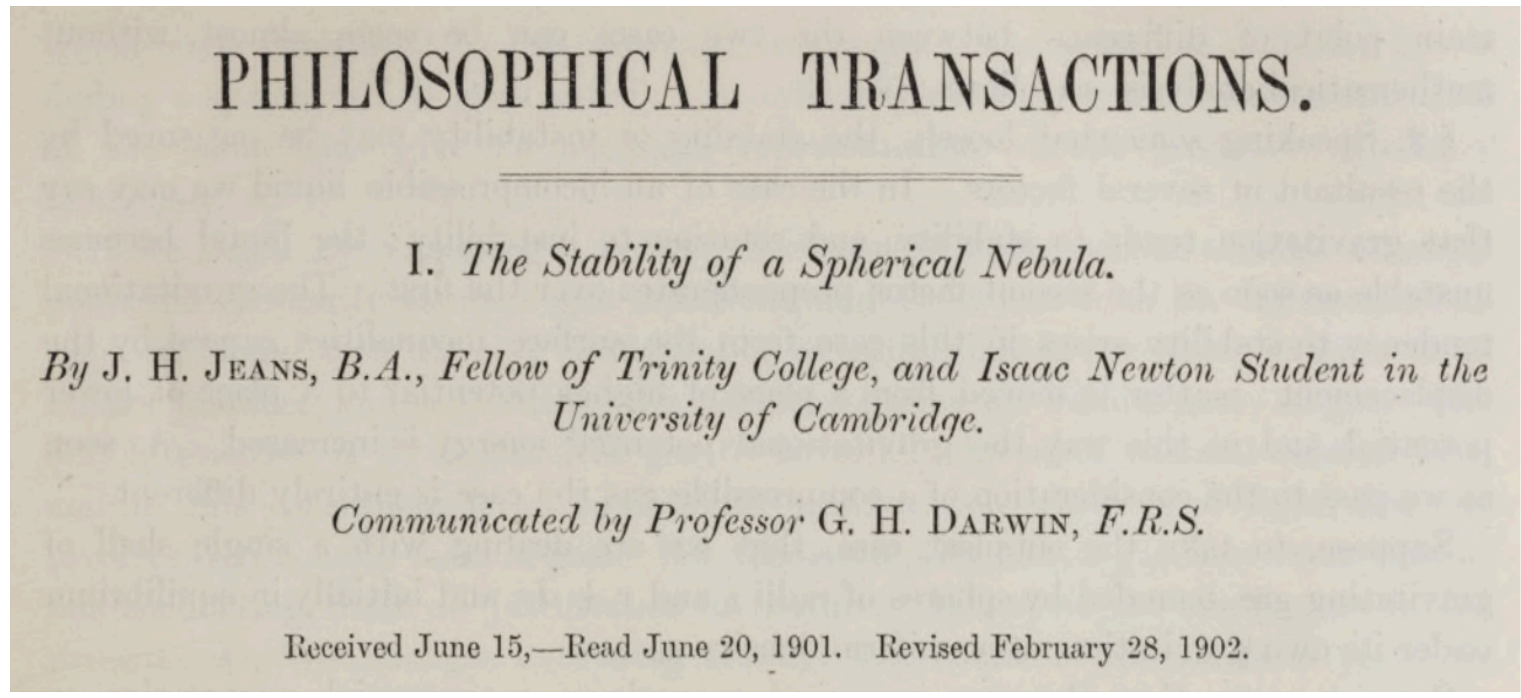
Temperature

Density

Mass of the cloud where thermal energy = gravitational energy



Jeans (1902)



PHILOSOPHICAL TRANSACTIONS.

I. *The Stability of a Spherical Nebula.*

By J. H. Jeans, *B.A., Fellow of Trinity College, and Isaac Newton Student in the University of Cambridge.*

Communicated by Professor G. H. Darwin, *F.R.S.*

Received June 15,—Read June 20, 1901. Revised February 28, 1902.

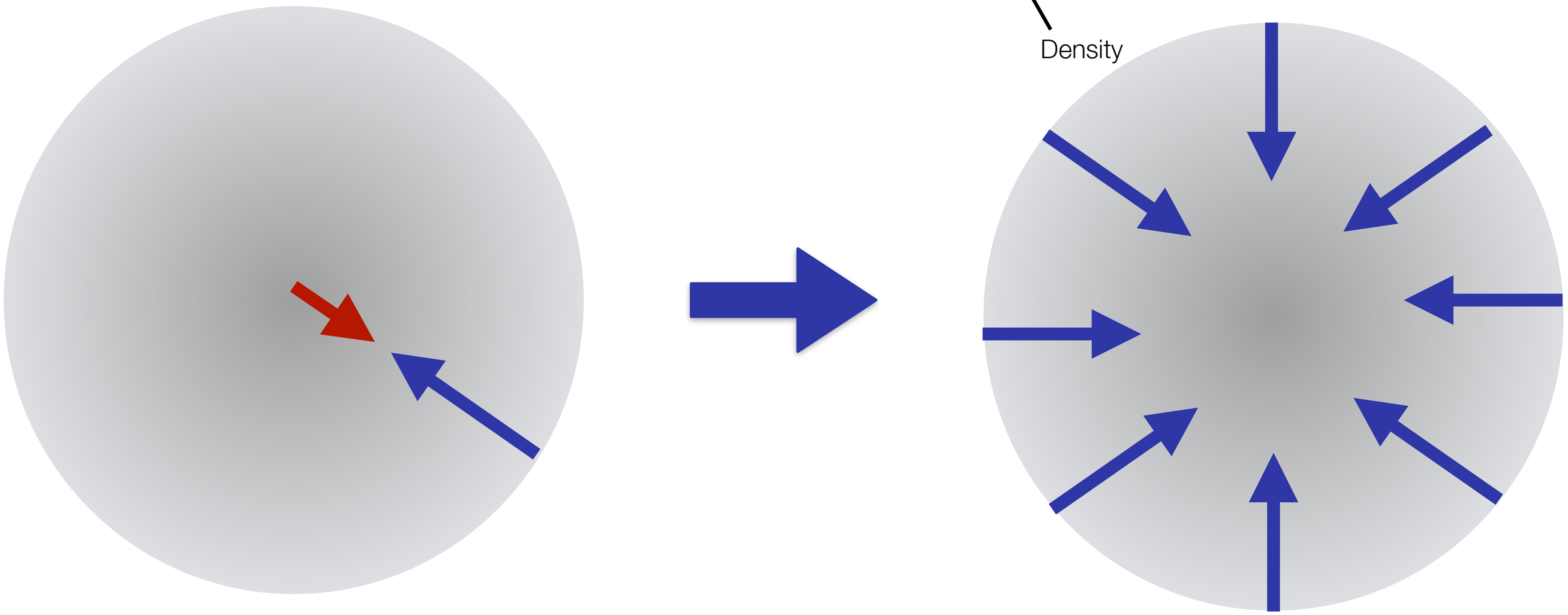
Reminder: Formation of stellar clusters

- As gas cools → pressure decreases → cloud becomes unstable to gravitational collapse

$$M_J = \rho R_J^3 \sim \sqrt{\frac{T^3}{\rho}}$$

Temperature

Density



Gas Pressure < Gravitational Attraction

Gravitational collapse

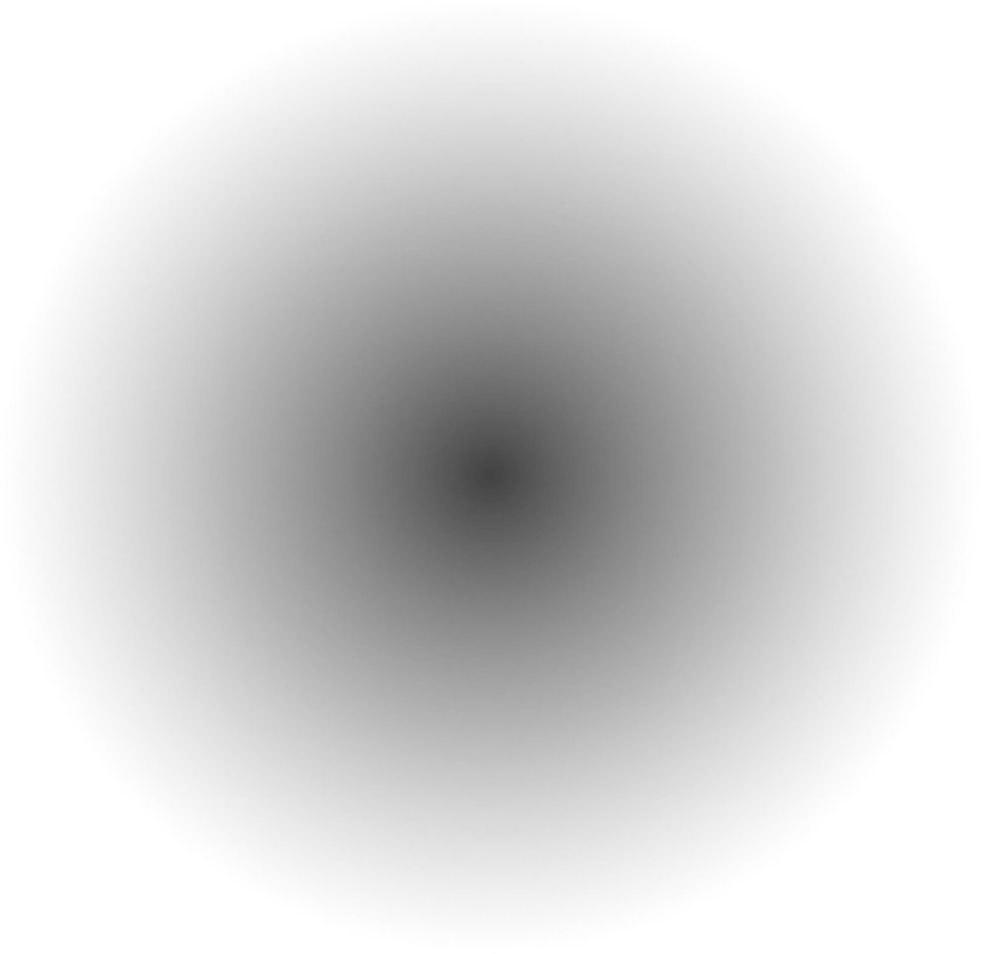
Reminder: Formation of stellar clusters

- Jeans mass decreases (as it decreases with increasing density) → local regions of collapsing material themselves collapse → fragmentation

$$M_J = \rho R_J^3 \sim \sqrt{\frac{T^3}{\rho}}$$

Temperature

Density



Higher density within collapsing cloud

Fragmentation of collapsing cloud into stars

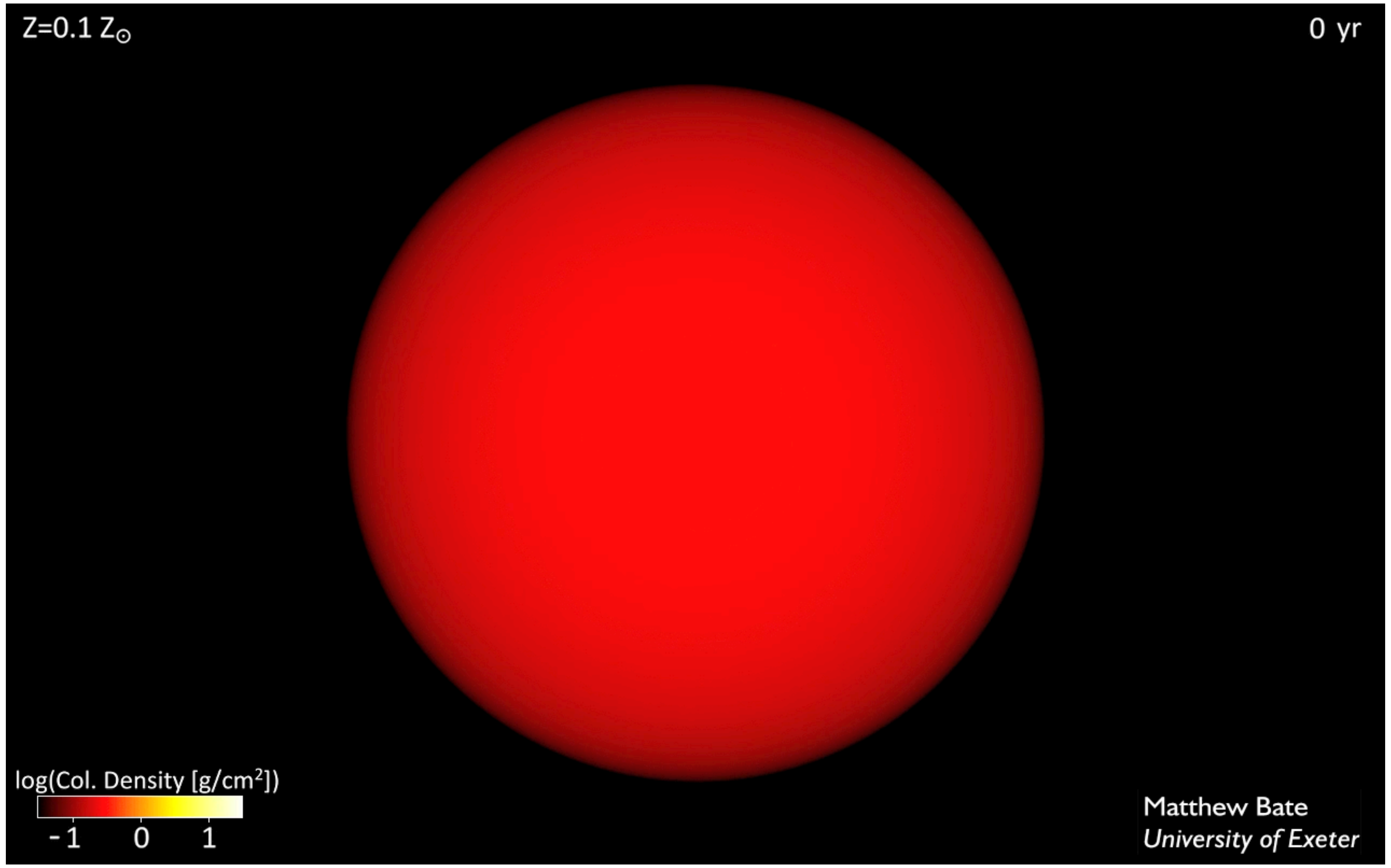
Video of star cluster formation

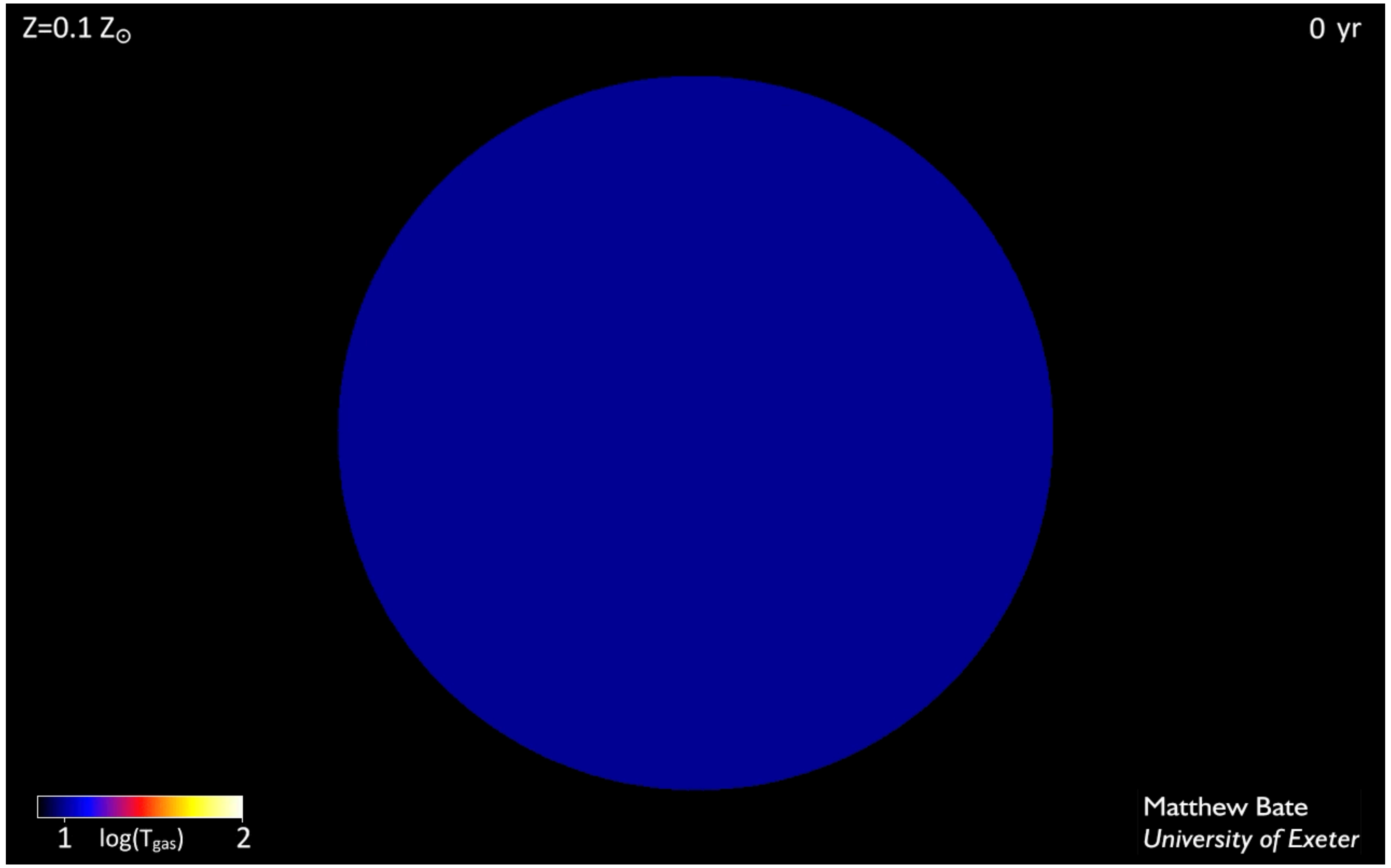
500 $M_\odot$ Gas Cloud

D = 0.8 pc or 2.6 lyrs

- gas
- gravity
- turbulence

- cooling
- fragmentation
- sink particles





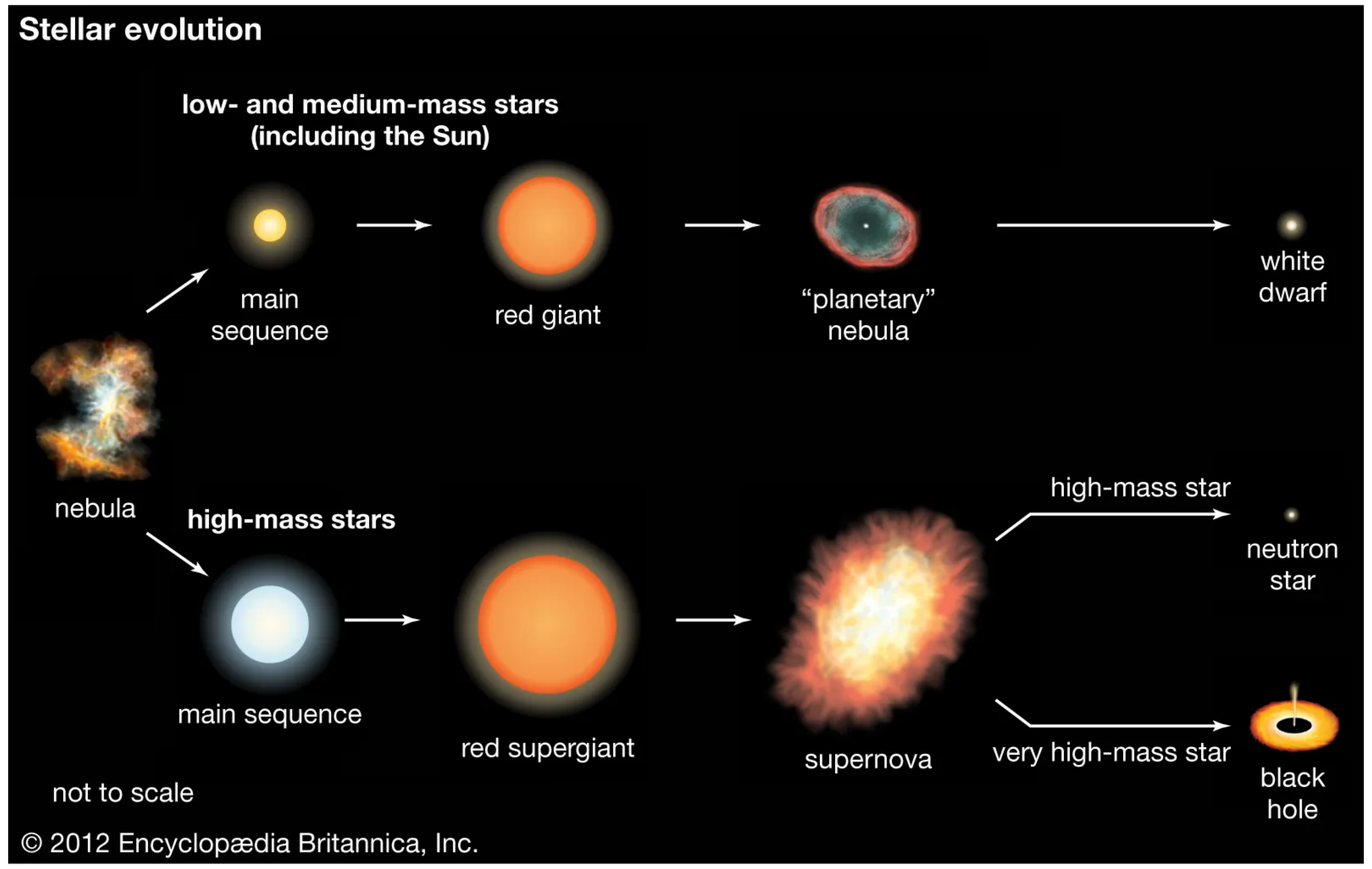
Bate (2019)
Videos available at: http://www.astro.ex.ac.uk/people/mbate/Research/Cluster/clusterMetallicity2.html

Smoothed-particle hydrodynamics simulation
Credit: Matthew Bate

Lifecycle of stars

- Gas cloud contracts under gravity. Temperature increases due to release of gravitational binding energy and increasing density.
- When the temperature is high enough ($\sim 10^7$ K) hydrogen fusion is ignited. The star is born (zero-age main sequence)
- When hydrogen is exhausted the star contracts further until the temperature is high enough to ignite helium fusion ($\sim 10^8$K) → until a carbon or iron core is formed (depends on initial mass and evolution)
- Gravitational collapse leads to a formation of a white dwarf, black hole or a neutron star (depends on initial mass and evolution)

Stellar life is just an intermediary stage between a gas cloud and a compact object

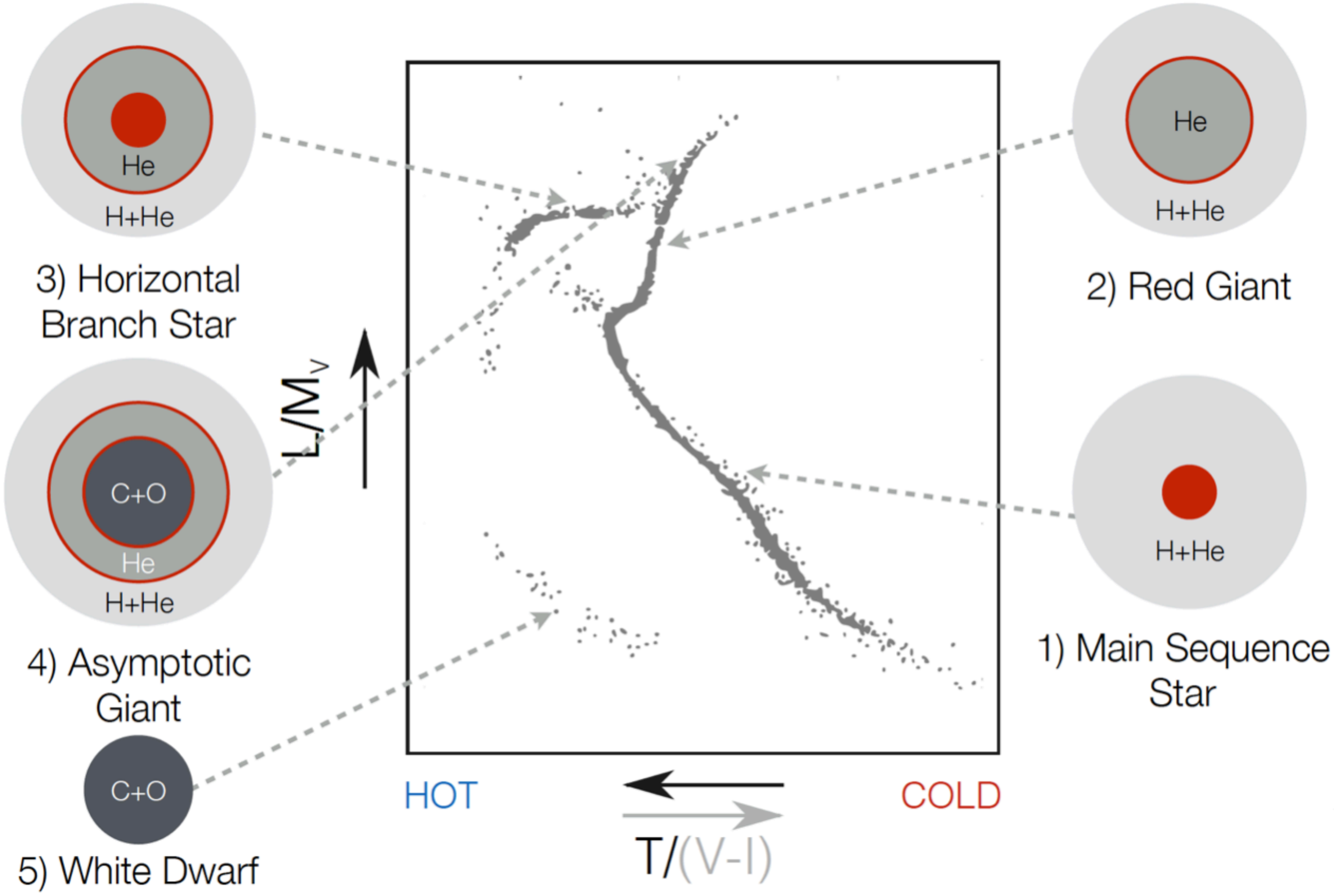


Important definitions and physical processes in stars

- Metallicity: Fraction of elements heavier than helium
- Main sequence: Stars that burn hydrogen in their core
- Post-main sequence star: that has left he main sequence and is no longer burning hydrogen in the core
- Stellar winds:
  - Photons in the atmosphere of a star transfer linear momentum to ions and unbind them
    - Particularly important for massive stars on the main sequence

$$X + Y + Z = 1.0$$

$$\text{Solar Values}: \ X \sim 0.73, \ Y \sim 0.25, \ Z \sim 0.02$$



Credit: N. Lüetzgendorf

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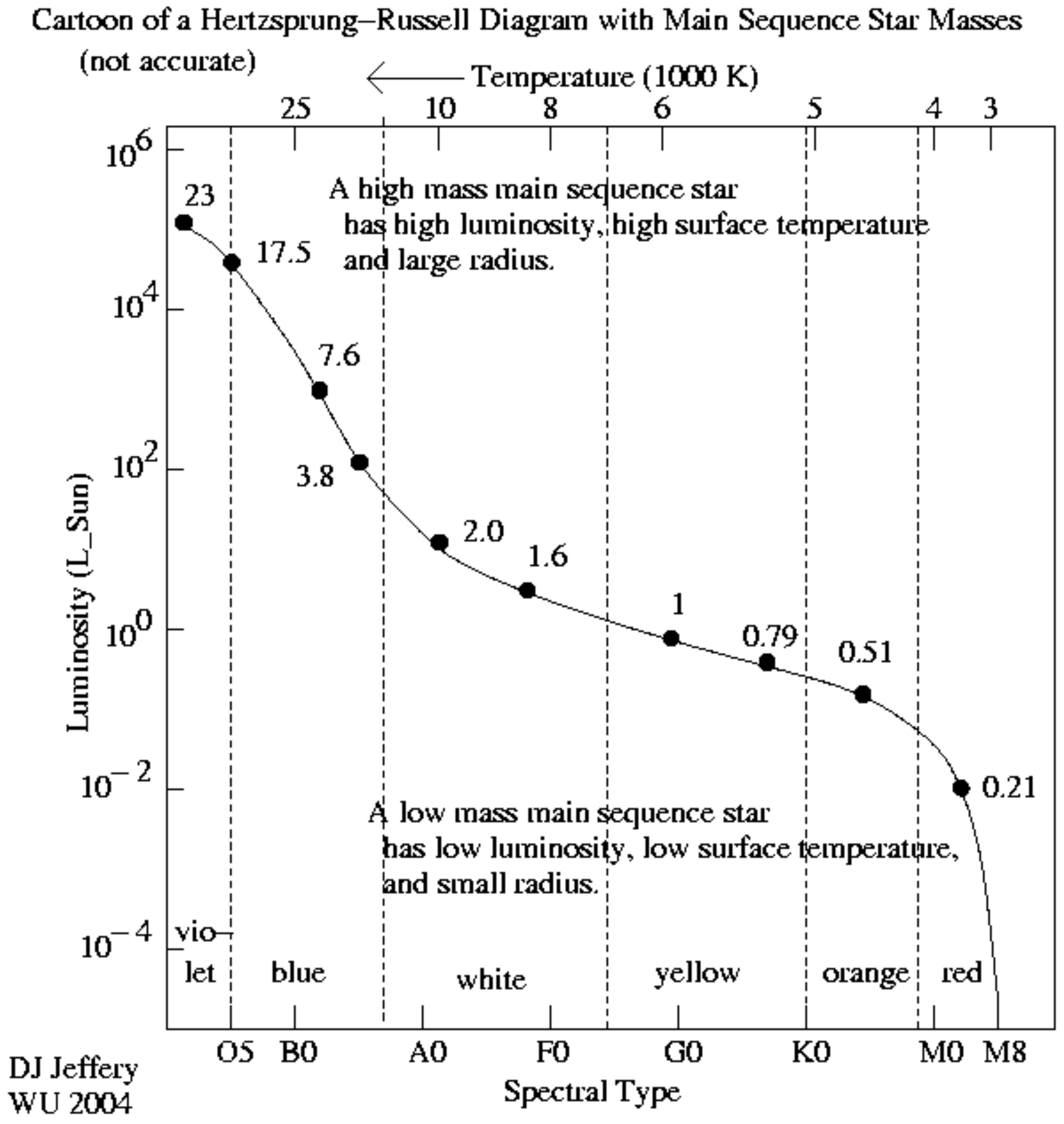
    - Particularly important for massive stars on the main sequence
    - Also important for post-main sequence stars of all masses
- Mass loss in final stages of evolution: e.g., AGB ejecta, supernova explosion

$$X + Y + Z = 1.0$$

Solar Values : $X \sim 0.73,\ Y \sim 0.25,\ Z \sim 0.02$



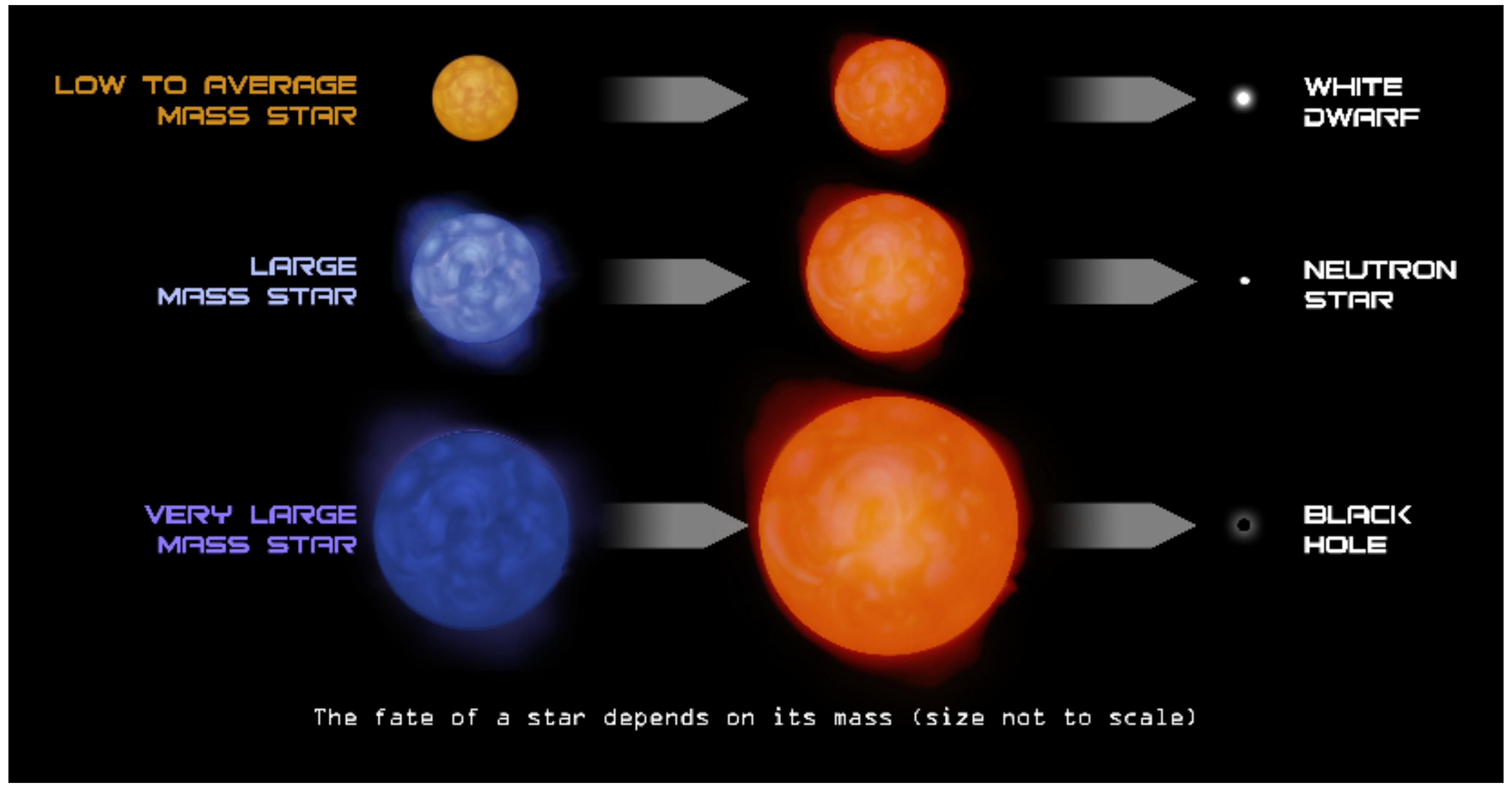
What determines the evolution of a star?

- ZAMS mass is the most important property that determines the evolution and final fate of a star
- It also sets the timescale of evolution of the star: $\tau_{\text{nuclear}} \propto M^{-2.5}$
  - where M is expressed in solar masses, time is given in terms of solar lifetimes, which is ~ 10 billion yr
- Metallicity is the second most important factor that determines the evolution of massive star
  - Important for mass loss through stellar winds: particularly for massive stars $\dot{m} \propto Z^{0.85}$ (Vink et al. 2021; Vink & de Koter 2005)

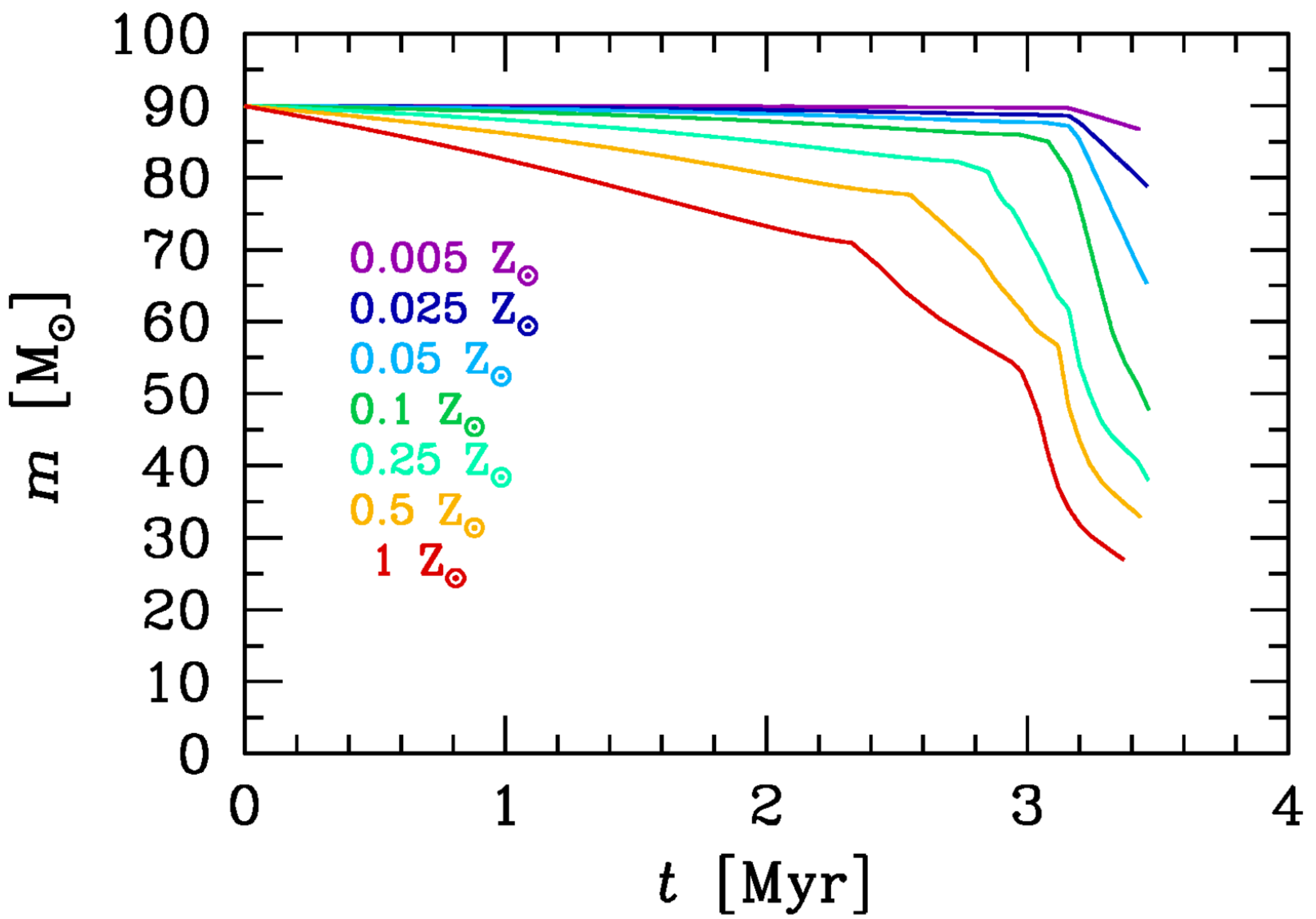
$M_{ZAMS} \lesssim 8\ M_\odot$ — ~ 10 billion yr

$8\ M_\odot \lesssim M_{ZAMS} \lesssim 20\ M_\odot$ — ~ 10 million yr

$M_{ZAMS} \gtrsim 20 M_\odot$ — ~ 1 million yr

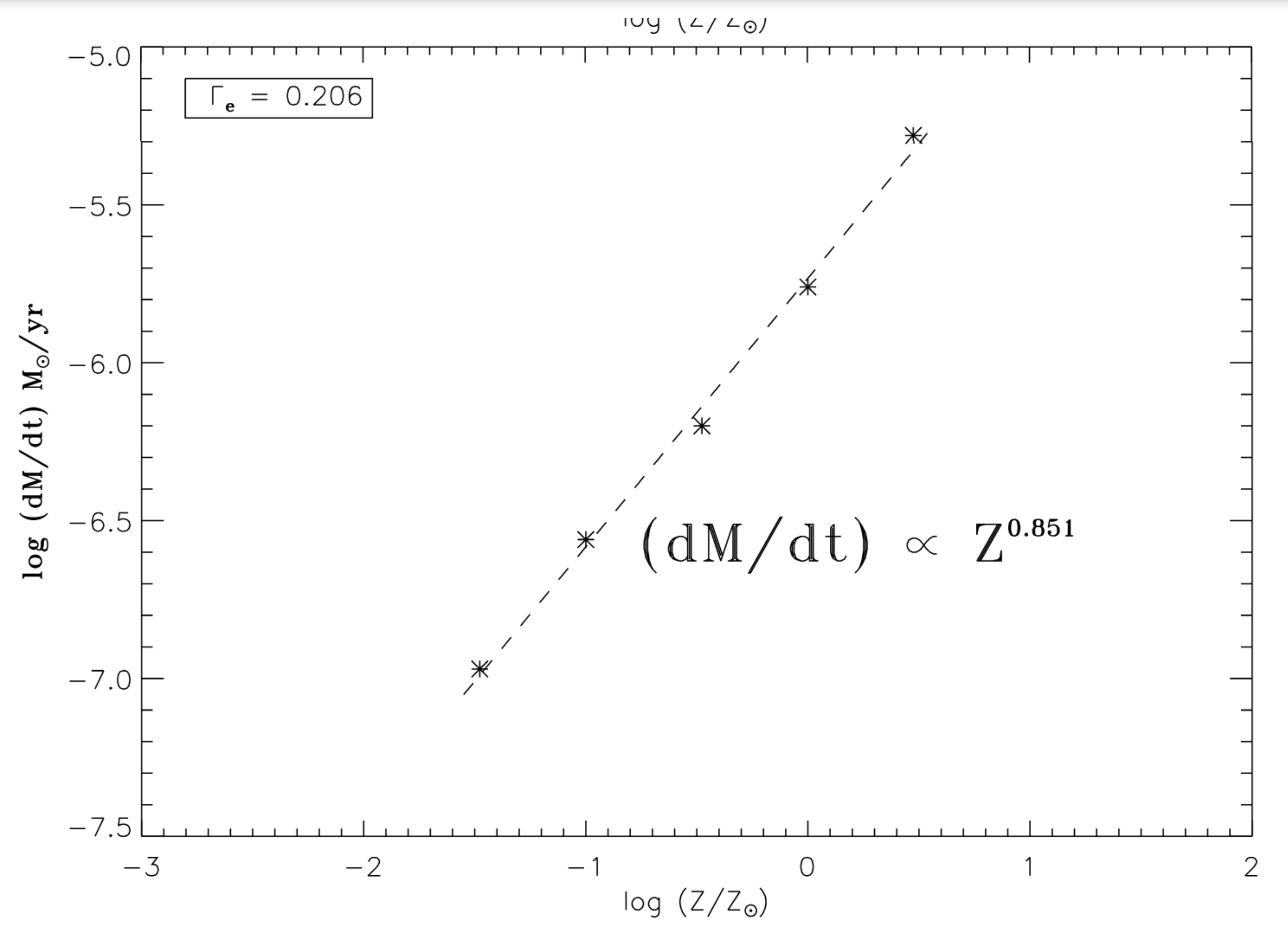


Mass loss due to stellar winds



Models from PARSEC stellar evolution code (Bressan+ 2012; Tang+ 2014; Chen, Bressan+ 2015)

Credit: Michela Mapelli



$\dot{m} \propto Z^{0.85}$

(Vink et al. 2021; Vink & de Koter 2005)

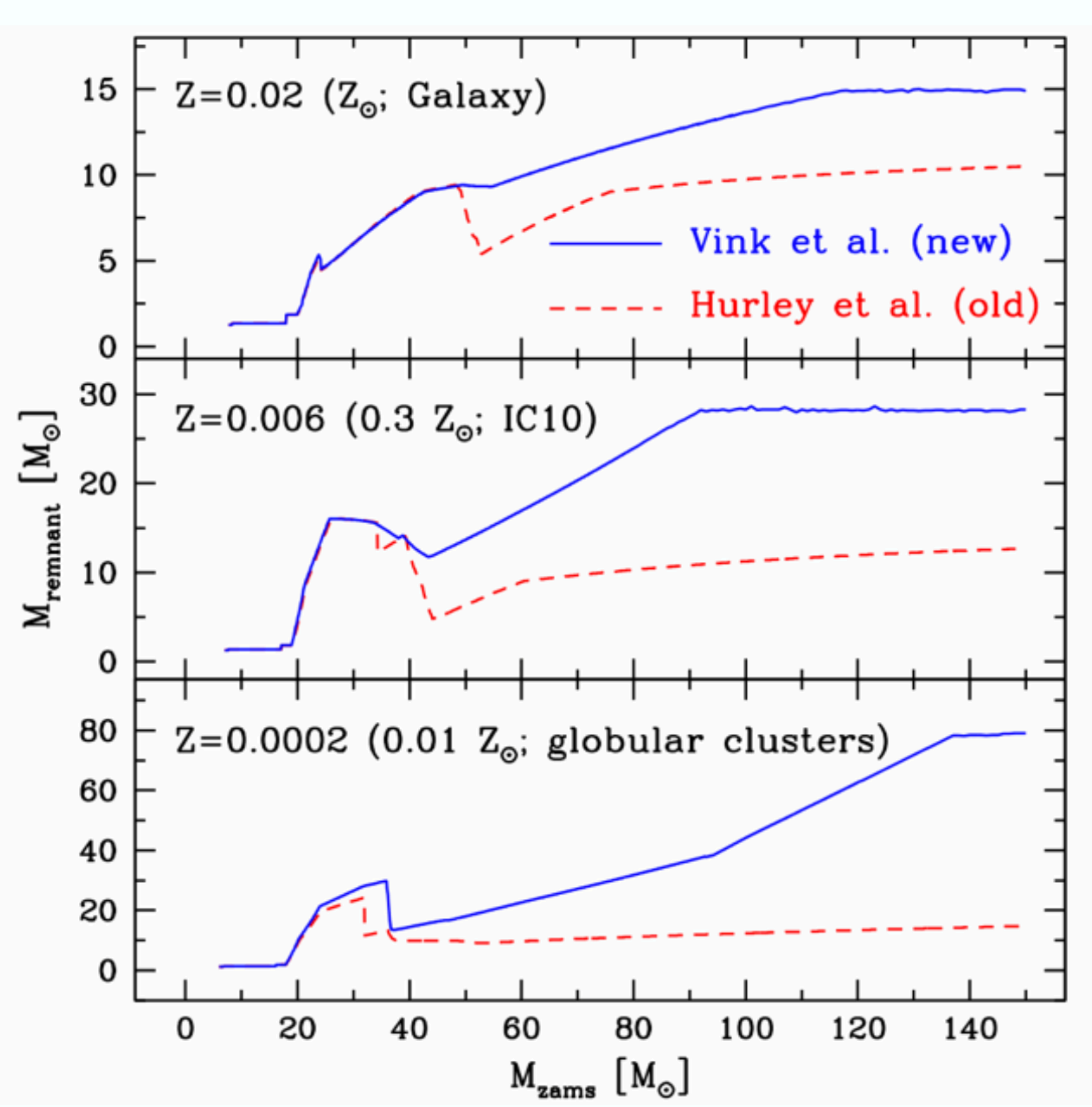
Mass loss due to stellar winds is still very uncertain: Sensitive to the prescriptions that are used and different assumptions

At solar metallicity, mass loss through stellar winds can be up to 60% of initial mass

Typical globular cluster metallicities ~0.05 $Z_\odot$

Supernova explosion mechanism, natal kicks and remnant masses?

