# Statistical Mechanics in the Galactic Center

## Mass segregation and phase transitions


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# Overview

# Young Stars in the Galactic center

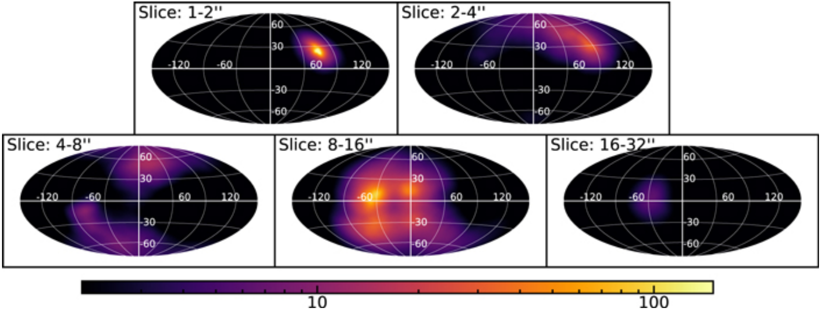


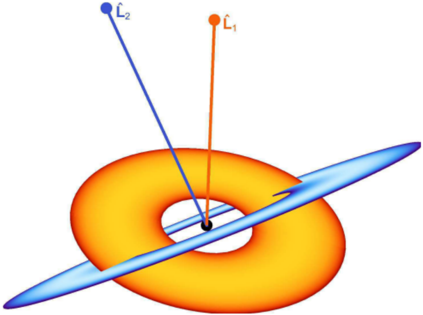

Figure: Overdensity of angular momentum distribution of the young stars in log scale at different radius slices[2].

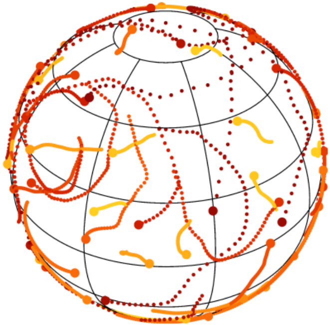
- More massive young stars form a coeval warped disk, with a possible counterrotating structure.

[1] Sebastiano D. von Fellenberg et al. "The Young Stars in the Galactic Center". In: ApJ 932.1, L6 (June 2022), p. L6. DOI: 10.3847/2041-8213/ac68ef. arXiv: 2205.07595 [astro-ph.GA].

[2] von Fellenberg et al., "The Young Stars in the Galactic Center".

# Vector Resonant Relaxation



- Averaged over orbit and precession.
- Fixed semi major axis and eccentricity.
- Efficiently change the inclinations of the stars. [3]

[3]Figures adopted from Kinetic theory notes on self-gravitating systems, Fouvry, 2022.

# Main objectives

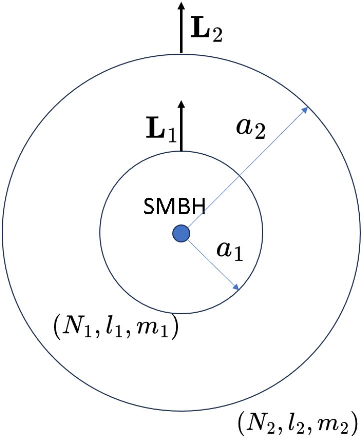
The main objectives of this presentation are:

- A statistical mechanical toy model for stars in the galactic center under vector resonant relaxtion (VRR);
- Heavier stars condense into a disk: mass segregation in the inclination of the stars;
- Order-disorder phase transition;
- Negative absolute temperatures;
- Compare thermal equilibrium with dynamical simulations;

# Statistical Mechanics Models

# Two-component Models

- Nuclear star clusters with two components denoted by $\mathcal{C}_1$ and $\mathcal{C}_2$ of $N_1$ and $N_2$ number of stars, respectively, orbiting around the same central SMBH, around the same axis of symmetry.



