Numerical Study of New Dark Force of Dark Matter (JHEP11(2023)105)

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Thank you

Table of Contents

Table of Contents

Particle Dark Matter

- CMB gives significant evidence for particle dark matter via the composition of universe.
- According to the Planck data [1] of CMB radiation, our universe contains 26.8% of nonbaryonic dark matter.
- Candidates: WIMPs, Axions, Asymmetric dark matter and others.

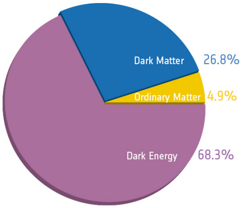


Figure 1: Energy density of universe. Image credit: ESA/Planck

WIMPs

WIMPs:

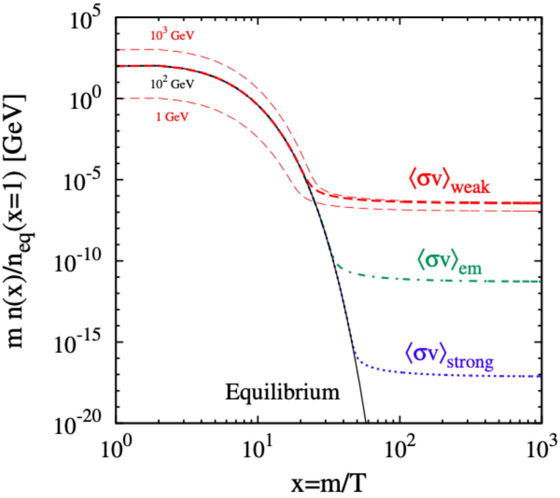
- Their mass are believe to be in range of some GeV to 10 TeV.
- Relic abundance:

Relic abundance:

$\langle \sigma_A v \rangle$: thermally averaged annihilation cross section σ_A multiplied with relative velocity v and h is the dimensionless Hubble parameter.

$$\Omega h^2 \simeq 0.1 \times \left(\frac{\langle \sigma_{\mathrm{A}} v \rangle}{3 \times 10^{-26} cm^3 s^{-1}} \right)^{-1} . \qquad (1)$$

WIMPs



Phys.Rev. D86 (2012) 023506

$$\frac{dn}{dt} + 3Hn = \frac{d(na^3)}{a^3 dt} = \langle \sigma v \rangle ({n_{eq}}^2 - n^2). \quad (2)$$

Dark matter detection

Dark matter detection:

Three methods: direct detection, indirect detection and at accelerators.

- Direct detection: observation of dark matter elastic scattering with nuclei.
- Indirect detection: detect SM particles produced via annihilation or decay of dark matter.
- Accelerator: At colliders [2], dark matter particles are simply referred to as missing energy. The detection of the visible counterpart of the signal, such as a jet or charged leptons, is the foundation of collider searches for dark matter.

Dark matter detection

Dark Matter excess through PAMELA

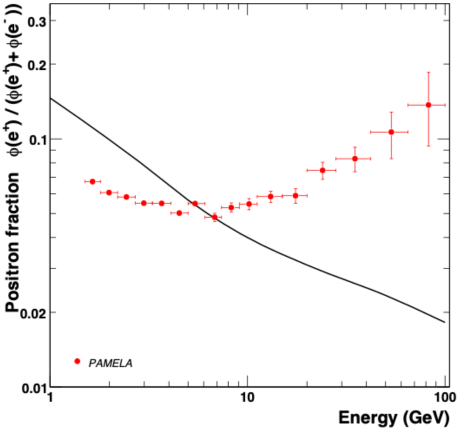


Figure 2: PAMELA positron fraction with theoretical models[3]

Dark matter detection

Dark Matter excess through ATIC

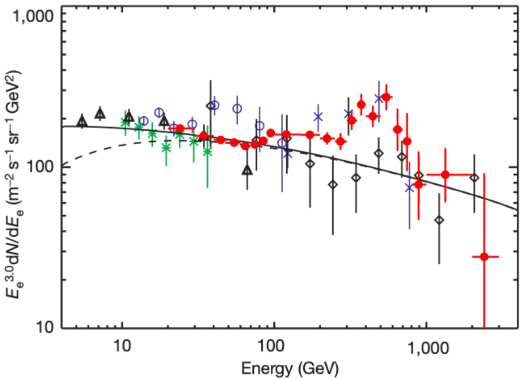


Figure 3: Energy spectrum of $(e^+ + e^-)$ obtained by ATIC[4]. The dashed line shows the background level expectation, solid line is a sum of a background and DM signal contribution.