- Final evolution of black hole progenitors is uncertain:
  - Supernova mechanism?
  - Direct collapse
- For very massive stars: pair and pulsational instability supernova?
  - Strongly depends on progenitor mass/metallicity (Belczynski et al. 2016)
- Natal kicks?
  - Mechanism
  - Velocity distribution
- Radial expansion during post main sequence evolution?
- **More about this when will cover black holes and gravitational wave sources in star clusters**



Belczynski et al. 2010

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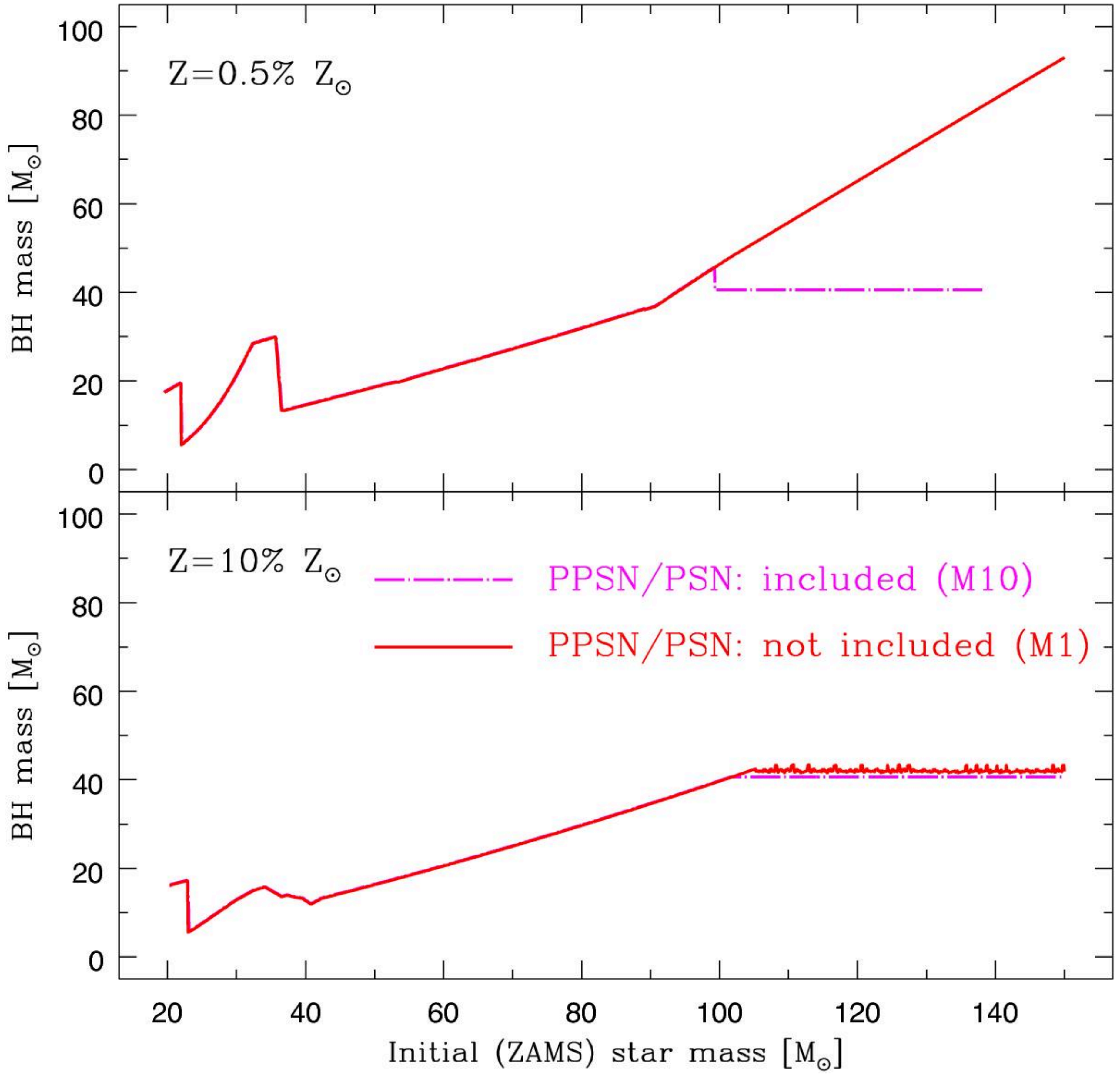
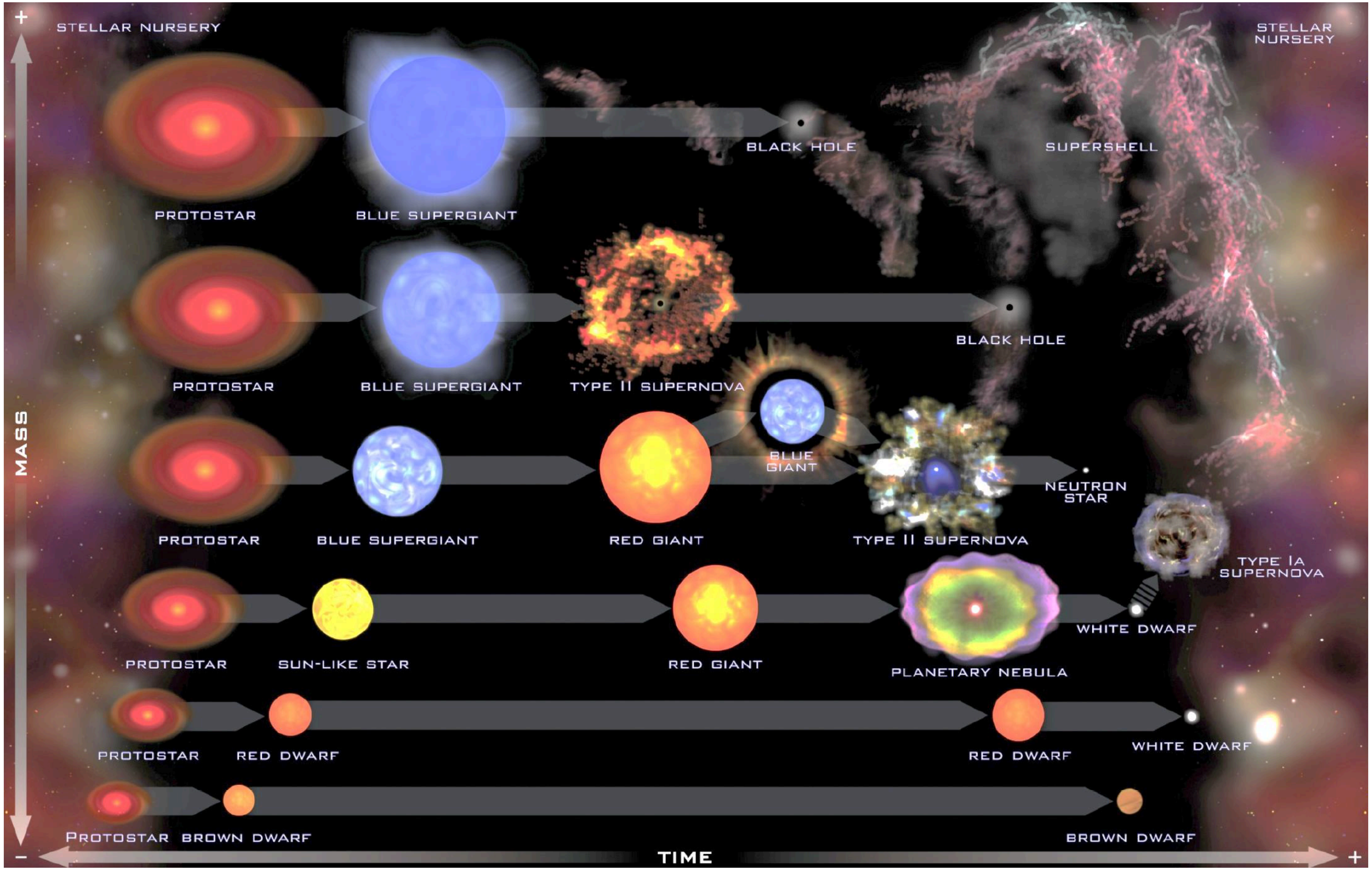


Belczynski et al. 2016

Stellar evolution in a nutshell



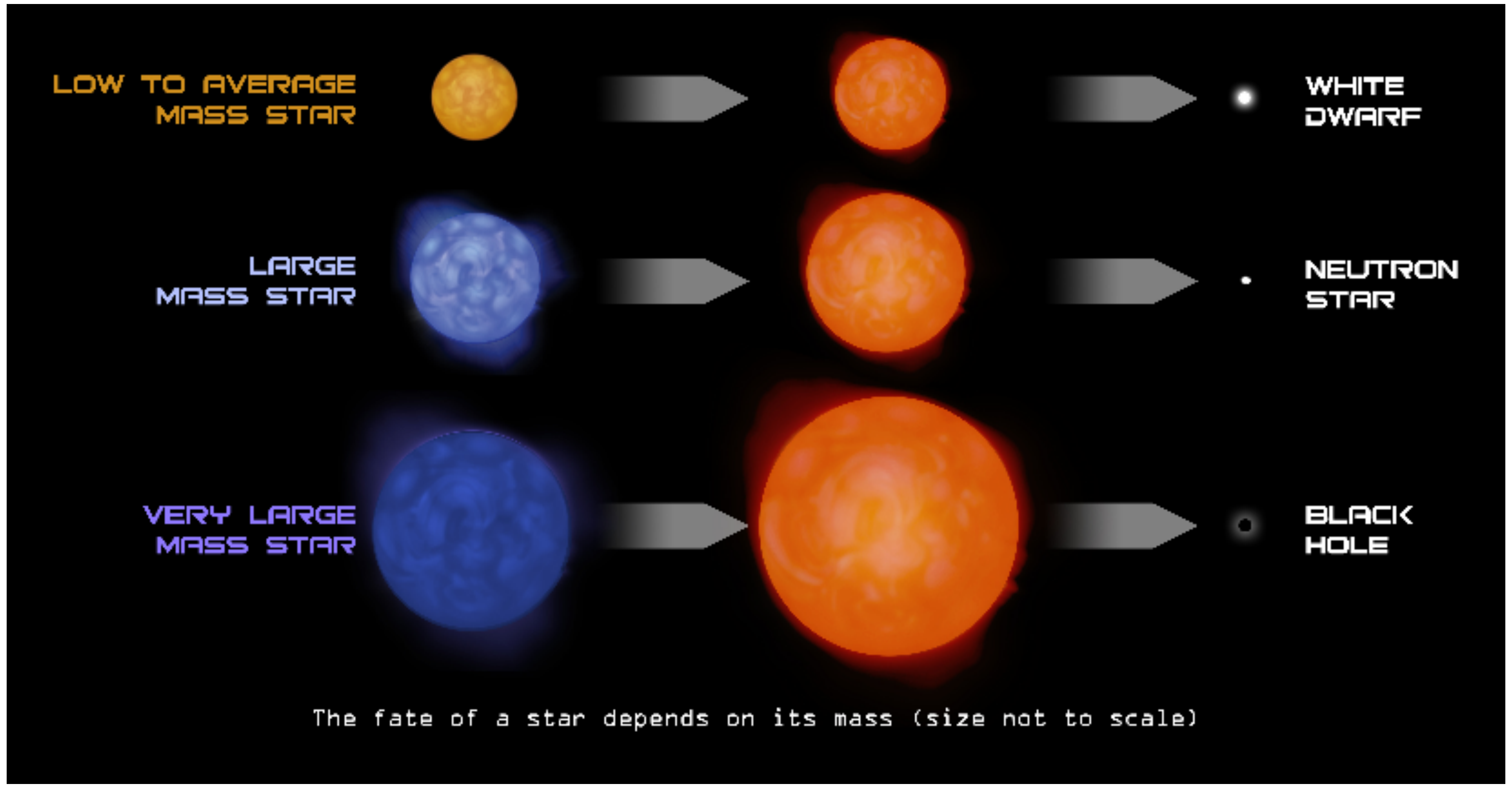
Credit: Thomas Tauris

Stellar evolution and its impact on star cluster evolution

$$M_{ZAMS} \lesssim 8\ M_\odot$$

$$8\ M_\odot \lesssim M_{ZAMS} \lesssim 20\ M_\odot$$

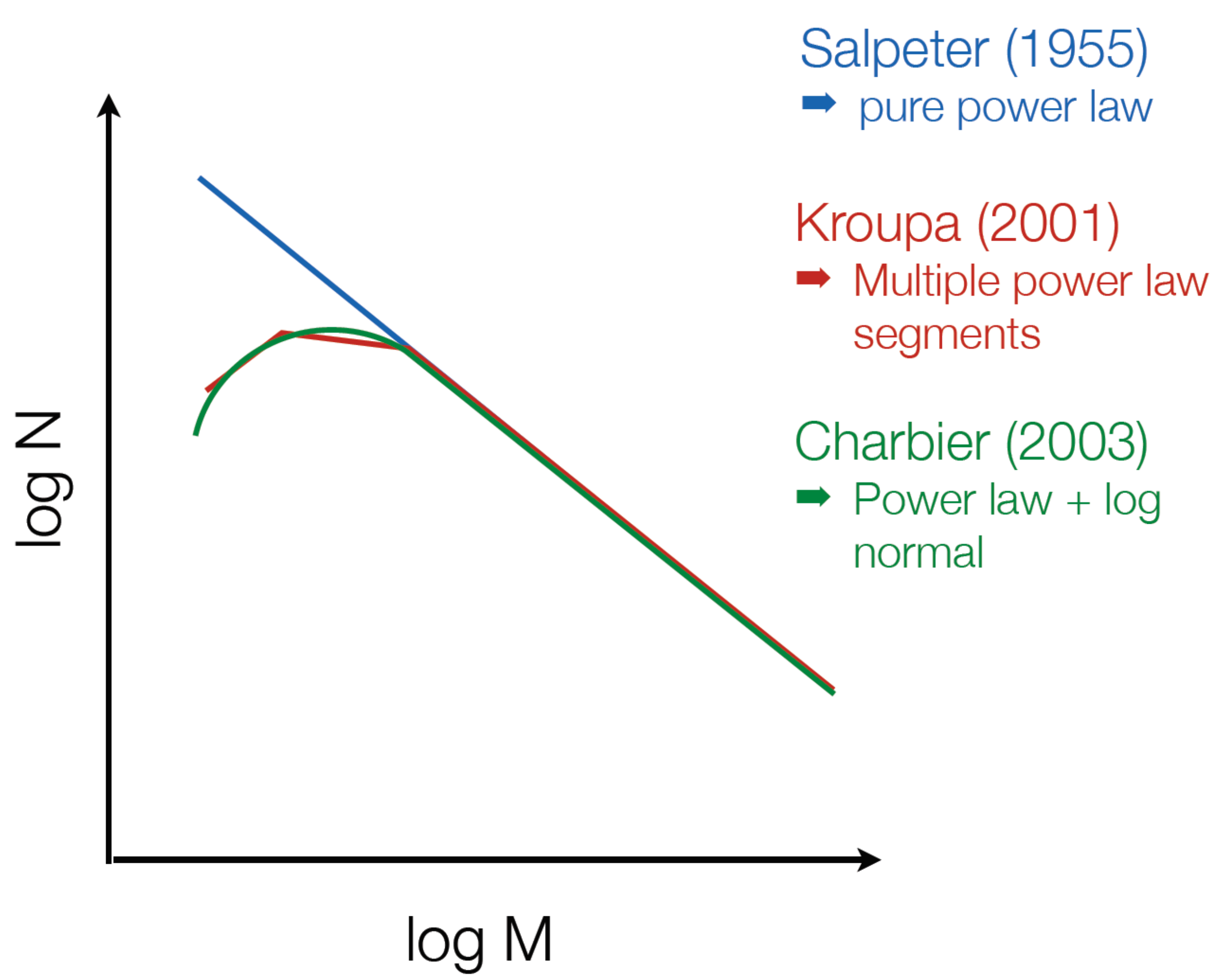
$$M_{ZAMS} \gtrsim 20 M_\odot$$



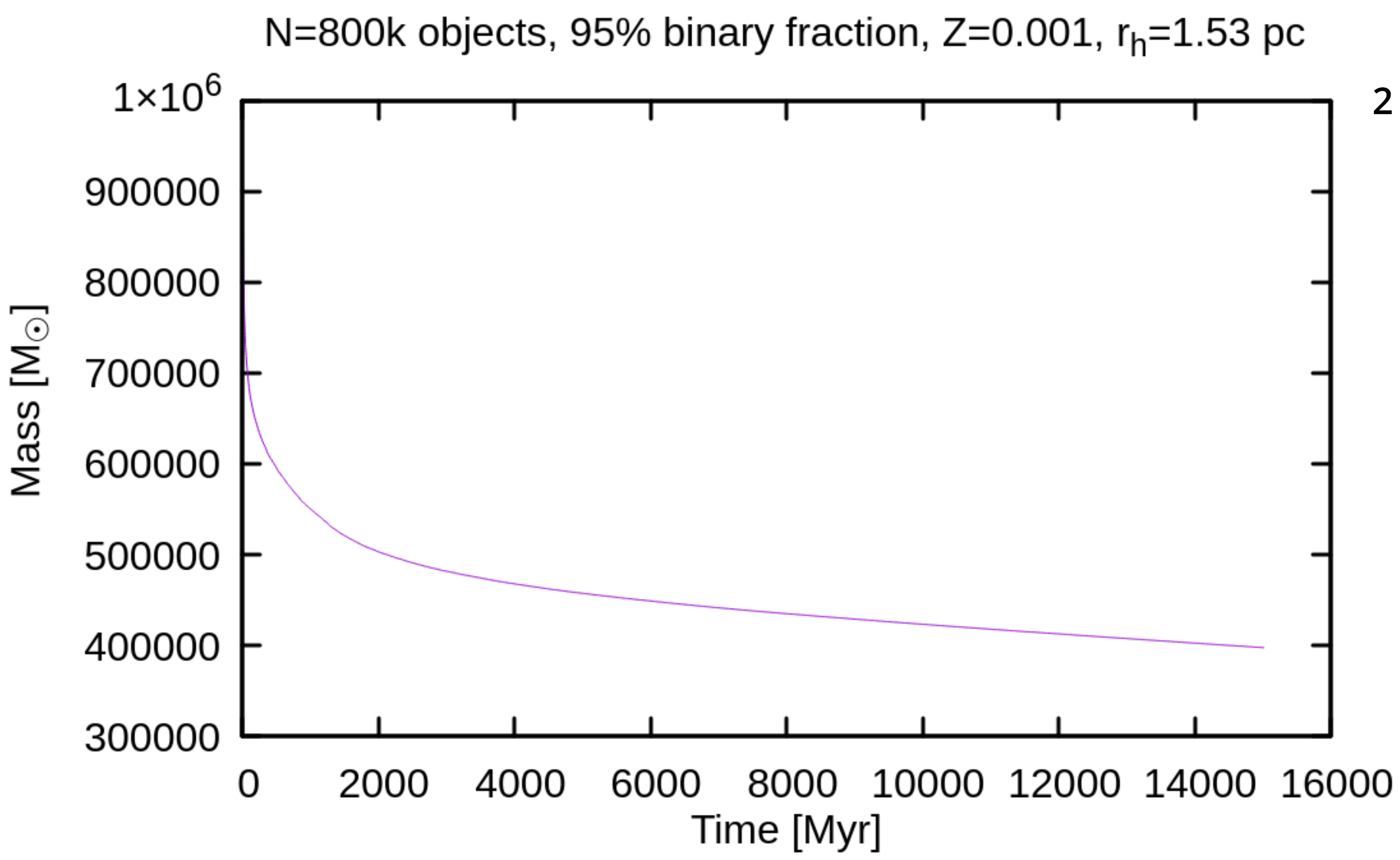
Initial Mass Function (IMF) of Stars in the Universe

Low mass stars are much more abundant than high mass stars

- ~ 2 black holes for every 1000 stars
- ~ 5 neutron stars for every 1000 stars

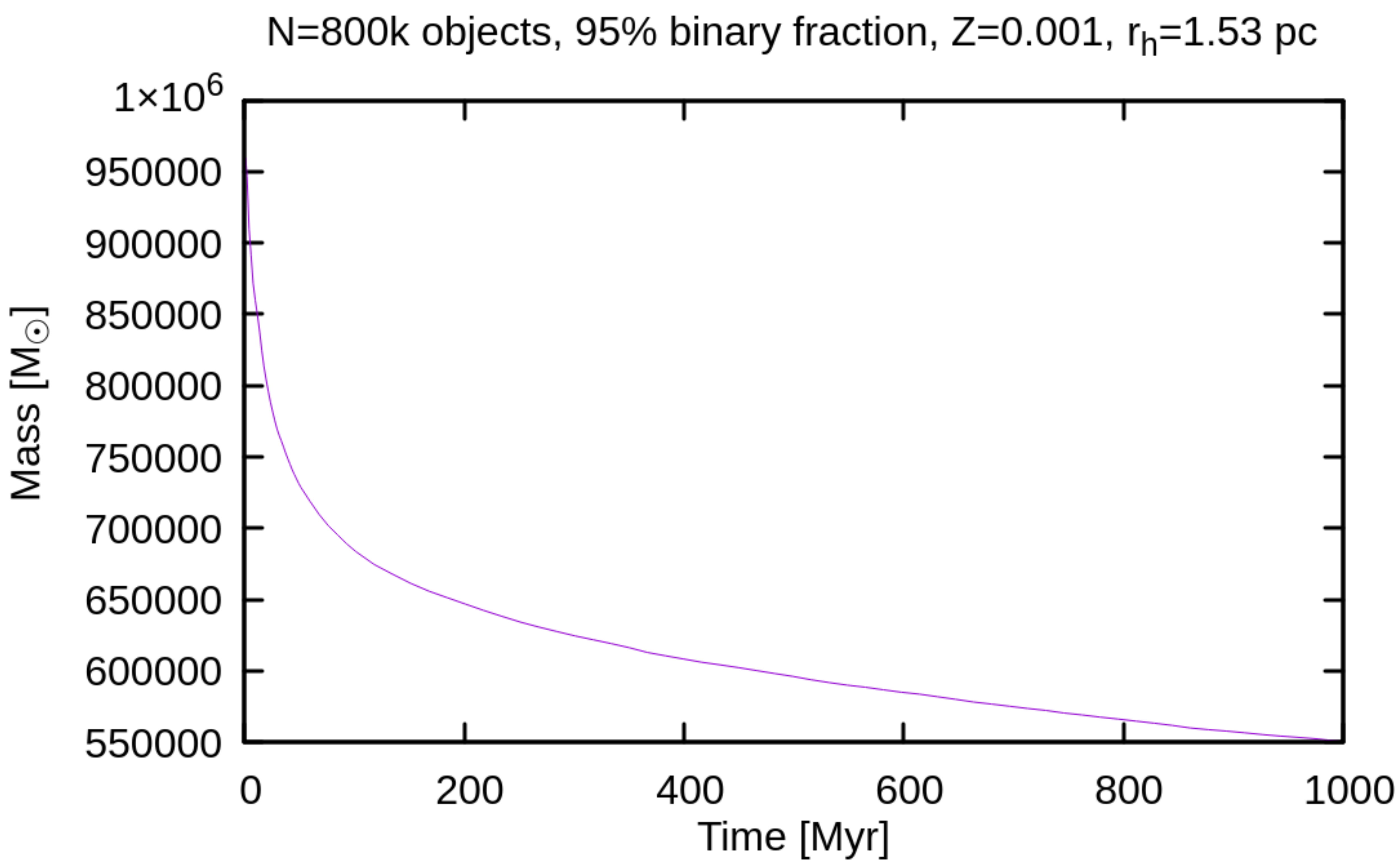


Stellar evolution and its impact on star cluster evolution



Models from MOCCA Survey 5 (Hypki et al. 2024)

2 component initial mass function (Kroupa 2001)
Sampled between $0.08\ M_\odot - 150\ M_\odot$



Cluster loses about 30% of its initial mass within ~100 Myr
Ignoring tidal field $\sim 45\,\%$ mass lost due to stellar evolution after 10 Gyr.

The viral theorem

- Systems evolve towards dynamical equilibrium:
  - No overall expansion or contraction of the system, or other bulk motion, even though all particles are in motion (on dynamical/crossing) timescale → **"virial equilibrium":**
  - **Relates the kinetic and potential energies of a gravitationally bound system in equilibrium.**

$$K = \sum_i \frac{1}{2} m_i v_i^2$$

$$U = -\sum_{i,j,i\neq j} \frac{G m_i m_j}{r_{ij}}$$

→

$$2K + U = 0$$

$$K + U = E$$

$$K = -E$$

$$U = 2E$$

$$M = \sum_{i=1}^{N} m_i$$

$$V^2 = \frac{2K}{M}$$

$$U = -\frac{GM^2}{2R}$$

What will be the evolution of the system when it loses mass $\Delta M < 0$ or kinetic energy $\Delta K < 0$?

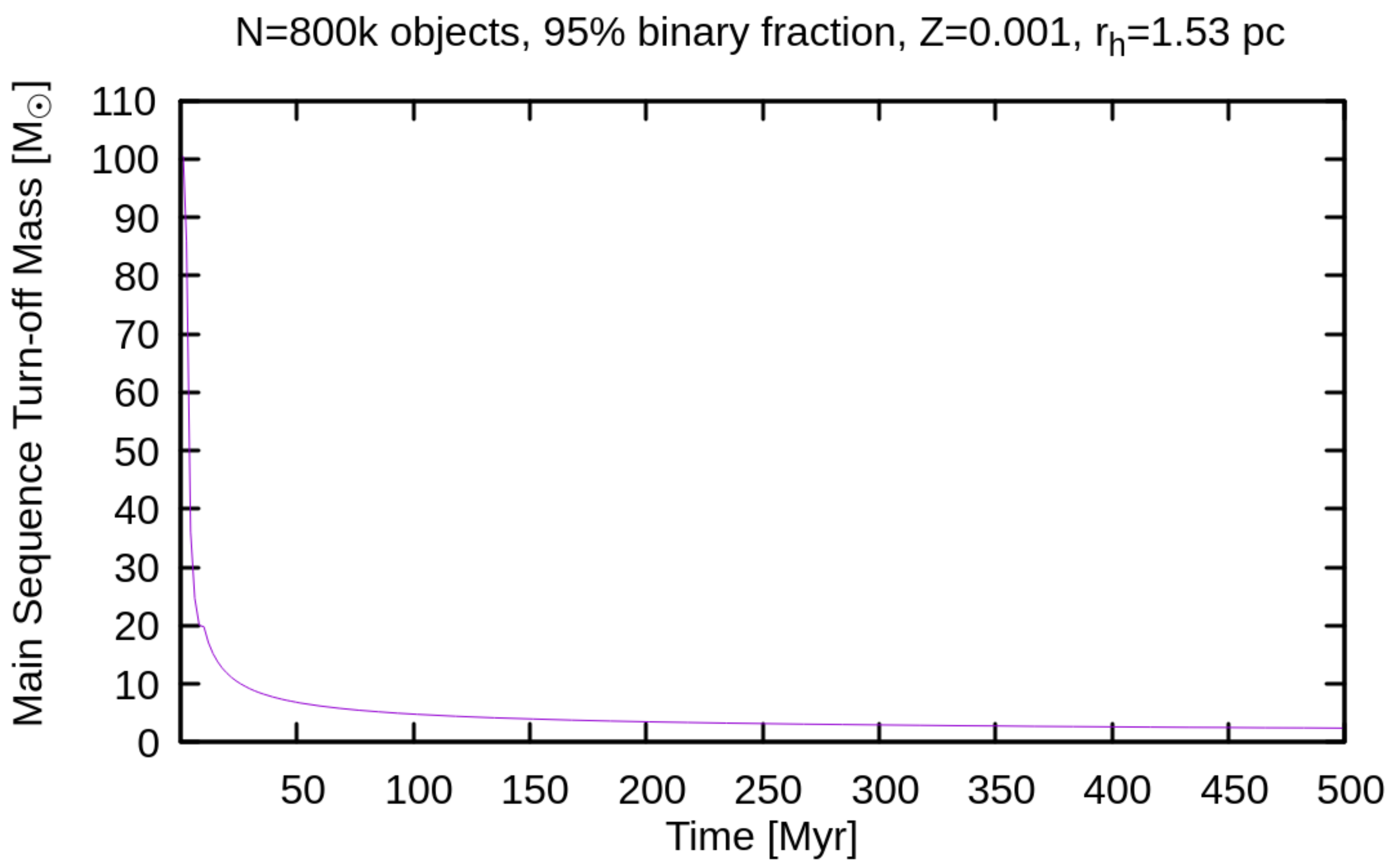
$$R_n = R\left(1 - \frac{\Delta M}{M}\right)$$ – R increasing!

$$R_n = R\left(1 - 2\frac{\Delta K}{U}\right)$$ – R decreasing!

$$V_n^2 = V^2\left(1 + 2\frac{\Delta M}{M}\right)$$ – V decreasing!

$$V_n^2 = V^2\left(1 - \frac{\Delta K}{K}\right)$$ – V increasing!

Stellar evolution and its impact on star cluster evolution

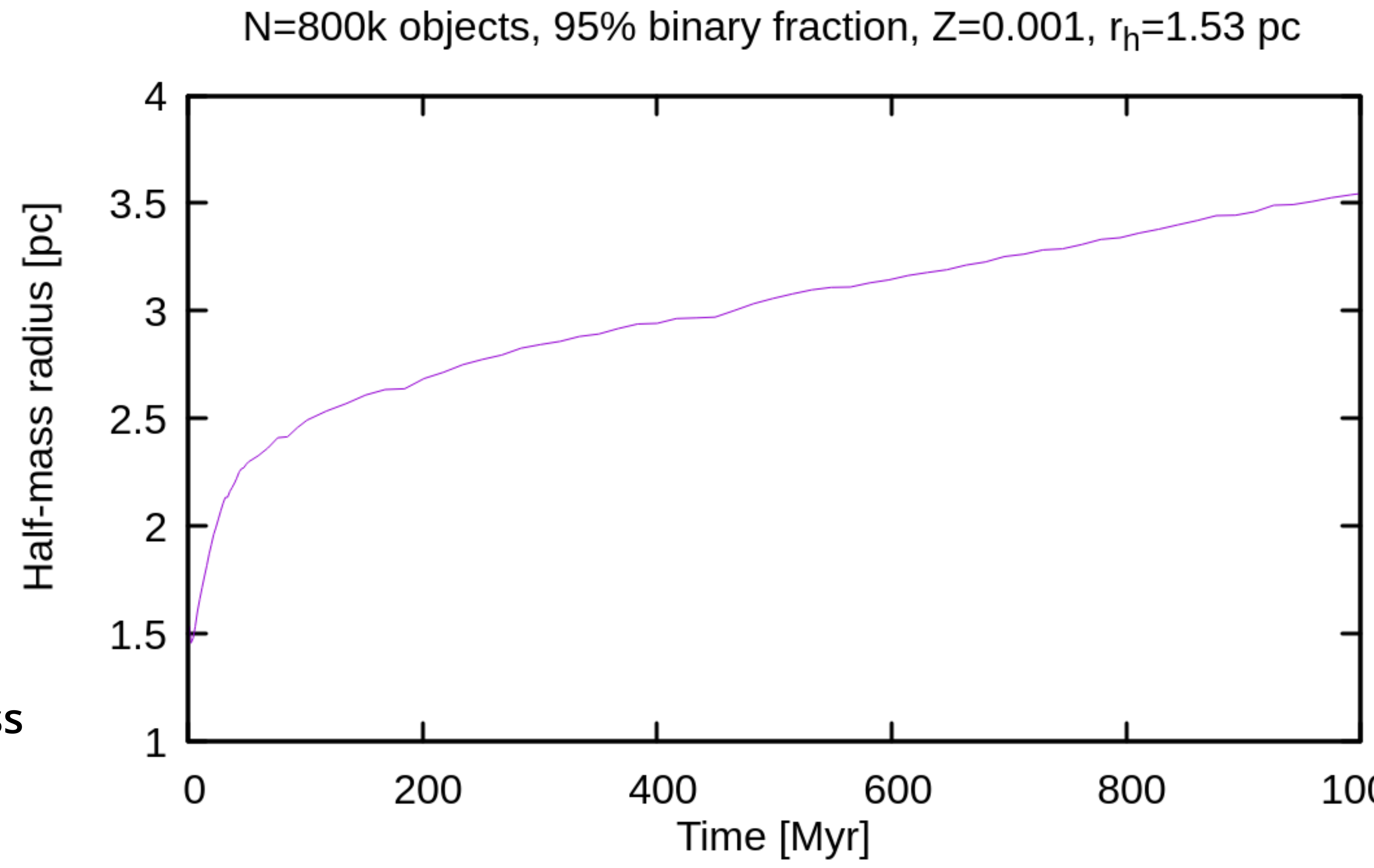


Models from MOCCA Survey 5 (Hypki et al. 2024)

Stellar evolution mass loss is adiabatic

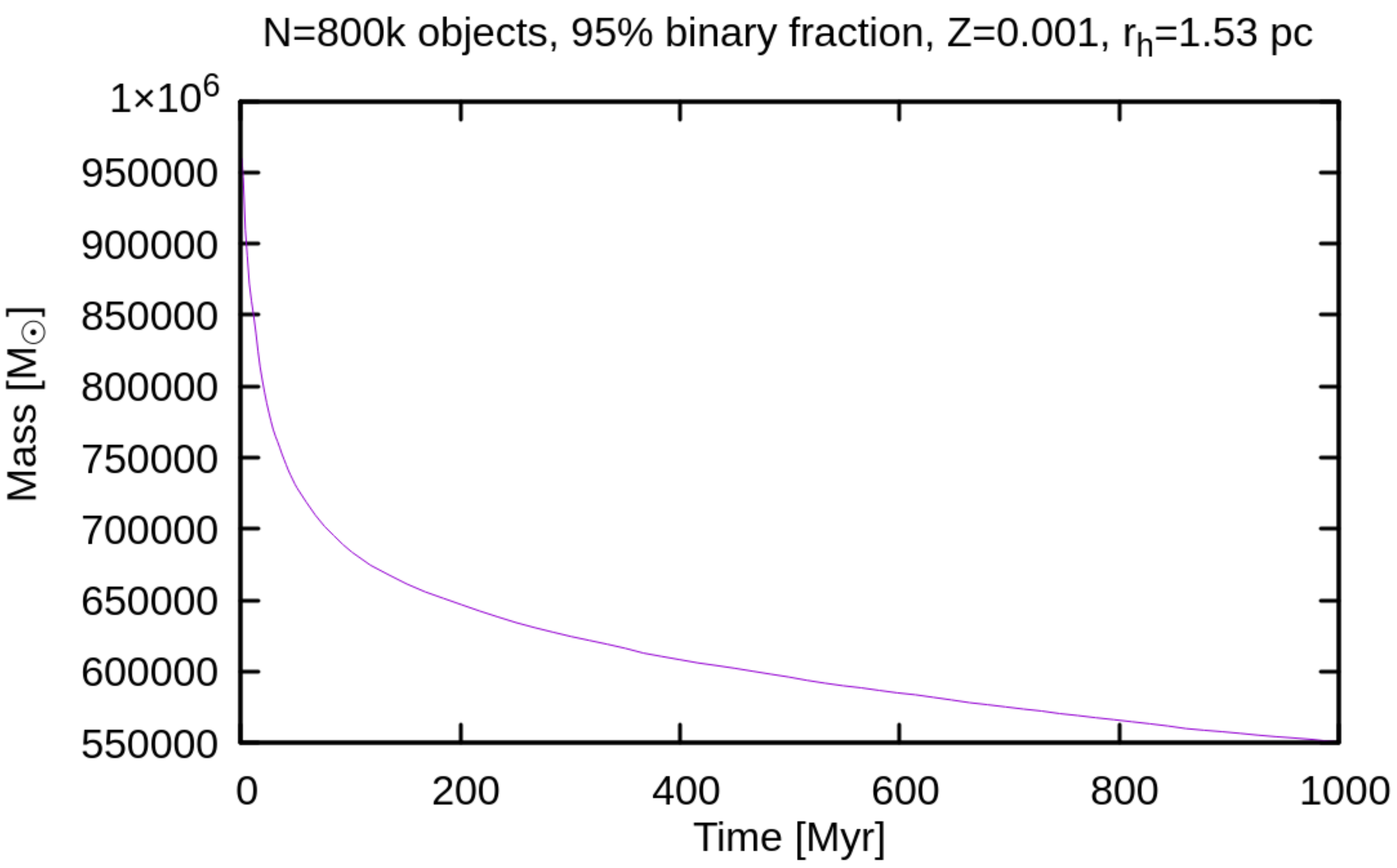
$$\frac{r_f}{r_i} = \frac{M_i}{M_f}$$

If no primordial mass segregation



Half-mass radius expands in response to mass loss from the system

Stellar evolution and its impact on star cluster evolution



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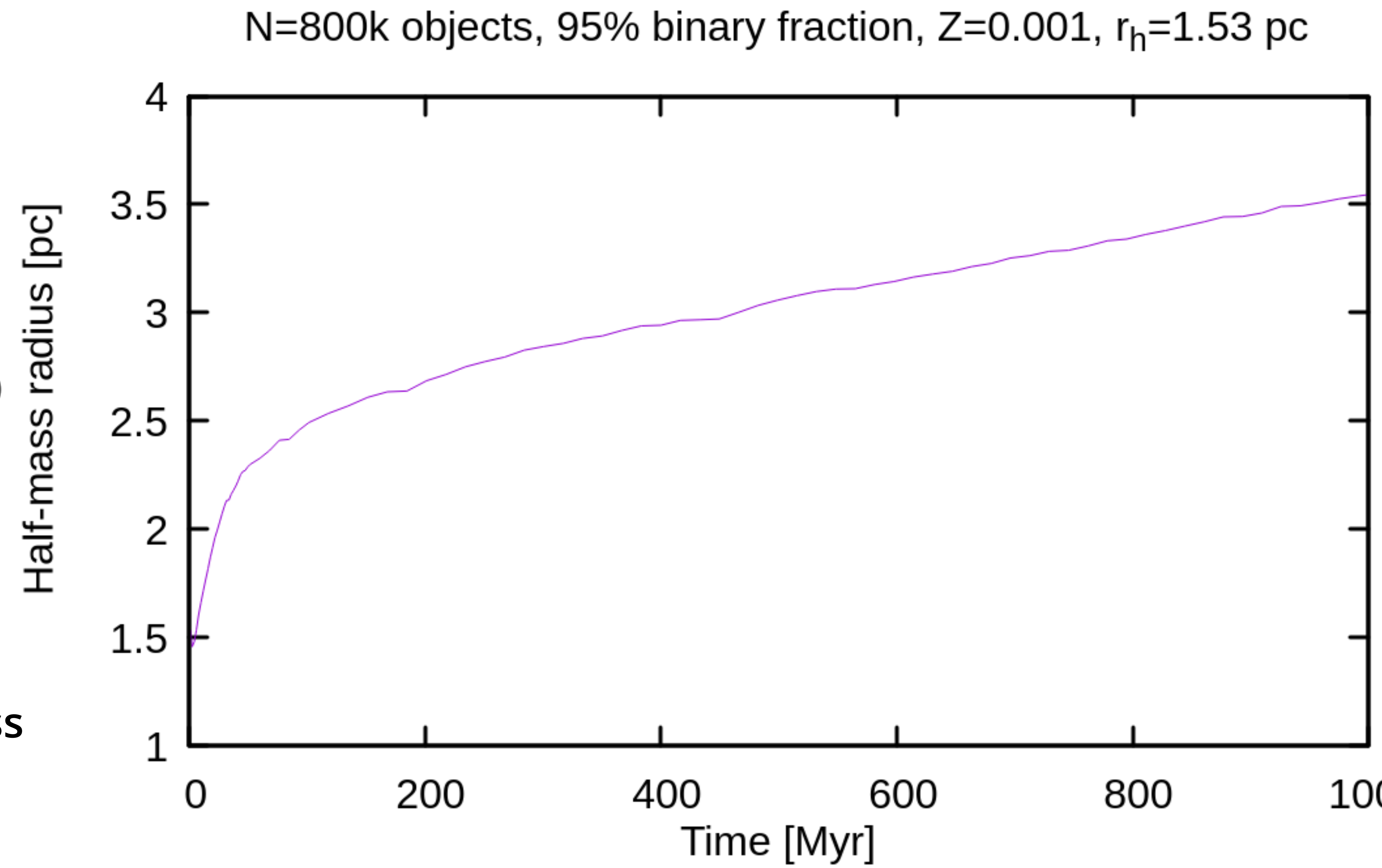

