Evolution of Star Clusters - Basic Considerations and Simple Models
Lecture V

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Star Cluster Dynamics and Evolution

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General Picture of Evolution

A star cluster evolves thanks to the relaxation process - uncorrelated, very distant gravitational interactions of stars with all other stars in the system. Relaxation alters the energy E and angular momentum A of each star. The time scale of the system evolution is proportional to the half-mass relaxation time: $t_{rh} = 0.138 N^{1/2} R_h^{3/2} / (m^{1/2} G^{1/2} ln(\gamma N))$ System regains equilibrium because Phase Mixing and Violent Relaxation on the crossing time scale: $t_{cr} \sim t_{rh} \ln(N)/N$

Evolution proceeds through a sequence of slowly changing equilibrium states described by viral equilibrium: E = T + W, where E is system binding energy and T and W are system kinetic and potential energies, respectively.

What will be the evolution of the system when it loses mass $\Delta M < 0$ or kinetic energy $\Delta T < 0$?

$R_n = R(1 - \frac{\Delta M}{M})$ – R increasing! $R_n = R(1 - 2\frac{\Delta T}{W})$ – R decreasing!

$V_n^2 = V^2(1 + 2\frac{\Delta M}{M})$ – V decreasing! $V_n^2 = V^2(1 - \frac{\Delta T}{T})$ – V increasing!

NEGATIVE HEAT CAPACITY

Gravothermal Catastrophe

We know from the previous Lectures that kinetic energy of the self-gravitating stellar system can be computed in similar way as for gas: $K = 0.5Nm < \sigma^2 >= 1.5NkT$ where $\sigma^2 = 3kT/m$.
So the specific heat capacity is equal: $dE/dT = c_v = -1.5kN$

Let's consider a gravitating isothermal gas confined by a sphere of radius, r_e, just less than $r_A = 0.335GM^2/(-E)$ (critical radius) and adiabatically expand the sphere. Work is done by the gas so E becomes more negative and r_A contract, $r_e > r_A$. The central parts are held in primarily by gravity so they expand less than the outer parts. Thus the adiabatic fall in temperature of the outer parts due to their expansion will initially be greater than the temperature fall of the inner parts. So a temperature gradient develops. The central parts contract and get hotter while the outer parts held in by the sphere behave like a normal gas so get hotter too. Do the outer parts get hotter faster than the inner one? Clearly if the outer parts have too great a positive heat capacity they will not respond enough and the inner parts will run away to ever higher temperatures losing more and more heat to the sluggishly responding outer parts. This is Antonov's gravothermal catastrophe.

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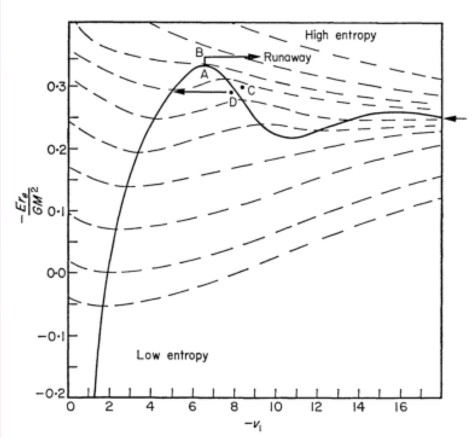
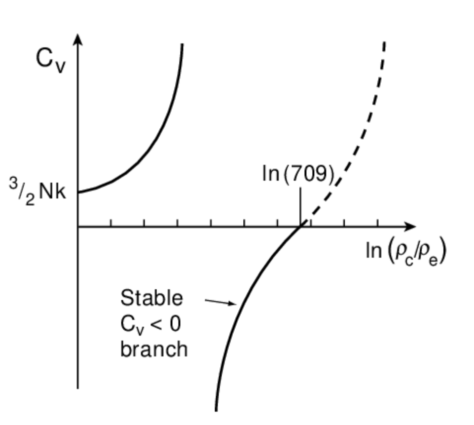
Gravothermal Catastrophe