Table of Contents

New Dark Force

Why New dark force?

If dark matter annihilates into standard model states, cosmic ray detectors may be able to detect it.

- According to PAMELA [5, 6]and ATIC [4], there are more $e^+ - e^-$ at energies between 10 and 100 GeV.
- one explanation for this is the annihilation of dark matter into e^+e^-.

★ Analysis of the PAMELA and ATIC signals demonstrates that the cross section required for the explanation of the e^+e^- excess is larger than thermal relic abundance of WIMP.

Relic abundance:

$$\Omega h^2 \simeq 0.1 \times \left(\frac{\langle \sigma_A v \rangle}{3 \times 10^{-26} cm^3 s^{-1}} \right)^{-1}. \tag{3}$$

Why New dark force?

Models that try to fit the observed PAMELA spectrum with positrons from DM annihilation usually consider boost factors of $10^2 - 10^5$ in order to match the strength of the observed excess with the annihilation rate [5, 6].

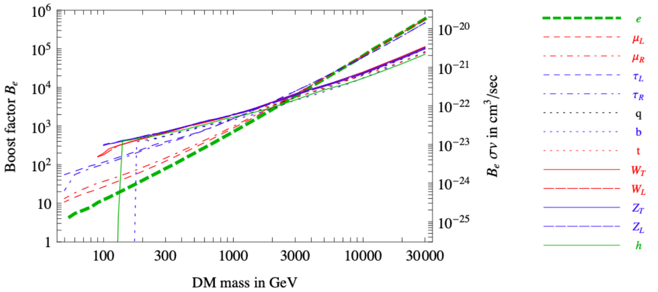


Figure 4: Values of $B_e \cdot \sigma v$ (right axis) and of the boost factor B_e (left axis, for the standard $\sigma\nu = 3 \times 10^{-26}\text{cm}^3/\text{sec}$) suggested by PAMELA excess[7].

Why New dark force?

To fit the observed ATIC spectrum with $e^- - e^+$ from KK dark matter annihilation, ATIC collaborator consider boost factor of ~ 200.

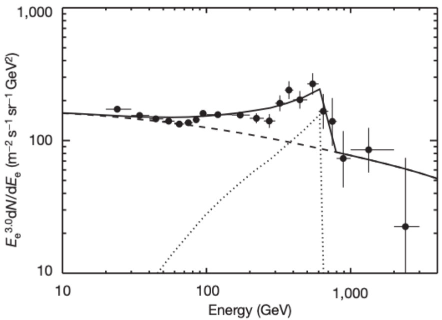


Figure 5: Assuming an annihilation of Kaluza-Klein dark matter with mass 620 GeV[4].

What is New dark force?

New dark force

if dark matter χ, annihilates via multiple exchange of new dark force ϕ, in the presence of potential which enhance annihilation cross section so called Sommerfeld enhancement, and this mediator particle then decay in standard model particles [8].

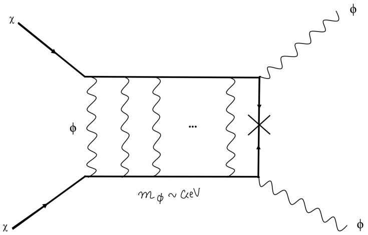


Figure 6: Annihilation diagram of $\chi\chi \to \phi\phi$. Annihilation process in the presence of Sommerfeld enhancement This diagram has been taken from article [8].

Properties of New dark force

- Mass sub-GeV scale: mass of the force carrier less than twice the pion mass;
- Some of these particles might be kinematically stable without couplings to the standard model.