# Two-component Models

- The VRR Hamiltonian with quadrupole approximation is given by

$$H_{\mathrm{VRR}} = -\frac{1}{2}\sum_{i,j\in\mathcal{C}_1} J_1 P_2(\boldsymbol{n}_i \cdot \boldsymbol{n}_j) - \frac{1}{2}\sum_{i,j\in\mathcal{C}_2} J_2 P_2(\boldsymbol{n}_i \cdot \boldsymbol{n}_j) - \sum_{i\in\mathcal{C}_1, j\in\mathcal{C}_2} J' P_2(\boldsymbol{n}_i \cdot \boldsymbol{n}_j), \tag{1}$$

- $\boldsymbol{n}_i$ is the unit angular momentum vector direction of the $i-$the star.
- $P_2$ is the second Legendre polynomials
- $J_1, J_2/J'$ are coupling constants.

# The equilibrium distribution function

- Extremising the total entropy subject to fixed total energy and total angular momentum.
- Using mean-field approximation, the equilibrium distribution function of the inclination $i$ is given by:

$$f_1(i|Q_1, Q_2) \propto e^{\frac{3}{2}\beta(J_1 N_1 Q_1 + J' N_2 Q_2)\cos^2 i + l_1 \gamma \cos i}. \qquad (2)$$

- Lagrange multipliers: $\beta$ (temperature), $\gamma$ (net rotation rate).
- Quadrupole moment

$$Q_{1,2} = \langle \cos^2 i - 1/3 \rangle \qquad (3)$$

# The order parameter Q

- Maximum value $Q = 2/3$, razor-thin disk, ordered. [a]

[a] https://galaxiesbook.org/chapters/II-01.-Flattened-Mass-Distributions.html

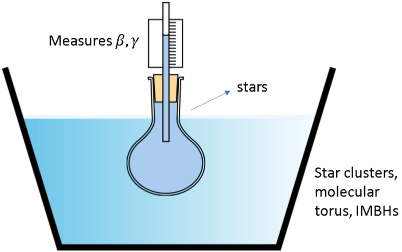
- $Q = 0$, isotropic distribution, disordered. [a]

[a] https://sci.esa.int/web/hubble/-/globular-cluster-ngc-1466

# Anisotropic Mass Segregation

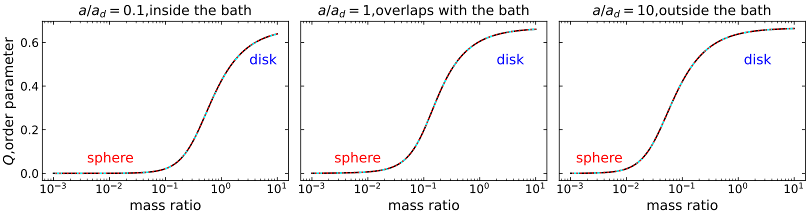
# The heat bath approximation

- One of the two components dominates the total energy and total angular momentum, acting as a heat bath, a reservoir of energy and angular momentum.
- The subdominant star cluster then determines its equilibrium state according to the temperatures $\beta, \gamma$ set by the heat bath.
- Similar to the canonical ensemble



# Vertical Mass Segregation

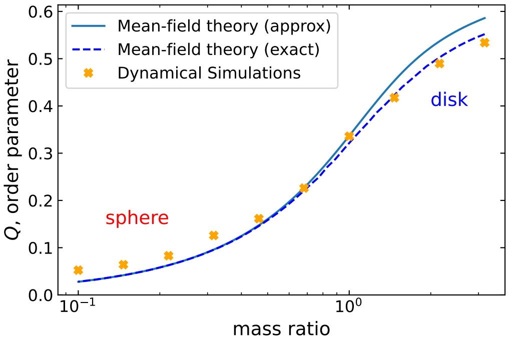
- Fix the distribution of the heat bath and vary the mass ratio of individual stars to the heat bath.
- Observe the change in the order parameter $Q$ of the subdominant component, Wang & Kocsis 2023[4].
- $Q$ close to 0: isotropic sphere, $Q$ close to $2/3$: disk-like.
- Heavier stars, larger $Q$ values, condense into a disk




[4] Hanxi Wang and Bence Kocsis. "Anisotropic mass segregation: Two-component mean-field model". In: Phys. Rev. D 108.10, 103004 (Nov. 2023), p. 103004. DOI: 10.1103/PhysRevD.108.103004. arXiv: 2302.12842 [astro-ph.GA].