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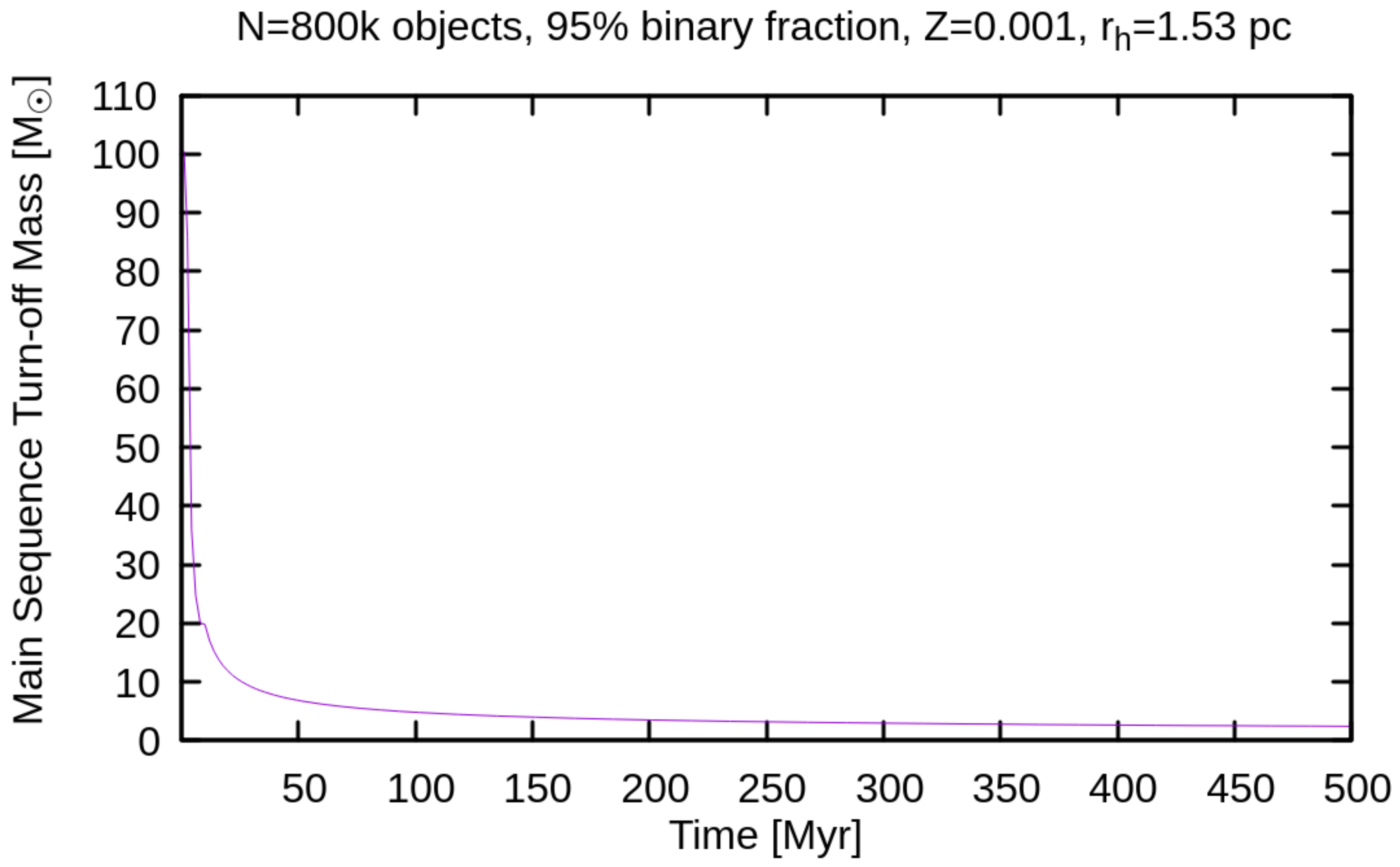
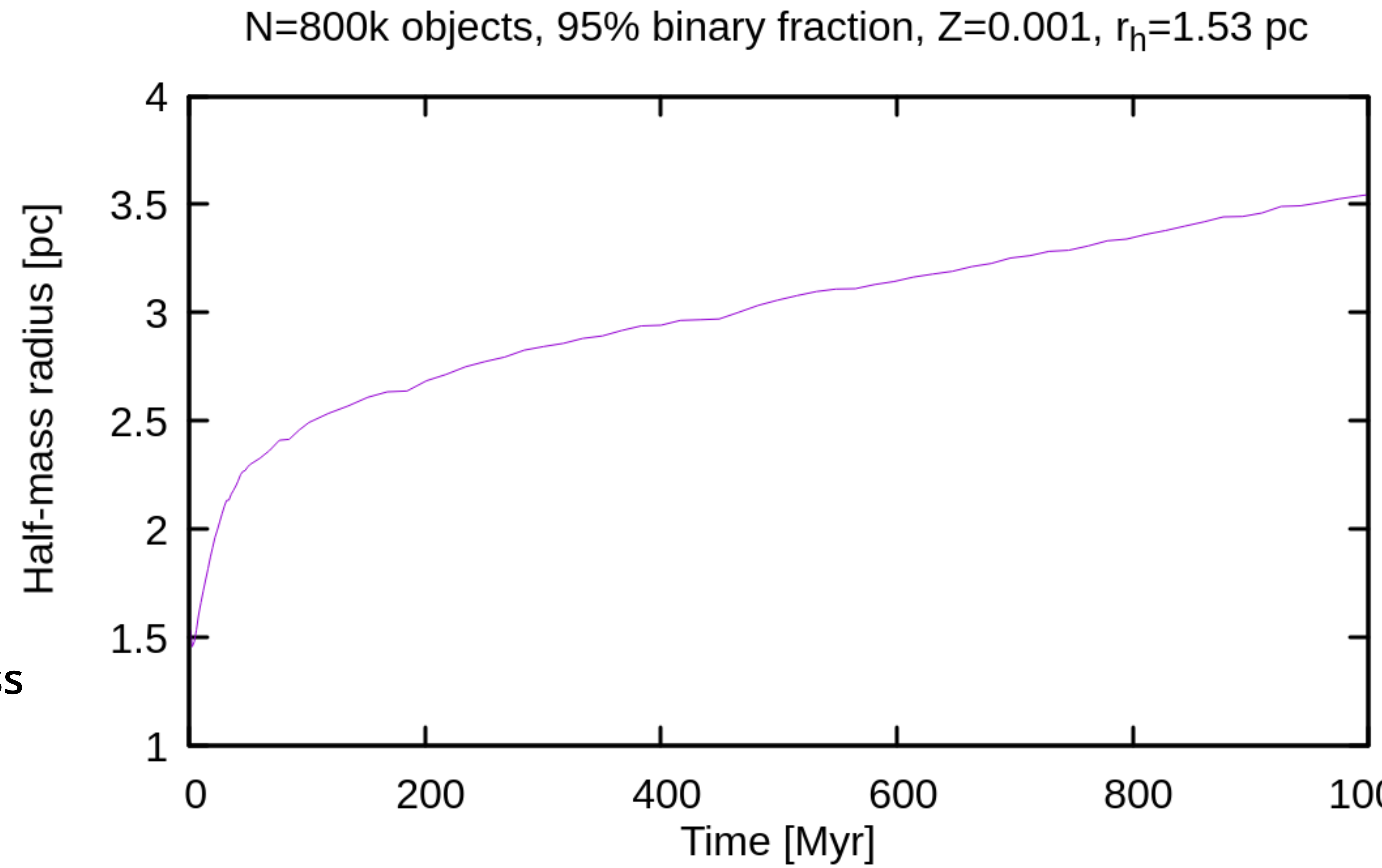


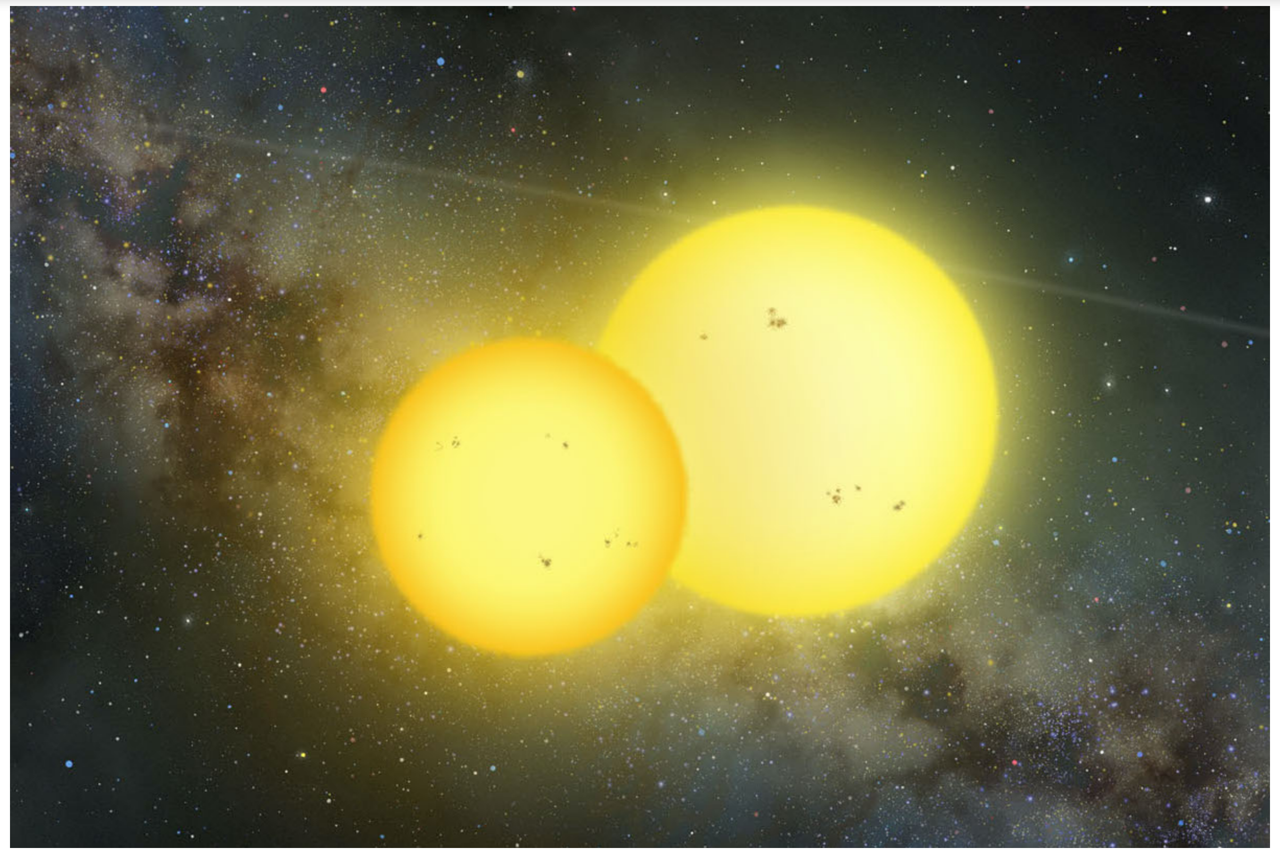
More about this when will cover black holes in star clusters

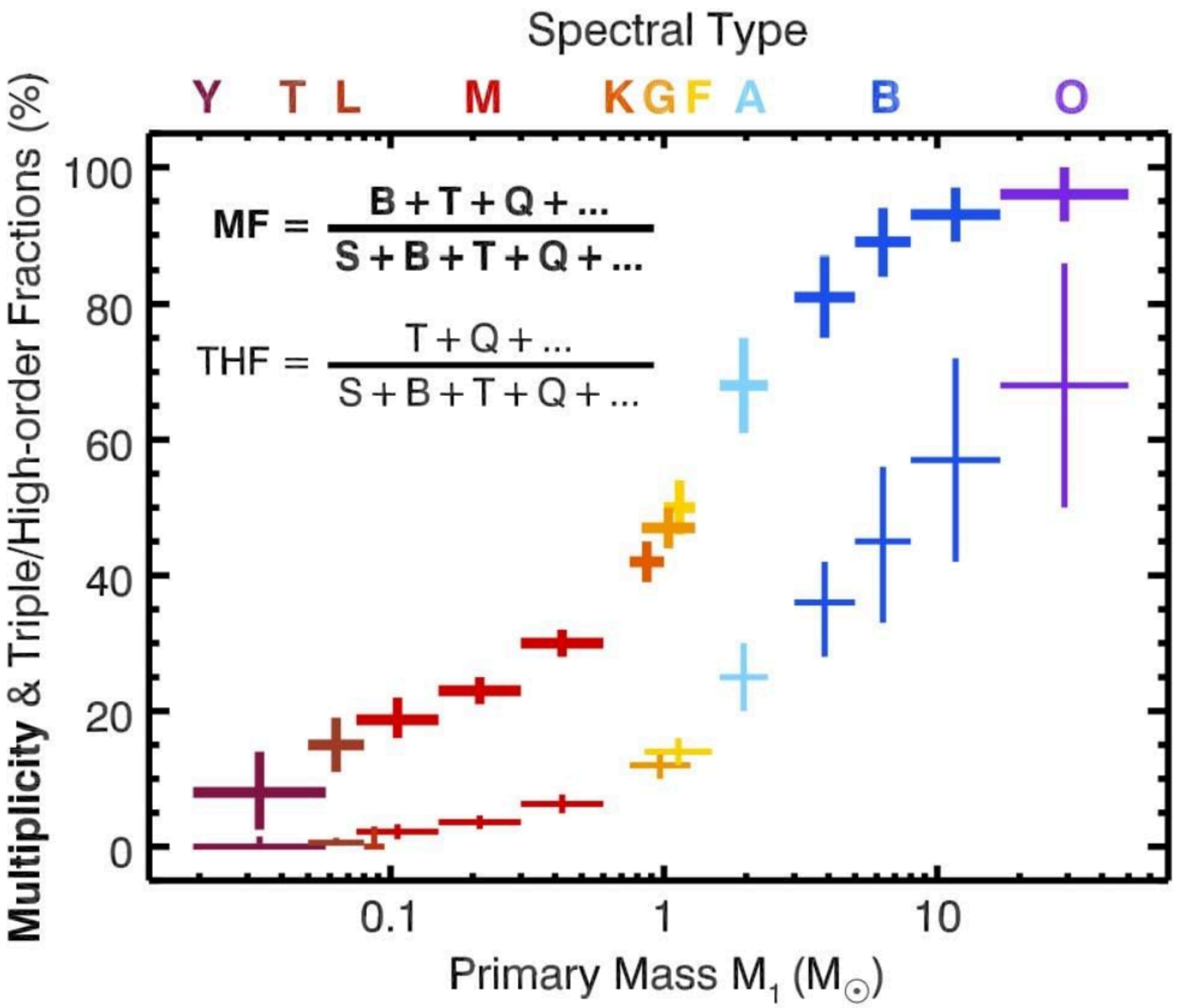
Models from MOCCA-Database Survey 5 (Hypki et al. 2024)

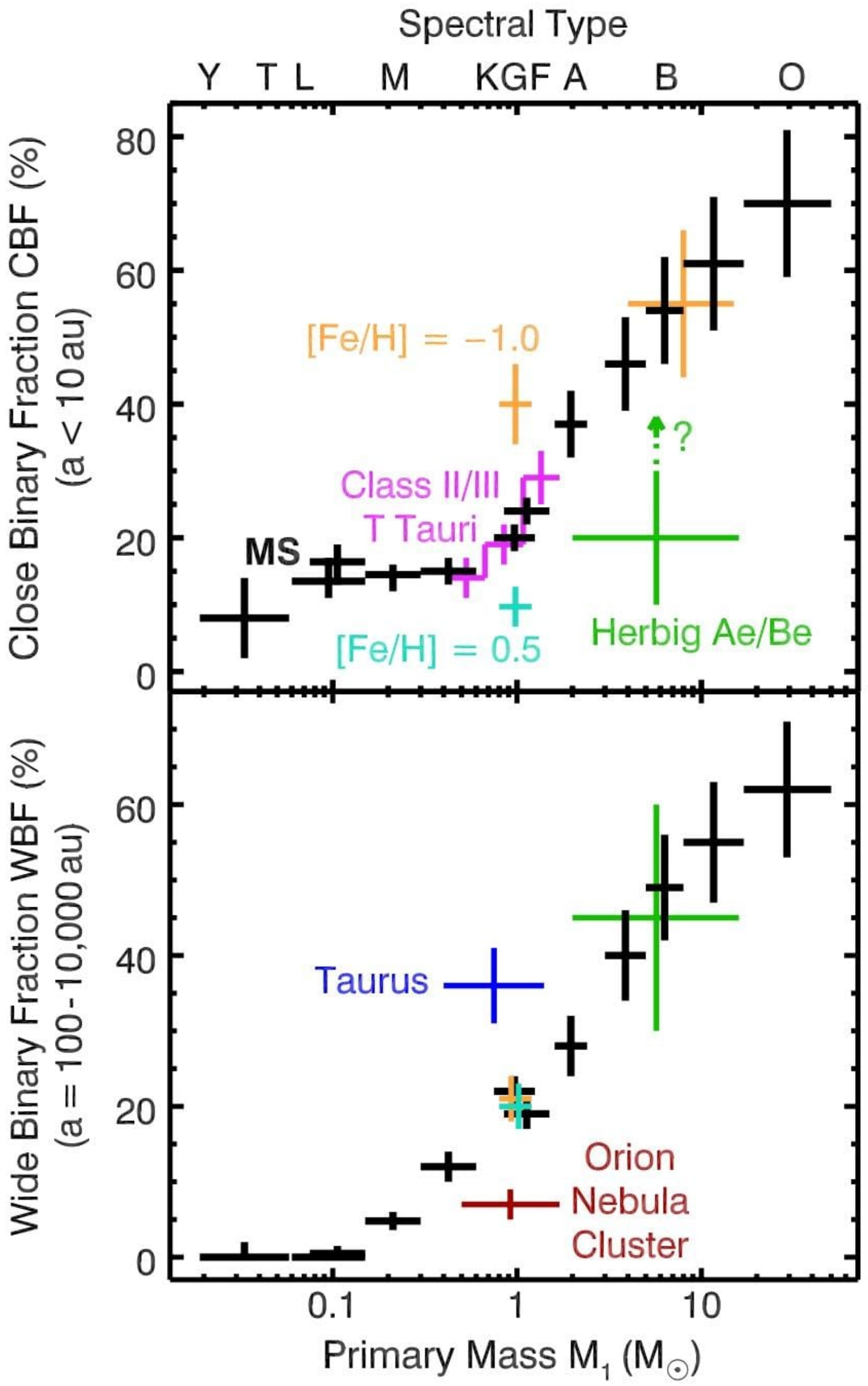


Half-mass radius expands in response to mass loss from the system

Binary stars

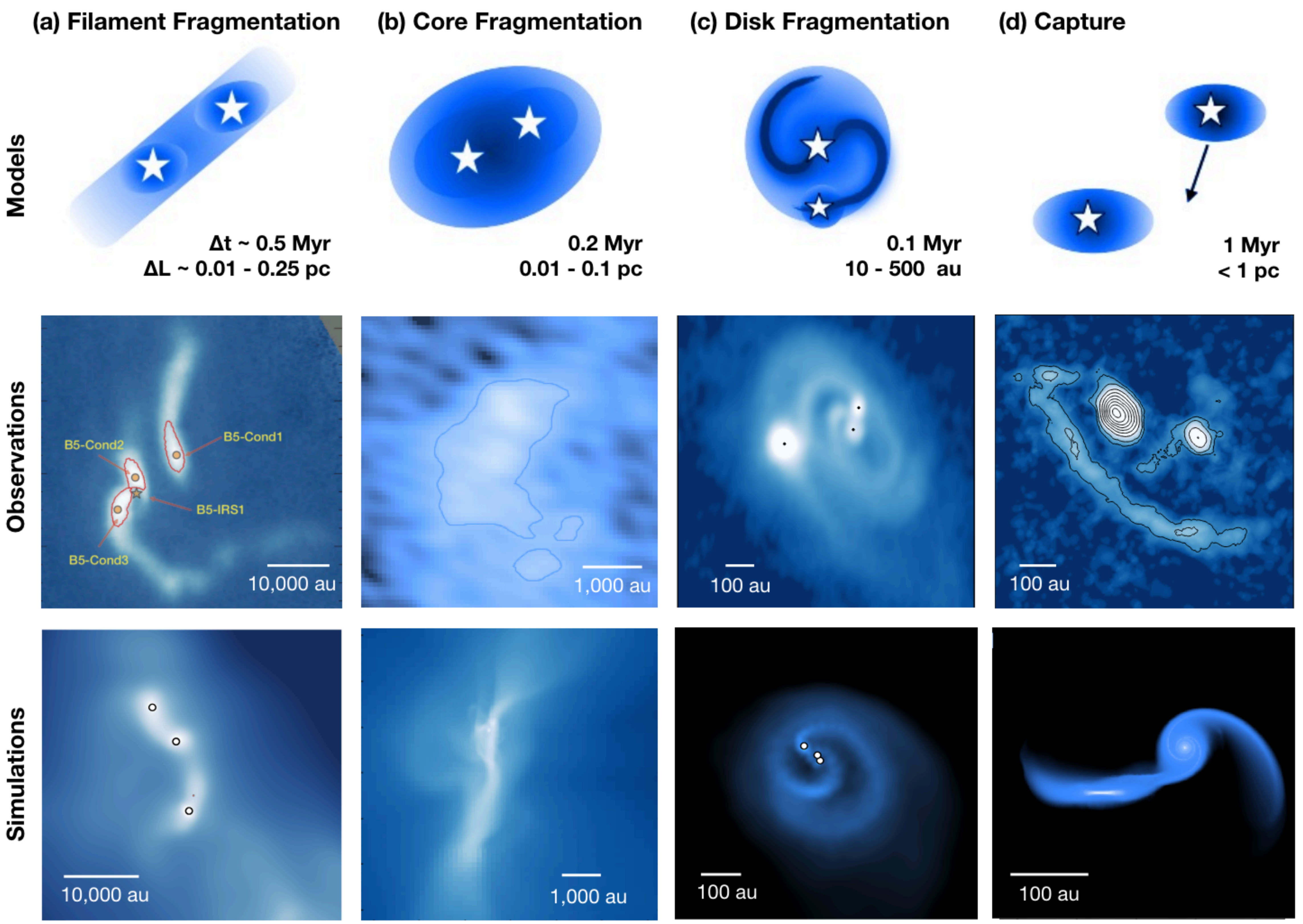






Offner et al. 2023: https://arxiv.org/abs/2203.10066

Formation of binaries



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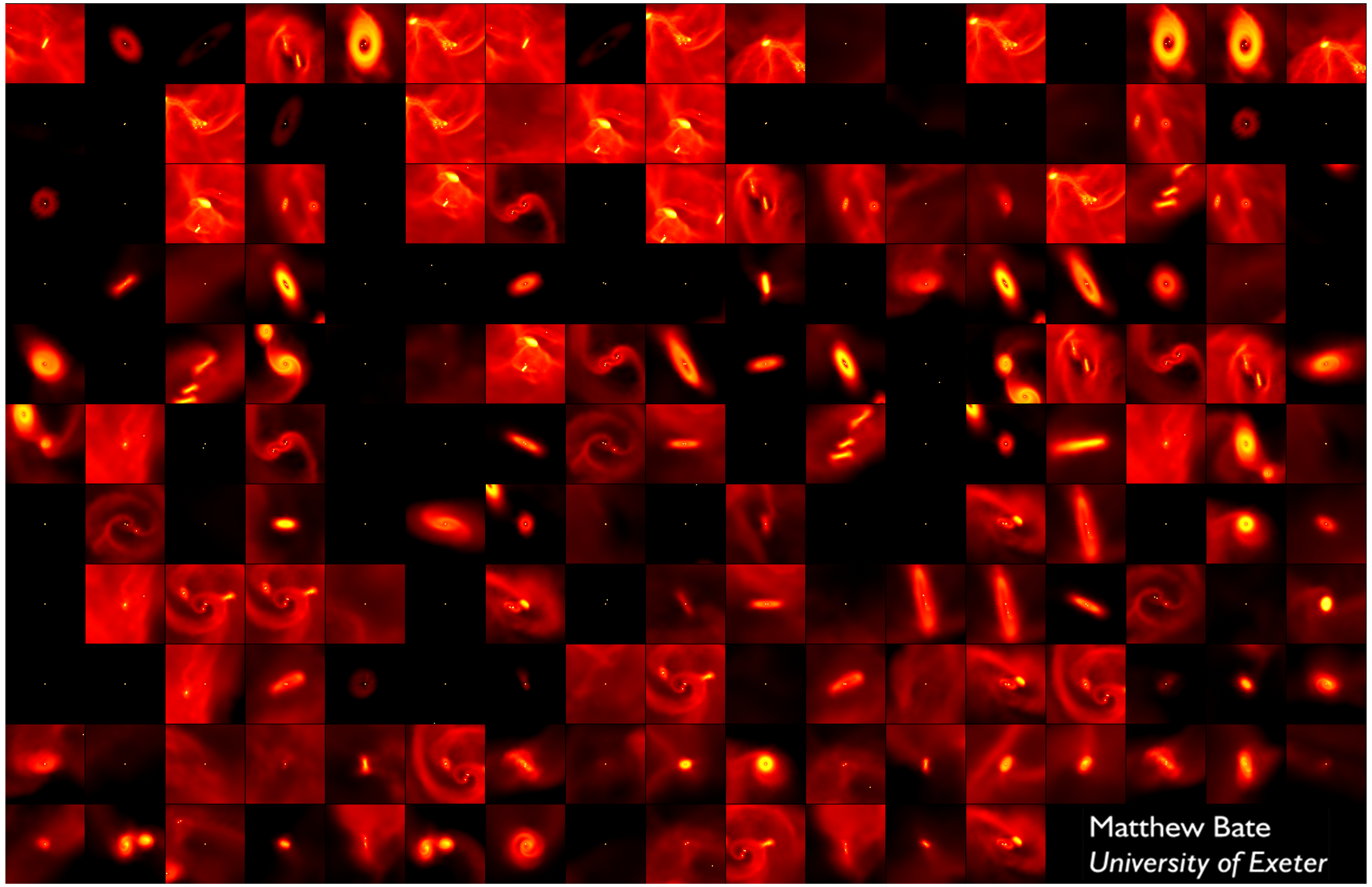
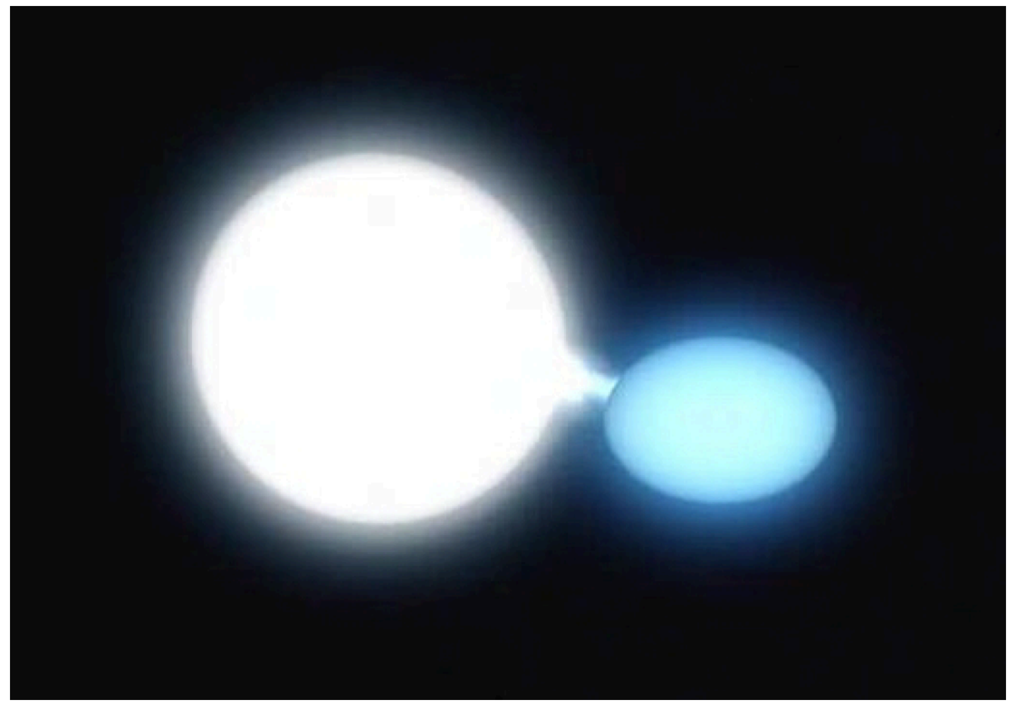


Bate (2018)

Smoothed-particle hydrodynamics simulation
Credit: Matthew Bate

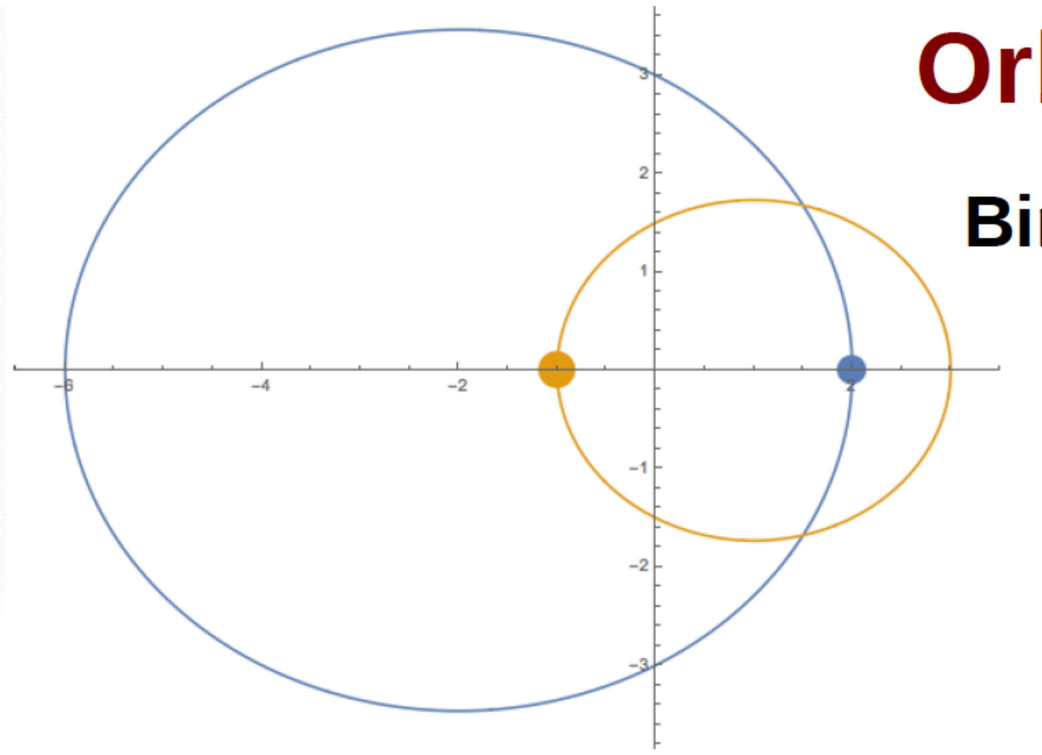
Binary evolution

- Depends strongly on
  - Initial masses, separation and eccentricity of a binary system
- All the complexities of single stellar evolution
  - Wind mass loss, supernova and natal kick can change orbital parameters
- Mass transfer
  - Wind mass transfer
  - Roche lobe overflow
  - Common envelope
- Tidal evolution
- Magnetic breaking
- All processes are difficult to model



Credits: ESO/ Calçada/M. Kornmesser/ S.E. de Mink

Binary orbital properties



Orbital properties

Binaries follow Kepler laws

Orbit as function of true anomaly

$$r(\theta) = \frac{a\,(1-e^2)}{1 + e\,\cos\theta}$$

Orbital period

$$P = 2\,\pi\left(\frac{a^3}{G\,(m_1+m_2)}\right)^{1/2}$$

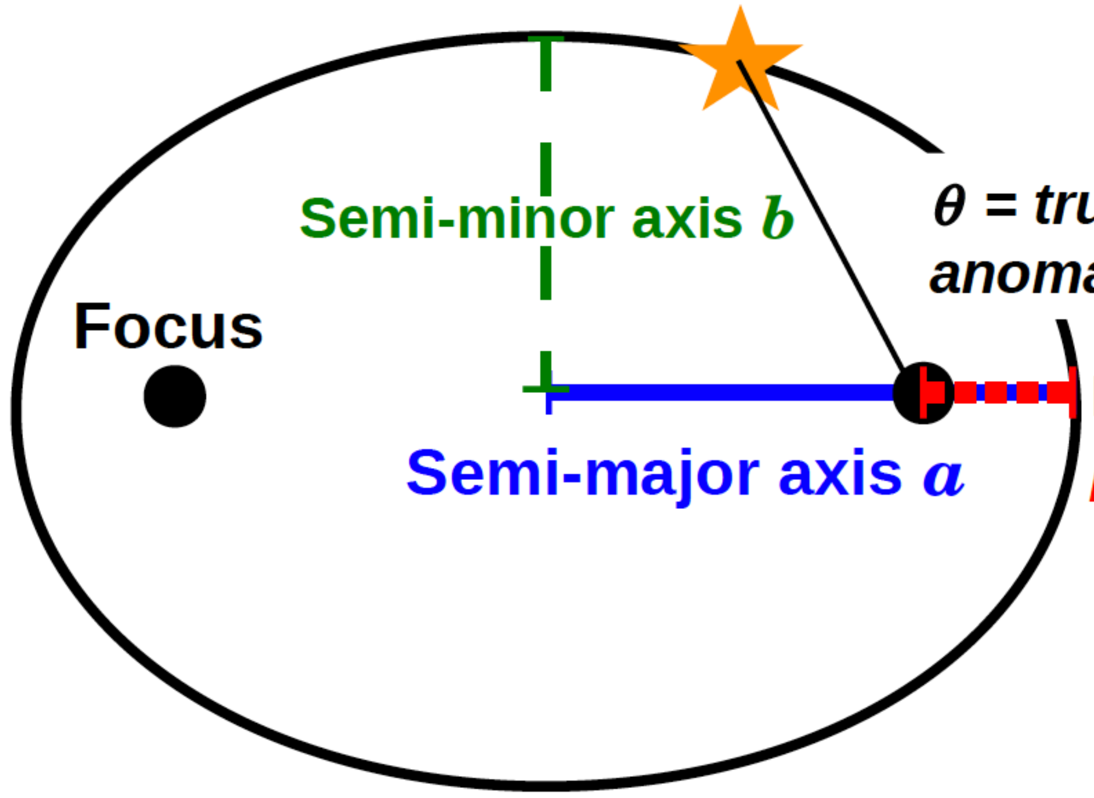
Energy

$$E = \frac{1}{2}\,\mu\,v^2 - \frac{G\,m_1\,m_2}{r}$$

Angular momentum

$$L = \mu\,\sqrt{G\,(m_1+m_2)\,a}$$

In the reduced particle + CoM frame

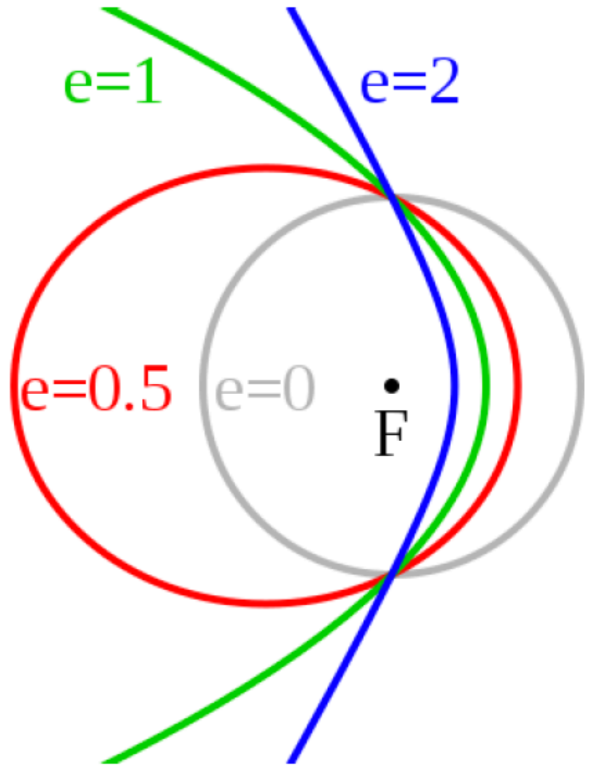


reduced mass:

$$\mu = \frac{m_1\,m_2}{(m_1+m_2)}$$

Eccentricity:

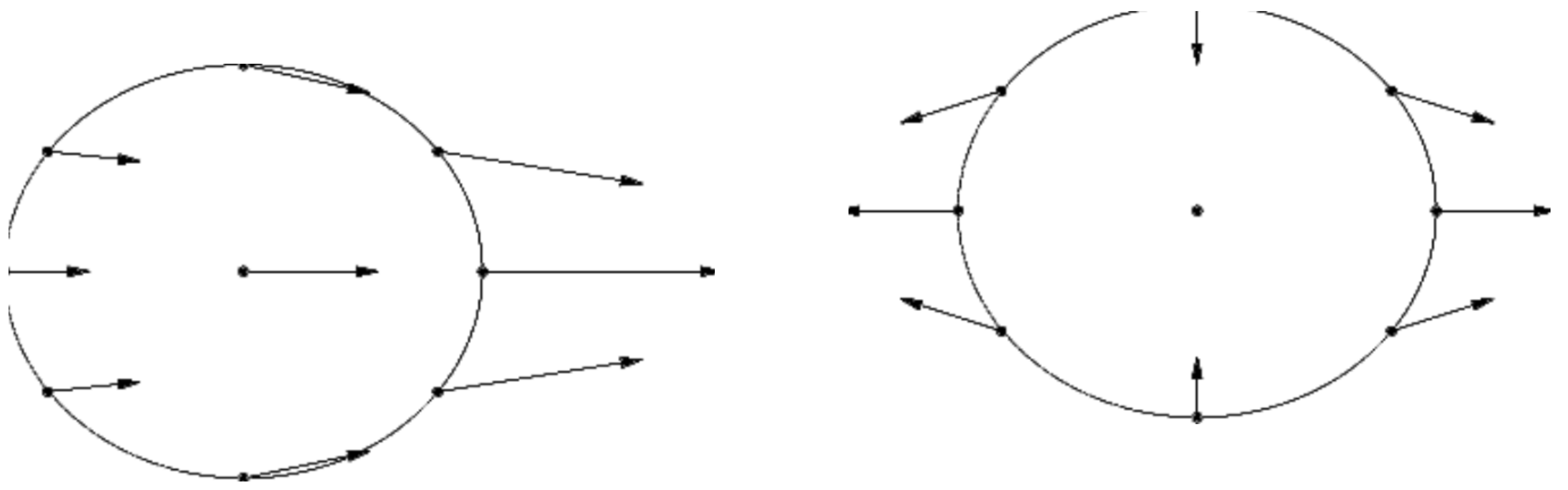
$$e = \sqrt{1 + \frac{2\,E\,L^2}{\mu^3\,G^2\,(m_1+m_2)^2}}$$