SIDM: New Force

Self-interacting dark matter (SIDM) [9, 10, 11], which Spergel and Steinhardt proposed as a solution to the problems of the core cusp and missing satellites problems. ★ In order to explain these astrophysical observations, the dark matter self scattering cross section must be:

$$\sigma \approx 2 \times 10^{-24}\ \mathrm{cm}^2 (m_\chi/\mathrm{GeV}). \tag{4}$$

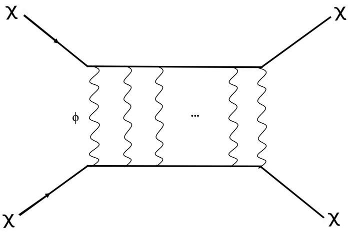


Figure 7: Self scattering of dark matter via exchange of mediator particle. Adapted from an article [8]

Table of Contents

The Sommerfeld enhancement

If DM annihilated in the ϕ through a contact interaction. The probability of finding the DM particles at the origin in this instance is simply $|\psi_k^{(0)}(0)|^2$, and if it annihilates via exchange of new force, so the probability of finding them $|\psi_k(0)|^2$.

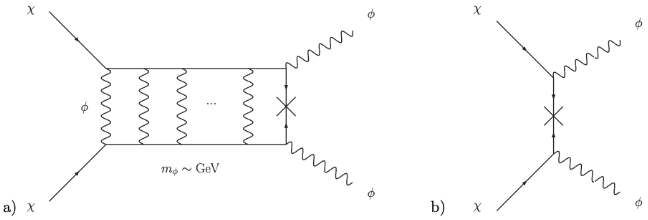


Figure 8: Annihilation of DM particles in ϕ particles. Taken from an article [8]

The Sommerfeld enhancement

quantitatively, the Sommerfeld enhancement factor,

Sommerfeld enhancement factor:

$$S_k = \frac{(\sigma_A v)}{(\sigma_A^0 v)} = \frac{|\psi_k(0)|^2}{|\psi_k^{(0)}(0)|^2}. \quad (5)$$

$\psi_k^{(0)}$ is the bare wavefunction, and $\psi_k(0)$ is the wavefunction under the effect of potential.

Sommerfeld enhancement factor and Self scattering cross section

In order to have Sommerfeld enhancement factor and self scattering cross section one have to solve quantum scattering theory.

Consider a free particle moving in a z direction, described by $\psi = e^{ikz}$, and scattered particle described by $f(\theta)e^{ikr}/r$. Thus the asymptotic form,

$$\psi_k \to e^{ikz} + \frac{f(\theta)e^{ikr}}{r} \text{ for } r \to \infty. \tag{6}$$

any solution to the Schr ödinger equation with rotation invariance around the z axis can be expanded as follows:

$$\psi_k = \sum_{\ell} A_\ell P_\ell(\cos\theta) R_{k\ell}(r) \tag{7}$$

here $R_{k\ell}(r)$ are the continuum radial functions associated with angular momentum ℓ which satisfy radial wave Schrödinger equation.

Radial wave Schrödinger equation

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR_{k\ell}}{dr}\right)+\left(k^2-\frac{\ell(\ell+1)}{r^2}-2\mu V(r)\right)R_{k\ell}=0, \tag{8}$$

where $\mu = m_\chi/2$ is a reduced mass and $k = \mu v$ is a momentum. And the asymptotic form of $R_{k,\ell}$ is,

$$\lim_{r\to\infty} R_{k\ell} = \frac{\sin(kr - \frac{1}{2}\ell\pi + \delta_\ell)}{r}. \tag{9}$$

Solving the ratio of the radial wave function close to the origin with and without potential is necessary to determine the ***Sommerfeld enhancement factor*** for partial wave annihilation

$$S_{k\ell} = \lim_{r\to 0}\frac{|R_{k\ell}(r)|^2}{|R^{(0)}_{k\ell}(r)|^2}. \tag{10}$$

Sommerfeld enhancement factor and Self scattering cross section

Sommerfeld enhancement factor

Sommerfeld enhancement factor for partial wave annihilation

$$S_{k\ell} = \lim_{r\to 0} \frac{|R_{k\ell}(r)|^2}{|R^{(0)}_{k\ell}(r)|^2}. \tag{11}$$

the behaviour of the solution R close to the origin for a potential in which:

$$\lim_{r\to 0}(V(r)r^2) = 0, \tag{12}$$

in the absence of potential $R^{(0)}_{k\ell}$ and in the presence of potential $R_{k\ell}$ is:

$$R^{(0)}_{k\ell} \approx \frac{k^{\ell+1}}{(2\ell+1)!!} r^{\ell}, \text{ and } R_{k\ell} \approx \mathscr{B} \times \frac{k^{\ell+1}}{(2\ell+1)!!} r^{\ell}. \tag{13}$$

$\mathscr{B}$ is a Constant.