Antonov 1966

Lynden-Bell and Wood (1968)



Lynden-Bell (1968)



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Gravothermal Catastrophe

- Two negative c_v systems in thermal contact do not attain thermal equilibrium - one gets hotter and hotter by losing energy, the other gets colder by gaining energy
- A negative c_v system can not achieve thermal equilibrium with a large heat bath. Any fluctuation that, e.g., makes it temporary energy too high will make its temporary temperature too low and the heat flow into it will drive it to even lower temperatures and higher energies.
- A negative c_v system can achieve a stable equilibrium in contact with a positive c_v system provided that their combined heat capacity is negative.
  Assume that the negative c_v system Minus is initially a little hotter than the positive c_v system Plus. Then heat will flow from Minus to Plus. On losing heat Minus will get hotter but on gaining that heat Plus will also get hotter. However, because Plus has a smaller absolute c_v value its temperature is more responsive to heat gain than Minus is to heat loss. Thus Plus will gain temperature faster than Minus and a thermal equilibrium will be attained with both.
  This stability is lost as soon as positive c_v has the same absolute c_v value as negative c_v; i.e., when their combined heat capacity reaches zero from below.

General Picture of Evolution

I would like to spend the next several minutes presenting a global picture of the cluster's evolution, a picture that will be derived from the knowledge gained so far on cluster models, relaxation and gravitational collapse, namely:

- Star systems collapse if the density gradient between the center and characteristic radius is greater than about **709**
- Star systems are close to isothermal - the velocity dispersion gradient is small in a significant part of the cluster, which contains a significant part of its mass
- According to the virial theorem $r_h \sim GM^2/|E|$
- The relaxation process does not change the total energy of the cluster, but it does change its redistribution among the stars
- Some stars will gain enough energy end escape from the cluster. The escape rate is constant per half-mass relaxation time end it is equal to **0.0074** for the isothermal model
- So the cluster dissolution time is proportional to the half-mass relaxation time -

$$t - t_{coll} \sim \frac{M^{1/2} r_h^{3/2}}{G^{1/2} m ln(\Lambda)}$$

General Picture of Evolution - Core Collapse

In 1975, Henon provided a theoretical argument that made it possible to break the deadlock in theoretical models of cluster evolution after its "collapse." The argument goes as follows: the rate of energy generated in the cluster's central regions (regardless of the physical process) must match the rate of energy flow through the cluster's half-mass radius

This argument will be used a lot in the rest of the lecture

We know that the rate of mass loss from star clusters is very small, and that the relaxation process does not change the total energy of the cluster, but only redistributes energy and angular momentum among the stars in the system. Then according the virial theorem the cluster characteristic radius should be constant ($r_h \propto GM^2/|E|$)

r_h = cost during the cluster collapse

Lynden-Bell 1968

Let's assume that the system is undergoing the gravothermal catastrophe with the central part contracting and getting hotter while the outer part could not raise its temperature fast enough to keep up. Since the center is now denser the critical point for the gravothermal instability is now inside the system rather than at its edge. Thus this point moves inwards through the mass. As the density increases, the timescale of heat flow becomes shorter and the outer system regions are too slow to respond other than adiabatically. Since the same process is occurring at ever smaller scales we expect a similarity solution with the density of the form:

$$\varrho(r,t) = \varrho_c(t)\varrho_\star(r_\star) \qquad where \qquad r_\star = \frac{r}{r_c(t)} \tag{1}$$

Since the halo is so slow that it is left behind in the every quickening evolution of the center we can put $\partial\varrho/\partial t = 0$ for large distance

General Picture of Evolution - Core Collapse

Differentiating over time Eq.(1) we get:

$$\dot{\varrho}_c \varrho_\star - \frac{\dot{r}_c}{r_c} \varrho_c \varrho'_\star = 0$$

