

# Formation and evolution of the Nuclear Star Cluster in the Milky Way and other spiral galaxies on the cosmological time scale.



**CAMK, Polish Academy of Sciences,  
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**MAO, National Academy of Sciences  
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CENTRE



Volkswagen**Stiftung**

**Dr. Sci. Peter Berczik**

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# Formation and evolution of the Nuclear Star Cluster in the Milky Way and other spiral galaxies on the cosmological time scale.

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April 14, 2023

# **Milky Way globular clusters on cosmological timescales. I. Evolution of the orbital parameters in time-varying potentials**

Maryna Ishchenko<sup>1,2</sup>, Margaryta Sobolenko<sup>1</sup>, Peter Berczik<sup>3,4,1,2</sup>, Sergey Khoperskov<sup>5,6</sup>, Chingis Omarov<sup>2</sup>,  
Olexander Sobodar<sup>1</sup>, and Maxim Makukov<sup>2</sup>

April 10, 2023











# **Milky Way globular clusters on cosmological timescales. II. Interaction with the Galactic centre**

Maryna Ishchenko<sup>1,2</sup>, Margaryta Sobolenko<sup>1</sup>, Dana Kuvatova<sup>2</sup>, Taras Panamarev<sup>3,2</sup>, and Peter Berczik<sup>4,5,2,1</sup>

June 19, 2024

# **Dynamical evolution of Milky Way globular clusters on the cosmological timescale**

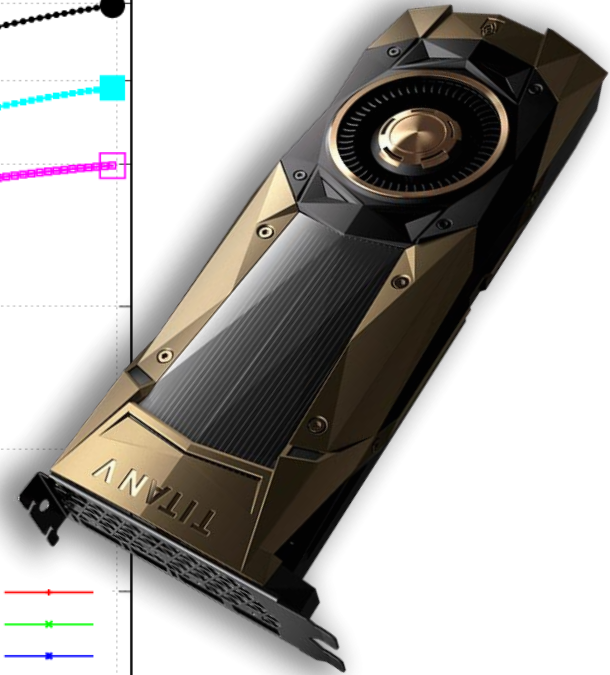
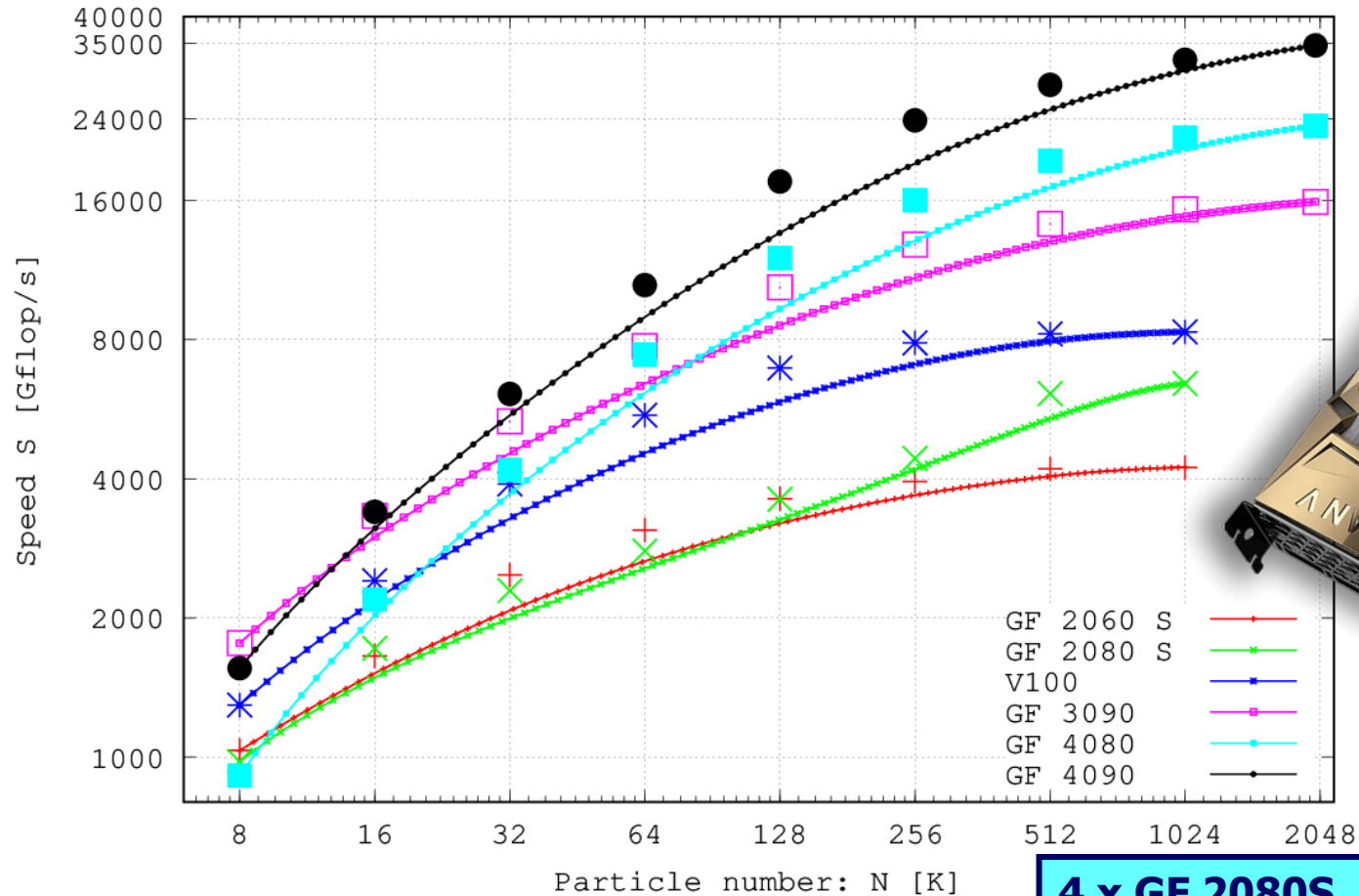
## **I. Mass loss and interaction with the nuclear star cluster**