Sommerfeld enhancement factor and Self scattering cross section

Self scattering cross section:

$$\psi_k = \sum_\ell A_\ell P_\ell(\cos\theta) R_{k\ell}(r) \tag{14}$$

general asymptotic form of ψ_k:

$$\psi_k \approx \textstyle\sum_{\ell=0}^{\infty}(2\ell+1)A_\ell P_\ell(\cos\theta)\frac{\sin(kr-\frac{1}{2}\ell\pi+\delta_\ell)}{kr}$$

$$= \sum_{\ell=0}^{\infty}(2\ell+1)A_\ell P_\ell(\cos\theta)\frac{i}{2kr}\{\exp[-i(kr-\frac{1}{2}\ell\pi+\delta_\ell)] - \exp[i(kr-\frac{1}{2}\ell\pi+\delta_\ell)]\}. \tag{15}$$

and the asymptotic form of e^{ikz} written as,

$$e^{ikz} \approx \sum_{\ell=0}^{\infty} i^\ell(2\ell+1)P_\ell(\cos\theta)\frac{i}{2kr}\{\exp[-i(kr-\frac{1}{2}\ell\pi)] - \exp[i(kr-\frac{1}{2}\ell\pi)]\}. \tag{16}$$

Sommerfeld enhancement factor and Self scattering cross section

Self scattering cross section:

$\psi_k - e^{ikz}$ represent an outgoing wave

$$\therefore f(\theta) = \frac{1}{2ik}\sum_{\ell=0}^{\infty}(2\ell+1)[e^{2i\delta_\ell} - 1]P_\ell(\cos\theta). \tag{17}$$

self scattering cross section,

$$\sigma = 2\pi \int_0^{\pi} |f(\theta)|^2 \sin\theta d\theta. \tag{18}$$

with Legendre polynomials, $\int_0^{\pi} P_\ell(\cos\theta)P_{\ell'}(\cos\theta)\sin\theta d\theta = \frac{2}{(2\ell+1)}\delta_{\ell\ell'}$.

$$\sigma = \frac{4\pi}{k^2}(2\ell+1)\sin^2\delta_\ell. \tag{19}$$

Table of Contents

Parameter space

$$V_Y(r) = -\frac{\alpha_\chi}{r}\, e^{-m_\phi r} \tag{20}$$

m_ϕ: mass of the new force or a mediator particle, and α_χ: coupling constant or fine structure constant of dark matter.

In the Yukawa potential, the term $1/m_\phi$ is a range of the Yukawa potential, and if we set $1/m_\phi = 0$, the potential coincides with the Coulomb potential.
Our numerical computations have two parameters a and b:

$$a = \frac{v}{2\alpha_\chi},$$
$$b = \frac{\alpha_\chi m_\chi}{m_\phi}.$$

Here a represents an approximate velocity, while b represents an approximate fraction of the dark matter mass and the mediator mass or the range of potential.

Numerically solved Sommerfeld enhancement factor and self-scattering cross section as a function of b for different values of a in the presence of Yukawa potential

Sommerfeld enhancement factor and Self-scattering cross section as a function of b

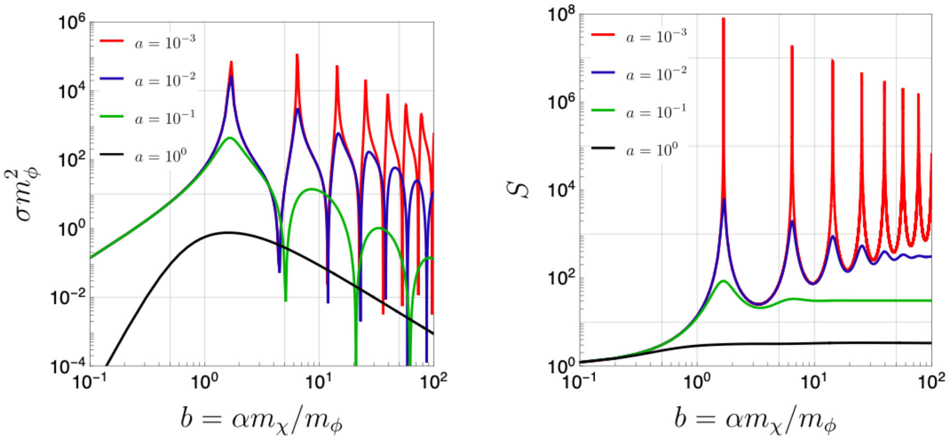


Figure 9: Self-scattering cross section σ (in unit of m_ϕ) and the Sommerfeld enhancement factor S for $s-$wave processes in the Yukawa potential as a function of b.

