Simulating the evolution of star clusters

Risks and rewards

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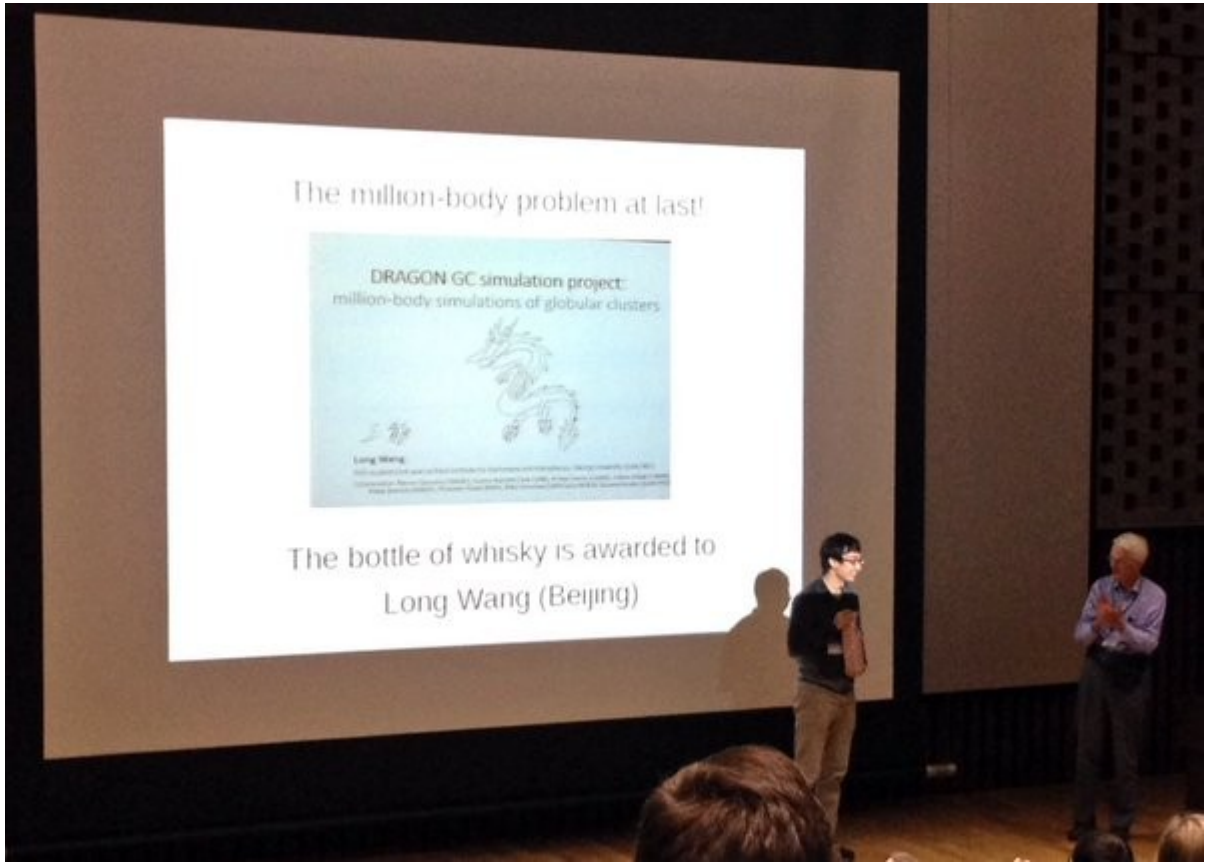
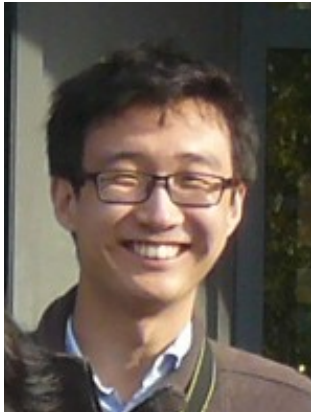
With a focus on N-body methods and Monte Carlo methods

Rewards of N-body modelling. I



- MODEST15-S (Kobe, Japan, December 2015)
- Long Wang receives a bottle of scotch whisky for the first long-term N-body simulation of the million-body problem



Rewards of N-body modelling. II

Sverre Aarseth

- Born 20 July 1934
- Writer of NBODY0,….,NBODY7

- Received the Brouwer Award (DDA of AAS) in 1998; see Aarseth 1999
- Asteroid 9836 Aarseth is named after him