Maryna Ishchenko<sup>1,2,3</sup>, Peter Berczik<sup>2,3,8,1</sup>, Taras Panamarev<sup>7,3</sup>, Dana Kuvatova<sup>3</sup>,  
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Sobodar<sup>1,2</sup>, and Chingis Omarov<sup>3</sup>

# MAO, Kiev 8 node GPU cluster

MAO 4+4 new nodes 4xGF 2080S + 4xGF 3070 + 4xGF 4080

phi-GPU: Plummer,  $G=M=1$ ,  $E_{\text{tot}}=-1/4$ ,  $\epsilon=10^{-4}$



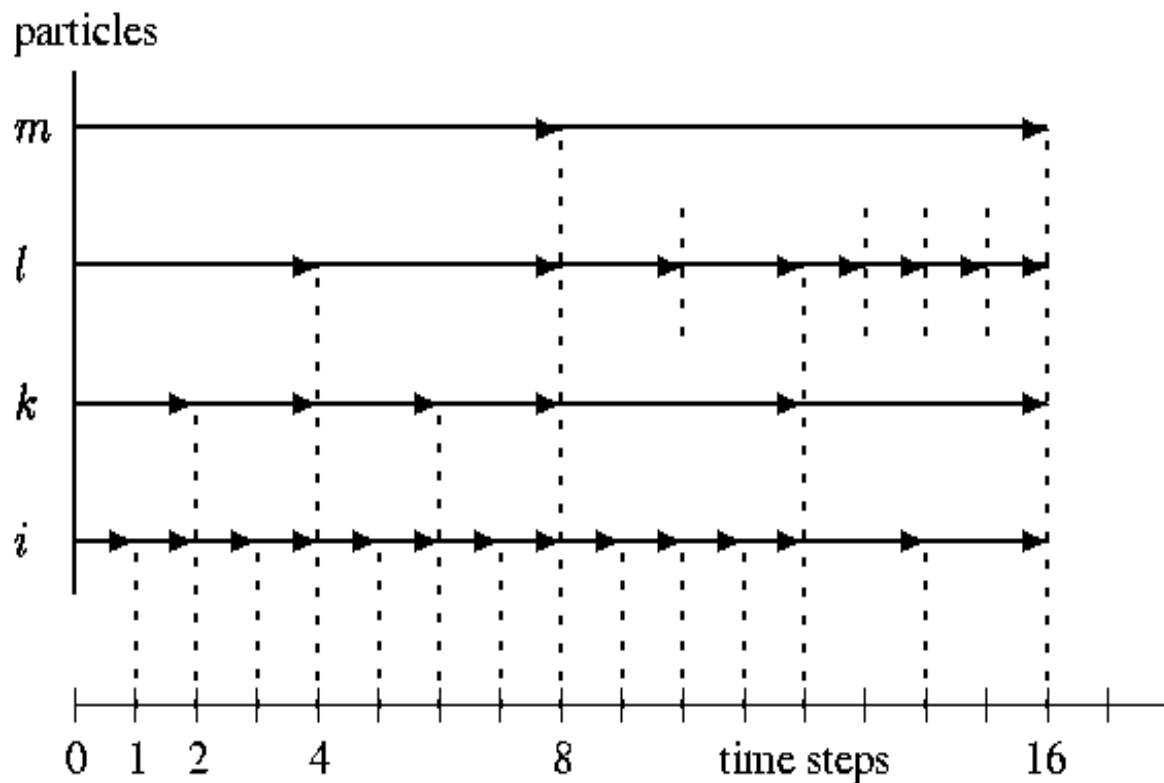