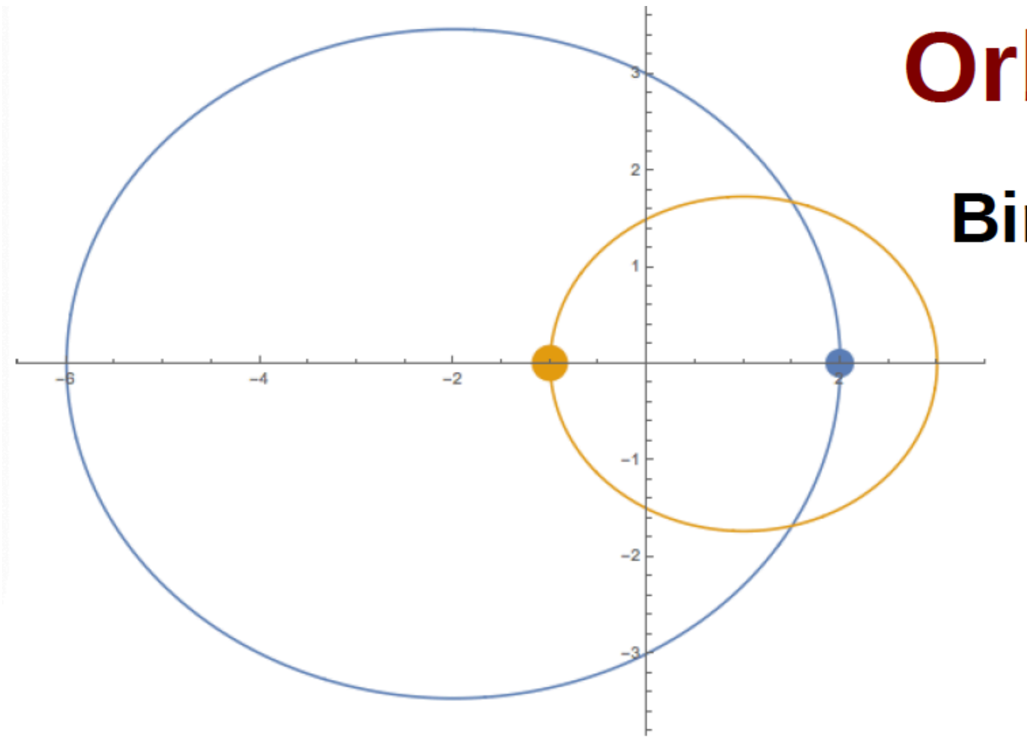
Binary Orbital Properties Cheat Sheet: Credit : Michela Mapelli

Binary orbital properties

- Gravitational tides:



- \- Tidal force is: proportional to mass of the primary inversely proportional to distance cubed $dF = \left(\frac{dF}{dr}\right) dr = \frac{2GMm}{r^3} dr$
- \- Tidal force circularizes orbits and synchronizes rotation
- constant of angular momentum
- minimum energy state



Orbital properties

Binaries follow Kepler laws

Orbit as function of true anomaly

$$r(\theta) = \frac{a\,(1 - e^2)}{1 + e\,\cos\theta}$$

Orbital period

$$P = 2\,\pi \left(\frac{a^3}{G\,(m_1 + m_2)}\right)^{1/2}$$

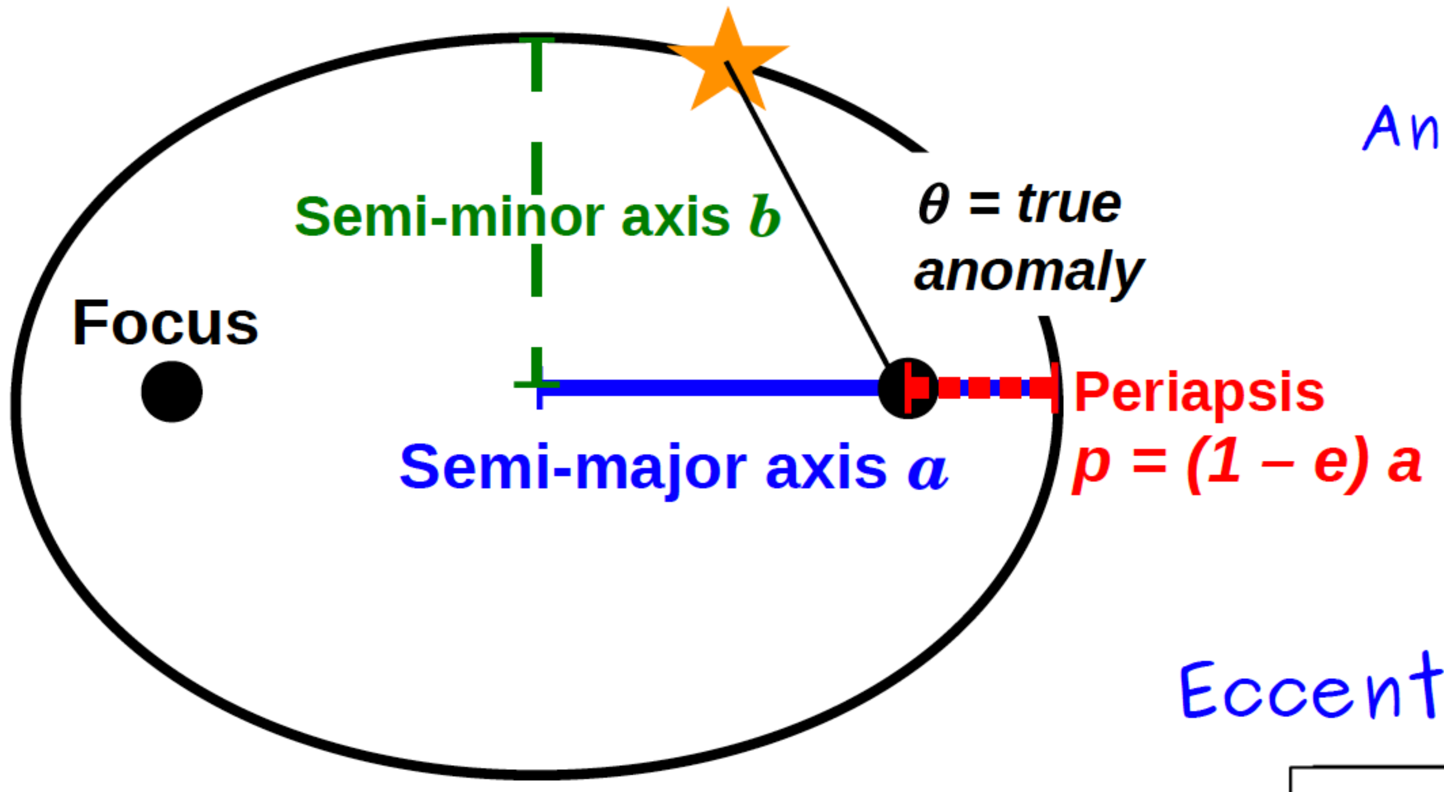
Energy

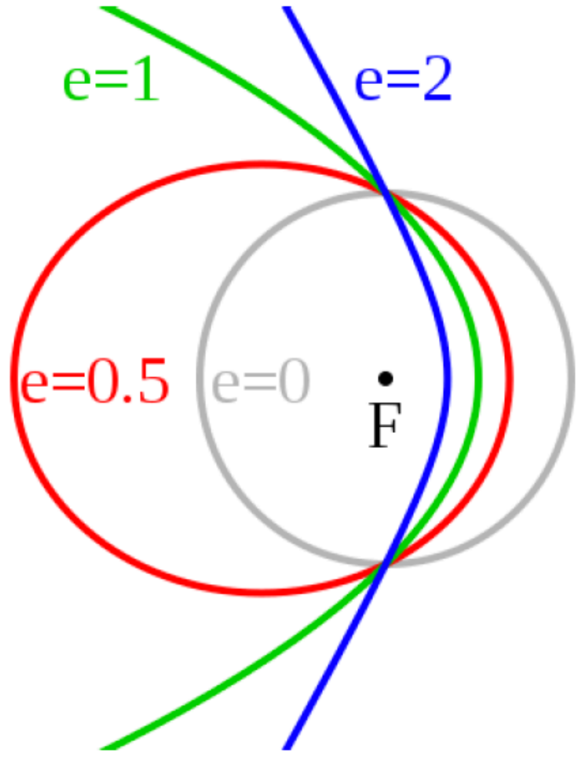
$$E = \frac{1}{2}\,\mu\,v^2 - \frac{G\,m_1\,m_2}{r}$$

Angular momentum

$$L = \mu\,\sqrt{G\,(m_1 + m_2)\,a}$$

In the reduced particle + CoM frame





Eccentricity:

$$e = \sqrt{1 + \frac{2\,E\,L^2}{\mu^3\,G^2\,(m_1 + m_2)^2}}$$

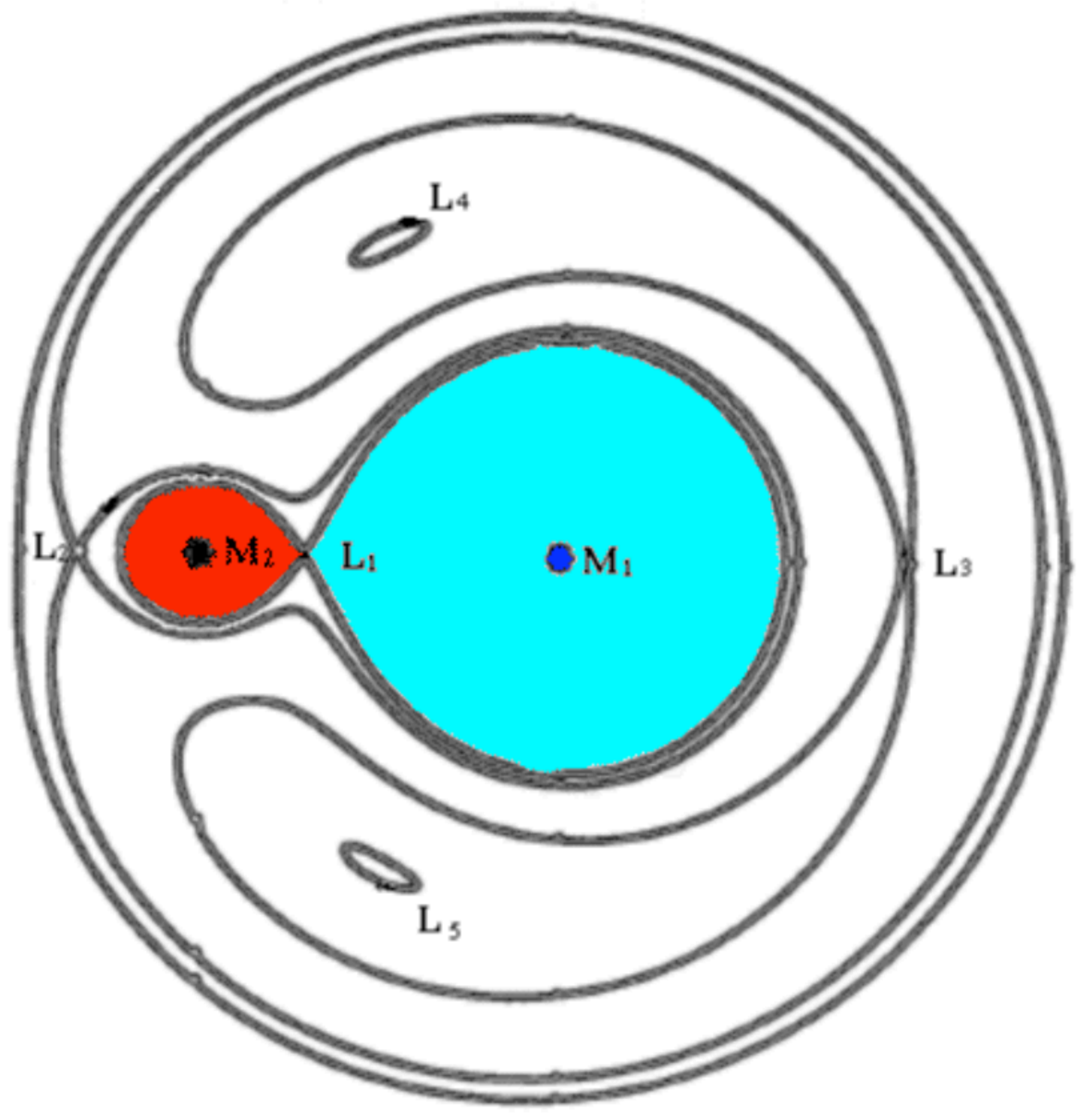
reduced mass:

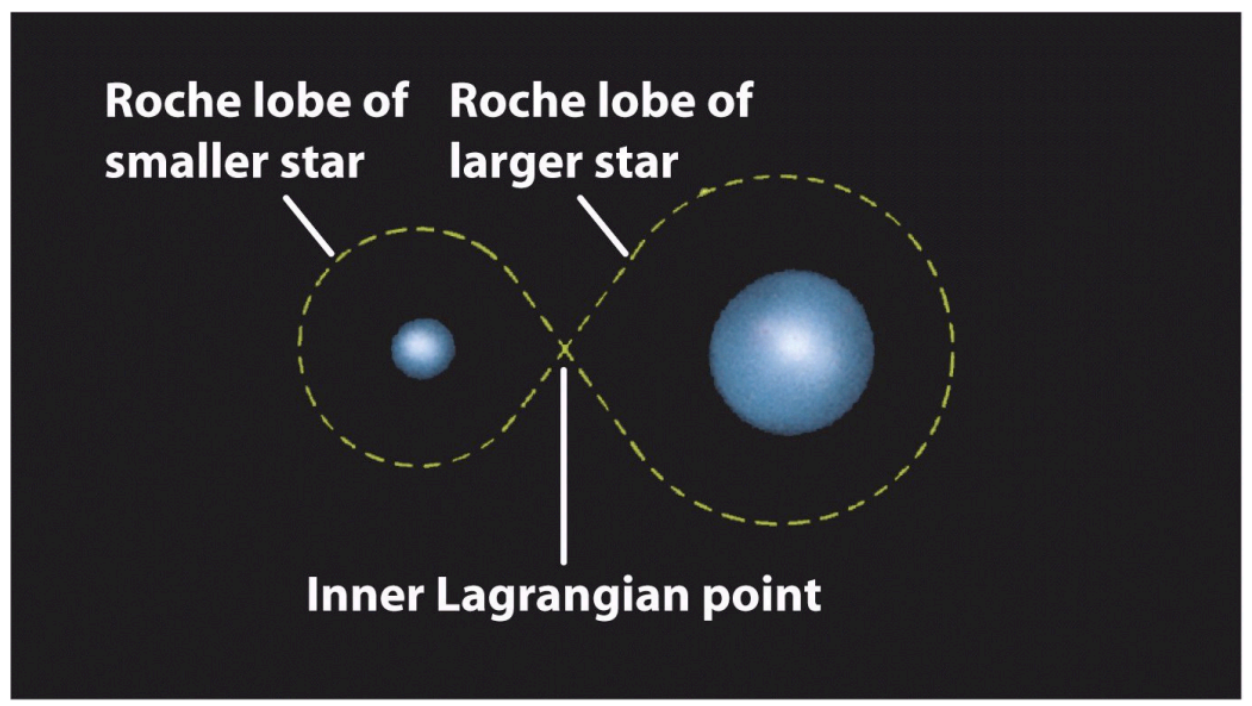
$$\mu = \frac{m_1\,m_2}{(m_1 + m_2)}$$

Binary Orbital Properties Cheat Sheet: Credit : Michela Mapelli

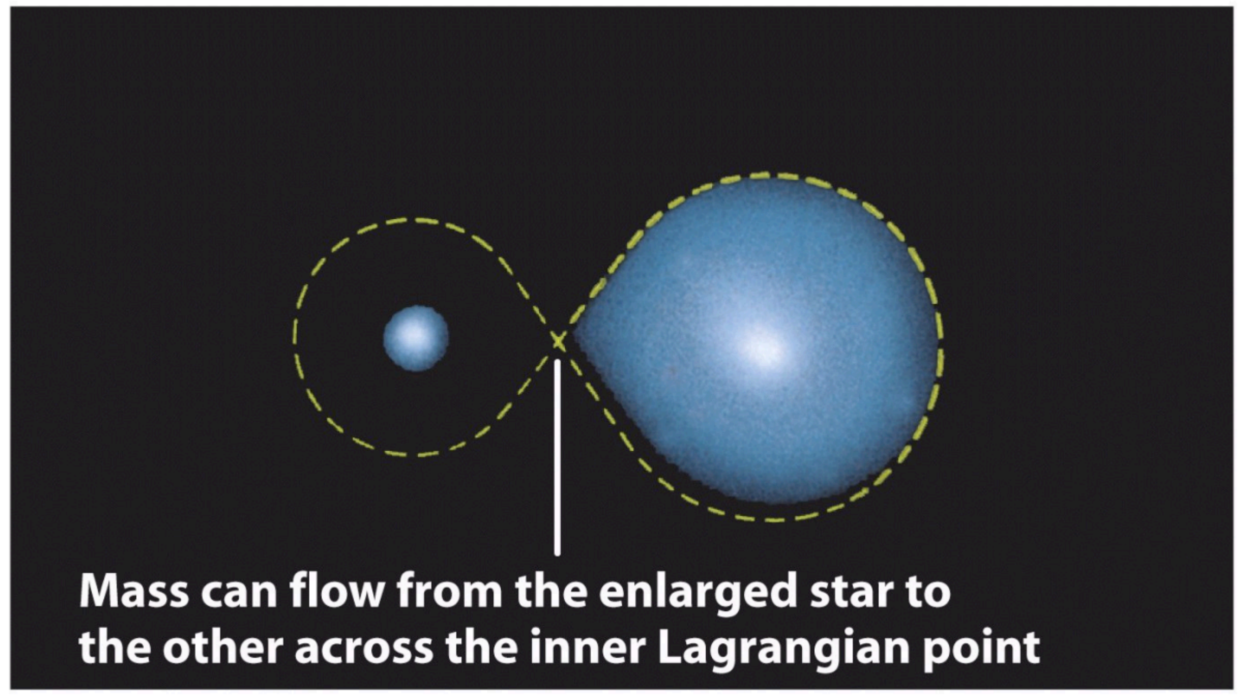
Mass transfer in binary systems

- Stars close enough to each other will have large gravitational influence
- Orbital parameters of a binary respond to mass transfer
- Roche Lobe
  - equipotential surface of gravitational force in a two body system
  - Material that crosses the Roche lobe can be transferred to the othe star, into orbit in thesystem, or ejected

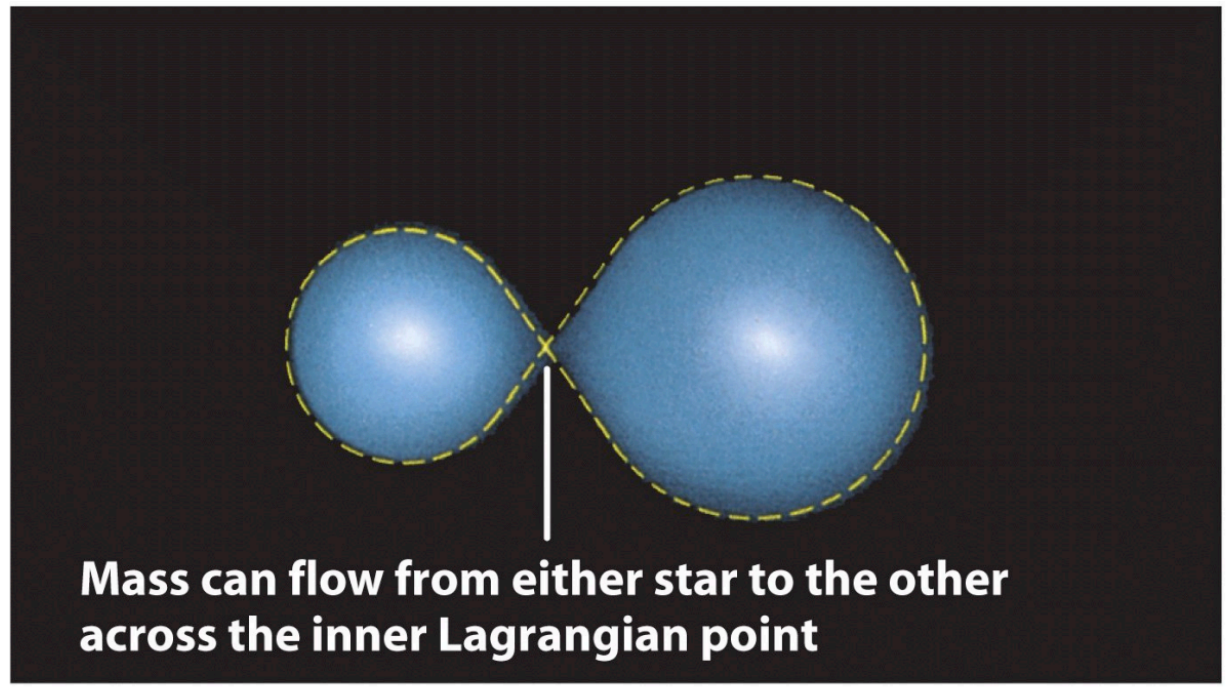




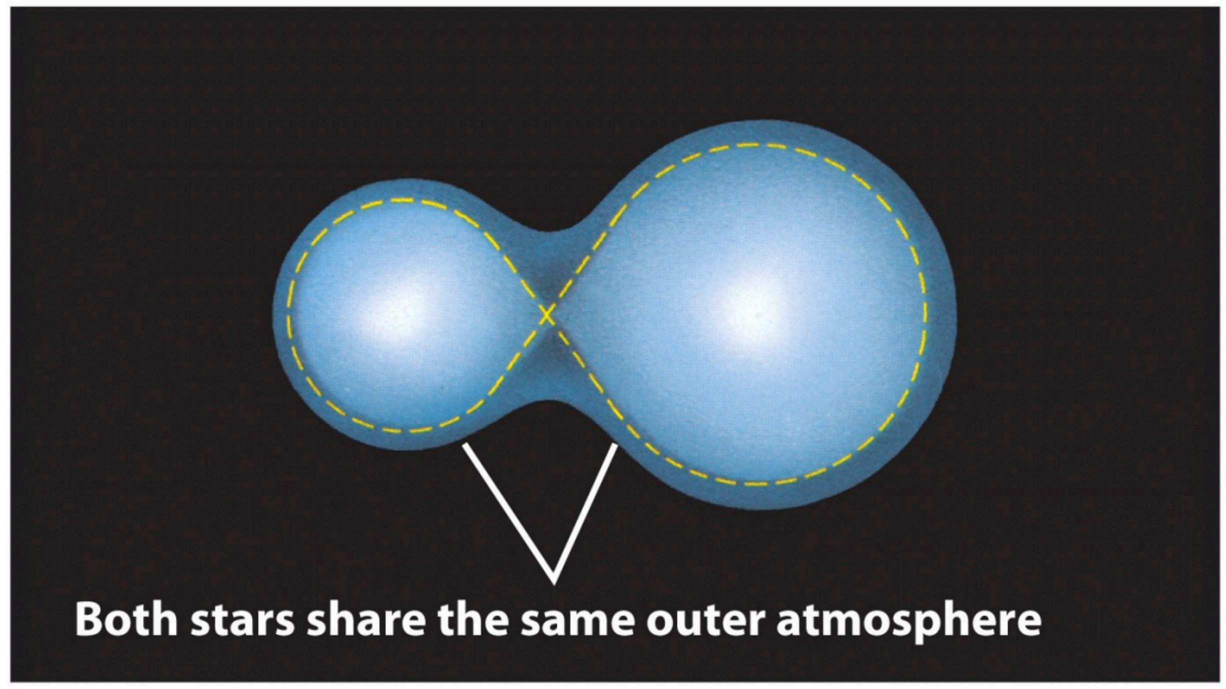
Detached binary: Neither star fills its Roche lobe.



Semi-detached binary: One star fills its Roche lobe.



Contact binary: Both stars fill their Roche lobes.

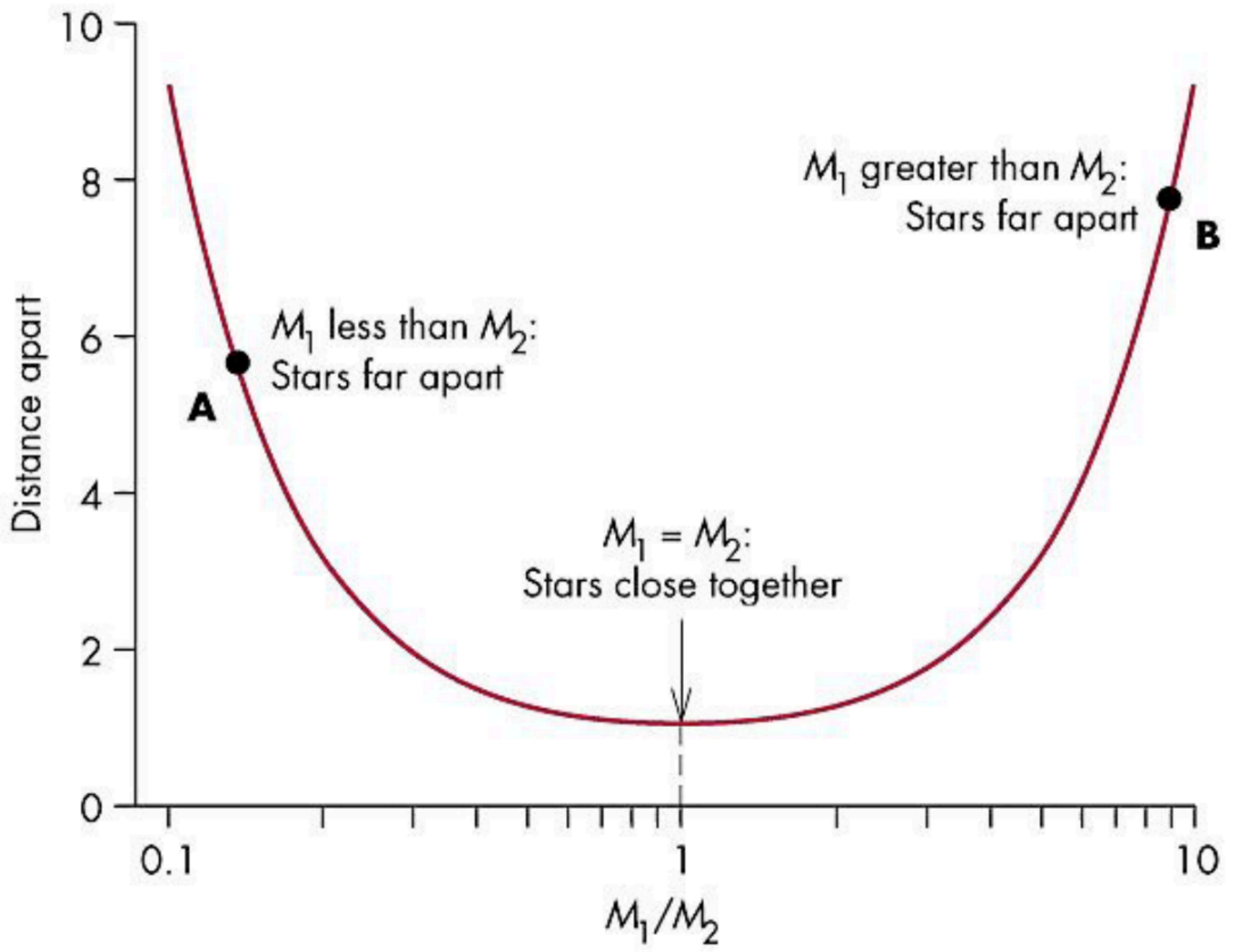


Overcontact binary: Both stars overfill their Roche lobes.

Credit: Reed L. Liddle

Mass transfer in binary systems

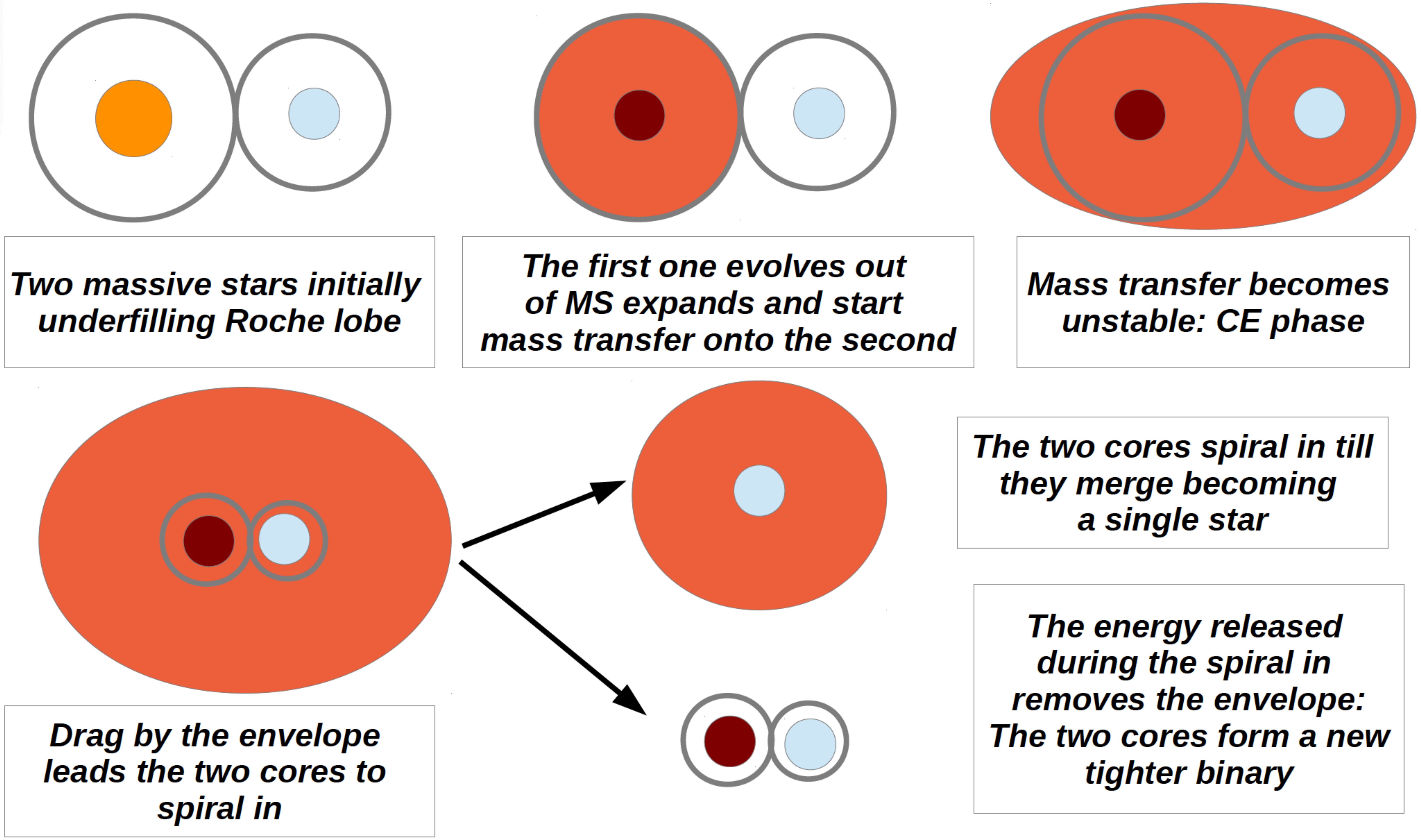
- Mass Transfer depends on
  - Distance between stars
  - Mass fraction of the stars
  - Dynamics of the binary system
  - Evolutionary stage of the star
- Mass transfer affects:
  - Separation
  - Mass ratio
  - Eccentricity
- Examples of different types of close binaries:
  - Massive binaries (O and B stars)
  - X-ray binaries (Star with a neutron star or black hole companion
  - Novae (Star with a white dwarf companion)



Credit: Reed L. Liddle

Common envelope evolution

If mass transfer becomes unstable (e.g. both stars fill Roche lobe), COMMON ENVELOPE (CE) phase = Two stars, one envelope



Credit : Michela Mapelli

- Proposed as a pathway to explain close binary systems (Paczyński 1976)
- Binary components can merge if initially too close
- Uncertainties:
  - Efficiency of removal of envelope
  - Final separation of binary components
  - Short living state: difficult to observe

Treating stellar/binary evolution in star clusters

- Binary evolution is modelled using population synthesis codes (also known as synthetic stellar evolution codes):
  - simplified models of stellar evolution (fitting formula to full stellar evolution tracks from codes like MESA, PARSEC)
  - include treatment of binary evolution processes (analytical/semi-analytical prescriptions for important stellar/binary evolution processes):
  - Tides, mass transfer, common envelope evolution!
  - include prescriptions for supernova explosions, remnant masses and natal kicks
- **- SSE/BSE (Hurley et al 2000; 2002)**: startrack (Belczynski 2008; 2010; 2016), binary_C (Izzard), MOBSE (Giacobbo & Mapelli 2018) Seba, COMPAS, biseps,
- Other codes: SEVN (based on PARSEC tracks) , MSE (Hamers et al.) for evolution of triples and quadruples
- Advantages:

  Fast and useful to see how a population of stars/binaries will evolve: explore initial parameter space

  Can be coupled with dynamical codes for star cluster evolution like NBODY6, MOCCA, PeTar etc

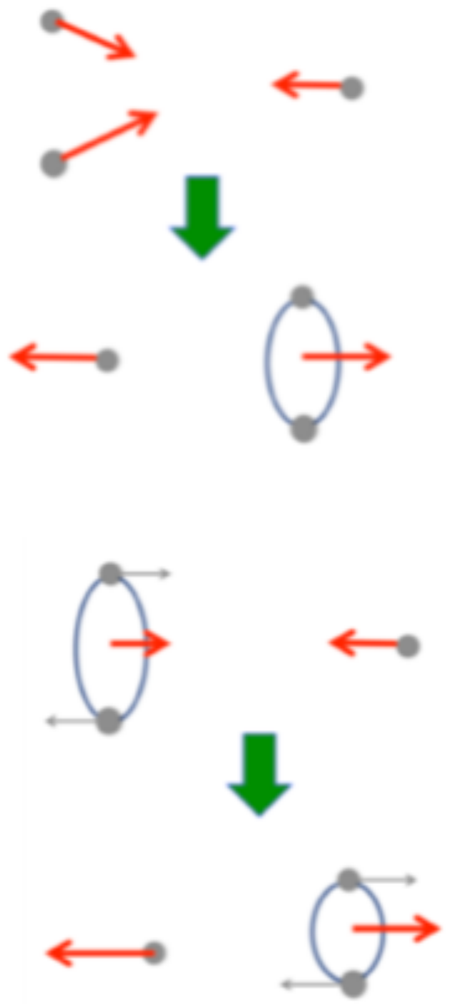
Recap: Counteracting core collapse with binaries

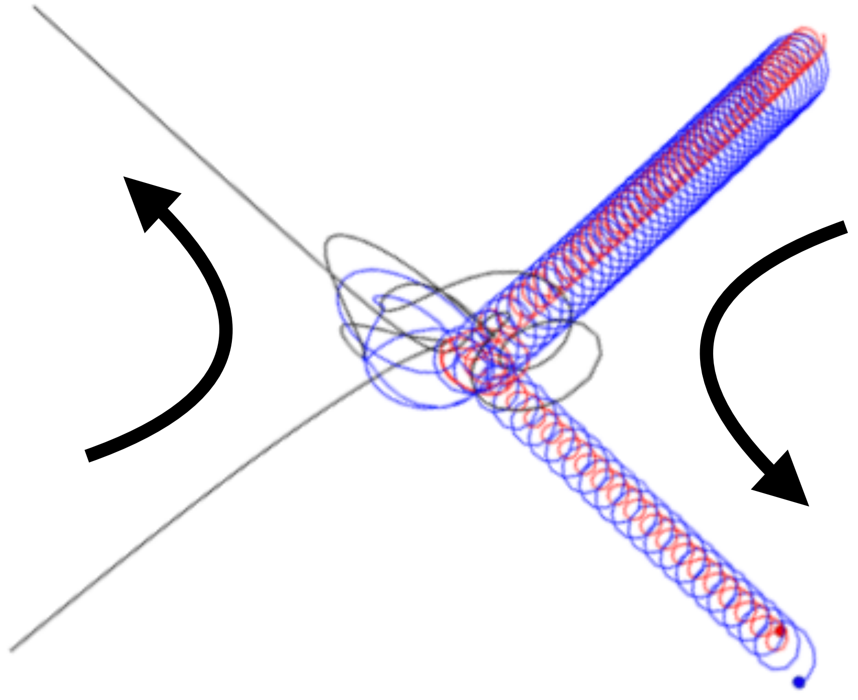
- Core collapse is a runaway process → would lead to an infinite core density and a null core radius → an energy source is needed to counteract core collapse
- 2 ways to counteract core collapse:
  - removing potential energy |U| from the core without removing much kinetic energy
    - Mass loss by stellar winds and supernovae → remove mass without changing K of other stars
  - **By injecting fresh kinetic energy into the core**
- **Binary Heating/Burning:** Binaries act as an energy source: their internal energy can be exchanged with other stars
  - Binaries can be primordial or can form in 3-body encounters
- Heating via binary-single encounters can prevent or delay core collapse.
  - Interactions will process binaries via encounters which can change membership and binary properties

$$E_{int} = \frac{1}{2}\mu v^2 - \frac{Gm_1m_2}{r}$$

$$E_{int} < 0$$

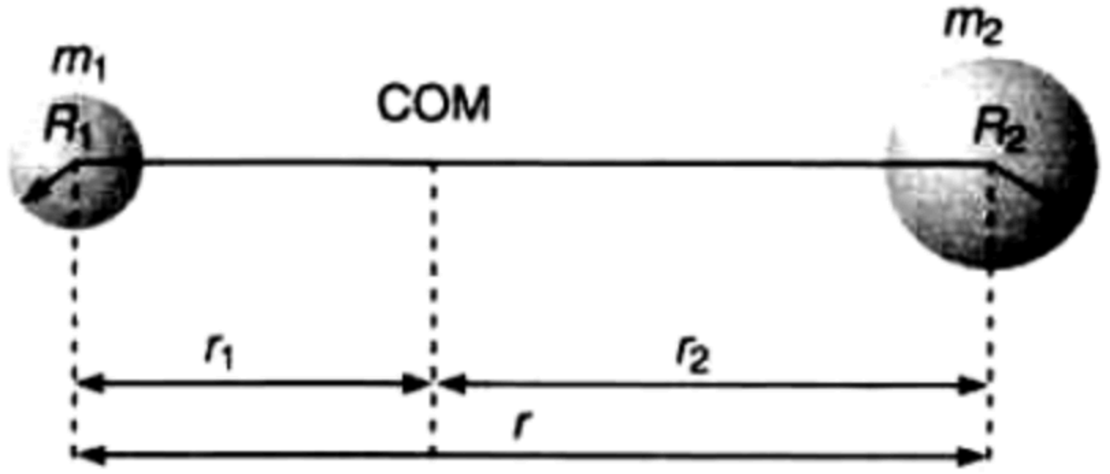
$$E_{int} = -\frac{Gm_1m_2}{2a} = -E_b$$





Binaries as energy sources

- Binaries can exchange their internal energy with other stars
  - Internal energy: total energy of the binary – kinetic energy of the center-of-mass



$$E_{int} = \frac{1}{2}\mu v^2 - \frac{Gm_1m_2}{r}$$

$$\mu = \frac{m_1m_2}{m_1+m_2}$$

- r and v are the relative separation to velocity

$E_{int} < 0$ if the binary is bound

- E_{int} can also be interpreted as the energy of a mass μ orbiting in the potential $-\frac{Gm_1m_2}{r}$

- For a bound binary, the orbit around the orbit of the reduced mass is elliptical with semi-major axis (a). Thus, the energy integral of motion is:

$$E_{int} = -\frac{Gm_1m_2}{2a} = -E_b$$

where E_b is the binding energy of the binary

Binaries as energy sources

$$E_{int} = -\frac{Gm_1m_2}{2a} = -E_b$$

where E_b is the binding energy of the binary

- Simple derivation of the above:

$$K = \frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 = \frac{m_1m_2}{2(m_1+m_2)}V^2$$

the speed V in the relative orbit is given by $V^2 = G(m_1+m_2)[(2/r) - (1/a)]$

vis-viva equation

$$U = -\int_r^\infty \frac{Gm_1m_2}{r^2}dr = -\frac{Gm_1m_2}{r}$$

$$E_{tot} = K + U = -\frac{Gm_1m_2}{2a} = -E_b \qquad E_b = \frac{Gm_1m_2}{2a}$$

- total energy: the larger the orbit (a ↑), the less negative, and hence larger E_{tot} ↑
- Binding energy: larger the orbit (a ↑), smaller the E_b

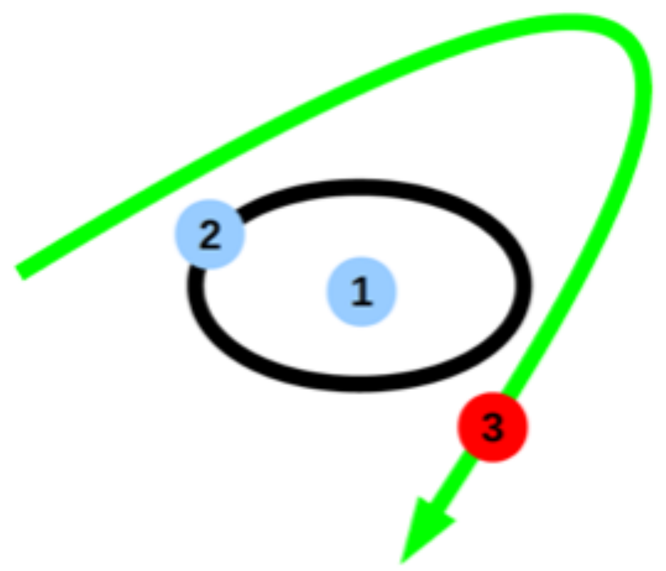
  tighter the binary (a ↓), larger the E_b

Binaries as energy sources

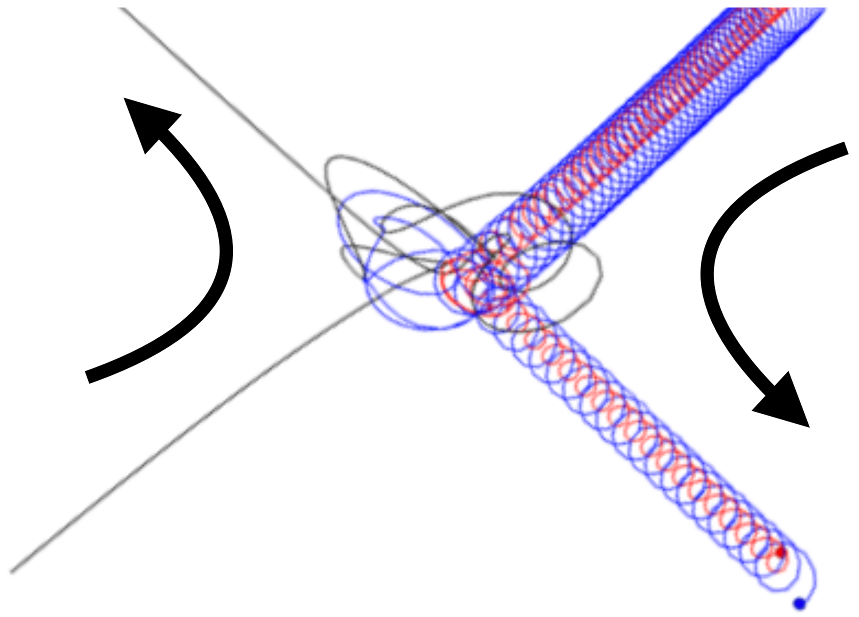
- The internal energy of the binary can be exchanged with stars in a cluster through close encounters

$$E_{int} = -\frac{Gm_1m_2}{2a} = -E_b \qquad E_b = \frac{Gm_1m_2}{2a}$$

- 3-body encounters
  - an interaction between a binary and a single star, the single star can either:
    - extract energy from the binary
    - or lose a fraction of its kinetic energy, which is converted into internal energy of the binary