# Dynamical Simulation Results

- N-body simulations of the VRR Hamiltonian.
- Fix the heat bath and total star number ratio ($N_1/N_2 = 30$).
- Start with a nearly isotropic subdominant component and let it evolve. 3100 total stars were used.
- Time-averaging the final states.





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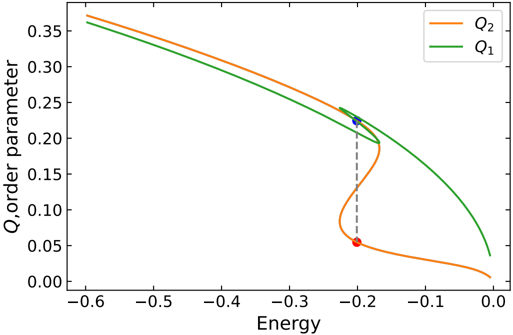
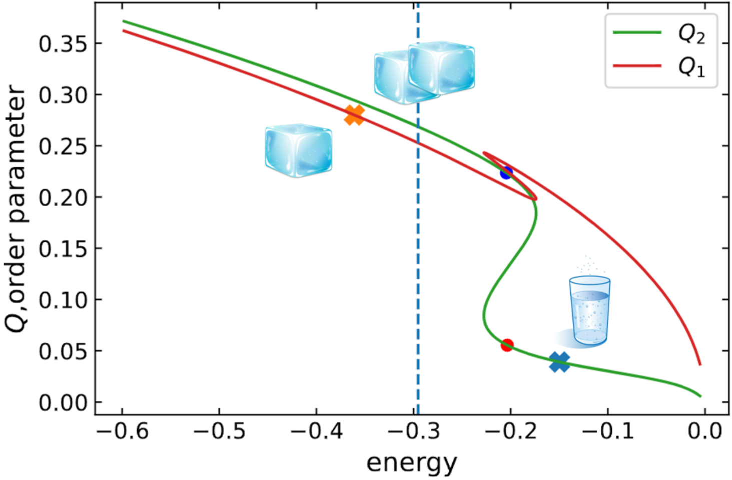
# Phase Transition

# Phase transition in the microcanonical ensemble

- A general two-component system (no heat bath).
- System will tend to the global maximum of entropy out of all equilibrium states $\delta S = 0$.
- Transition between water ($Q$ near 0, disordered) and ice ($Q$ large, ordered disk), Wang & Kocsis, in prep.

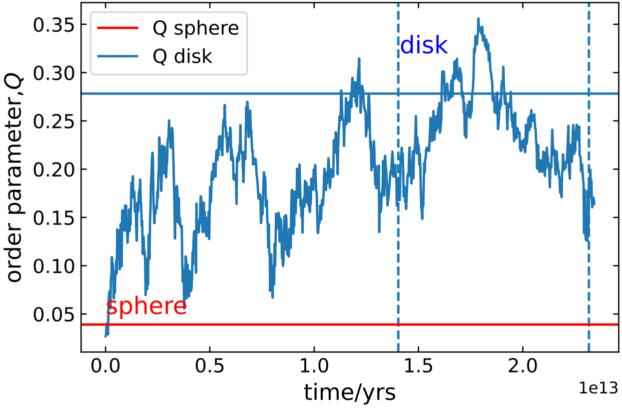


# Throwing ice in water.



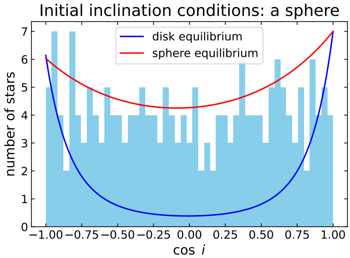
# Water becomes ice as well.