**4 x GF 2080S, 3072 SP @ 1.81 GHz**  
**4 x GF 3070, 5888 SP @ 1.77 GHz**  
**4 x GF 4080, 9728 SP @ 2.54 GHz**



# Our $\phi$ GPU (parallel) N-body code

Old GRAPE implementation: Harfst et al, NewA, 12, 357 (2007)

## Hierarchical Individual Block Time Steps



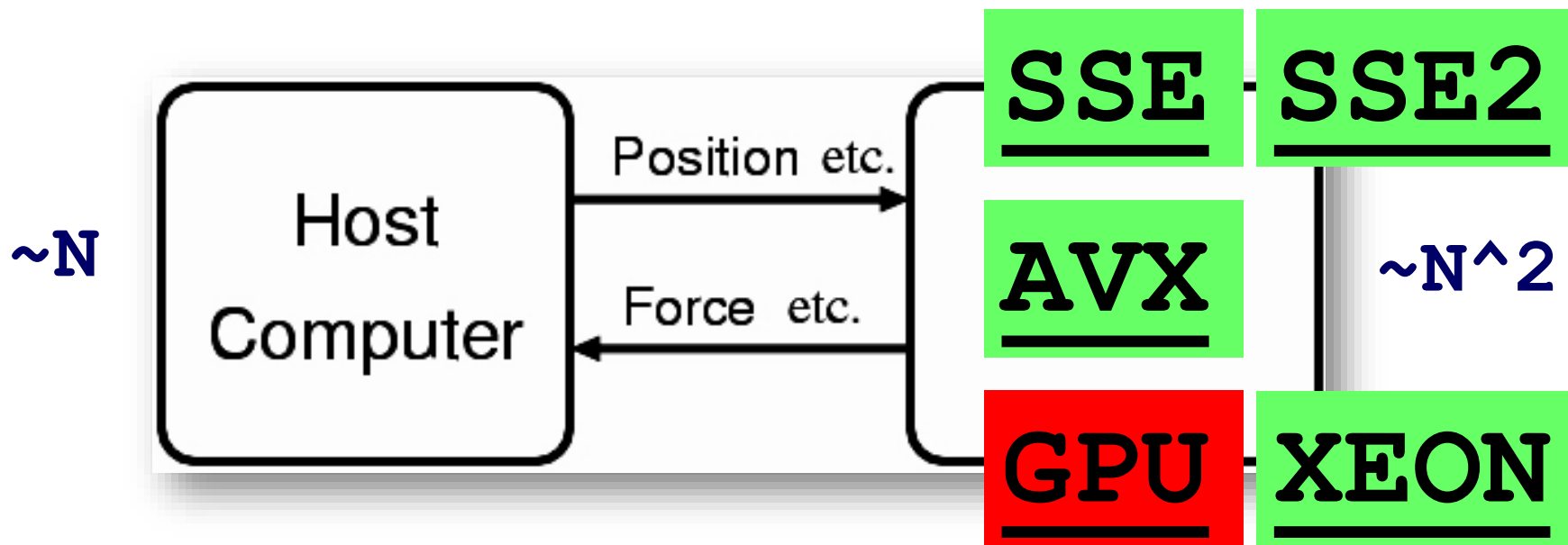
$$\Delta t = \sqrt{\eta \frac{|\vec{a}| |\vec{a}^{(2)}| + |\vec{a}|^2}{|\vec{a}| |\vec{a}^{(3)}| + |\vec{a}^{(2)}|^2}}$$