Unsolved questions

I believe there are still unsolved questions in this research:

- Why we observed exact same positions of resonances for Sommerfeld enhancement factor and Self scattering cross section? (JHEP11(2023)105)
- Why we observed weird behaviour for first resonance and anti-resonance in scattering cross section as a function of *a* plot for *d*-wave (current work)?

Thank you

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Table of Contents

Why New dark force?

Models that try to fit the observed PAMELA spectrum with positrons from DM annihilation usually consider boost factors of $10^2 - 10^5$ in order to match the strength of the observed excess with the annihilation rate [5, 6].

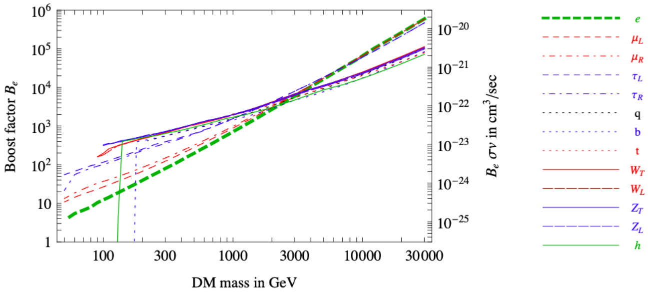


Figure 10: Values of $B_e \cdot \sigma v$ (right axis) and of the boost factor B_e (left axis, for the standard $\sigma \nu = 3 \times 10^{-26}\text{cm}^3/\text{sec}$) suggested by PAMELA excess[7].

Why New dark force?

To fit the observed ATIC spectrum with $e^- - e^+$ from KK dark matter annihilation, ATIC collaborator consider boost factor of ~ 200.

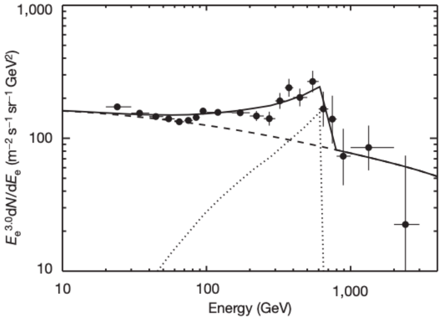


Figure 11: Assuming an annihilation of Kaluza-Klein dark matter with mass 620 GeV[4].

SIDM: New Force

- Core-cusp problem:
  Discrepancy between density profile of numerical simulations and observation is called core-cusp problem.
- Missing satellite problem:
  Numerical simulation predict that, milky way should have several hundreds subhalos with $v_{\rm max} \sim 10 - 30 \text{km}/\text{s}$ within its virial radius, however only 11 dwarf satellite galaxies were known in Milky way
- Too big to fail problem: This problem states that many of the satellites predicted by simulation are simply so massive that there is no way they couldn't have visible stars. Another way of looking at it: the observed satellites of the Milky Way are not massive enough to be consistent with predictions from CDM simulation.

★ In order to explain these astrophysical observations, the dark matter self scattering cross section must be:

$$\sigma \approx 2 \times 10^{-24} \text{ cm}^2 (m_\chi/\text{GeV}). \tag{21}$$

Effective range theory [18, 19]

The differential cross section of any collision determines the scattering rate and describes the velocity dependence of the process.

Effective range formalism

$$k^{2\ell+1}\cot\delta_\ell(k) \simeq -\frac{1}{a_\ell^{2\ell+1}} + \frac{1}{2r_{e,\ell}^{2\ell-1}}k^2 \tag{22}$$

where a_ℓ: scattering length and $r_{e,\ell}$:effective range. At sufficiently low energies, this approximation accurately describes phase shifts.

★Therefore, if one partial wave dominates the scattering process, only two factors can be used to estimate how the cross section depends on the velocity.