  - If the star extracts E_{int} from the binary, its final kinetic energy $\left(K_f\right)$ is higher than the initial kinetic energy $\left(K_i\right)$
  - More precisely: K_f of the center-of-mass of the single star and of the binary is higher than their K_i
  - As a result of the encounter, the interacting star and the binary acquire recoil velocity

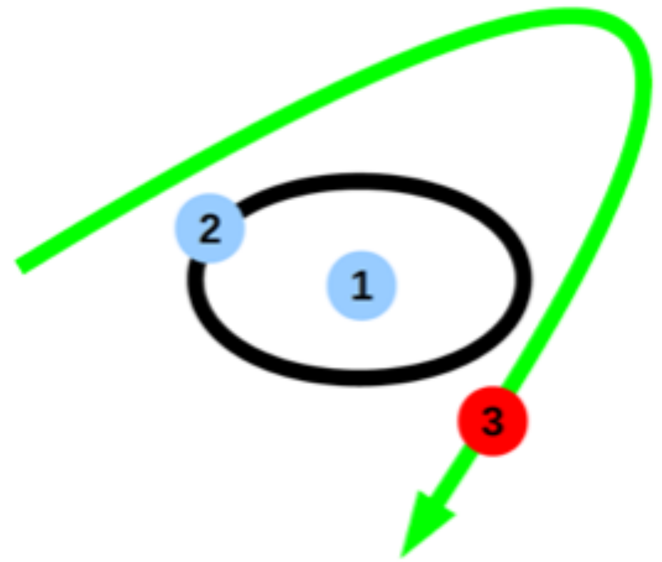


Binaries as energy sources

- The internal energy of the binary can be exchanged with stars in a cluster through close encounters

$$E_{int} = -\frac{Gm_1m_2}{2a} = -E_b \qquad E_b = \frac{Gm_1m_2}{2a}$$

- 3-body encounters
  - an interaction between a binary and a single star, the single star can either:
    - **extract energy from the binary**
    - or lose a fraction of its kinetic energy, which is converted into internal energy of the binary



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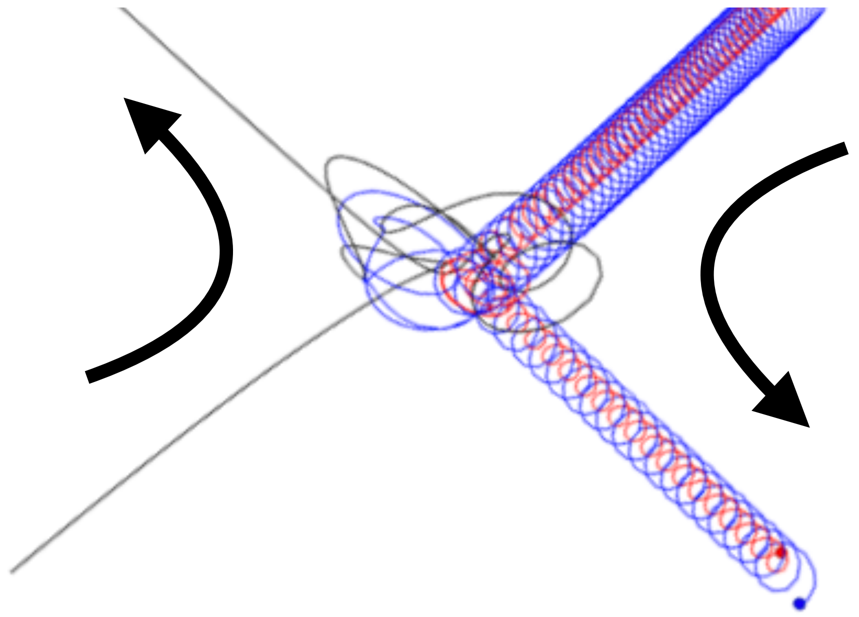
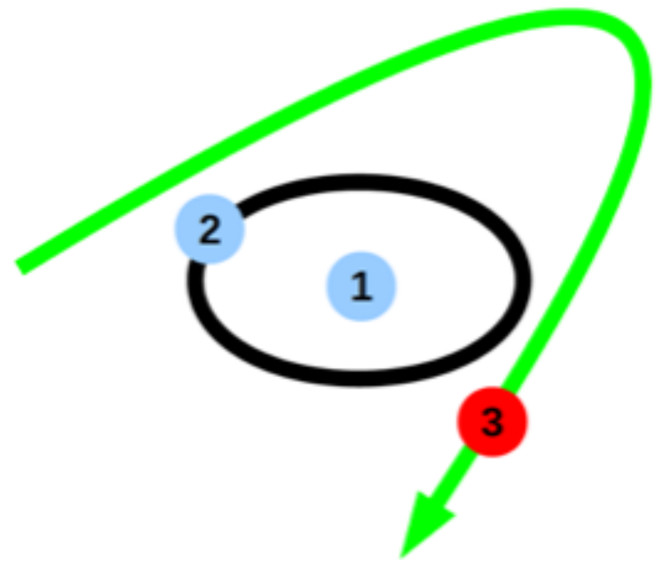


3-body encounters: flyby resulting in binary hardening

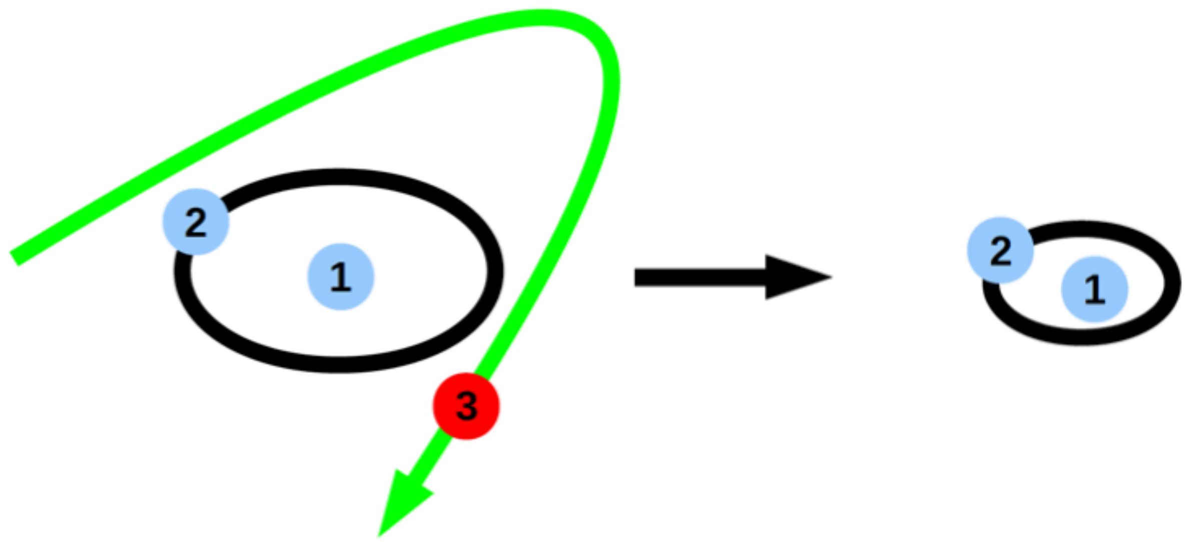
- An interaction between a single star and a binary: the single star extracts E_{int} from the binary

$$E_{int} = -\frac{Gm_1m_2}{2a} = -E_b$$



- E_{int} ↓, becomes more negative and E_b increases ↑ :
  - the binary becomes more bound (a ↓)

$$E_b = \frac{Gm_1m_2}{2a_f} > \frac{Gm_1m_2}{2a_i} \qquad a_f < a_i$$



Credit: Michela Mapelli
Lecture notes on Collisional Dynamics
http://web.pd.astro.it/mapelli/
2014colldyn3.pdf

- Single star gains kinetic energy
- Binary system loses energy and becomes more bound!

3-body encounters: flyby resulting in binary hardening

$m_1 = 1\ M_\odot$
$m_2 = 1\ M_\odot$
$a = 1$ AU
$e = 0$

$m_3 = 1\ M_\odot$
$b = 10$ AU
$v_\infty = 10$ km/s
$D = 25 \times a$ [AU]

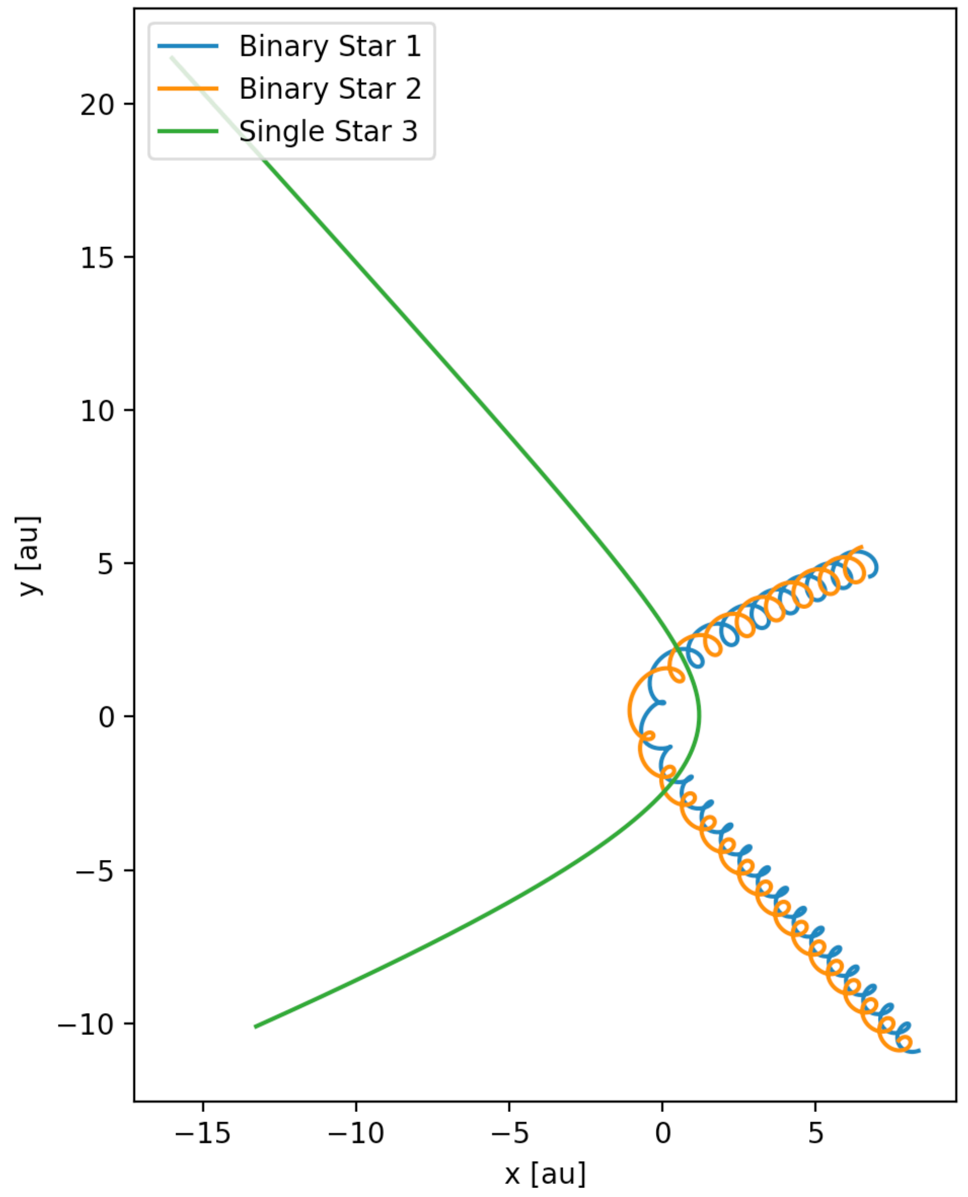
Final Integration Time = 10.68 yr
$E_{tot} = -0.46 \quad \Delta E = 2.42E-14$

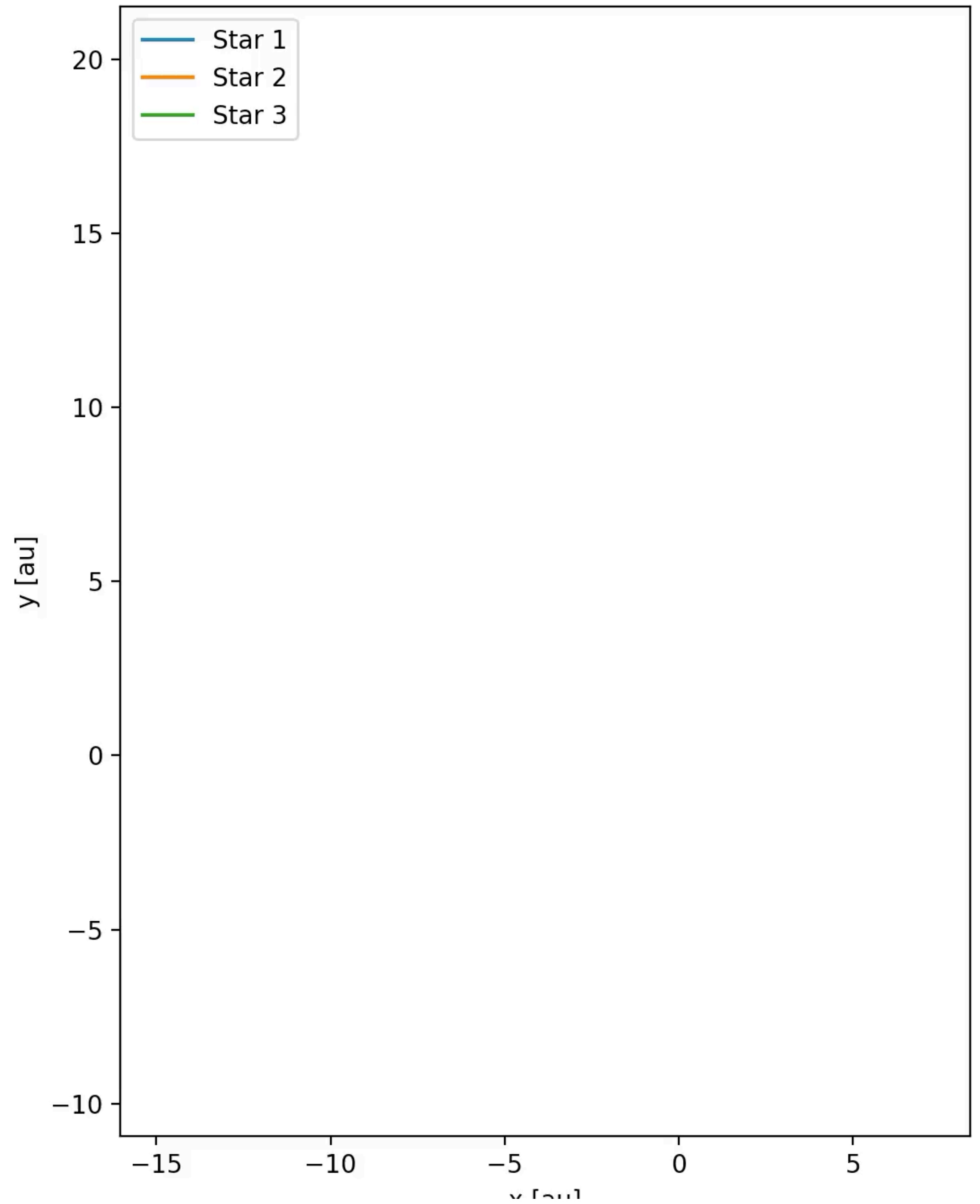
$m_1 = 1\ M_\odot$
$m_2 = 1\ M_\odot$
$a = 0.70$ AU
$e = 0.44$

$m_3 = 1\ M_\odot$
$v_\infty = 17.3$ km/s

$$v_\infty^2 = -\frac{GM}{a}$$

Binary phase is randomly sampled between 0 and 2π





Simulations carried out using the *TSUNAMI* code (Trani et al. 2022, 2023; Hellström et al. 2022)
TSUNAMI is a "few-body" (N-body problem for a few objects) code: planetary systems, multiple star systems, and especially for encounters between a single star and a multiple star system.

3-body encounters: flyby resulting in binary hardening

$m_1 = 1\ M_\odot$
$m_2 = 1\ M_\odot$
$a = 0.5$ AU
$e = 0$

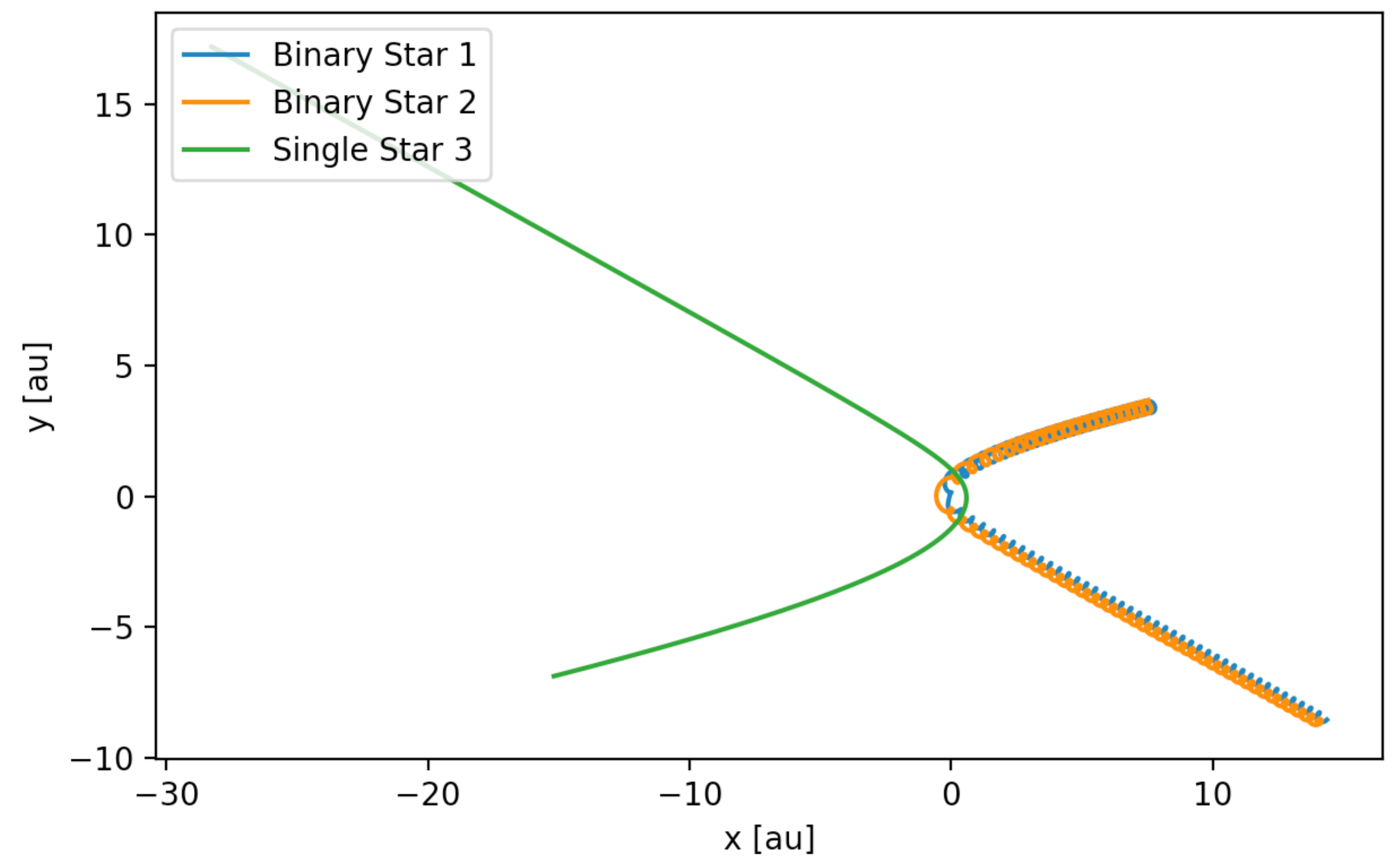
$m_3 = 1\ M_\odot$
$b = 10$ AU
$v_\infty = 7$ km/s
$D = 50 \times a$ [AU]

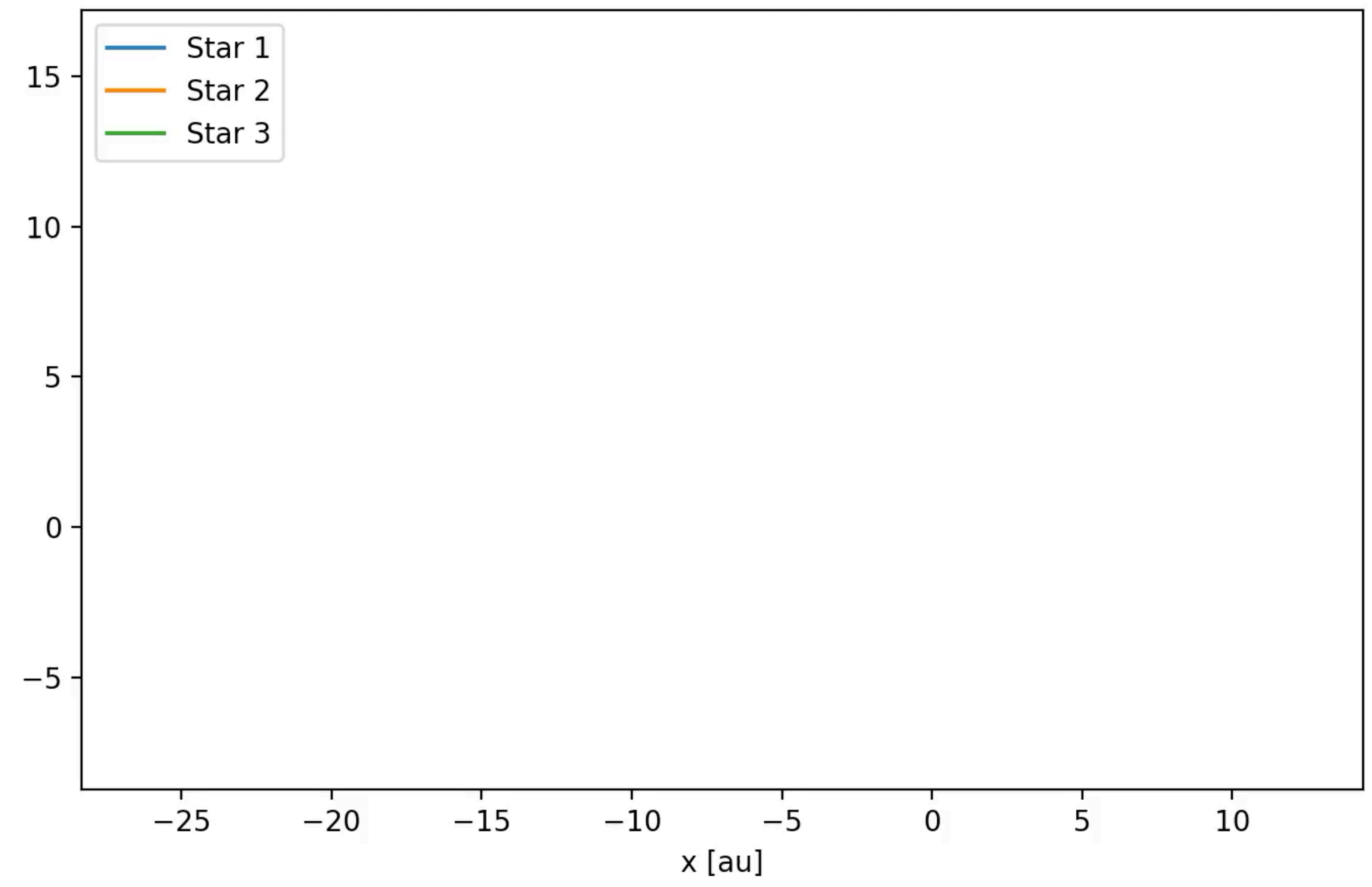
Final Integration Time = 11.01 yr
$E_{tot} = -0.98 \quad \Delta E = 5.5E-13$

$m_1 = 1\ M_\odot$
$m_2 = 1\ M_\odot$
$a = 0.33$ AU
$e = 0.59$

$m_1 = 1\ M_\odot$
$v_\infty = 24.93$ km/s

$$v_\infty^2 = -\frac{GM}{a}$$





Simulations carried out using the *TSUNAMI* code (Trani et al. 2022, 2023; Hellström et al. 2022)

3-body encounters: resonant flyby

$m_1 = 1\ M_\odot$
$m_2 = 1\ M_\odot$
$a = 0.5\ \mathrm{AU}$
$e = 0$

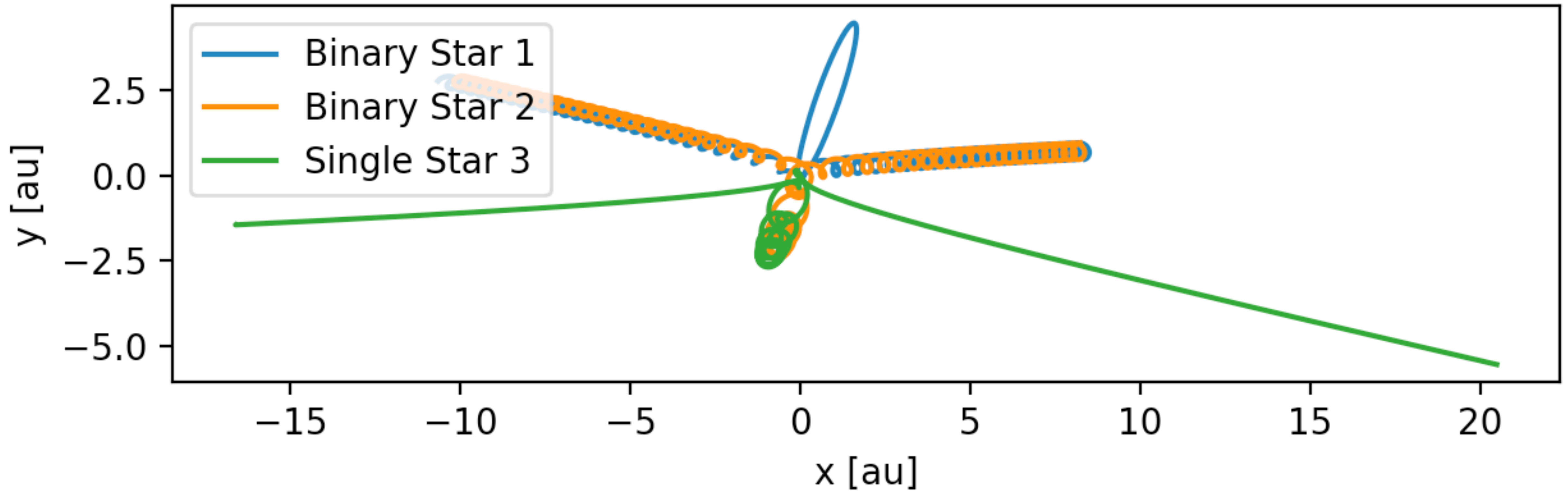
$m_3 = 1\ M_\odot$
$b = 3\ \mathrm{AU}$
$v_\infty = 5\ \mathrm{km/s}$
$D = 50 \times a\ [\mathrm{AU}]$

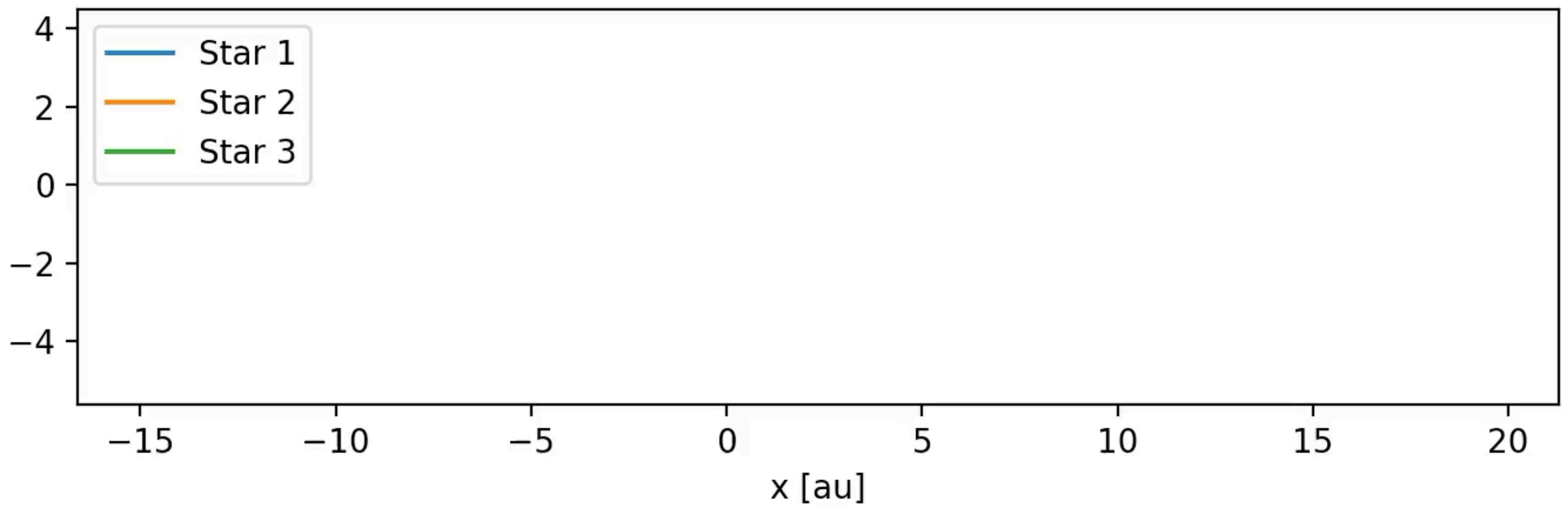
Final Integration Time = 15.41 yr
$E_{tot} = -0.99 \quad \Delta E = 9.5E-14$

$m_1 = 1\ M_\odot$
$m_2 = 1\ M_\odot$
$a = 0.47\ \mathrm{AU}$
$e = 0.68$

$m_1 = 1\ M_\odot$
$v_\infty = 8.60\ \mathrm{km/s}$

$$v_\infty^2 = -\frac{GM}{a}$$





Simulations carried out using the *TSUNAMI* code (Trani et al. 2022, 2023; Hellström et al. 2022)

3-body encounters: exchange

- Another way for a binary to transfer internal energy to field stars and to increase its binding energy E_b is via exchange encounter
- Exchange interactions: single star replaces one of the binary components (m_1 or m_2 is replaced by m_3) - More likely when $m_3 > m_1$ or $m_3 > m_2$

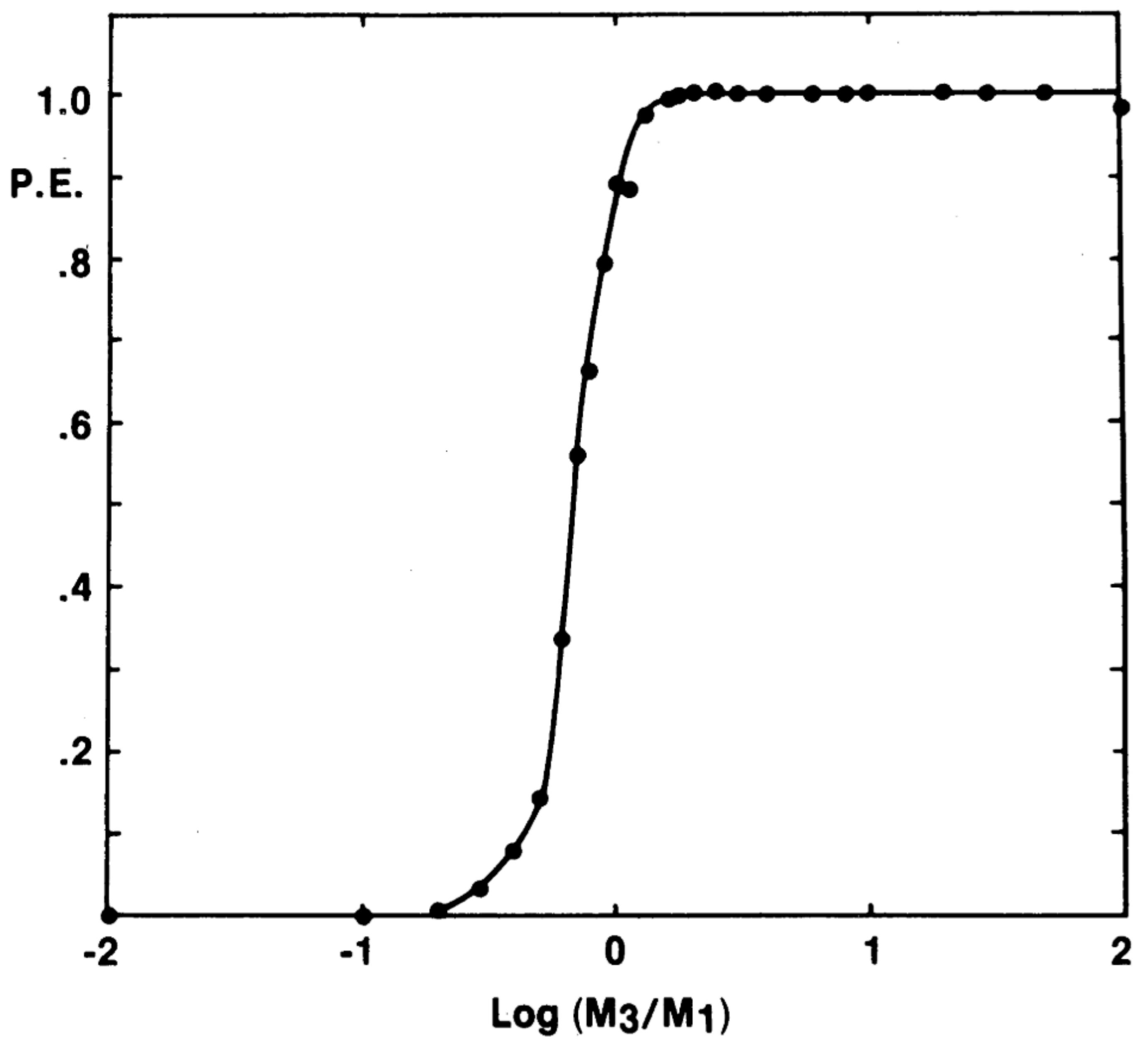
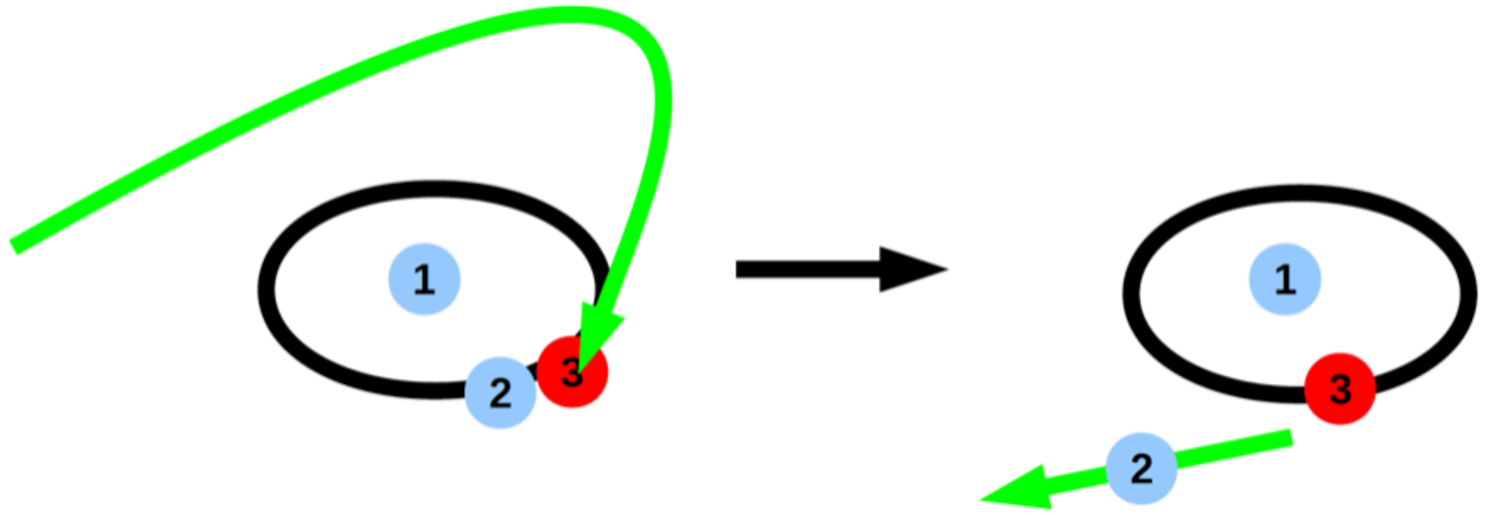


FIG. 4. The exchange probability (the fraction of the post-encounter binaries containing the original impacting star as one of its stellar components).