Sverre (left, with Pavel Kroupa), IAU 246, Capri, 2007

Rewards of Monte Carlo modelling

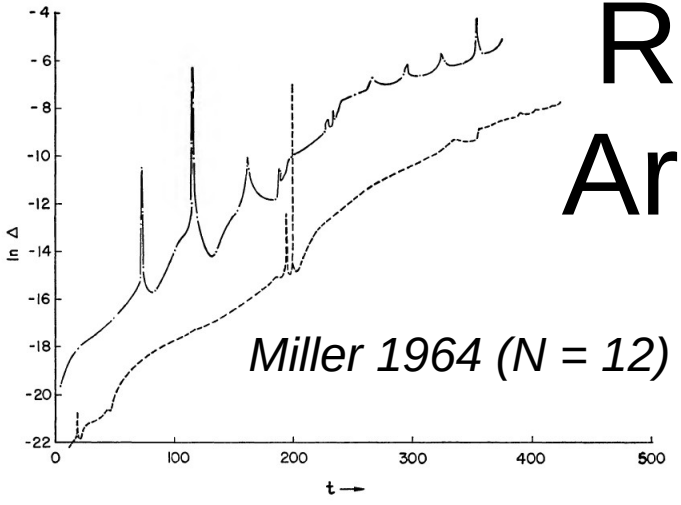
IAU Colloquium 10, Cambridge, UK, 1970

Michel Hénon

- 23 July 1931 – 7 April 2013
- Invented and developed MC modelling: 1967 – 1975
- See 2014arXiv1411.4936H

- Received the Prix Jean Ricard in 1978
- Received the Brouwer Award in 1983
- Salle Michel Hénon named in his memory at IAP (Paris)

Risks of N-body modelling
Are N-body results correct?



Miller 1964 (N = 12)

- Exponential growth of the difference between two solutions starting from nearly equal initial conditions (Butterfly effect)
- e-folding time $\simeq t_{cr}/10$ (large N, Plummer initial conditions, Goodman+ 1993)
- Double precision error reaches O(1) after a few t_{cr}
- After this time we can be sure that the calculation is wrong
- We *assume* that the statistical properties of the calculation are correct
- For chaotic 3-body systems, this can be established by “arbitrary”-precision calculations (Portegies Zwart & Boekholt 2015)

Dick Miller (1926-2020)

Risks of Monte Carlo modelling
Is Chandrasekhar* theory correct?

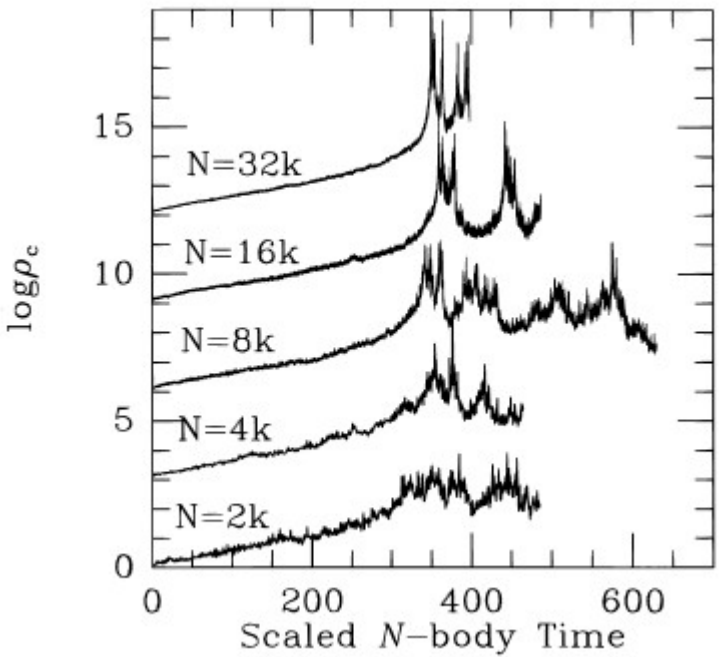
An empirical test – time to core collapse (Plummer, equal masses)

Numerical (Pavlik & Subr 2018)
- $t_{cc} = 2297 \pm 52$ (N = 10^4), 9347 ± 150 (N = 5×10^4)