4<sup>th</sup> order Hermite scheme

$$\frac{d^2 \vec{r}_i}{dt^2} = \vec{a}_i$$

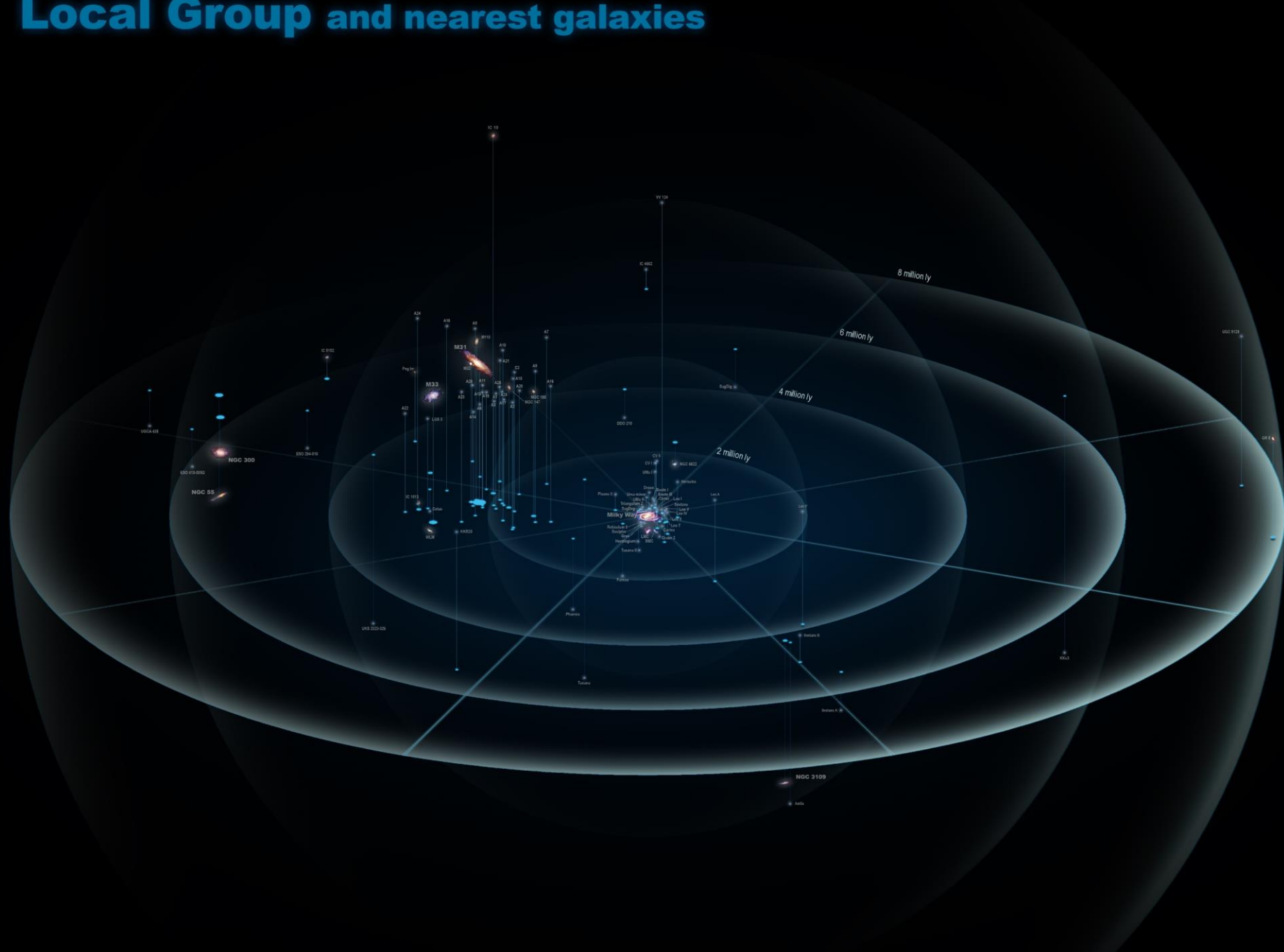
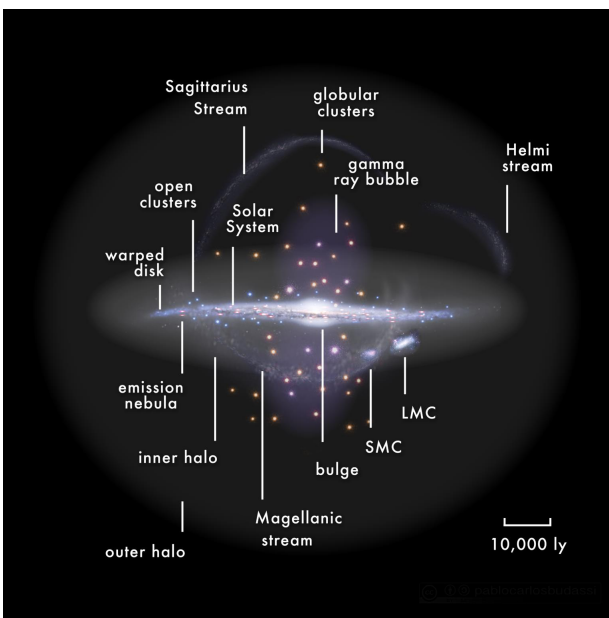
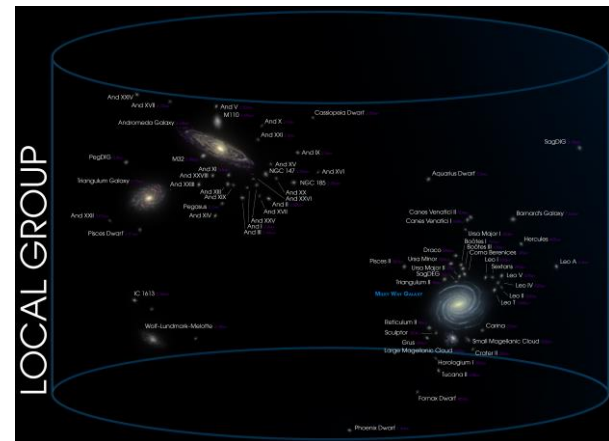
<https://github.com/berczik/phi-GPU-mole>