Hills & Fullerton (1980)

$$E_b = \frac{Gm_1m_3}{2a} > \frac{Gm_1m_2}{2a}$$



Credit: Michela Mapelli
Lecture notes on Collisional Dynamics

3-body encounters: exchange

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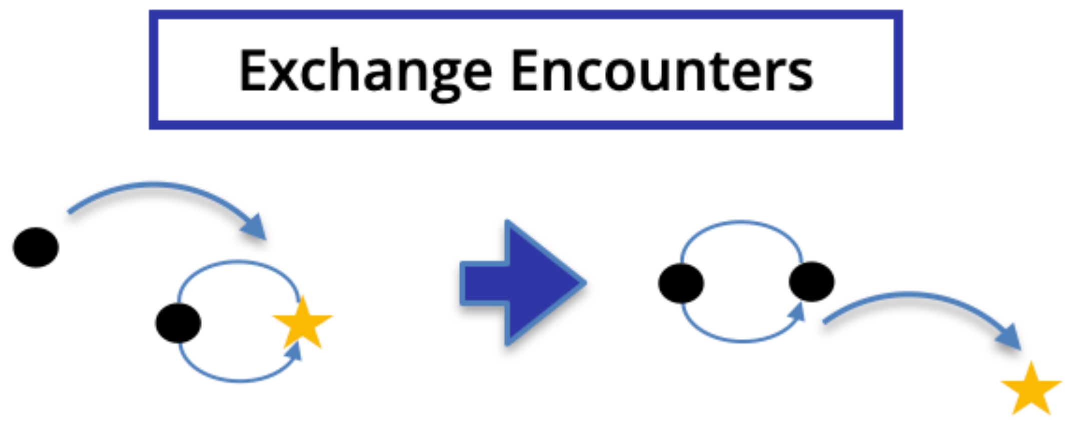
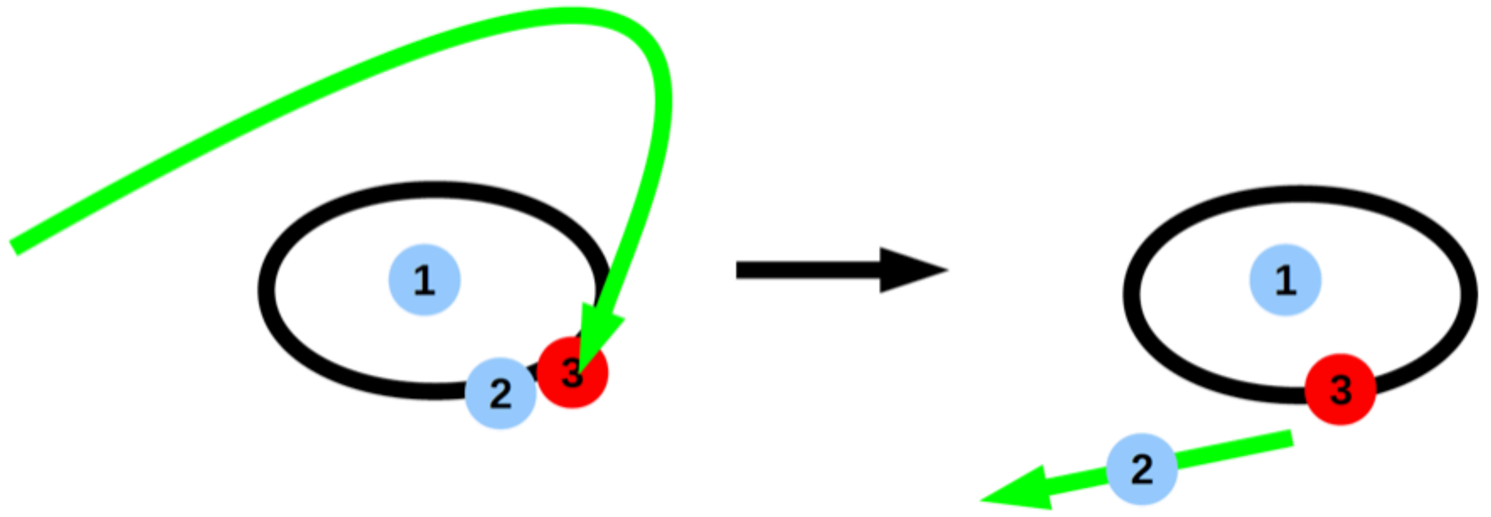
$$E_b = \frac{Gm_1m_3}{2a} > \frac{Gm_1m_2}{2a}$$





Credit: Michela Mapelli
Lecture notes on Collisional Dynamics

- These close encounters can change binary membership
  - Important in the context of forming close binary systems e.g, X-ray binaries, gravitational wave sources

3-body encounters: exchange

$m_1 = 1\ M_\odot$

$m_2 = 1\ M_\odot$

$a = 0.5\ \mathrm{AU}$

$e = 0$

$m_3 = 1\ M_\odot$

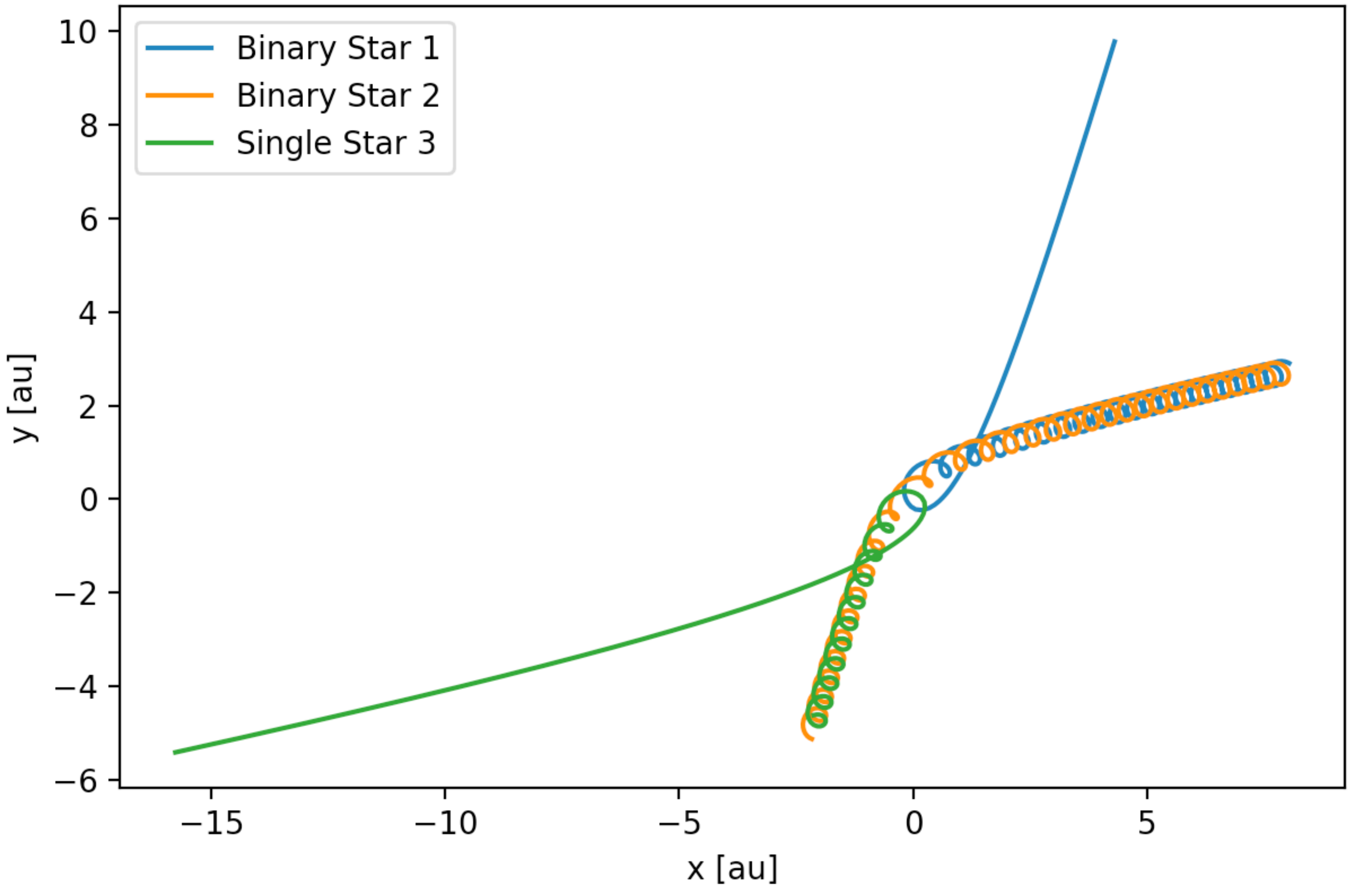
$b = 5\ \mathrm{AU}$

$v_\infty = 10\ \mathrm{km/s}$

$D = 50 \times a\ [\mathrm{AU}]$

$\mathrm{Final\ Integration\ Time} = 7.11\ \mathrm{yr}$

$E_{tot} = -0.96 \quad \Delta E = 3.8E-14$



m_3 promptly exchanged with m_1

$v_\infty^2 = -\frac{GM}{a}$

$m_3 = 1\ M_\odot$

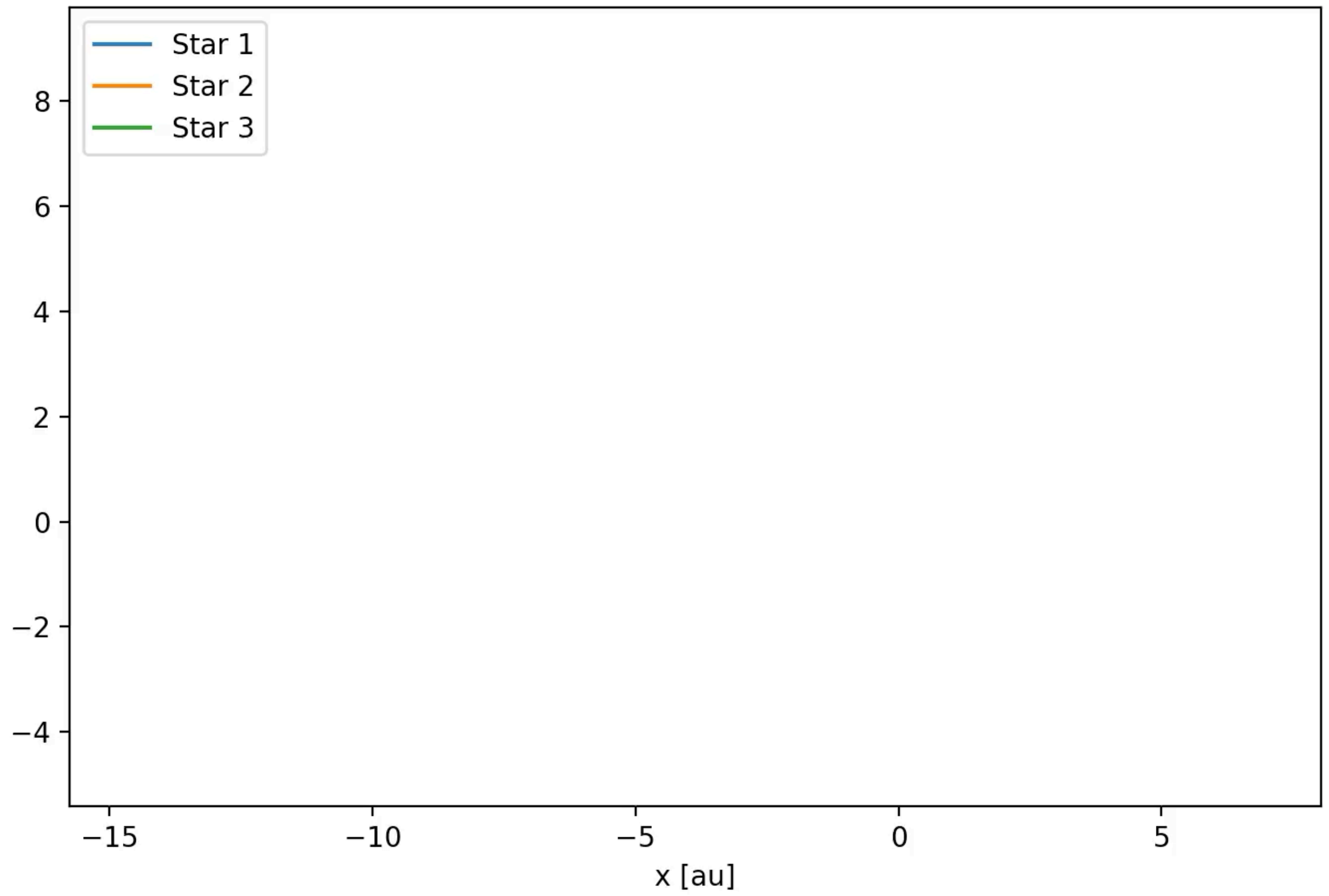
$m_2 = 1\ M_\odot$

$a = 0.45\ \mathrm{AU}$

$e = 0.18$

$m_1 = 1\ M_\odot$

$v_\infty = 13.7\ \mathrm{km/s}$



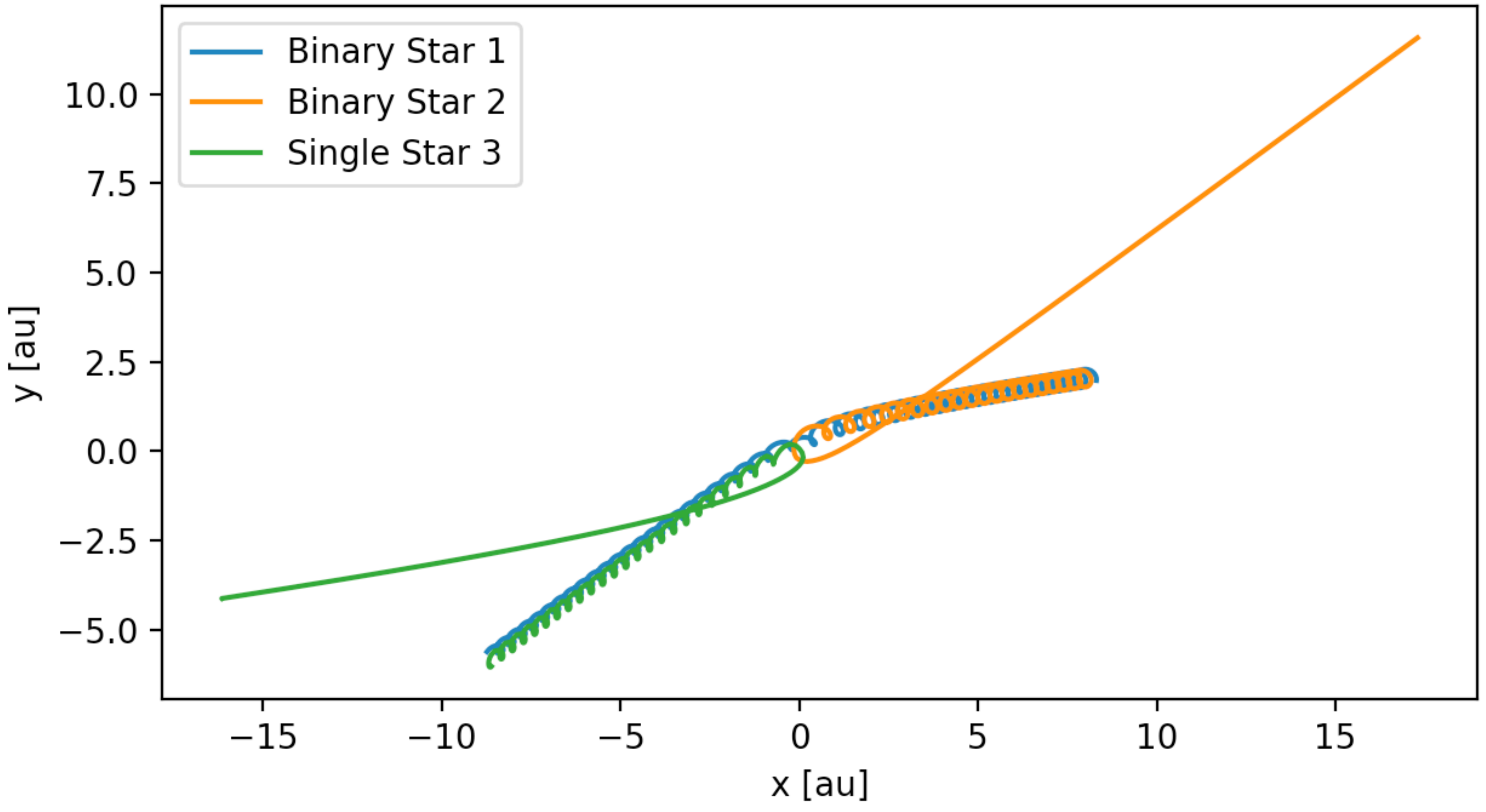
Simulations carried out using the *TSUNAMI* code (Trani et al. 2022, 2023; Hellström et al. 2022)

3-body encounters: exchange

$m_1 = 1\ M_\odot$
$m_2 = 1\ M_\odot$
$a = 0.5$ AU
$e = 0$

$m_3 = 1\ M_\odot$
$b = 5$ AU
$v_\infty = 8$ km/s
$D = 50 \times a$ [AU]

$\text{Final Integration Time} = 8.89$ yr
$E_{tot} = -0.98 \quad \Delta E = 1.3E-13$

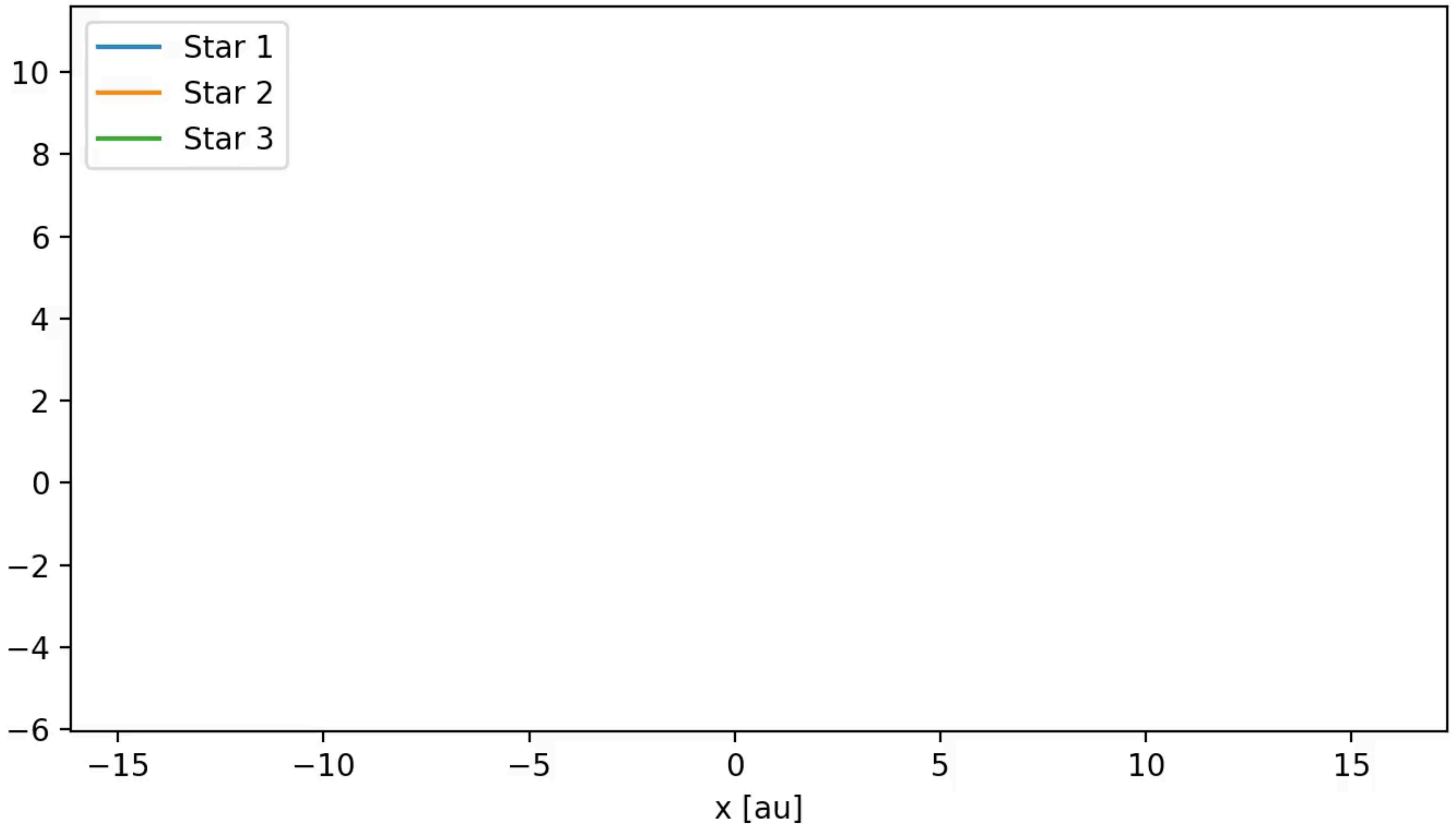


m_3 promptly exchanged with m_2

$m_1 = 1\ M_\odot$
$m_3 = 1\ M_\odot$
$a = 0.37$ AU
$e = 0.68$

$m_2 = 1\ M_\odot$
$v_\infty = 21.08$ km/s

$$v_\infty^2 = -\frac{GM}{a}$$



Simulations carried out using the *TSUNAMI* code (Trani et al. 2022, 2023; Hellström et al. 2022)

3-body encounters: unequal mass ratio

$m_1 = 1\ M_\odot$

$m_2 = 0.5\ M_\odot$

$a = 0.5\ \mathrm{AU}$

$e = 0$

$m_3 = 1\ M_\odot$

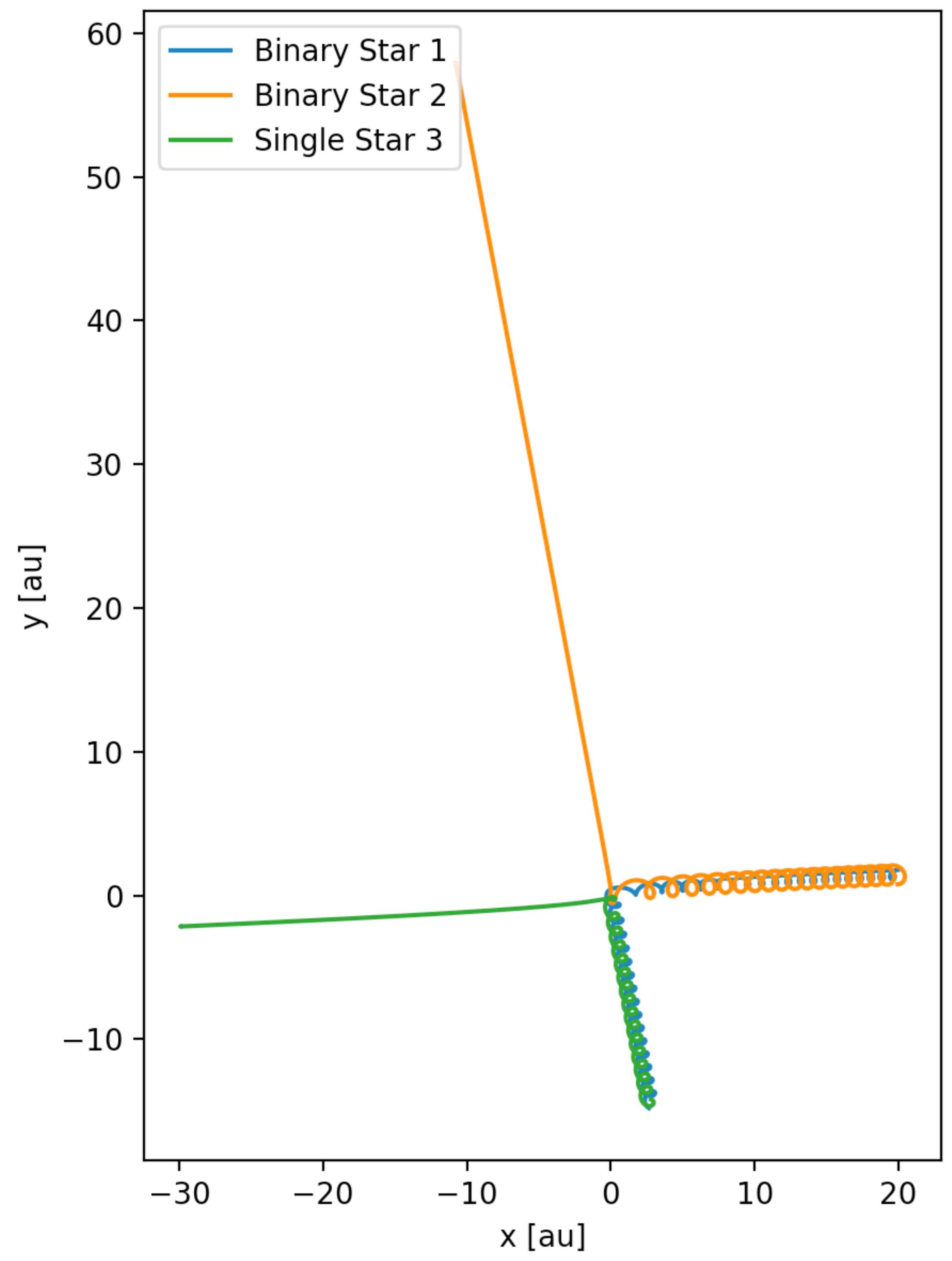
$b = 3\ \mathrm{AU}$

$v_\infty = 5\ \mathrm{km/s}$

$D = 50 \times a\ [\mathrm{AU}]$

Final Integration Time = 23.7 yr

$E_{tot} = -0.24 \quad \Delta E = 1.37E-14$



m_3 promptly exchanged with m_2

$m_1 = 1\ M_\odot$

$m_3 = 1\ M_\odot$

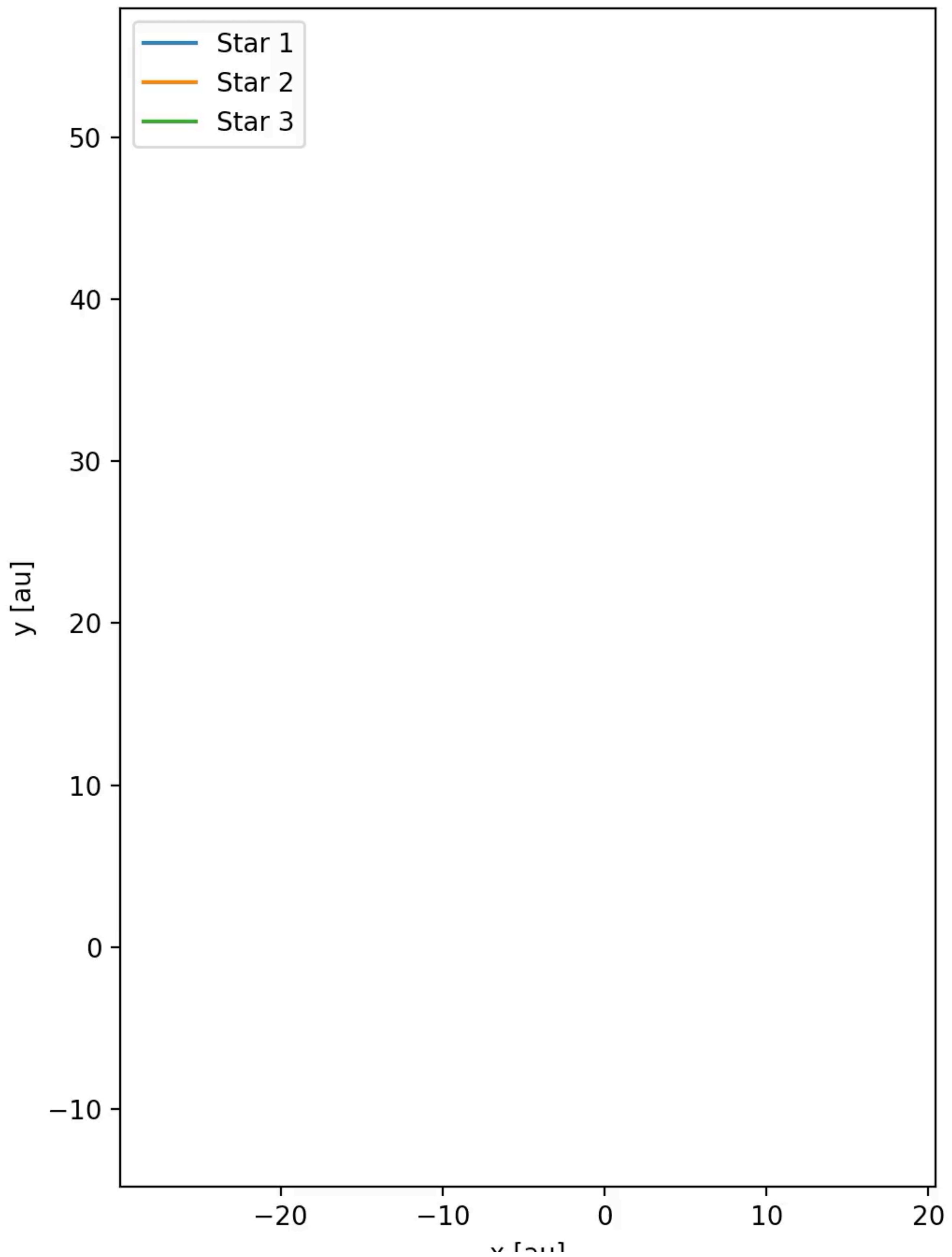
$a = 0.82\ \mathrm{AU}$

$e = 0.30$

$m_2 = 0.5\ M_\odot$

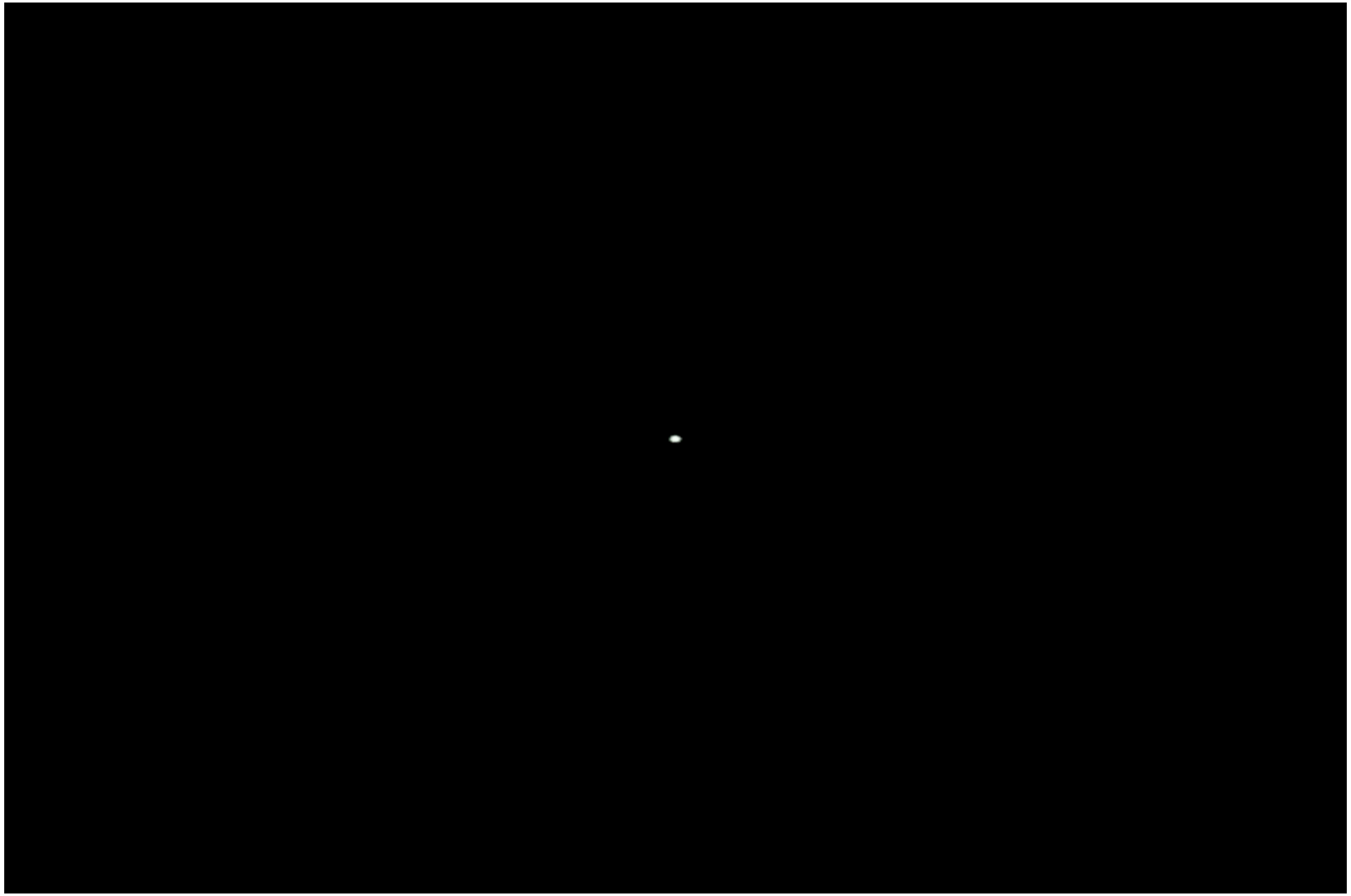
$v_\infty = 32\ \mathrm{km/s}$

$$v_\infty^2 = -\frac{GM}{a}$$



Simulations carried out using the *TSUNAMI* code (Trani et al. 2022, 2023; Hellström et al. 2022)

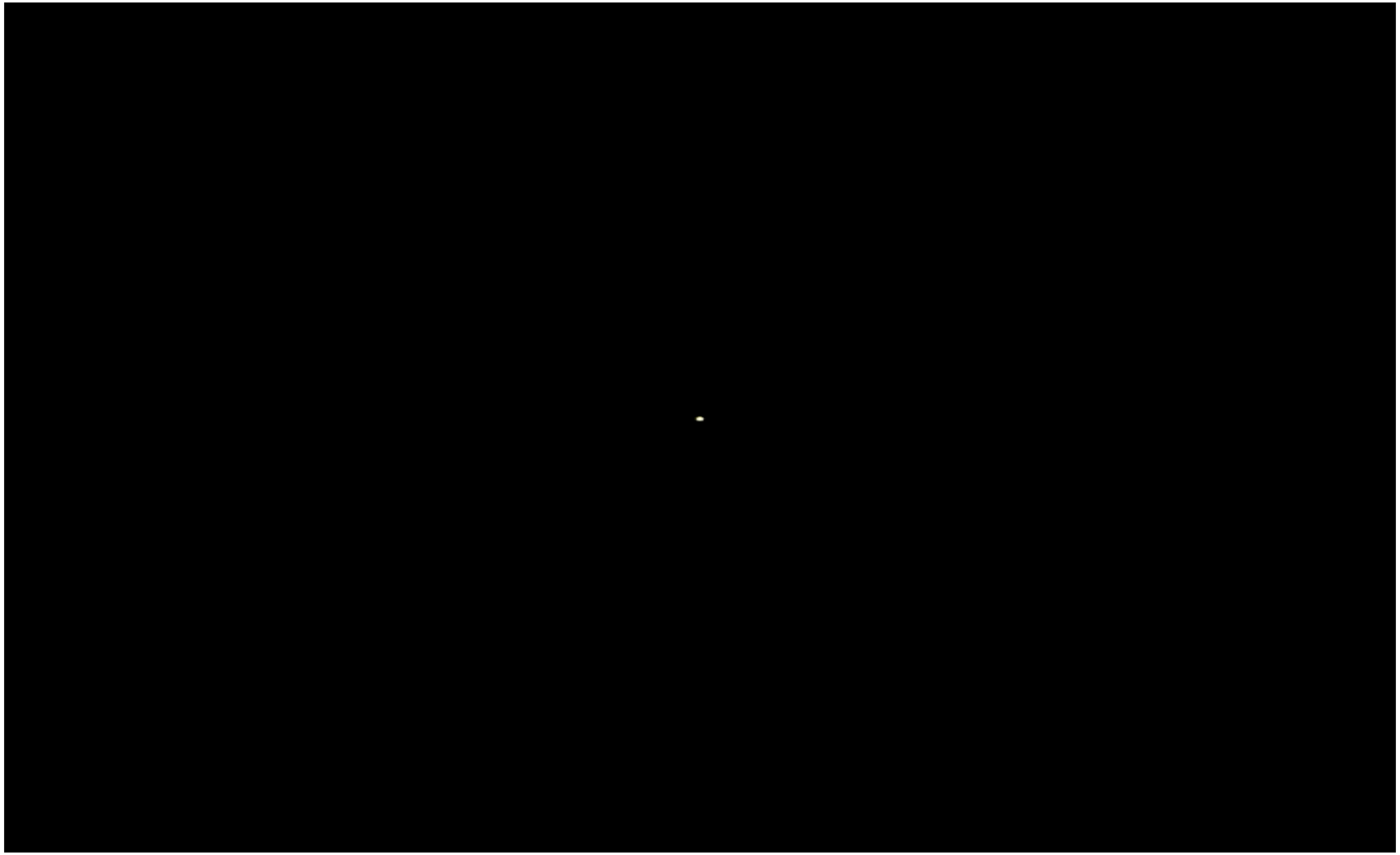
3-body encounters: resonant exchange



Fewbody can be downloaded from: https://sourceforge.net/projects/fewbody/files/

Exchange during binary-single interaction Credit: Aaron Geller & Fewbody code (Fregeau et al. 2004)
https://www.youtube.com/watch?v=hA4BpXK5hec

3-body encounters: collision



During the chaotic phase of three-body evolution, two of the three bodies sometimes approach very close to each other.

At this time, their speeds increase to values which are much higher than the average speed.

In astrophysical systems, the bodies have finite sizes which means that in close approaches the two bodies may collide.

Collision during binary-single interaction Credit: Aaron Geller & Fewbody code (Fregeau et al. 2004)
https://www.youtube.com/watch?v=qCQhP2syAXk

https://faculty.wcas.northwestern.edu/aaron-geller/visuals.html

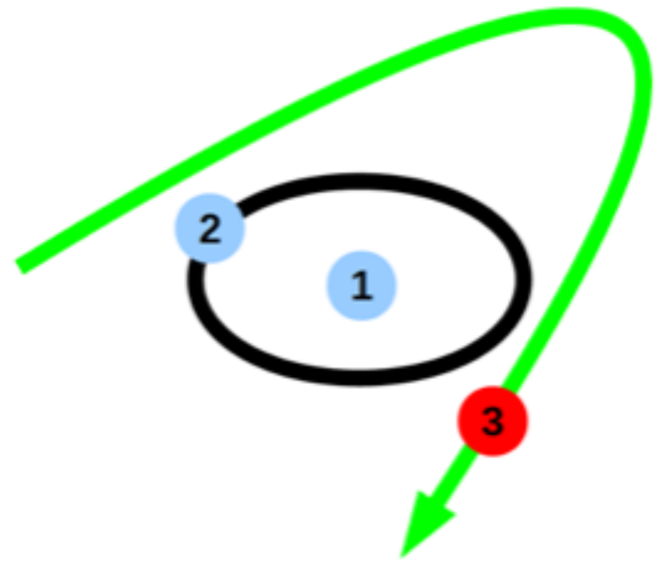
3-body encounters: flyby resulting in binary softening

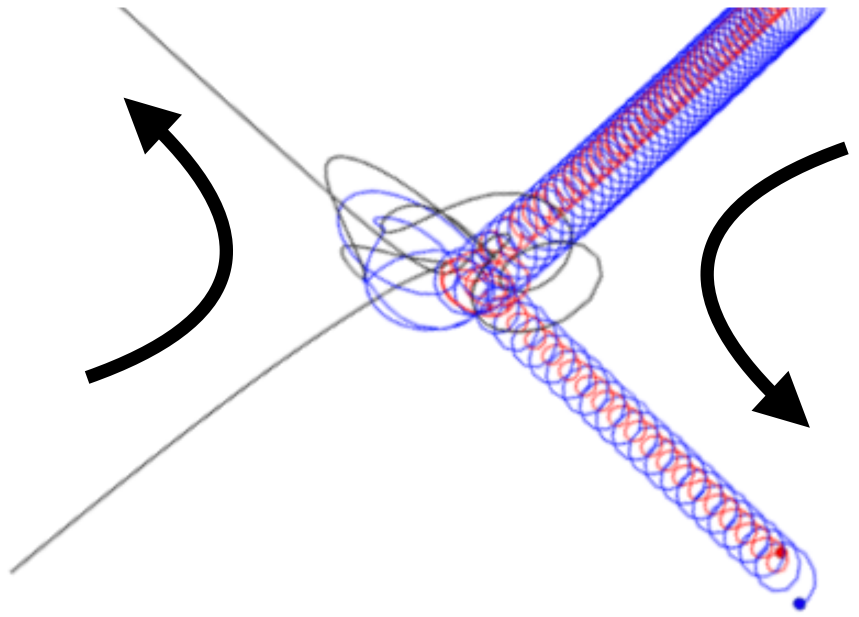
- The internal energy of the binary can be exchanged with stars in a cluster through close encounters

$$E_{int} = -\frac{Gm_1m_2}{2a} = -E_b$$

$$E_b = \frac{Gm_1m_2}{2a}$$

- 3-body encounters
  - an interaction between a binary and a single star, the single star can either:
    - extract energy from the binary
    - **or lose a fraction of its kinetic energy, which is converted into internal energy of the binary**

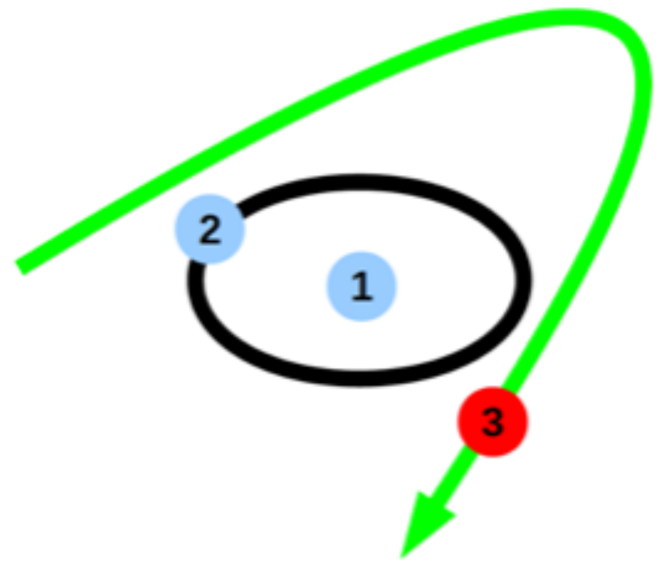




  - If the star transfers kinetic energy to the binary, its final kinetic energy $\left(K_f\right)$ will be lower than the initial kinetic energy $\left(K_i\right)$
  - More precisely: K_f of the center-of-mass of the single star and of the binary is lower than their K_i

3-body encounters: flyby resulting in binary softening

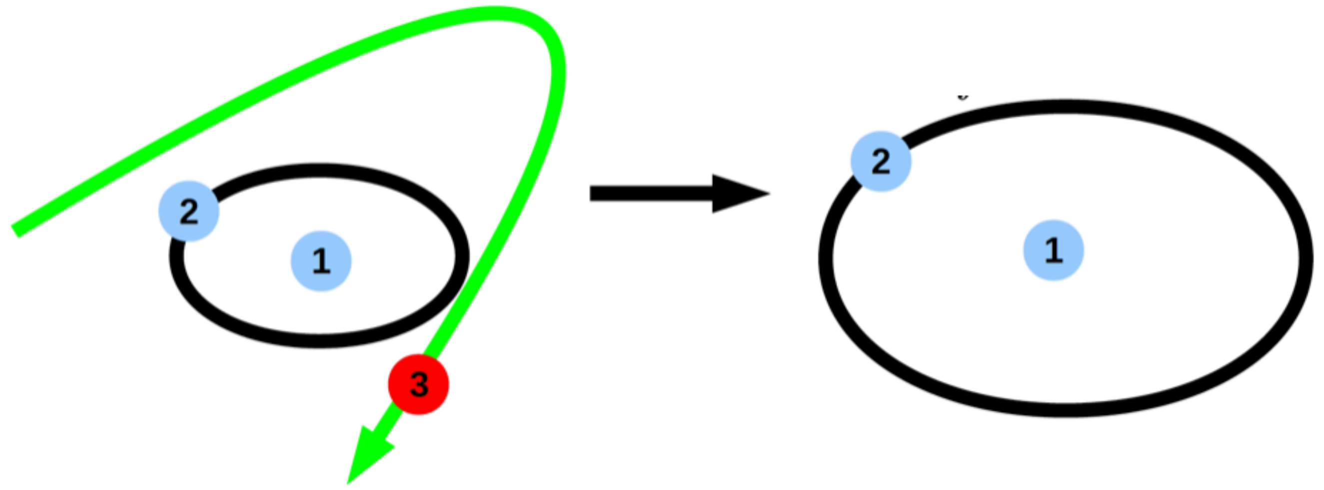
- An interaction between a single star and a binary: the single star gives its kinetic energy to the binary $E_{int} = -\frac{Gm_1m_2}{2a} = -E_b$



- E_{int} ↑, becomes less negative and E_b decreases ↓ :
  - the binary becomes less bound (a ↑)

$$E_b = \frac{Gm_1m_2}{2a_f} < \frac{Gm_1m_2}{2a_i}$$

$$a_f > a_i$$



Credit: Michela Mapelli
Lecture notes on Collisional Dynamics

- Single star loses kinetic energy
- Binary system gains energy and becomes less!
- Can also result in to ionization of the binary: binary becoming unbound

3-body encounters: flyby resulting in binary softening

$m_1 = 1\ M_\odot$
$m_2 = 1\ M_\odot$
$a = 3\ \text{AU}$
$e = 0$

$m_3 = 1\ M_\odot$
$e_2 = 1.25$
$v_\infty = 30\ \text{km/s}$
$D = 50 \times a\ [\text{AU}]$

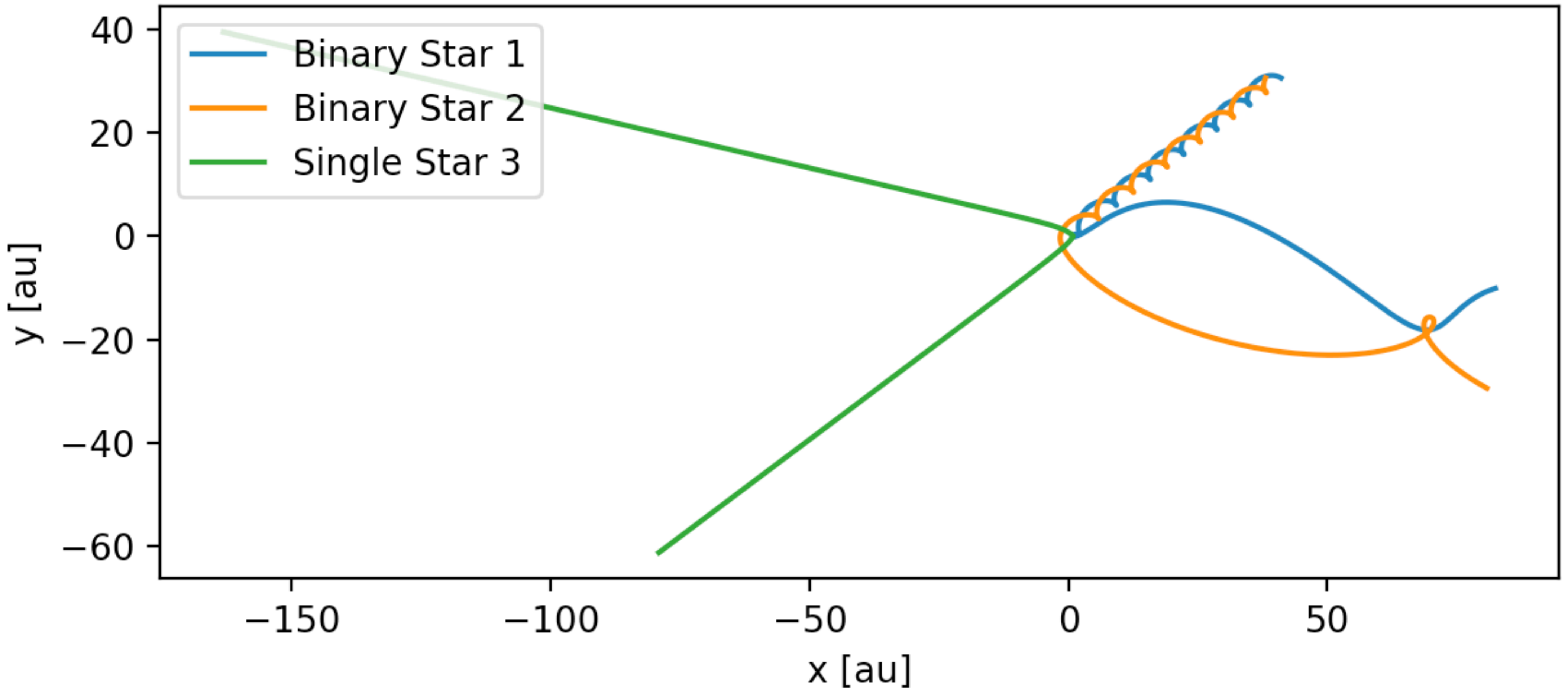
Final Integration Time = 70 yr
$E_{tot} = 0.171 \quad \Delta E = 1.66E - 14$