Chandrasekhar (Takahashi 1995)
- $t_{cc} = 17.6\ t_{rh}$

Hence
- N-body/Chandra = $1.026 \pm 0.023,\ 1.019 \pm 0.016$

Makino 1966

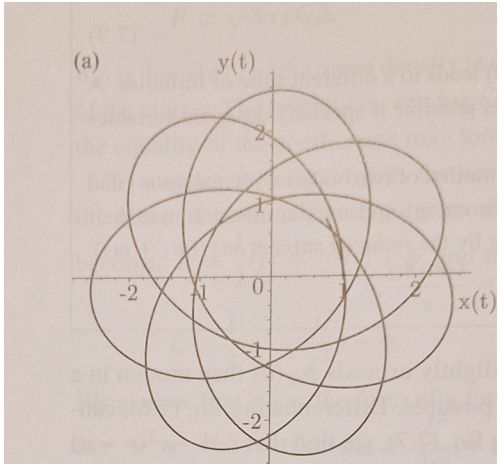


"Traditional" view: Chandra's theory is good to a few percent

*Based on Chandrasekhar (1942) plus
- Coulomb logarithm $\ln(\lambda N^{2/3}) \rightarrow \ln(\lambda N)$ (Hénon 1958)
- Orbit-averaged Fokker-Planck equation (Kuzmin 1957)

A critique of Chandrasekhar theory

- In the theory, successive encounters between two stars are independent. But in a star cluster, they are continuously interacting with each other, and can do so *resonantly,* with enhanced effect.



- The theory neglects effects of self-gravity of the system:
  - Excitation of weakly damped modes (Weinberg 1994)
  - Polarisation cloud (dressed potentials)
  - See Hamilton+ (2018)

Lau & Binney 2019 concluded:

“The predictions of [Chandrasekhar] theory are qualitatively wrong because [it] neglects self-gravity. These results imply that [Chandrasekhar] theory is of little value. Future work on cluster evolution should employ either N-body simulation or the Balescu-Lenard equation. However, significant code development will be required to make use of the BL equation practicable.”

This is the “revolutionary” point of view.

Traditionalists v. Revolutionaries

Tep+ 2022 measured the rate of relaxation (df/dt at t = 0) in N-body simulations of equal-mass Plummer models, and compared with Chandrasekhar.

	Inner 10%	Intermediate 40%	Outer 50%	Global	
Chandra/ N-body	1.4	1.4	1.0	1.4	Tep+ 2022 Finite differences
	1.12	1.01	0.93	1.03	Tep+H in prep Interpolation

Kerwann Tep

In significantly radially anisotropic models*, global ratio 1.4 → 1.10
In significantly tangentially anisotropic models*, global ratio 1.7 → 1.38

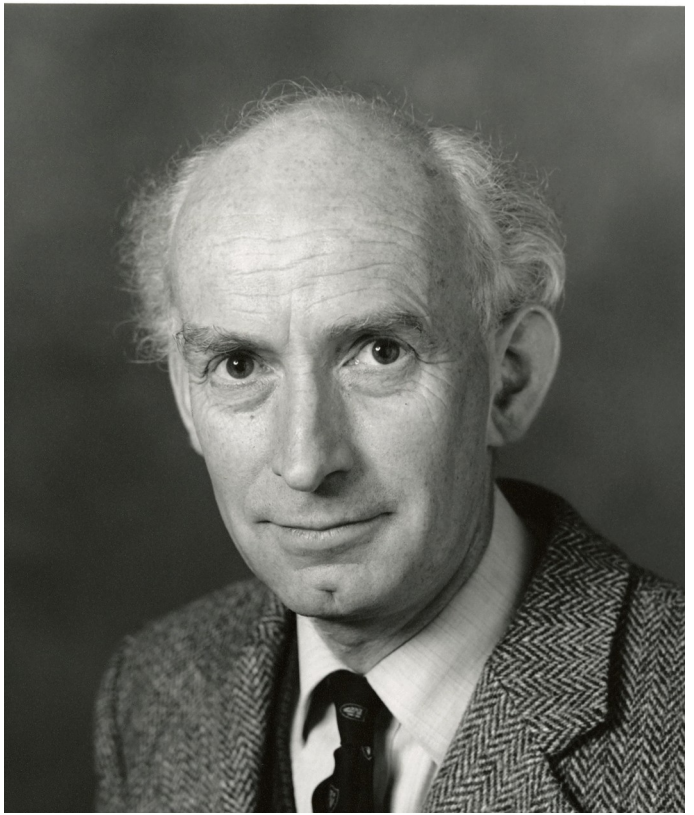
- *Plummer-Dejonghe models with q = 1, -2
- Provisional corrected data

Summary

1. Globally, Chandrasekhar performs well (a few percent) for systems which have
- Spherical symmetry
- Equal masses
- Little anisotropy

2. Open questions
- Anisotropic systems
- Unequal masses
- Rotating systems

3. How to do research in stellar dynamics
- “Find out things and have fun”
- Join the MODEST community



Donald Lynden-Bell 1935-2018