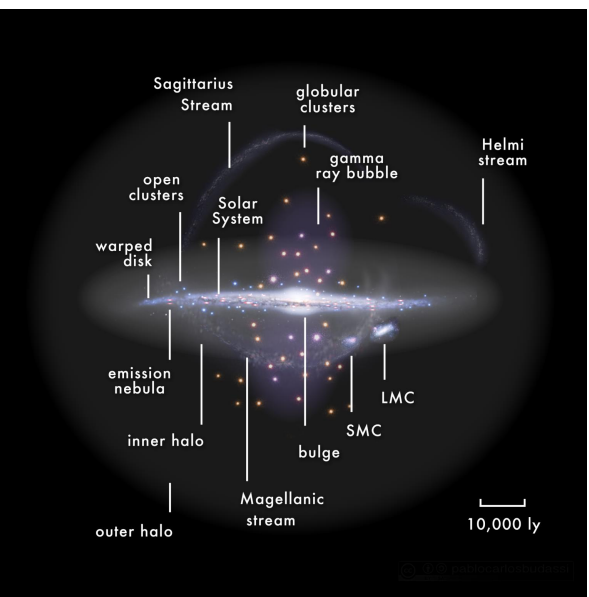
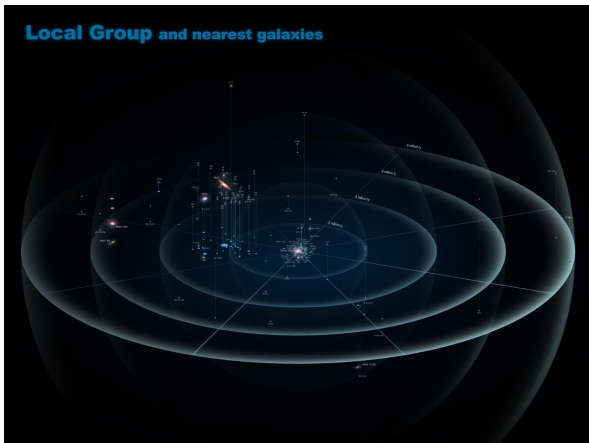
# Our $\phi$ GPU (parallel) N-body code



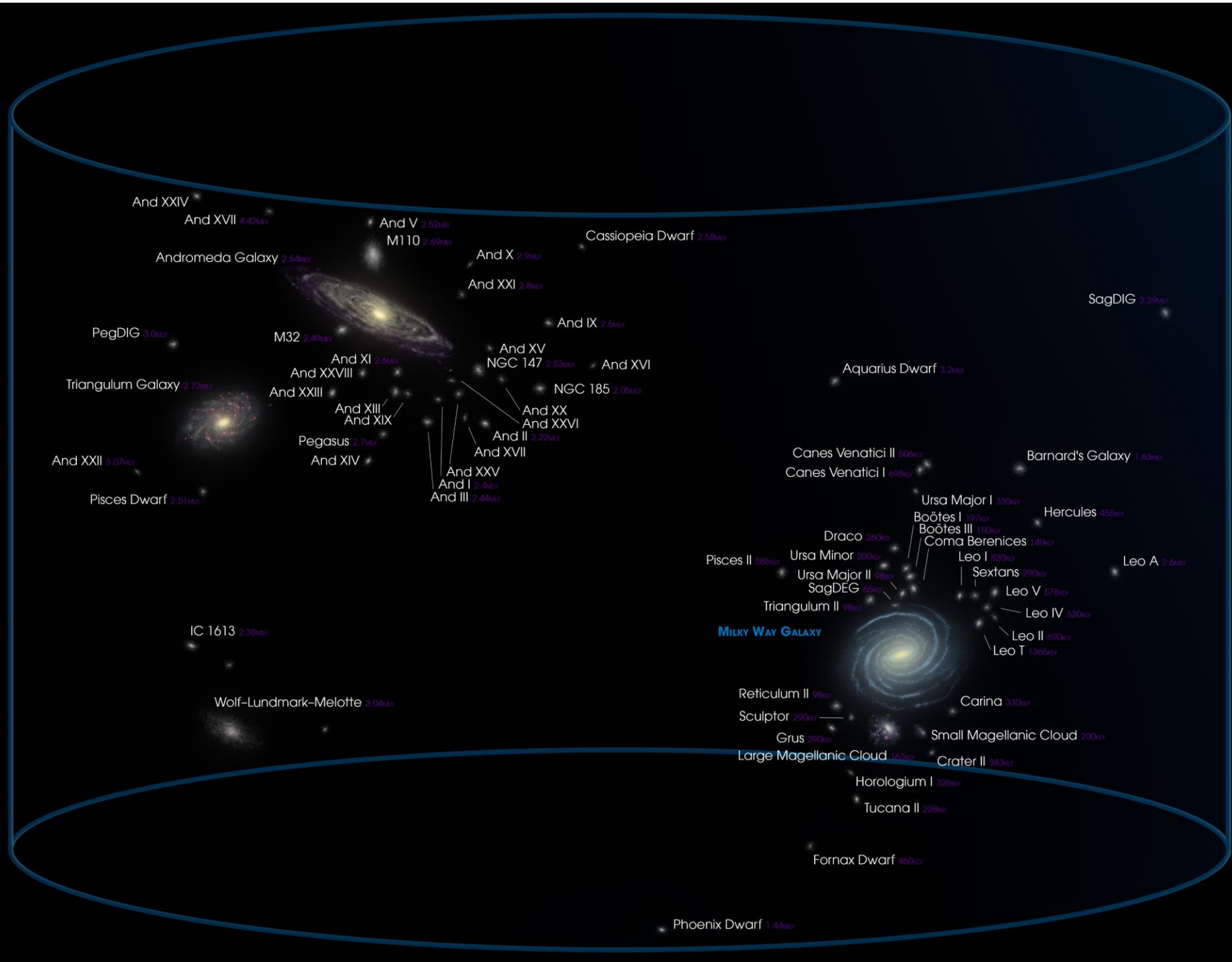
$$\vec{a}_i = \sum_{j=1; j \neq i}^N \vec{f}_{ij} \quad \vec{f}_{ij} = - \frac{G \cdot m_j}{(r_{ij}^2 + \epsilon^2)^{3/2}} \vec{r}_{ij}$$