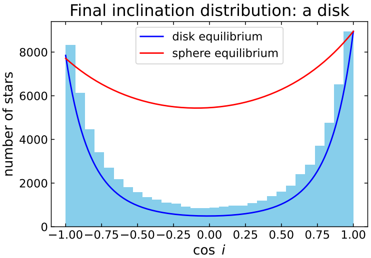
- We evaluate the time-averaged order parameter $Q$ as time evolves in the simulations.
- $Q$ value increases from the sphere value to the disk value.



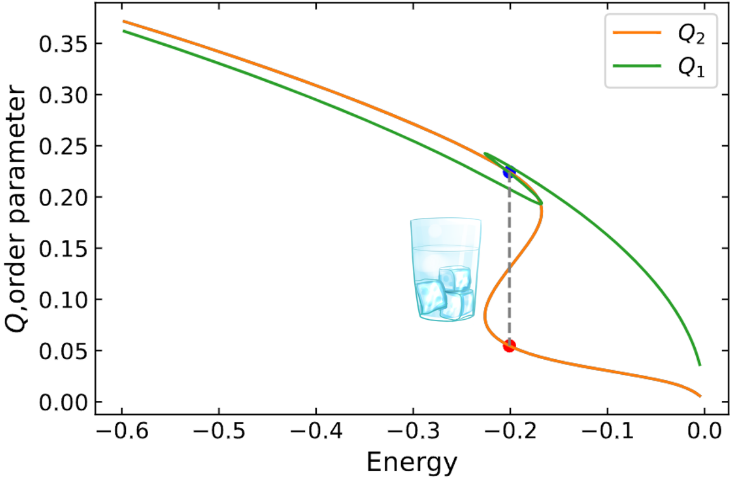
# Initial and final states

- Initially a spherical state
- Final state follows the thermal equilibrium of the disk.



# A water and ice mixture



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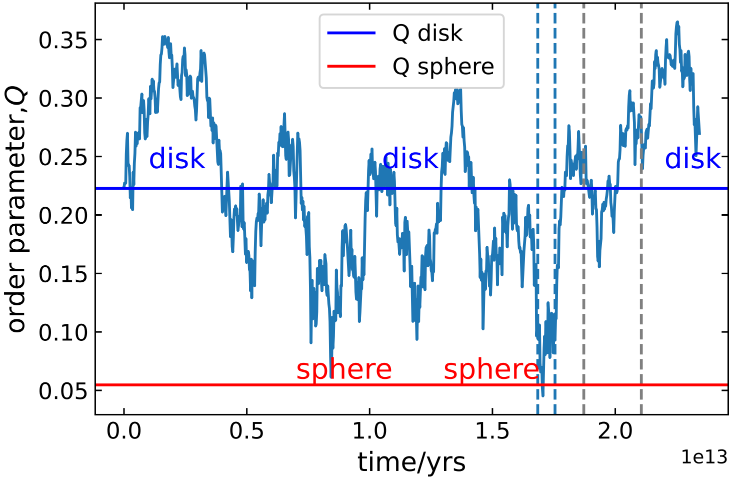
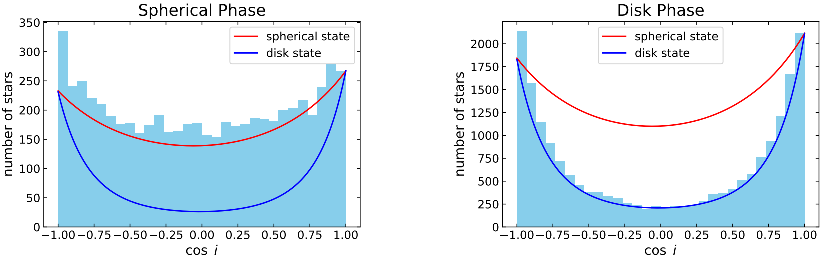


# Sphere and disk states

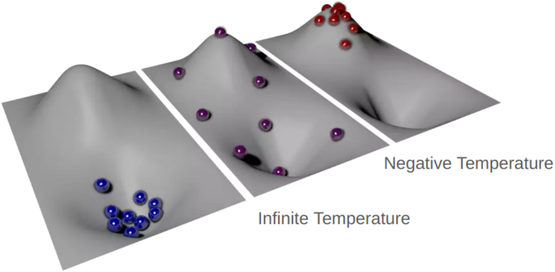
- Changes between disk and spherical states.