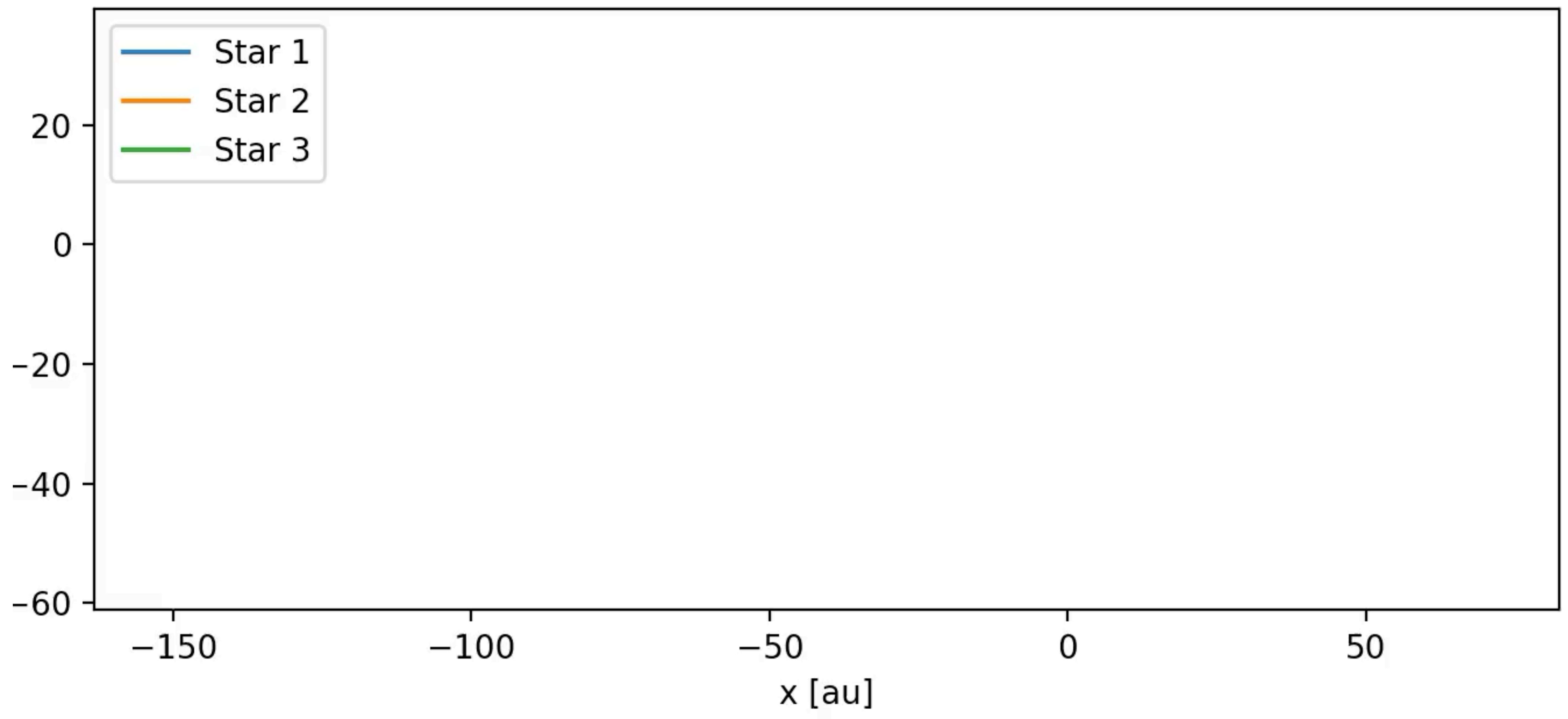
$m_1 = 1\ M_\odot$
$m_2 = 1\ M_\odot$
$a = 15.09\ \text{AU}$
$e = 0.84$

$m_3 = 1\ M_\odot$
$v_{inf} = 15.6\ \text{km/s}$

$$v_\infty^2 = -\frac{GM}{a}$$

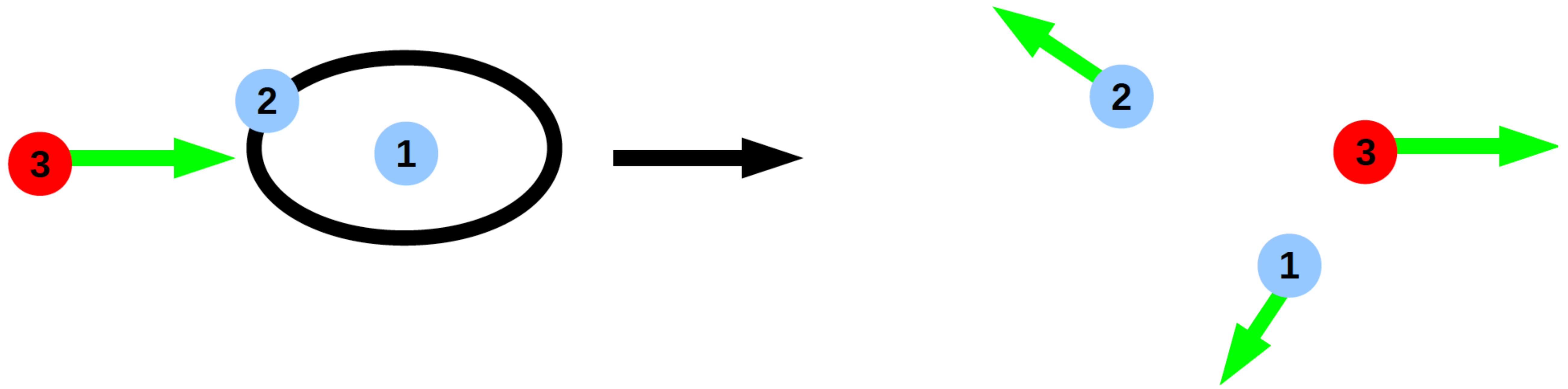
$$v_{orb} = \sqrt{\frac{G(m_1 + m_2)}{a_{bin}}}$$





Simulations carried out using the *TSUNAMI* code (Trani et al. 2020; 2022, 2023; Hellström et al. 2022)

3-body encounters: binary dissolution/ionization



Credit: Michela Mapelli
Lecture notes on Collisional Dynamics

- A single star can ionize the binary only if its v_∞ (when it is far from the binary, thus unperturbed by the binary) exceeds the critical velocity (Hut & Bahcall 1983)

$$v_c = \sqrt{\frac{G m_1 m_2 (m_1 + m_2 + m_3)}{m_3 (m_1 + m_2) a}}$$

- This critical velocity was derived by imposing that the K of the reduced particle of the 3-body system is equal to E_b:

$$\frac{1}{2} \frac{m_3 (m_1 + m_2)}{(m_1 + m_2 + m_3)} v_c^2 = \frac{G m_1 m_2}{2a}$$

3-body encounters: binary dissolution/ionization

$m_1 = 0.5\ M_\odot$
$m_2 = 0.5\ M_\odot$
$a = 1\ \text{AU}$
$e = 0$

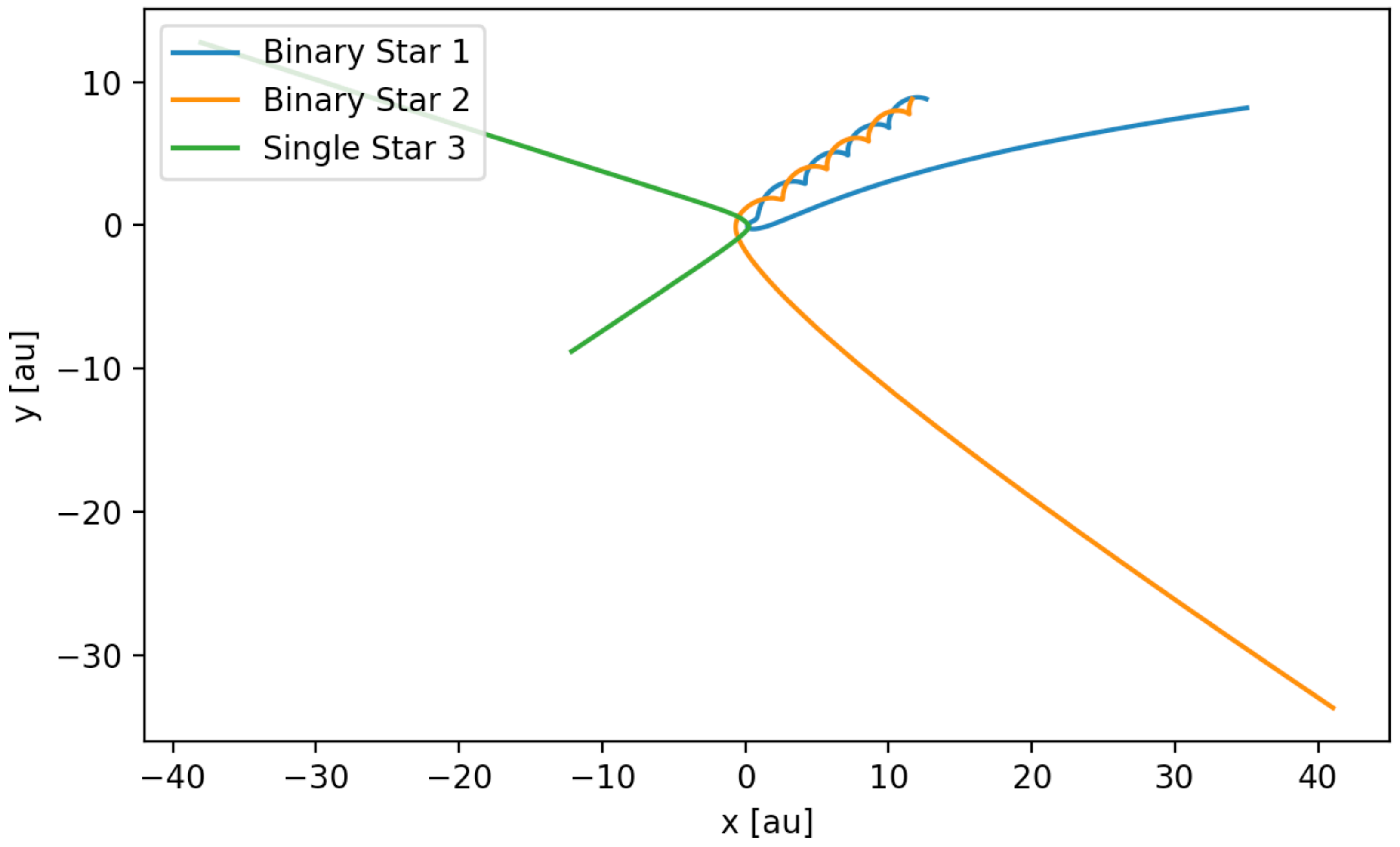
$m_3 = 1\ M_\odot$
$e_2 = 1.25$
$v_\infty = 30\ \text{km/s}$
$D = 30 \times a\ [\text{AU}]$

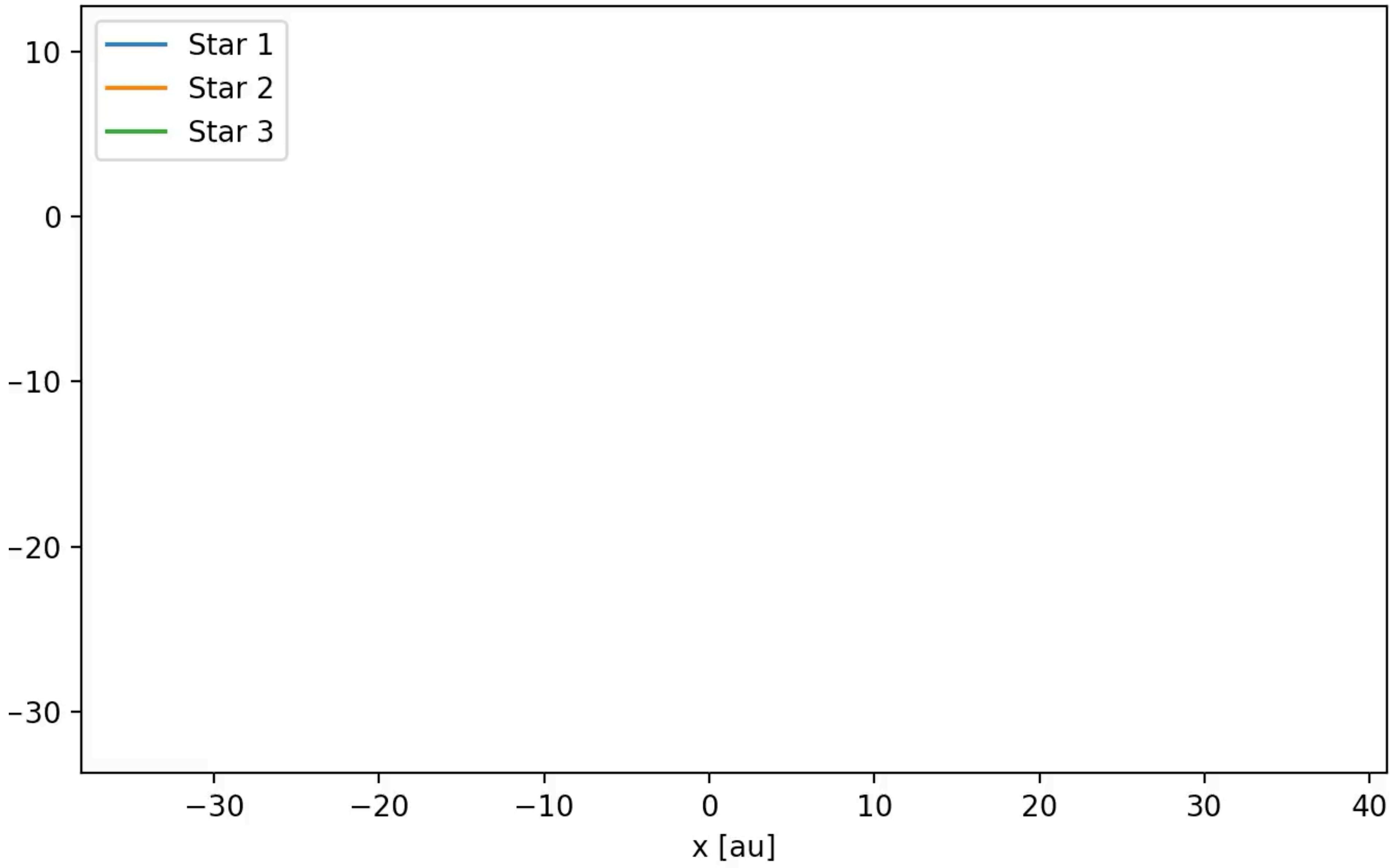
$$v_{\text{orb}} = \sqrt{\frac{G(m_1+m_2)}{a_{bin}}}$$

Final Integration Time = 20 yr
$E_{tot} = 0.13 \quad \Delta E = 1.1E-13$

$$v_c = \sqrt{\frac{Gm_1m_2(m_1+m_2+m_3)}{m_3(m_1+m_2)a}}$$

$v_c \approx 29.8\ \text{km/s}$





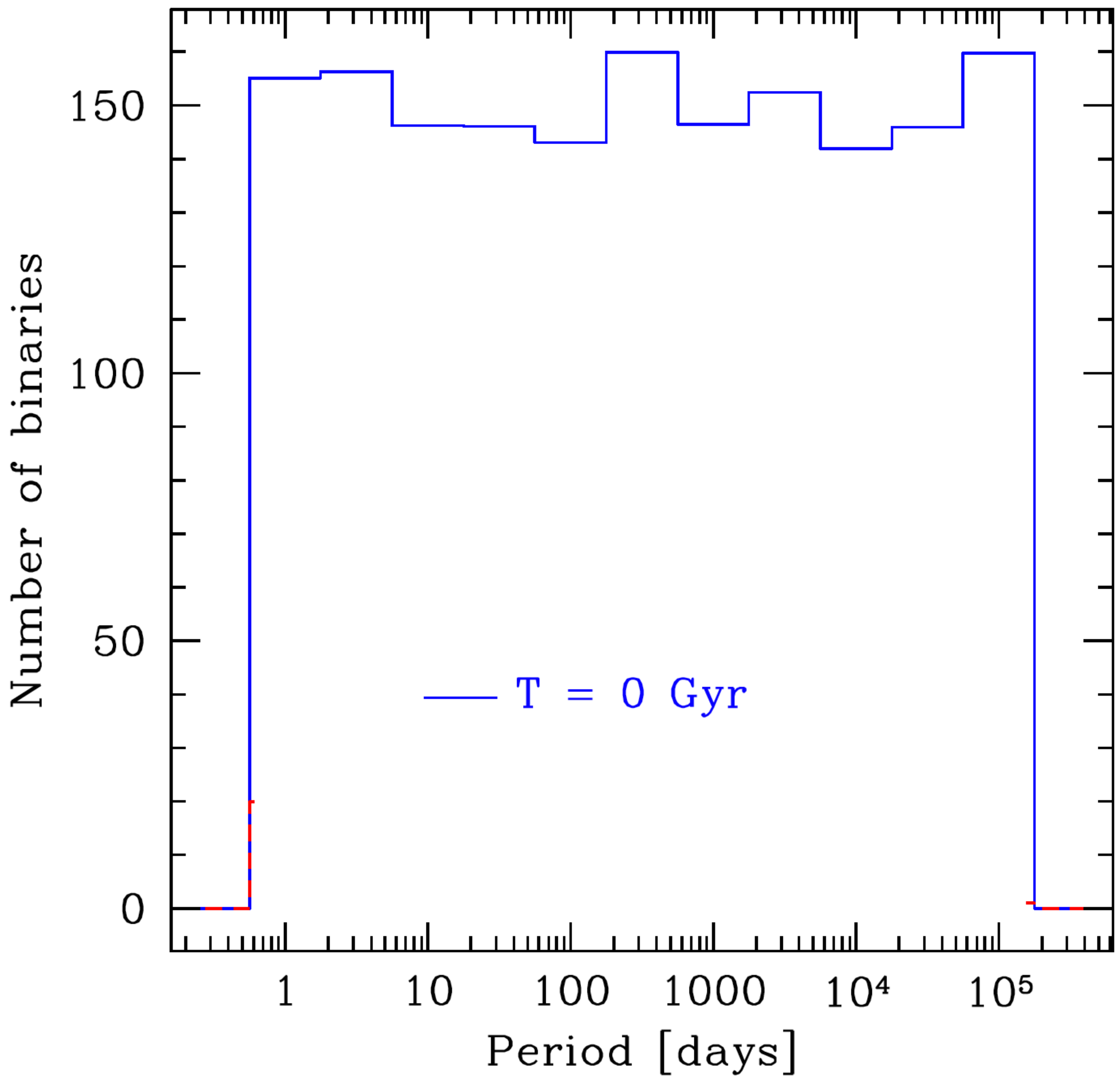
Simulations carried out using the *TSUNAMI* code (Trani et al. 2020; 2022, 2023; Hellström et al. 2022)

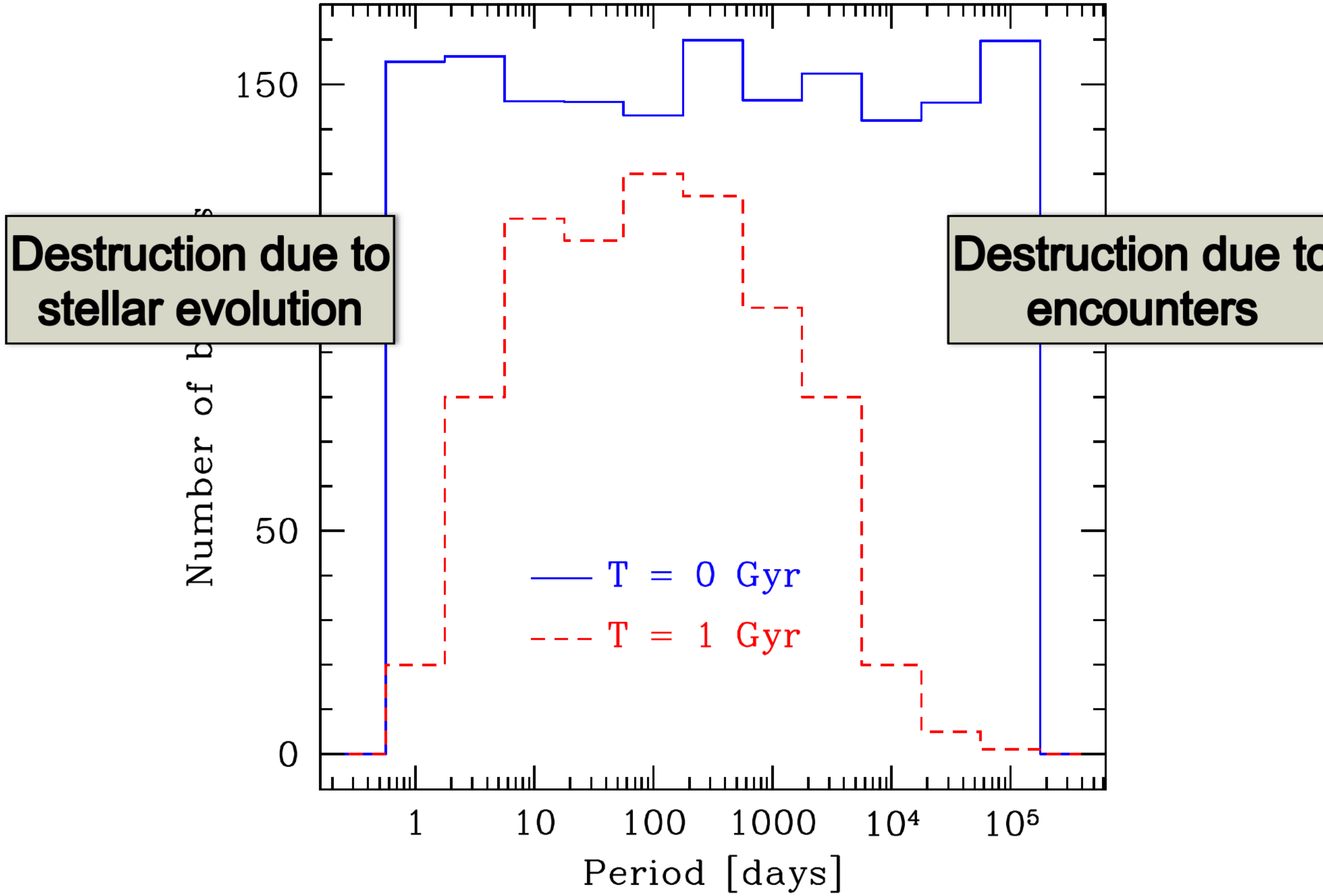
Outcome of 3-body interactions: will a binary gain or lose binding energy?

- Only in a statistical sense
- Hard binary: $\frac{Gm_1m_2}{2a} > \frac{1}{2}\langle m\rangle\sigma^2$ binding energy higher than the average kinetic energy of a star in the cluster
- Soft binary: $\frac{Gm_1m_2}{2a} < \frac{1}{2}\langle m\rangle\sigma^2$ binding energy lower than the average kinetic energy of a star in the cluster
- All types of interactions can occur, but on average, hard binaries (those bound more tightly than the average binding energy in the cluster) give away more energy than they absorb, and become ever more tightly bound (binary hardening/shrinking)
- Soft binaries more likely to absorb energy and become more loosely bound or unbound (binary disruption/ionization)

Heggie/Hill's (1975) Law: In 3-body encounters hard binaries become harder (increase E_b) and soft binaries tend to become softer (decrease E_b)

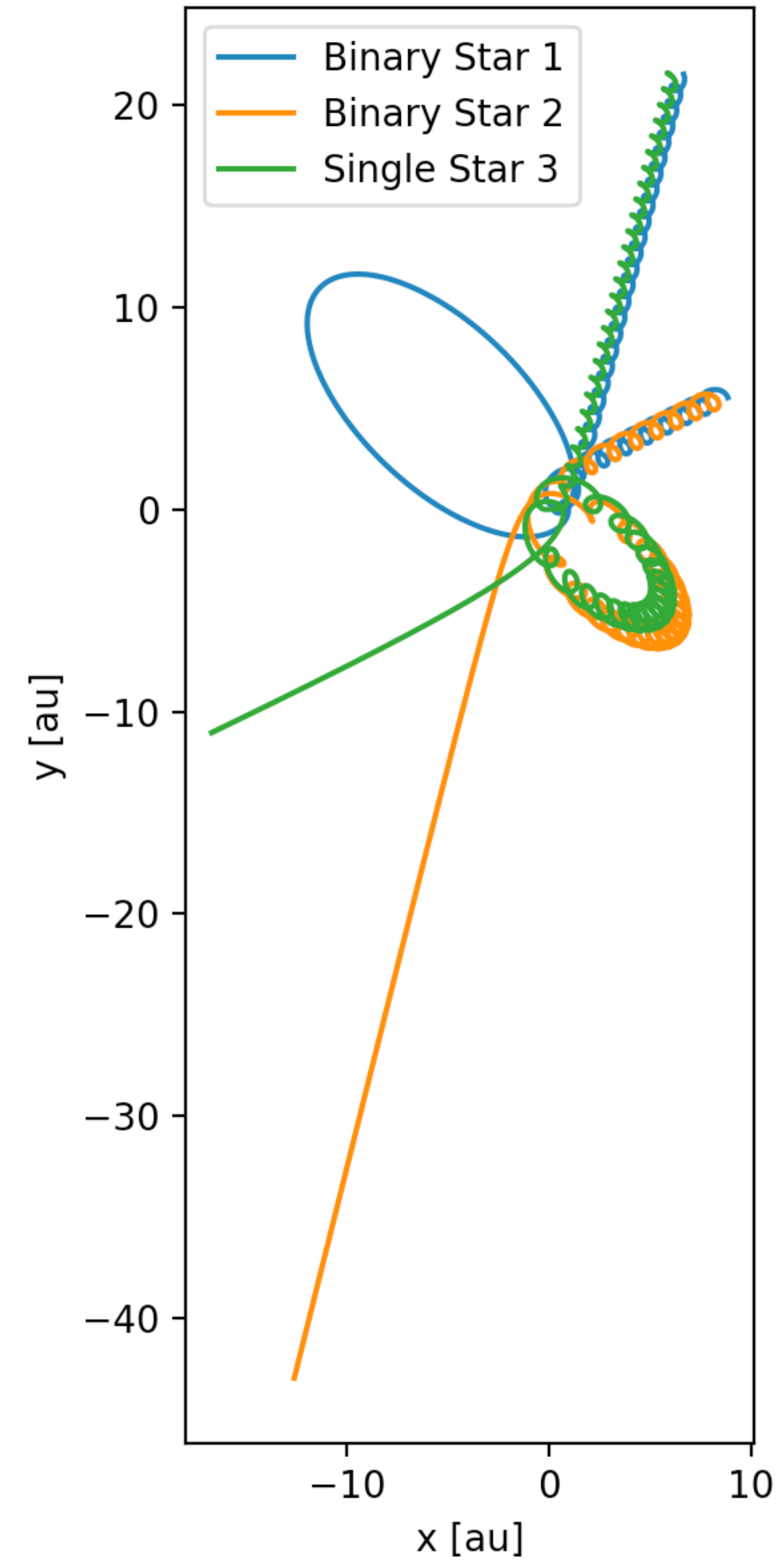
Evolution of the binary population in a cluster



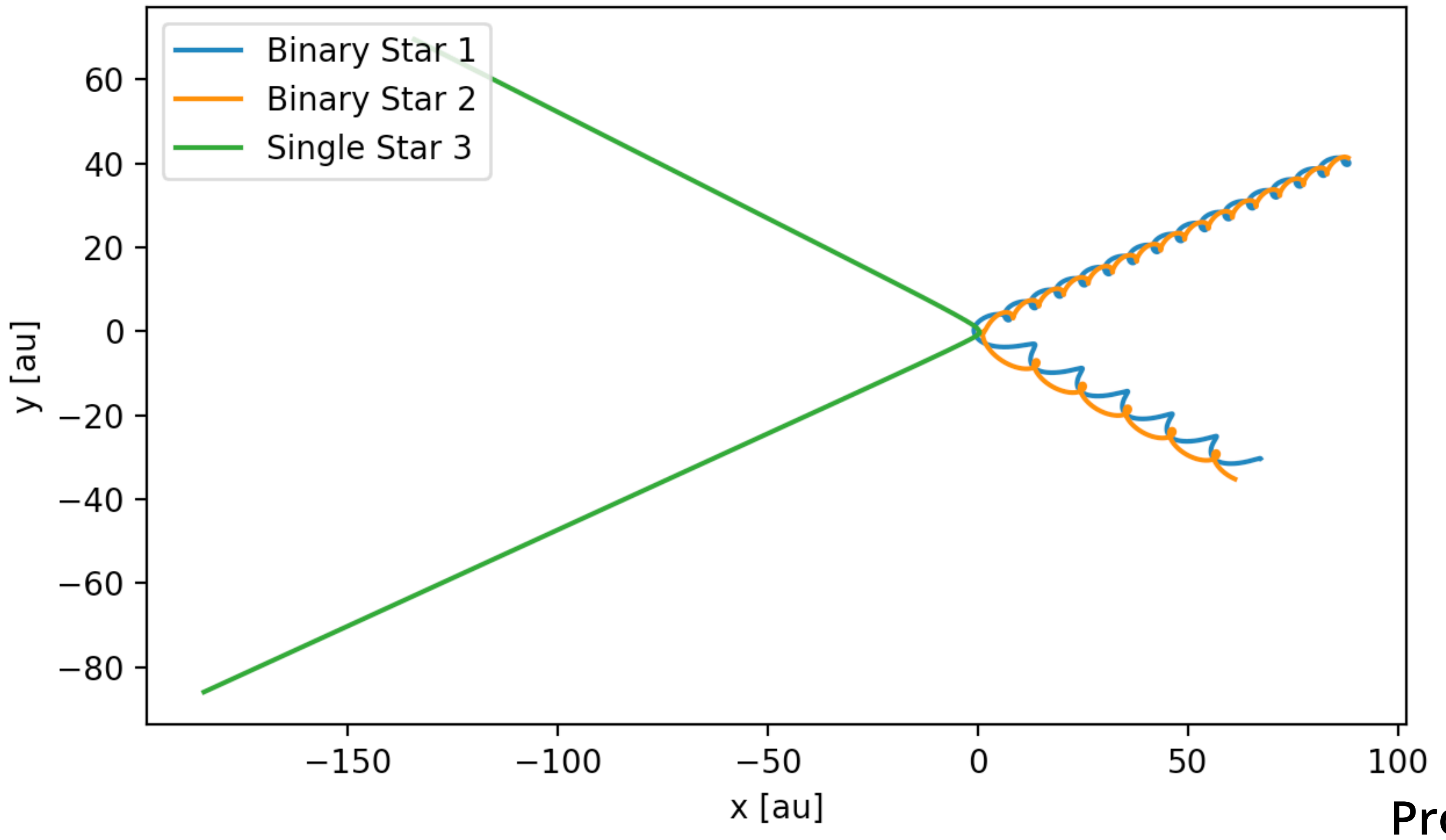


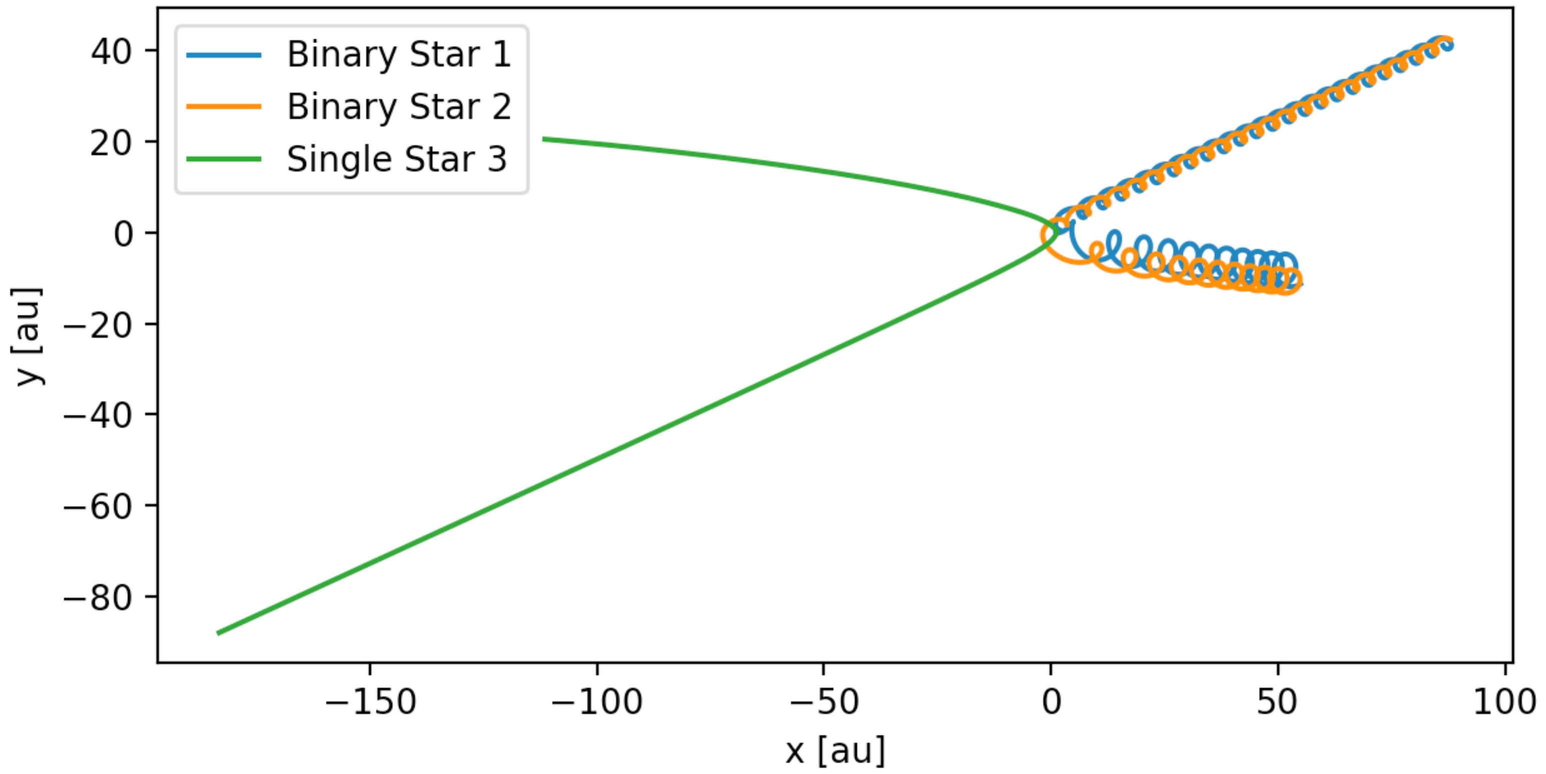
Credit: Holger Baumgardt

Some more simulated 3-body encounters

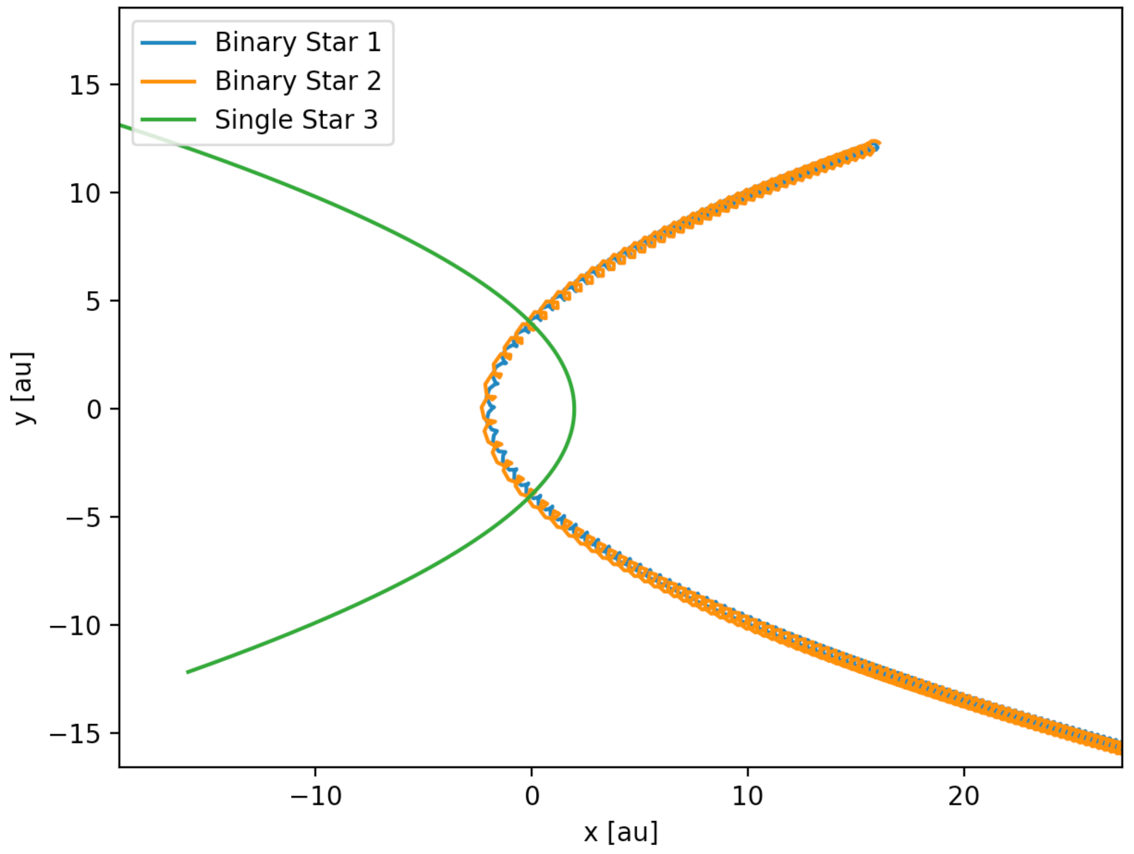


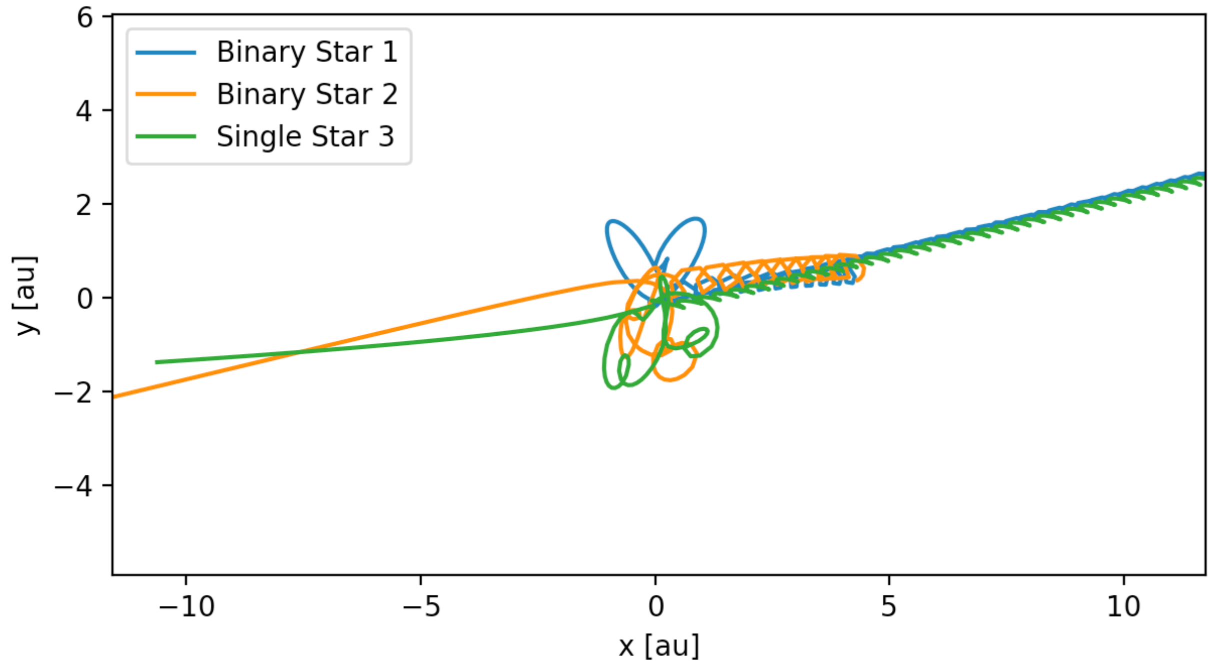
Resonant exchange



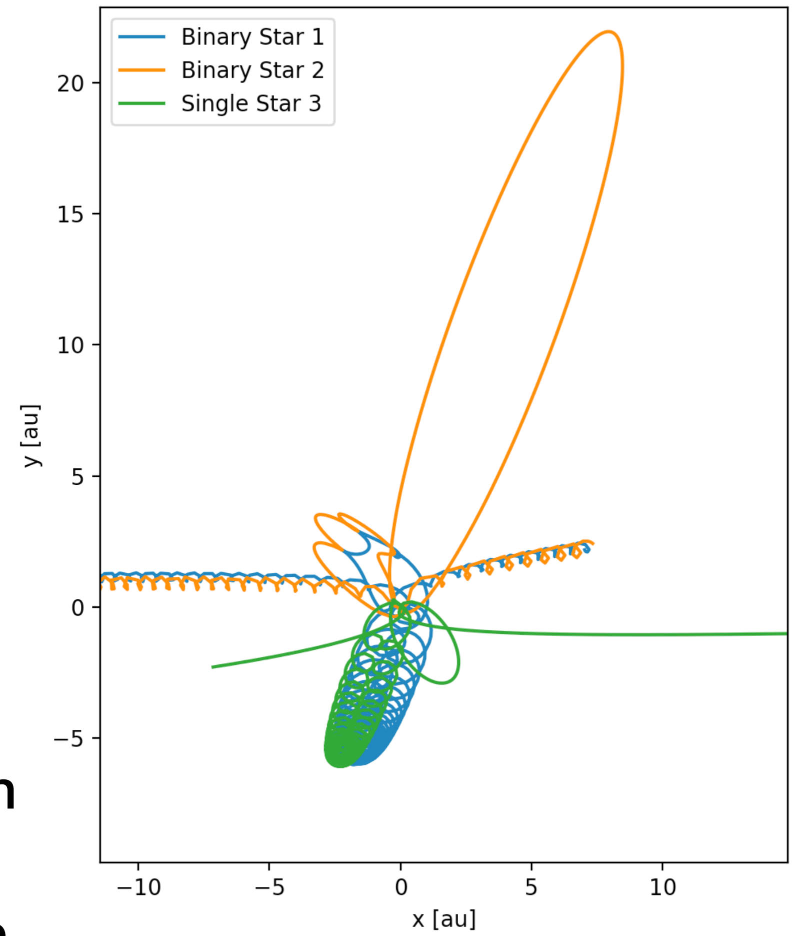


Prompt flyby leading to softening





Resonant interaction resulting in binary hardening/exchange

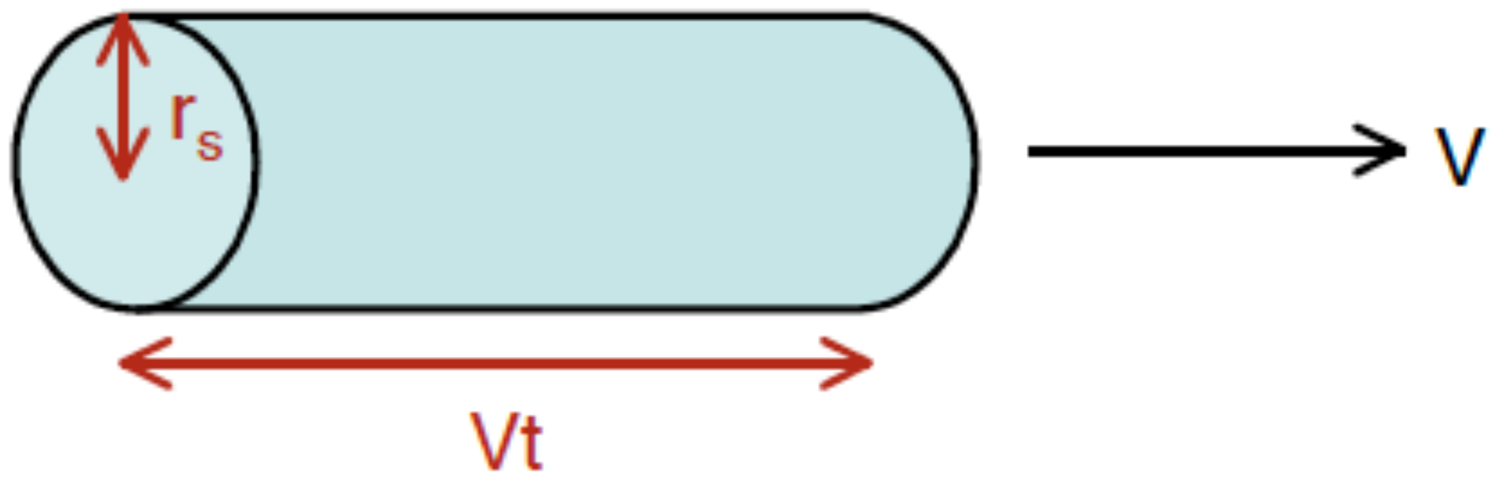


Simulations carried out using the *TSUNAMI* code (Trani et al. 2020; 2022, 2023; Hellström et al. 2022)