# Local Group and nearest galaxies



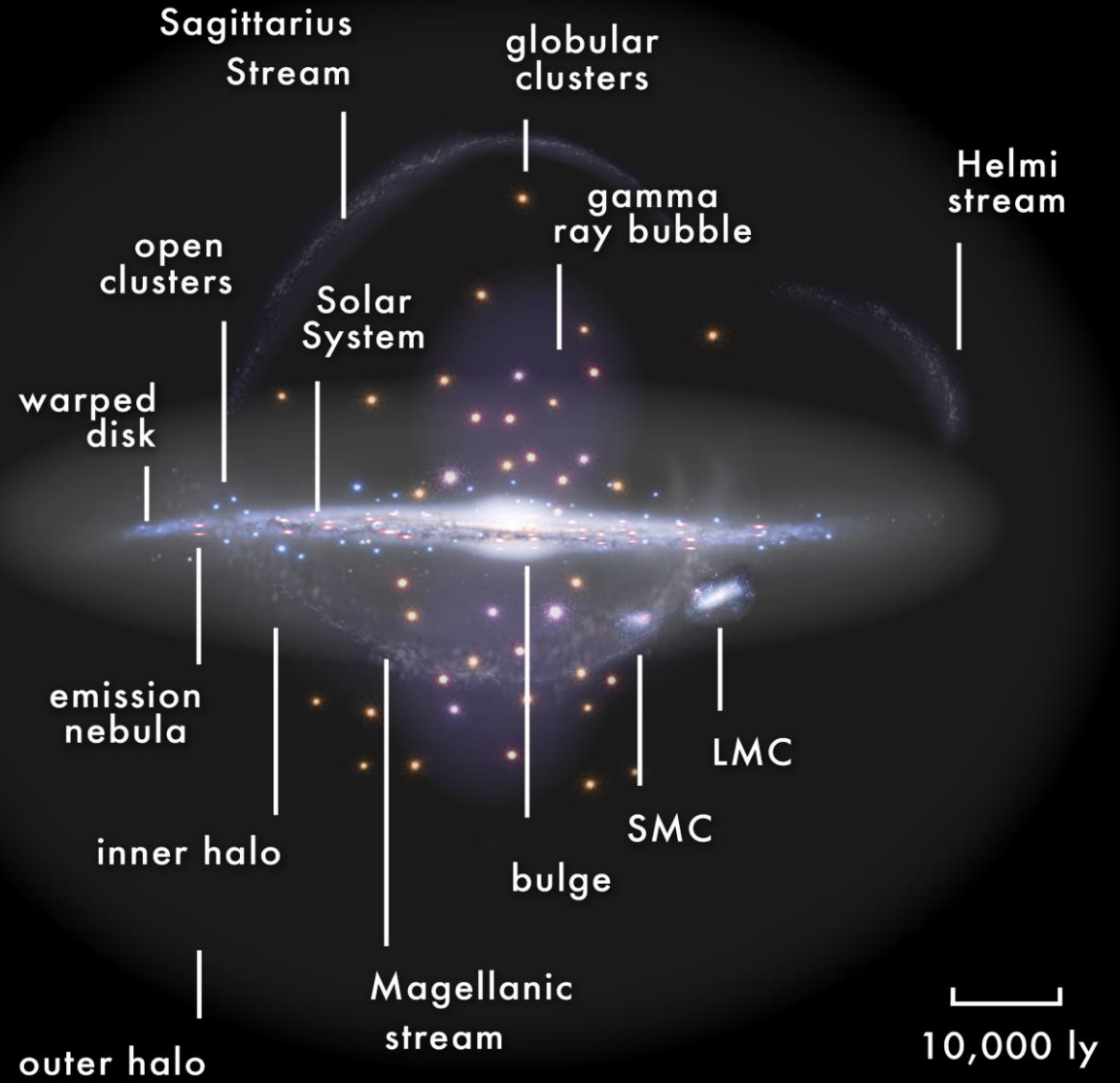