So for large r after separating t and $r_\star$ me have

$$\frac{r_\star \varrho'_\star}{\varrho_\star} = \frac{r_c \dot{\varrho}_c}{\dot{r}_c \varrho_c} = -\alpha$$

Thus $\varrho_\star = A r_\star^{-\alpha}$ at large $r_\star$ and $\varrho_c \propto r_c^{-\alpha}$ for all t

$$\frac{\dot{\varrho}_c}{\varrho_c} = \frac{1}{t_{rc}} \qquad where \qquad t_{rc}^{-1} = \frac{\varrho_c}{v_c^3}(4\pi G^2 m \ln(N)) \qquad v_c^2 \propto G \frac{4}{3} \frac{\pi \varrho_c r_c^3}{r_c} \propto G \varrho_c^{1-\frac{2}{\alpha}} \qquad v_c^2 \propto r_c^{(2-\alpha)}$$

$$\frac{\dot{\varrho}_c}{\varrho_c} = \varrho_c^{\frac{3}{\alpha} - \frac{1}{2}}$$

General Picture of Evolution - Core Collapse

After integrating over time equations for ϱ_c and v_c we get:

$$\varrho_c \propto |t_{coll} - t|^{\frac{2\alpha}{6-\alpha}} \qquad r_c \propto |t_{coll} - t|^{\frac{2}{6-\alpha}}$$

$$M_c \propto \varrho_c r_c^3 \propto |t_{coll} - t|^{\frac{2(3-\alpha)}{6-\alpha}}$$

$$v_c^2 \propto \frac{GM}{r_c} \propto |t_{coll} - t|^{\frac{(4-2\alpha)}{6-\alpha}}$$

The collapse of the core is driven by a negative temperature gradient it follows that $\alpha > 2$

The energy within the core has to be finite during collapse - $\varrho_c r_c^3 v_c^2 \propto r_c^{5-2\alpha}$ so $2 < \alpha < 2.5$

Numerical simulations give: $\alpha \cong 2.21 - 2.23$

Credits: Cohn 1980

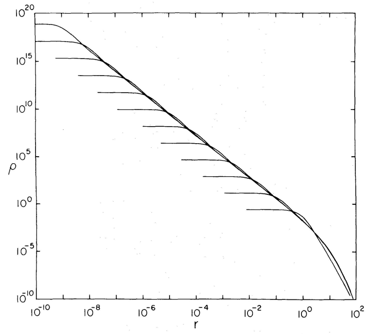

FIG. 1.—Evolution of the cluster density profile. The central density increases in time while the radius of the central core decreases. The curves correspond, in order of increasing central density, to the epochs specified by $x_0 = 6.00, 9.05, 10.58, 11.69, 12.46, 12.97, 13.31, 13.53, 13.67, 13.82$, and 13.85, where $x_0 \equiv 3\phi(0)/v_m^2(0)$. (The time dependence of the scaled escape energy x_0 is shown in Fig. 4) Note that the time interval between the first two epochs is about 80% of the time to complete core collapse.

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General Picture of Evolution - Core Expansion

Of course, a gravitational collapse cannot lead to infinite densities in an actual star system. Much sooner, processes related to strong gravitational interactions between stars and to the finite size of stars will turn on. These processes have so far been neglected in the description of cluster evolution. Let's first consider the process of formation of binary systems

One note of caution before we move on. Binary systems in dynamic interactions generate extra kinetic energy at the expense of the binary system's binding energy. This is "external" energy to the cluster's binding energy. A decrease in the cluster's binding energy will result in an increase in the cluster's half radius.