# Negative Temperature States

# Negative absolute temperature states

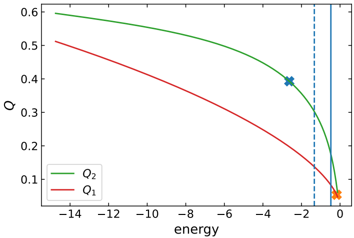
- Energy must be bounded.
- Entropy decreases when energy increases. $1/T \sim dS/dE$
- Boltzmann distribution inverted.
- Observed in magnetic systems, cold atoms in optical lattices, laser systems etc. [5]

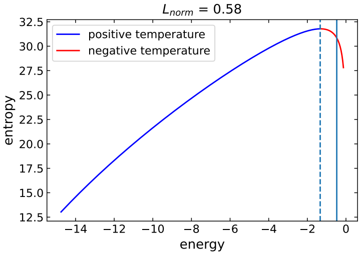



[5] Figure from https://www.quantum-munich.de/119947/Negative-Absolute-Temperatures

# Negative temperature states of stellar orbits.

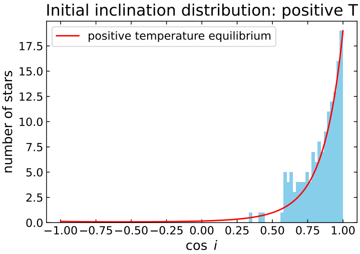
- Bounded total energy for VRR, negative temperature states allowed. $1/T \sim dS/dE$
- Higher total angular momentum systems have more negative temperature states. (Roupas et al. 2017, Wang & Kocsis 2023)
- disordered and spherical, more energetic microstates relatively more populated.
- $N_1/N_2 = 15, m_1/m_2 = 0.5, a_1 = a_2, L_{\mathrm{norm}} = 0.57.$

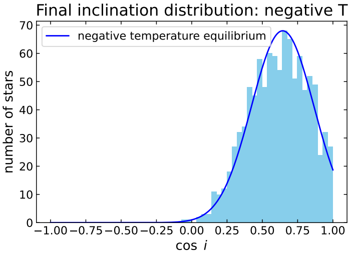


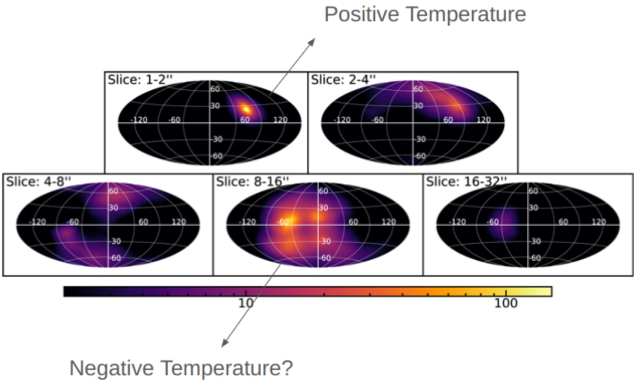
# Negative temperature state simulations

- Component 2 changes from a positive temperature to negative temperature
- Component 1 keeps its negative temperature.



# Negative temperatures in the galactic center?



# Summary

- We have built a toy model of the statistical mechanics of star orbits in the galactic center.
- The thermal equilibrium under VRR displays a mass segregation in the inclination. Higher-mass stars condense into a flattened disk under the heat bath.
- We explore the possibility of phase transition: the coexistence of 'water' and 'ice'.
- We demonstrate gravitating systems with negative temperatures.
- The dynamical simulations show good agreement with the thermal theory.