Computational Method: STEP 1

we change the initial point from 0 to $x_i > 0$ we apply a different initial condition for $\tilde{\chi}_{k\ell}$ which is $x = x_i$ (close to the origin). When x_i is small enough ($x_i \ll b, \sqrt{(\ell+1)}/a, (\ell+1)$), the Yukawa and a^2 terms in equation **??** become subdominant compared to the angular momentum term, and the solution is approximately $\chi_{k\ell}(x) \propto x^{\ell+1}$. We apply the initial condition, disregarding the global normalisation of $\chi_{k\ell}(x)$ because it is not important to determine δ_ℓ:

$$\tilde{\chi}_{k\ell}(x_i) = 1, \tag{23a}$$

$$\tilde{\chi}'_{k\ell}(x_i) = \frac{(\ell+1)}{x_i}. \tag{23b}$$

Based on trial and error within parameter range in numerical computation, we select following condition, which states that x_i must be small enough for equation 23a and 23b to hold:

$$x_i \approx 10^{-2} \times \text{Numerical minimum of } (b, \sqrt{(\ell+1)}/a, \ell+1) \tag{24a}$$

This is our initial estimate of a sufficiently small x_i.

STEP 2

With the initial conditions (equations 23a and 23b) applied, we solve equation **??** for $x \in [x_i, x_m]$. Here, the end point x_m is chosen so that the Yukawa term is insignificant compared to the term a^2 and the exponential term according to condition $a^2 \gg e^{(-x_m/b)}/x_m$. The phase shift δ_ℓ will be obtained by matching the solution at x_m for a potential that is precisely zero for $x \geq x_m$.

The condition specified for x_m in the numerical computation is

$$x_m = 10 \times \text{Numerical maxima of } (b, x) \tag{25a}$$

$$\text{where, } a^2 = \exp(-x/b)/x. \tag{25b}$$

STEP 3

we determine Phase shift by, matching $\chi_{k\ell}(x)$ to the asymptotic form at x_m to obtain the phase shift, which is

$$\chi_{k\ell}(x) = ax(\cos\delta_\ell j_\ell(ax) - \sin\delta_\ell n_\ell(ax)) \text{ for } x \to \infty, \tag{26a}$$

It comes from

$$\lim_{r\to\infty} R_{k\ell}(r) = kr[\cos\delta_\ell j_\ell(kr) - \sin\delta_\ell n_\ell(kr)]. \tag{27}$$

here j_ℓ and n_ℓ are the spherical Bessel and Neumann functions respectively.
Phase shift is determined by inverting equation 26a,

$$\delta_\ell = \tan^{-1}\left(\frac{ax_m j'_\ell(ax_m) - \beta_\ell j_\ell(ax_m)}{ax_m n'_\ell(ax_m) - \beta_\ell n_\ell(ax_m)}\right). \tag{28a}$$

where, $\beta_\ell = \frac{x_m \tilde{\chi}'_{k\ell}(x_m)}{\tilde{\chi}_{k\ell}(x_m)} - 1$

STEP 3 Continue

we determine self scattering cross section by substituting obtained phase shift:

$$\sigma \times m_{\phi}^{2} = \frac{4\pi}{(ab)^{2}}(2\ell + 1)\sin^{2}\delta_{\ell} \tag{29a}$$

STEP 4

To obtain the Sommerfeld enhancement factor for partial wave annihilation it's necessary to solve the equation 19, with and without potential near to the origin,

$$S_{k\ell} = \frac{|\chi_{k\ell}(0)|^2}{|\chi^{(0)}_{k\ell}(0)|^2} = |\mathscr{B}|^2. \tag{30}$$

where,

$$\chi_{k\ell} \approx \mathscr{B} \times \frac{(ax_i)^{\ell+1}}{(2\ell+1)!!} \tag{31}$$

To find a value of $\mathscr{B}$ (constant), we introduce another constant $\mathscr{D}$ and computed values of phase shifts. for $x = x_m$, the asymptotic form of $\tilde{\chi}_{k\ell}$,

$$\tilde{\chi}_{k\ell}(x_m) = \mathscr{D}(ax_m)(\cos\delta_\ell j_\ell(ax_m) - \sin\delta_\ell n_\ell(ax_m)); \tag{32}$$

STEP 4 Continue

Now, if we assume $\frac{\tilde{\chi}_{k\ell}(x_i)}{\mathscr{D}}$ is similar with $\mathscr{B} \times \frac{(ax_i)^{\ell+1}}{(2\ell+1)!!}$ and taking $\tilde{\chi}_{k\ell}(x_i) = 1$, as we have a central potential. As a result, the final equation for the Sommerfeld enhancement factor for partial waves is as follows.

$$S_{k\ell} = |\mathscr{B}|^2 = \left| \frac{1}{\mathscr{D}\frac{(ax_i)^{\ell+1}}{(2\ell+1)!!}} \right|^2 . \quad (33)$$

Plugging a determining value of phase shift δ_ℓ, into equation 32, we can find value of constant $\mathscr{D}$ and through this value we can determine a Sommerfeld enhancement factor.