Cross section for 3-body encounters

- Compared to single stars, binaries have a geometric larger cross section!

$$\Sigma_\star = \pi R_*^2 \qquad \Sigma_b = \pi a^2$$



- The difference is even larger if we take a more realistic definition of cross section for 3-body encounters

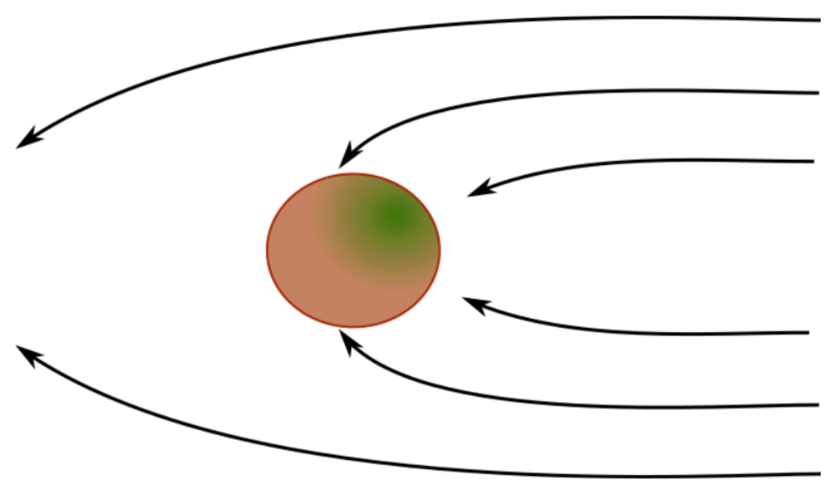
$$\Sigma = \pi b_{\max}^2$$

- $b_{\max}$ is the maximum impact parameter for a non-zero energy exchange between star and binary

Cross section for 3-body encounters

$$\Sigma = \pi b_{\max}^2$$

- $b_{\max}$ is the maximum impact parameter for a non-zero energy exchange between star and binary
- Gravitational focussing
  - In the 2-body frame: binary is significantly more massive than a single → the trajectory of the single is deflected by the binary when it approaches pericenter (p)
  - The relationship between impact parameter (b) and effective pericenter (p) can be derived the conservation of energy and angular momentum



$$p = \frac{Gm_T}{v_\infty^2}\left[\sqrt{1 + \left(\frac{v_\infty^2}{Gm_T}\right)^2 b^2} - 1\right]$$

$$m_T \equiv m_1 + m_2 + m_3$$

if $\frac{Gm_T}{v_\infty^2 b} >> 1 \Rightarrow p \sim b^2 \frac{v_\infty^2}{2Gm_T}$

Credit: Alice Quillen Notes on Hyperbolic Encounters: https://astro.pas.rochester.edu/~aquillen/ast233/lectures/lecture2.pdf

Cross section for 3-body encounters and interaction rate

- $b_{\max}$ can be expressed in terms of the pericenter distance to obtain a useful expression for the 3-body cross section

$$\Sigma = \pi \left(\frac{2Gm_T}{v_\infty^2} \right) p_{\max}$$

See: Michela Mapelli
Lecture notes on Collisional Dynamics
http://web.pd.astro.it/mapelli/2014colldyn3.pdf

- For a strong encounter, the cross section can be found setting the pericenter distance to for example: the binary semi-major axis

$$\Sigma = 2\pi G \frac{m_T a}{v_\infty^2}$$

- Interaction rate can be computed from the cross-section

$$R = \frac{dN}{dt} = n \Sigma v_\infty$$

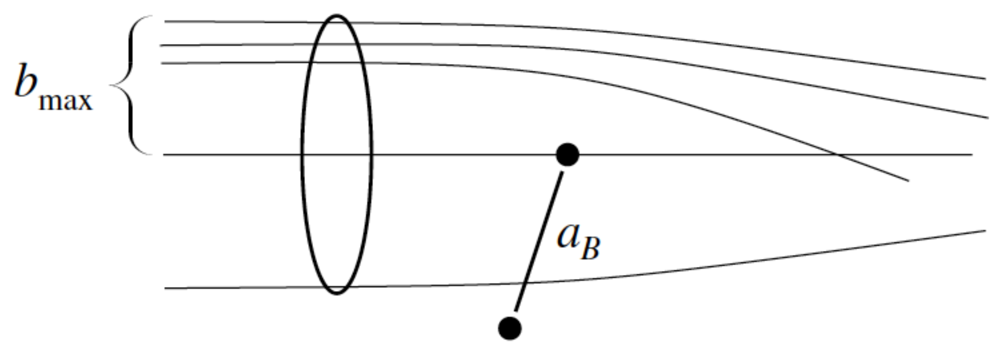
where n is the local density of stars

$$R = 2\pi G \frac{m_T n a}{v_\infty}$$

Interaction rate and timescale for 3-body encounters

- Interaction rate for strong encounters depends on:
  - on the semi-major axis of the binary (wider binaries have a larger cross section)
  - on the total mass of the interacting objects (more massive objects interact more)
  - on the local density: denser environments have higher interaction rates
  - on the local velocity field: systems with smaller velocity dispersion have higher interaction rate ($v_\infty \sim \sigma$)

$$R = 2\pi G \frac{m_T n a}{v_\infty}$$



- Interaction timescale for 3-body encounters

$$t_{3b} = \frac{\sigma}{2\pi G m_T n a}$$

Energy exchanges in 3-body interactions

- Hills (1983) defined the post-encounter energy parameter

$$\xi = \frac{m_1 + m_2}{m_3} \frac{\langle \Delta E_b \rangle}{E_b}$$

$< \Delta E_b >$ is the average binding energy variation per encounter

- From numerical simulations, it has been shown that:
  - ξ depends strongly on impact parameter (a factor of >200 between b=0 and b=20a)
  - ξ depends slightly on binary eccentricity ($\xi \sim 2$ if $e = 0$, $\xi \sim 6$ if $e = 0.99$)
  - ξ depends on binary mass ratio ($\xi \sim 2$ if $m_1 = m_2$, $\xi \sim 4$ if $m_1/m_2 = 10 - 30$)
- Averaging over impact parameters: $\xi = 0.2 - 1$

$$\langle \Delta E_b \rangle = \xi \frac{m_3}{m_1 + m_2} \frac{G m_1 m_2}{2a}$$

Hardening rate in 3-body encounters

- Rate of change of binding energy for a hard binary

$$R = 2\pi G \frac{m_T n a}{\sigma}$$

$$\frac{dE_b}{dt} = \left\langle \Delta E_b \right\rangle \frac{dN}{dt} = \xi \frac{m_3}{m_1 + m_2} E_b \frac{dN}{dt}$$

$\frac{dN}{dt}$ is the rate of encounters

$$t_{3b} = \frac{\sigma}{2\pi G m_T n a}$$

$$\frac{dE_b}{dt} = \xi \frac{\langle m \rangle}{m_1 + m_2} E_b \frac{2\pi G \left(m_1 + m_2\right) n a}{\sigma}$$

$\langle m \rangle$ is the average mass of stars and

$\langle m \rangle n$ is the local mass density of stars: ρ

$$E_b = \frac{G m_1 m_2}{2a}$$

$$\frac{dE_b}{dt} = \pi \xi G^2 \frac{\rho}{\sigma} m_1 m_2$$

- Binary hardening rate depends on the binary mass and the local cluster properties (density and velocity dispersion)
- Hardening is constant in time if cluster properties are not changing

Hardening rate in 3-body encounters

- Expressing in terms of E_b (and assuming no exchanges) $$\frac{dE_b}{dt} = \pi\xi G^2 \frac{\rho}{\sigma} m_1 m_2$$

$$\frac{d}{dt}\left(\frac{1}{a}\right) = \frac{2}{Gm_1m_2}\frac{dE_b}{dt} = 2\pi G\xi\frac{\rho}{\sigma}$$

- The time average evolution of the semi-major axis of a hard binary

$$\frac{da}{dt} = -2\pi G\xi\frac{\rho}{\sigma}a^2$$

- Rate of binary hardening depends on the square of the semi-major axis
- The smaller the a, the more difficult is for the binary to shrink further (because cross section becomes smaller).
- Timescale for a binary to harden by 3-body encounters: $$t_h = \left|\frac{a}{\dot{a}}\right| = \frac{1}{2\pi G\xi}\frac{\sigma}{\rho}\frac{1}{a}$$

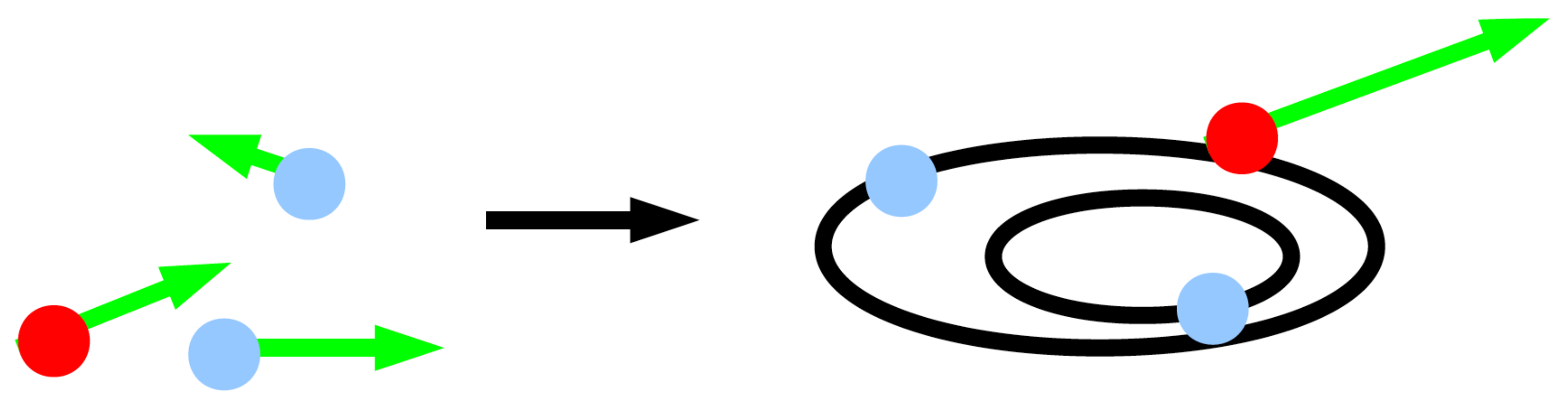
Formation of binary systems

- Can be primordial
  - We know from observations that binary fraction is high in the field (~40-50%)
- Dynamically formed in 3-body interactions
  - Need high local stellar density (e.g., during core-collapse)
- Tidal captures

(Fabian, Pringle & Rees 1975; Press & Teukolsky 1977)

$$r_t = r_2 \left(\frac{2m_1}{m_2} \right)^{1/3}$$

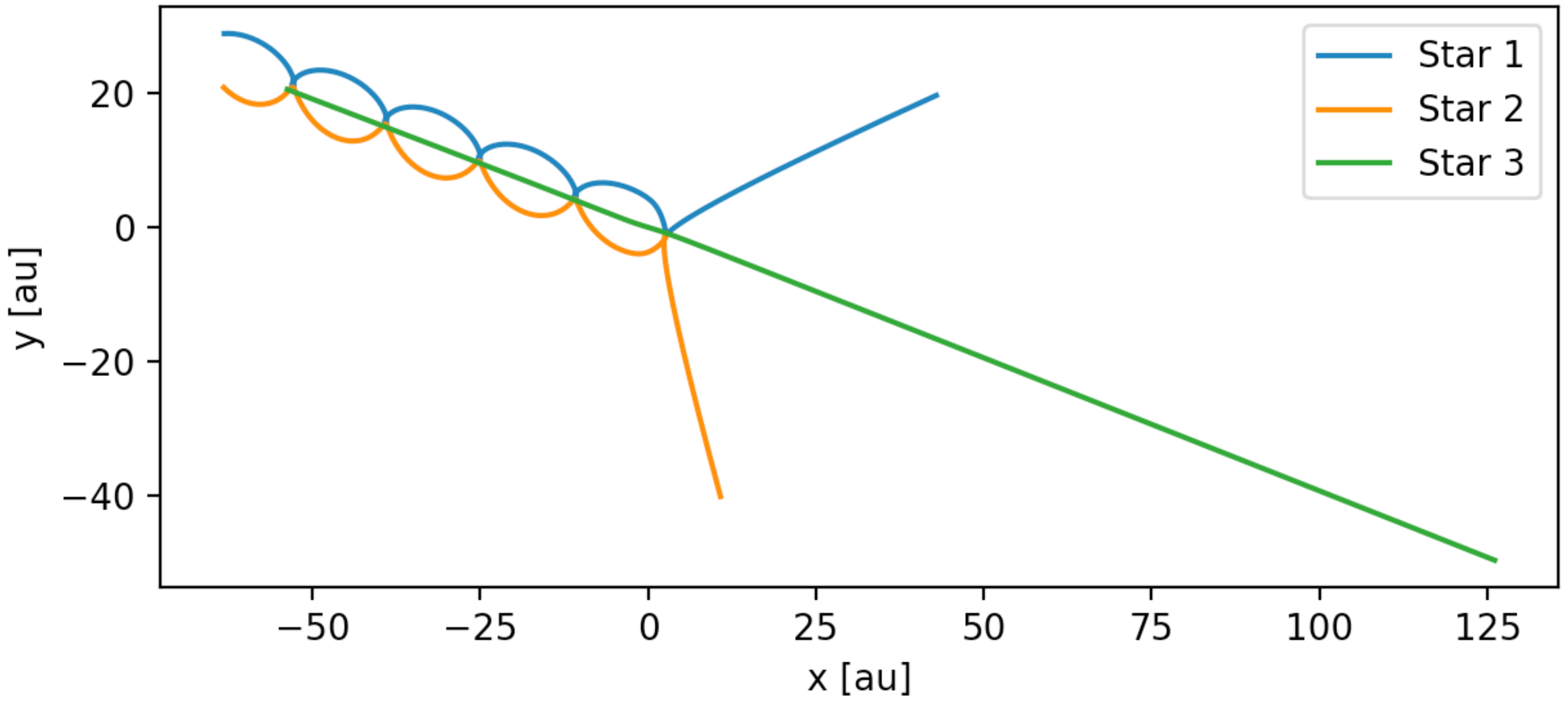
  - Stars have to pass close to each other: energy dissipation produces bound star
  - Requires high stellar density (e.g., during core-collapse)



Credit: Michela Mapelli

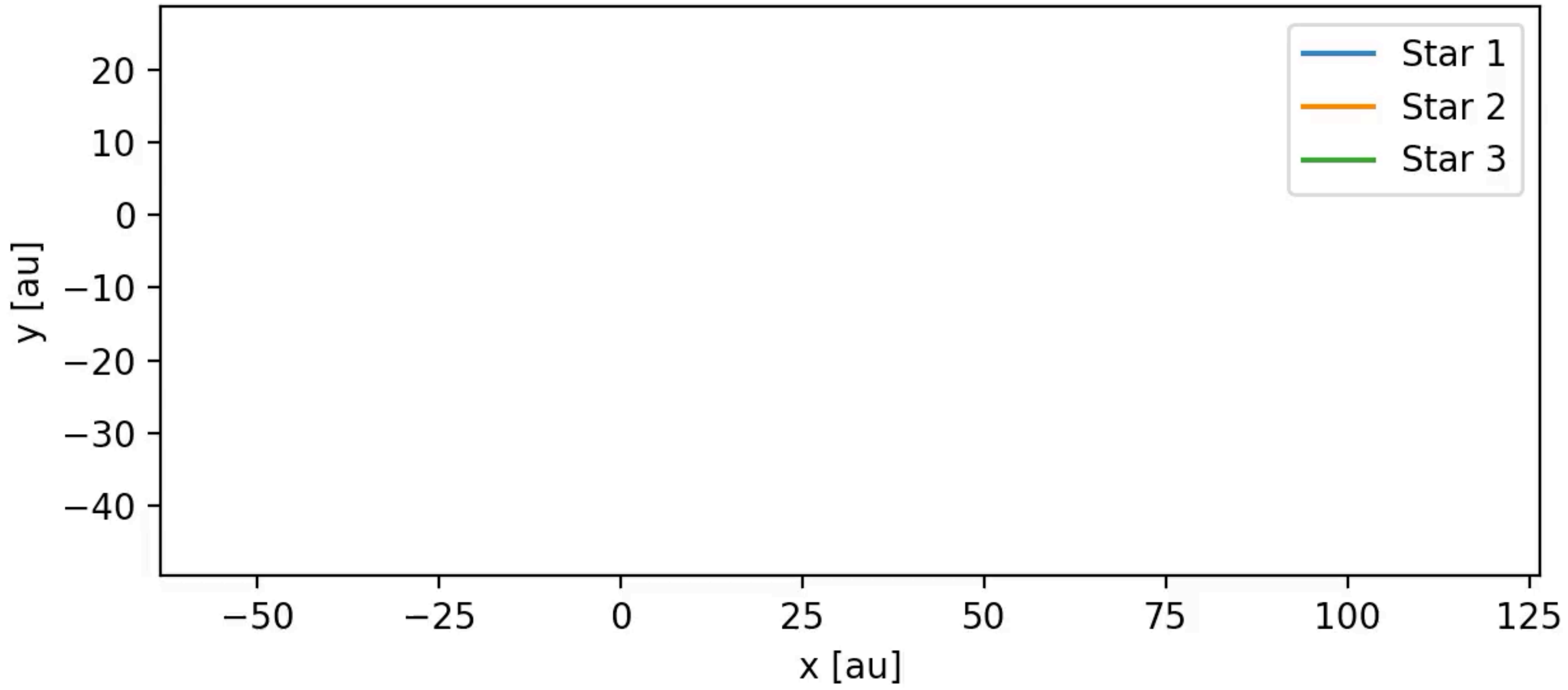
Formation of binary systems

$m_1 = 1\ M_\odot \quad m_2 = 1\ M_\odot \quad m_3 = 1\ M_\odot$



$m_1 = 1\ M_\odot \quad a = 4.75\ \mathrm{AU}$

$m_2 = 1\ M_\odot \quad e = 0.98$



Formation of binary systems

- Dynamically formed in 3-body interactions

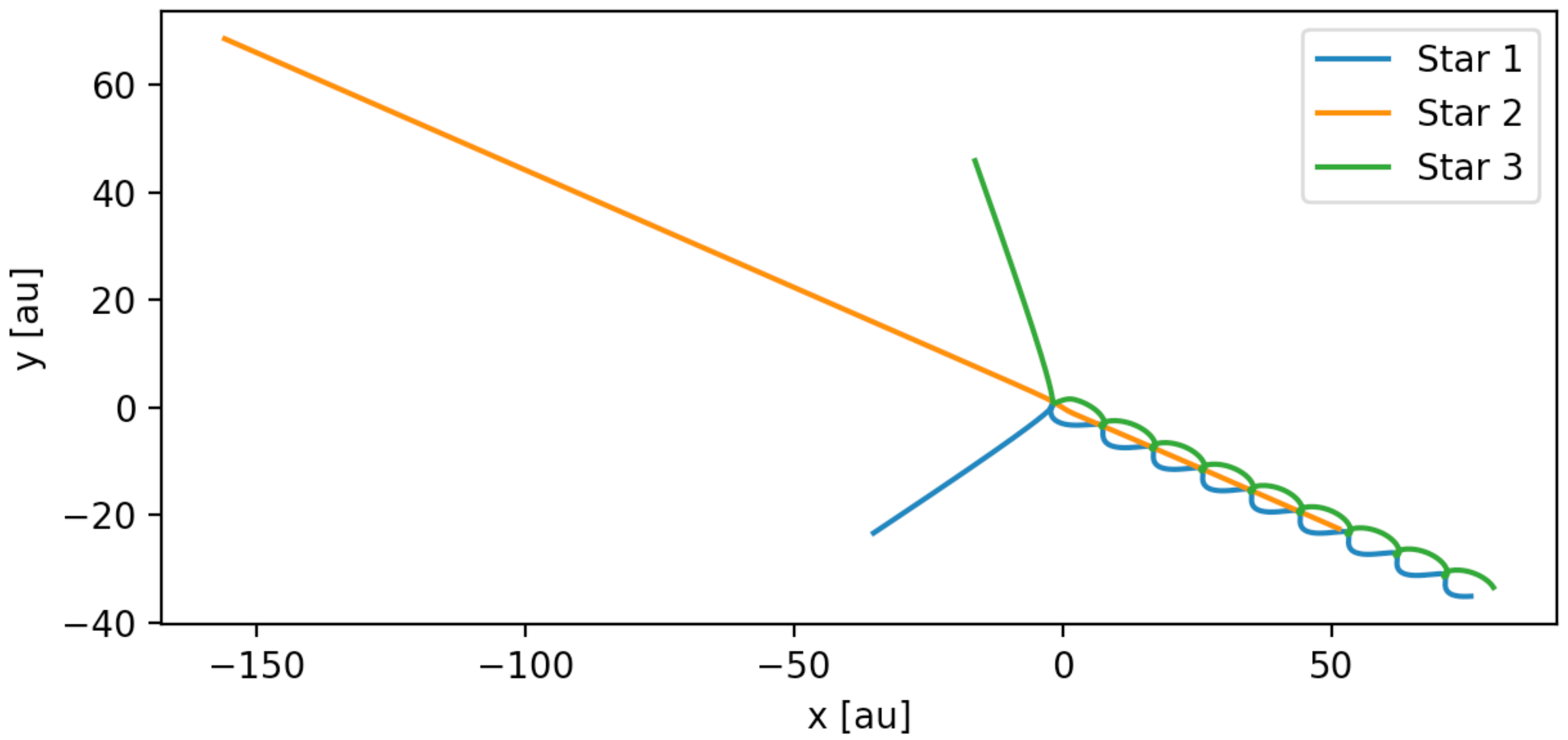
$m_1 = 1\ M_\odot$ $a = 3.21\ \mathrm{AU}$

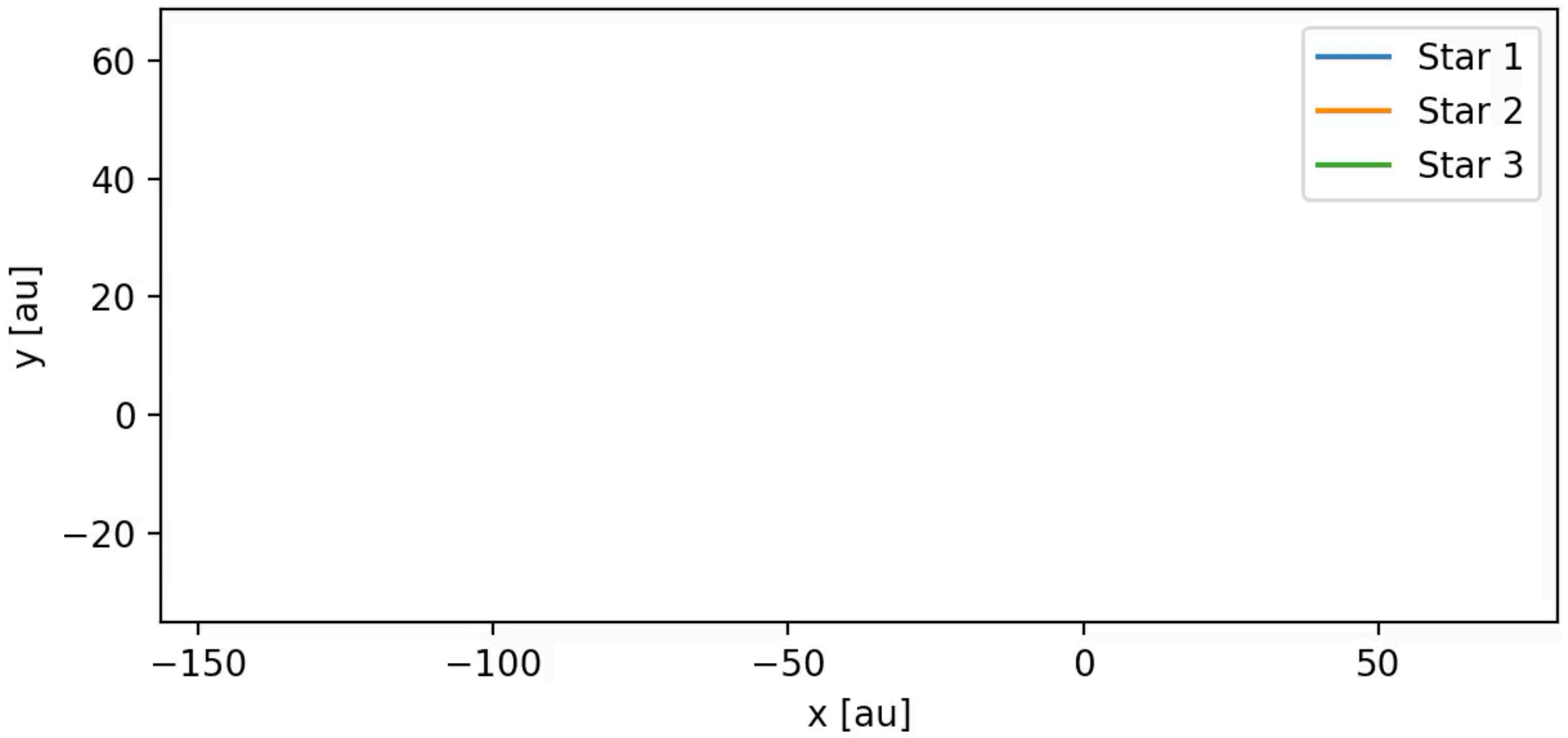
$m_3 = 1\ M_\odot$ $e = 0.88$

$m_1 = 1\ M_\odot$

$m_2 = 1\ M_\odot$

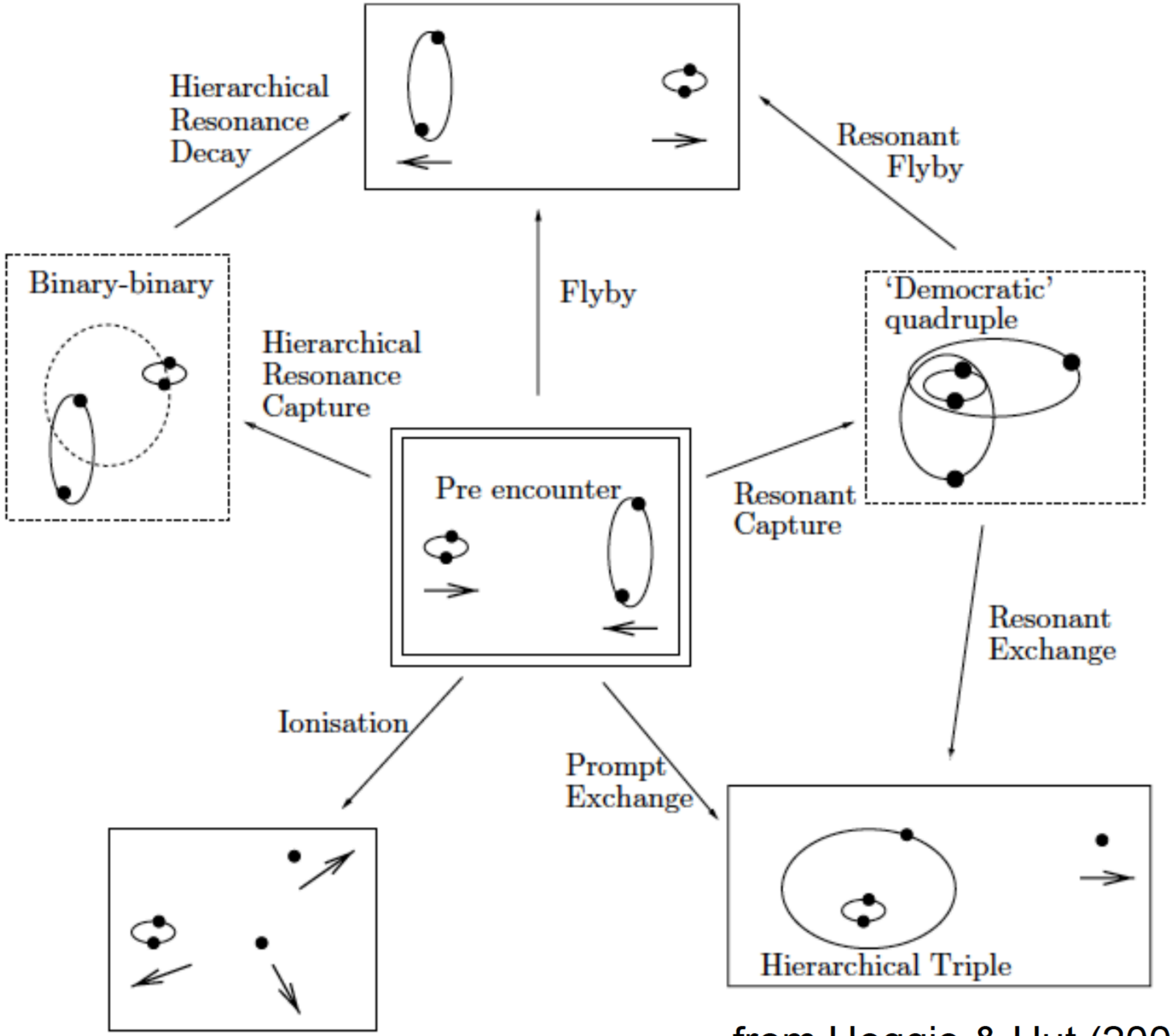
$m_3 = 1\ M_\odot$





Binary - Binary interactions

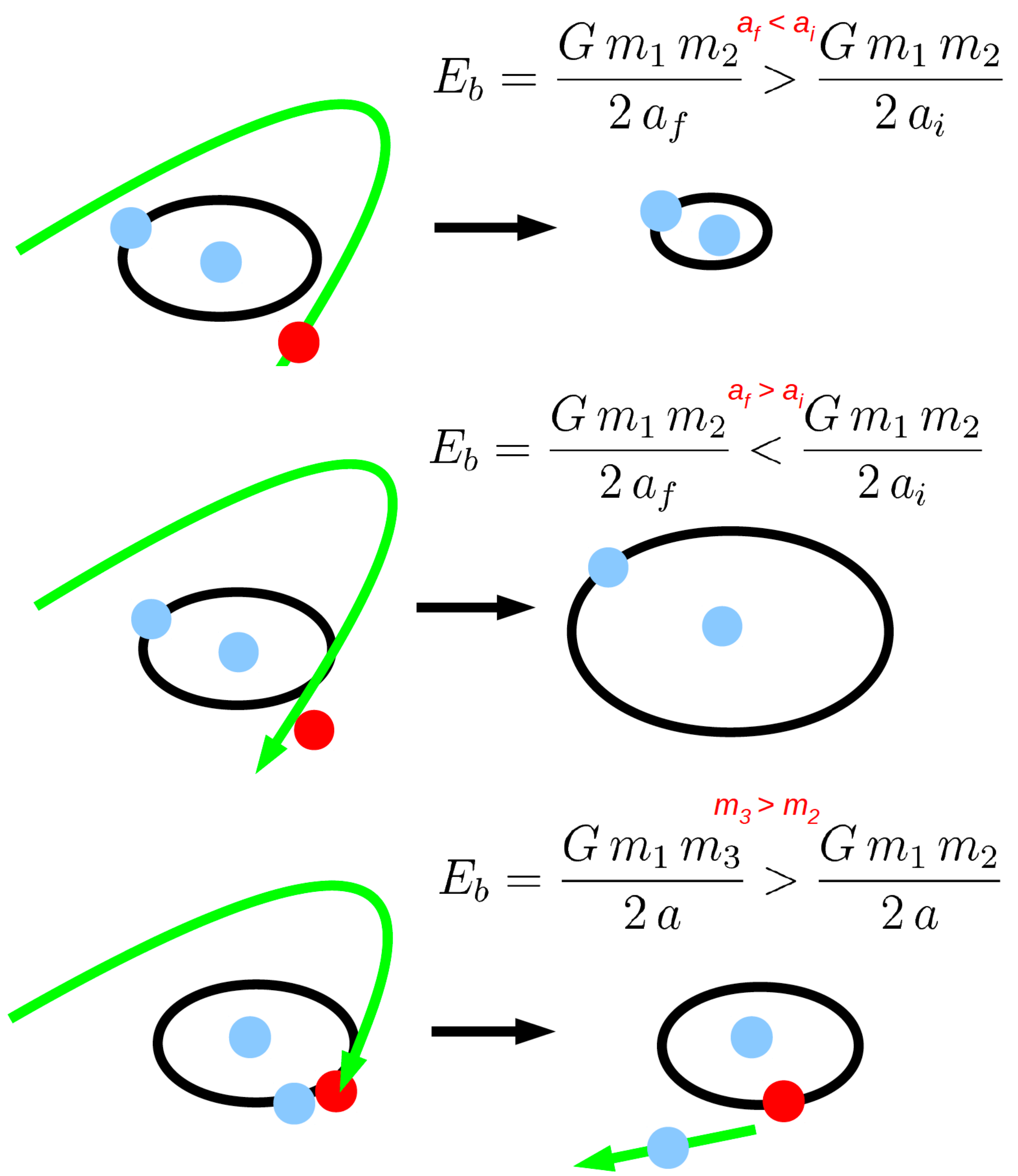
Everything we learned still holds, but higher level of complexity



from Heggie & Hut (2003)

Summary: Internal heating from binaries

- **Binary Interactions (3-body interactions)**
  - For a bound binary system: $E_{int} = -\dfrac{G\, m_1\, m_2}{2\, a} = -E_b$
  - Binary shrinking/hardening:
    - Single star extracts internal energy of the binary (stars final kinetic energy is higher than initial) - Flyby
    - Binary softening or Ionization:
      - Single star transfers kinetic to the binary
    - Exchange interaction:
      - Single star can replace one of the binary companions



Credit: Michela Mapelli
Lecture notes on Collisional Dynamics

Heggie/Hill's (1975) Law: In 3-body encounters hard binaries become harder and soft binaries tend to become softer

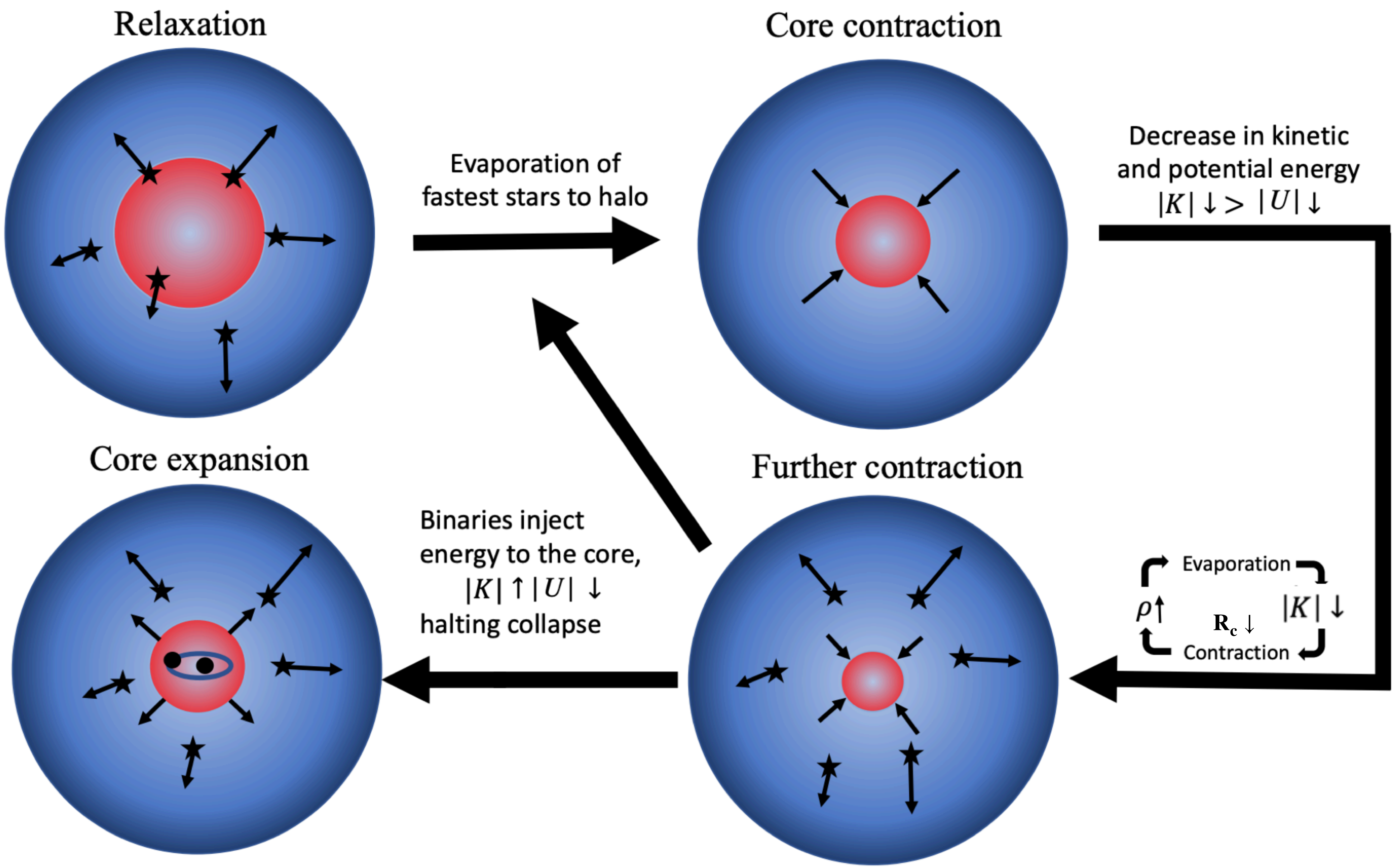
$$\frac{Gm_1m_2}{2a} > \frac{1}{2}\langle m\rangle\sigma^2$$

Hard Binaries

$$\frac{Gm_1m_2}{2a} < \frac{1}{2}\langle m\rangle\sigma^2$$

Soft Binaries

Summary: counteracting core collapse



- An external energy source (K_{ext}) is need to counteract core collapse
- 3-body encounters: Energy extracted from binaries $|U|$ and increases K
- Mass loss by stellar winds also decreases $|U|$
- Binaries harden by further encounters and provide an energy source for the core.
- If a binary is ejected or merged, the core can re-collapse until a new binary is formed.

Key takeaways

- Stellar/binary evolution has important consequences for cluster evolution
  - Significant mass loss from the fast evolution (~10-100 Myr) of massive stars
    - Influences the structure of the cluster and its radius
  - Binary evolution is important: influences separation/semi-major axis of binaries and their fraction
    - Many uncertainties in physical processes
- Binaries can generate energy by exchanging their binding energy with cluster stars
  - Binaries generate energy in the cluster which is then dissipated via relaxation:
  - Main source of energy in the cluster
  - Encounters with stars or other binaries can lead to a variety of different outcomes
    - Can lead to the formation of close binary systems, gravitational wave sources (more in future lectures)