$t - t_o \sim \frac{M^{1/2} r_h^{3/2}}{G^{1/2} m ln(\wedge)}$

For constant cluster mass $r_h(t) = r_h(o)(t - t_o)^{2/3}$

Taking into account mass loss from the system in the form $M(t) = M(o)(t - t_o)^{\nu}$ we get $r_h(t) = r_h(o)(t - t_o)^{(2+\nu)/3}$, $\nu \sim 0.08$

The system expansion is self-similar - Henon 1961, 1965

Three-Body Binary Formation

The cross section for a collision between a star and a binary with a semi-major axis is: $\sigma = \pi a^2(1 + 2GM_{123}/aV^2) \approx 2\pi GM_{123}a/V^2$ - for hard binaries $a < Gm/v^2$

The rate at which a given star encounters another at the distance "a" is of order G^2m^2n/v^3, where n is the number of stars per unit volume

The probability that a third star is within this volume (of order a^3) at the same time is of order na^3, and so the rate of formation of triple systems involving a given star is of order $G^5m^5n^2/v^9$

Since there are n stars per unit volume, the rate is of order $G^5m^5n^3/v^9$

$$\dot{N}_b = \frac{105G^5m^5n^3}{v^9} \qquad \textit{Goodman and Hut } (1993)$$

General Picture of Evolution - Core Bounce

Hard binaries become harder and soft binaries become softer

Soft binary when its binding energy is smaller than the average kinetic energy of neighboring stars.
Hard binary when its binding energy is greater than the average kinetic energy of neighboring stars.

The energy added to the core (per binary) may be only of order a few times the mean kinetic energy of one core member, i.e. of order a few times mv_c^2. So the energy rate generated by binaries per unit volume is: $\varepsilon \propto G^5 m^6 n^3 / v_c^7$

$$\varepsilon = \dot{E} = \frac{4200 G^5 m^3 \varrho^3}{v^7} \qquad Cohn\ (1985)$$

By comparing the change in the kinetic energy of the core with the energy generated by binaries, we can get a limit on the number of stars in the core when the energy generated by binaries becomes dominant.

The number is about 50 - 80. For unequal masses 25

General Picture of Evolution - Core Expansion

The flux of energy from the core is given by $\varepsilon = G^5 m^6 n^3 r^3 / v^7$ and in steady expansion must be comparable with the flux at r_h, which is of order $\propto |E|/t_{rh}$

Assuming that the velocity dispersion v^2 does not vary much inside r_h - system is nearly isothermal. We get the following relation: $G^2 m^2 \rho_c^3 r_c^3 r_h \sim M^2 v^4 \rho_h \ln(N)$ and using relation for the core radius $G \rho_c r_c^2 \sim v^2$

$$\left(\frac{m}{M}\right)^2 \frac{\rho_c}{\rho_h} \frac{r_h}{r_c} \frac{1}{\ln(N)} \sim 1$$

For the nearly isothermal sphere $\rho \propto r^{-2}$. Neglecting the Coulomb logarithm dependence on N we get $\mathbf{r_h / r_c \propto N^{2/3}}$

Since the mass inside a given radius is nearly proportional to r in an isothermal model, it follows also that the number of stars inside the core in this post-collapse expansion varies as $\mathbf{N_c \propto N^{1/3}}$ and number of binaries as $\mathbf{N_b \propto N^{-1/3}}$ (Goodman 1984)

For post-collapse evolution driven by the energy generated by three-body binaries, the number of stars in the core is small - a few dozens. Also, the number of binaries in the nucleus is small, on the order of 1

General Picture of Evolution - Gravothermal Oscillations

A short summary of what we have learned so far

- Gravothermal collapse is self-similar and is consequence of the negative heat capacity for self-gravitating systems
- The energy released during the core collapse is balanced by energy flow through the half-mass radius
- The central parts of the system have to be nearly isothermal. The central relaxation time is orders of magnitude smaller than the half-mass relaxation time
- The collapse proceed until central density is large enough to form binaries. Binaries start to supply energy needed to sustain energy flow through half-mass radius - Number of stars in the core is about few dozens and is independent on N
- The evolution during post-collapse is self-similar and number of stars in the core is proportional to $Nc \propto N^{1/3}$, which is larger number than at the core bounce time
- The energy generation in the core is self-regulated