Results: Sommerfeld enhancement factor

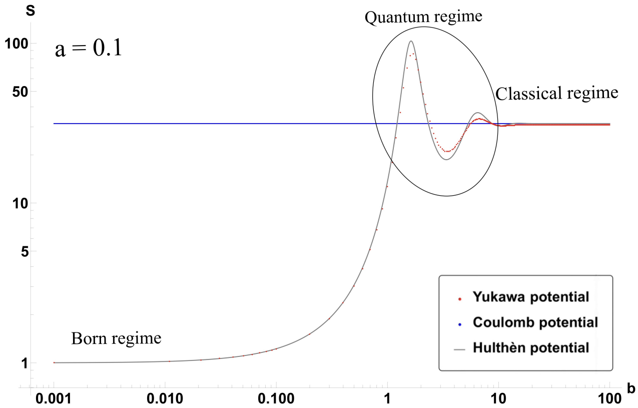


Figure 12: Sommerfeld enhancement factor as a function of b for s-wave annihilation. (red: numerically solve Sommerfeld enhancement factor for Yukawa potential, blue and grey are analytically solved sommerfeld enhancement factor for Coulomb and Hulthén potential respectively.)

Results: Self scattering cross section on anti-resonance

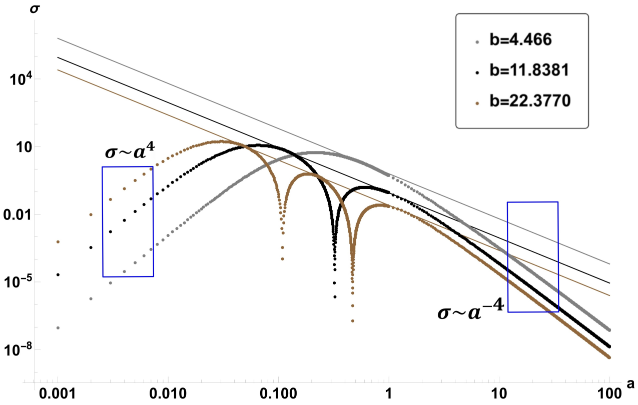


Figure 13: Scattering cross-section as a function of a for first (black), second (gray) and third (brown) anti-resonance positions for s-wave.

Results: Self scattering cross section on anti-resonance

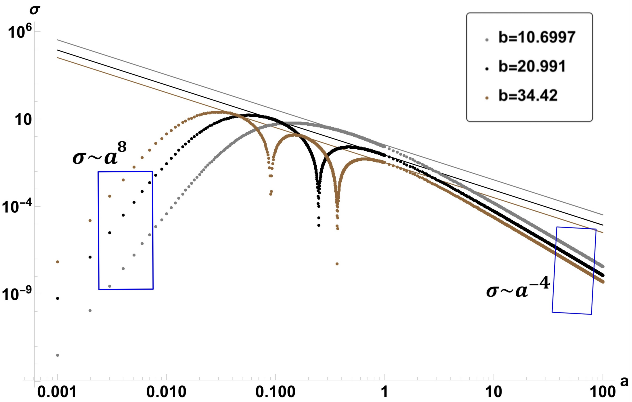


Figure 14: Self scattering cross section as a function of *a*, for first three anti-resonance positions for *p*-wave.

Results: Self scattering cross section on anti-resonance

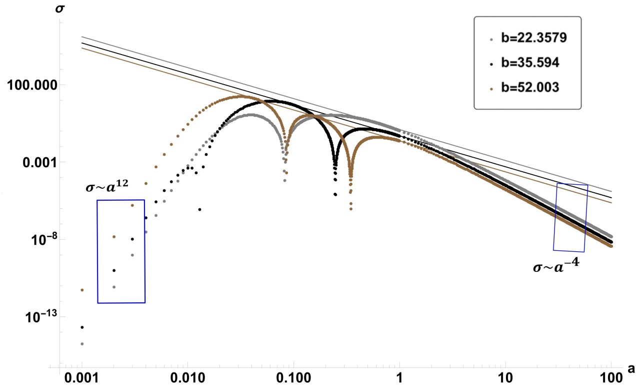
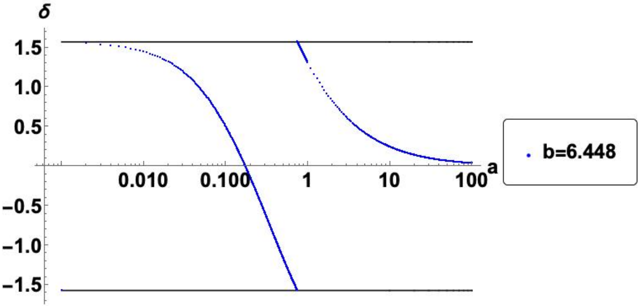


Figure 15: Self scattering cross section as a function of a for anti-resonance positions for d-wave. (The observed wiggling behaviour on the first anti-resonance in this case isn't a mathematica artefact; rather, it has an unidentified physical significance.)

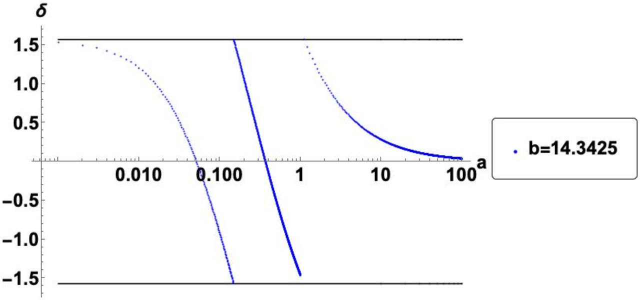
Why sharp behaviour on 2nd and 3rd resonances?

Phase shift as a function of velocity on 2nd resonance $b = 6.448$ for s-wave:



Why sharp behaviour on 2nd and 3rd resonances?

Phase shift as a function of velocity on 3nd resonance $b = 14.3425$ for s-wave:



Effective range formalism: on resonance behaviour of scattering cross section for small velocities

These higher partial wave behaviors can be evaluated using the effective range formalism [18, 19]:

$$k^{2\ell+1}\cot\delta_\ell(k) \simeq -\frac{1}{a_\ell^{2\ell+1}} + \frac{1}{2r_{e,\ell}^{2\ell-1}}k^2,$$

here, *Cotangent* diverges at the origin and $a \to \infty$ as $k \to 0$, so in order find δ_ℓ,

$$k^{2\ell+1}\cot\delta_\ell(k) \approx \frac{1}{2r_{e,\ell}^{2\ell-1}}k^2,$$
$$\delta_\ell(k) \sim 2r_{e,\ell}^{2\ell-1}k^{2\ell-1}$$

using the self scattering cross section formula $\sigma = \frac{4\pi}{k^2}\sin^2\delta_\ell$,

$$\sigma \sim \frac{4\pi}{k^2}(\ 2r_{e,\ell}^{2\ell-1}k^{2\ell-1})^2,$$
$$\therefore \sigma \sim 4\pi(\ 2r_{e,\ell}^{2\ell-1})^2k^{4\ell-4}$$

This equation describes on resonance behavior of the self-scattering cross sections for higher partial waves.

Effective range formalism: on anti-resonance behaviour of scattering cross section for small velocities

To find self-scattering cross-section behavior at anti-resonance positions for s-wave and higher partial waves, one cannot directly use equation ?? because $a \to 0$ on anti-resonances. However, we can use,

$$k^{2\ell+1}\cot\delta_\ell(k) \approx \frac{1}{\mathscr{C}^{2\ell+3}k^2},$$
$$\delta_\ell \sim \mathscr{C}^{2\ell+3}k^{2\ell+3},$$

and substituting it in self scattering cross section formula

$$\sigma \sim \frac{4\pi}{k^2}\left(\mathscr{C}^{2\ell+3}k^{2\ell+3}\right)^2$$
$$\therefore \sigma \sim 4\pi\mathscr{C}^{4\ell+6}k^{4\ell+4}$$

here $\mathscr{C}$ is a length parameter.