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General Picture of Evolution - Gravothermal Oscillations

The structure of the system after the core bounce is nearly isothermal central part with large density gradient between the center and half-mass radius

We know from the description of the gravothermal collapse that such configuration is unstable

The process of energy generation in the core is self-regulating - The larger energy generation the larger core expansion and consequently the smaller energy generation. Conversely, when energy generation is too small

The energy generation by binaries is strongly stochastic and as it was pointed out by Sugimoto and Bettwieser (1983, 1984) such system should be unstable not smoothly expanding

At the end of core bounce the core is producing enough energy to halt the collapse, and this is too much for the subsequent expansion of the cluster as a whole. Therefore the thermal energy generated builds up in and around the core faster than it can be conducted away. This causes an expansion and cooling of the core and its immediate surroundings

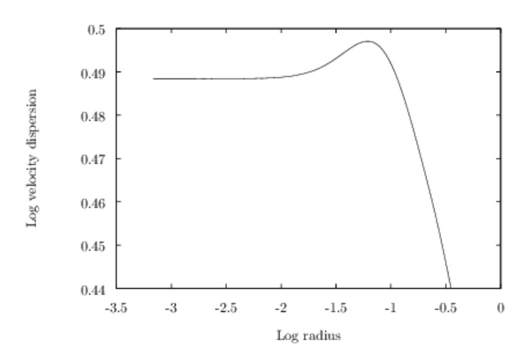
General Picture of Evolution - Gravothermal Oscillations

If core collapse is sufficiently deep, which simply requires sufficiently large N, the expanding core can actually cool to temperatures below that of its surroundings. At this point there is a temperature inversion, i.e. there is a range of radii in which the temperature increases outwards

In this region the thermal flux is actually inwards. This flux enhances the expansion of the core and its cooling

It is the gravothermal instability again, this time working in reverse. Eventually, however, the expanding core comes in thermal contact with the cooler parts of the cluster around r_h. Now the heat generated in the core can flow out as required, and the temperature inversion disappears. Again system start to collapse

Credit: Heggie et al. (1994)



The strength of system instability depends strongly on N. The larger N the gravothermal oscillations are more chaotic. Limiting $N = 8000$

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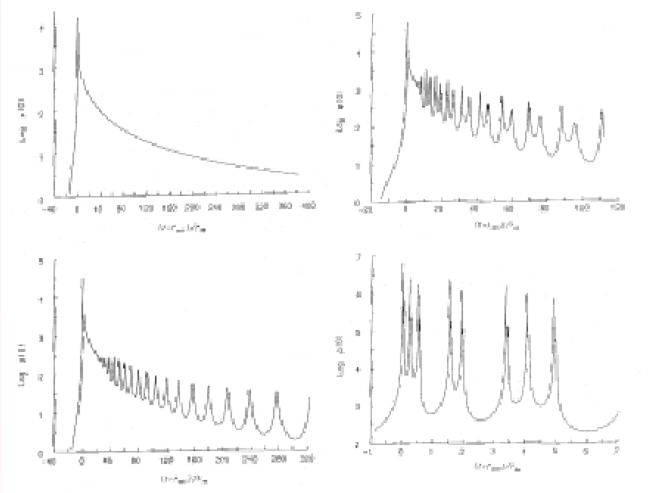
General Picture of Evolution - Gravothermal Oscillations

Credit: Heggie and Ramamani (1989)



N=6000, 8000, 10000, 50000

The collapse is more-or-less like the first core collapse: the core overshoots, it is too dense, creates too much energy, and a temperature inversion, and the cycle starts again.

Gravothermal oscillations were confirmed in many types of single mass star cluster simulations: gaseous, Fokker-Planck, N-body and Monte Carlo

General Picture of Evolution - Black Hole Subsystem

Let's consider two-component system of mass M_1 and M_2 with individual stellar masses m_1 and m_2. $M_2 \ll M_1$ and $m_2 > m_1$

Massive component (black holes, BH) will mass segregate, form massive binaries and start to generate energy in binary interactions

The energy exchange between massive and light components has to balance the energy flow through the half-mass radius: $E/t_{rh} = \dot{E}_{ex}$

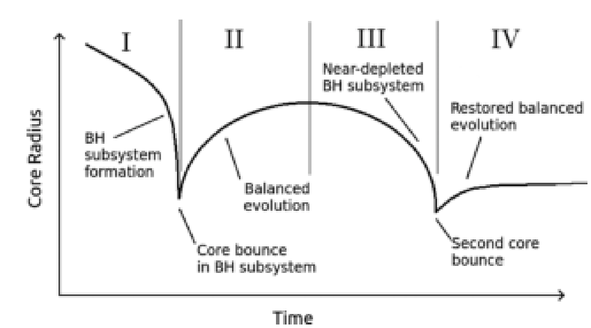
The flax generated by BHS is E_2/t_{rh2} and mus be equal to the energy exchange between light and massive components

$$\frac{E}{E_2} \sim \frac{t_{rh}}{t_{rh2}}$$

So

$$\frac{t_{rh}}{t_{rh2}} \approx \frac{M^{1/2} r_h^{3/2} m_2 \ln \Lambda_2}{M_2^{1/2} r_{h2}^{3/2} m \ln \Lambda}$$

Credit: Breen and Heggie (2013)



According to the virial theorem system absolute value of energy is equal to the system kinetic energy

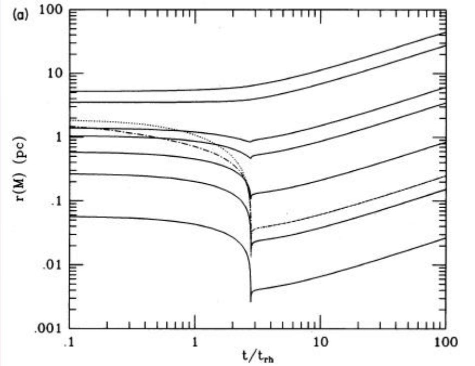
$$\frac{|E|}{|E_2|} \approx \frac{Mv^2}{M_2 v_2^2} \approx \frac{M^2 r_{h2}}{M_2^2 r_h}$$

Finally we get

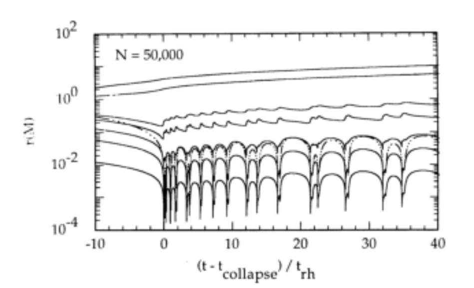
$$\frac{r_{h2}}{r_h} \sim \frac{M_2^{3/5} m_2^{2/5} (\ln \Lambda_2)^{2/5}}{M^{3/5} m^{2/5} (\ln \Lambda)^{2/5}}$$

General Picture of Evolution

Credit: Murphy et al. (1989)

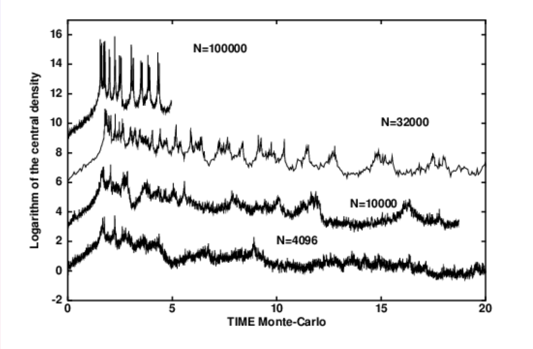


Breeden et al. (1994)



General Picture of Evolution

Giersz (1998) (Monte Carlo)



Giersz + (2019)