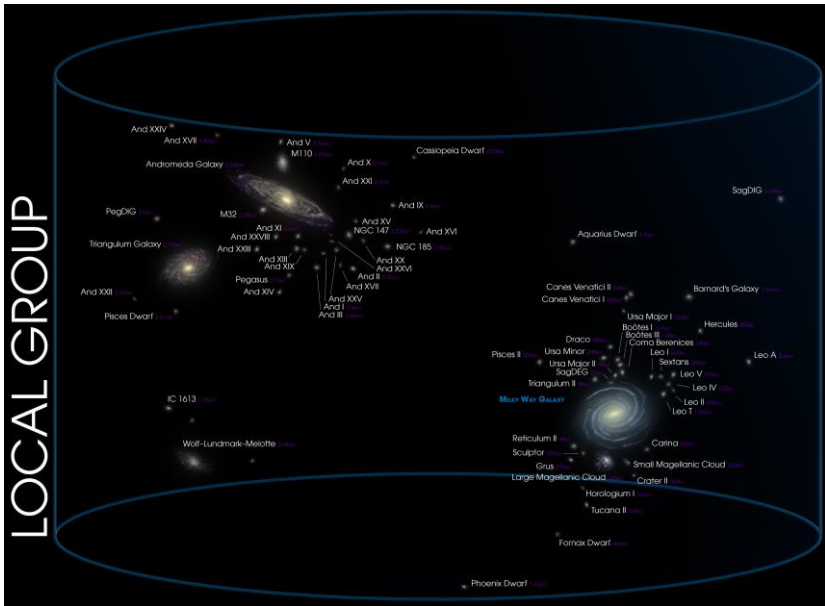
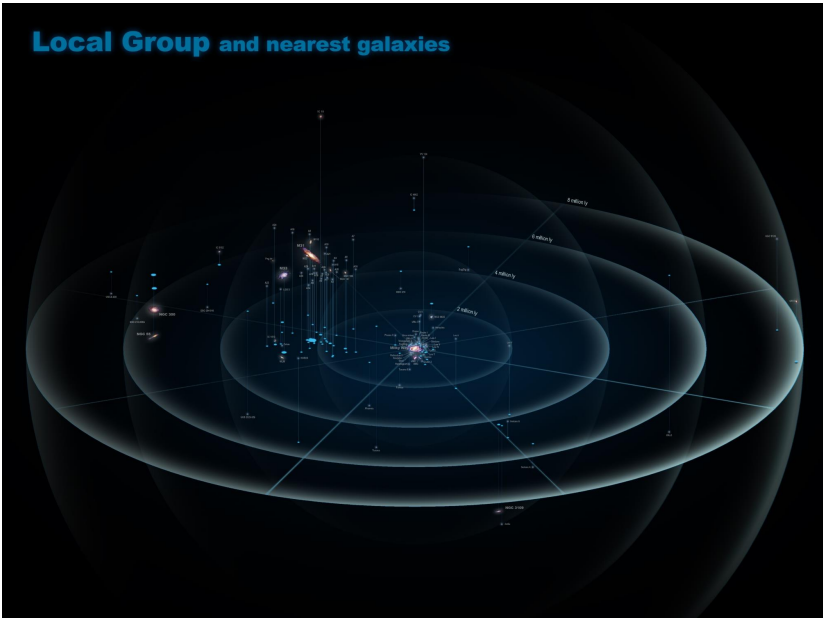


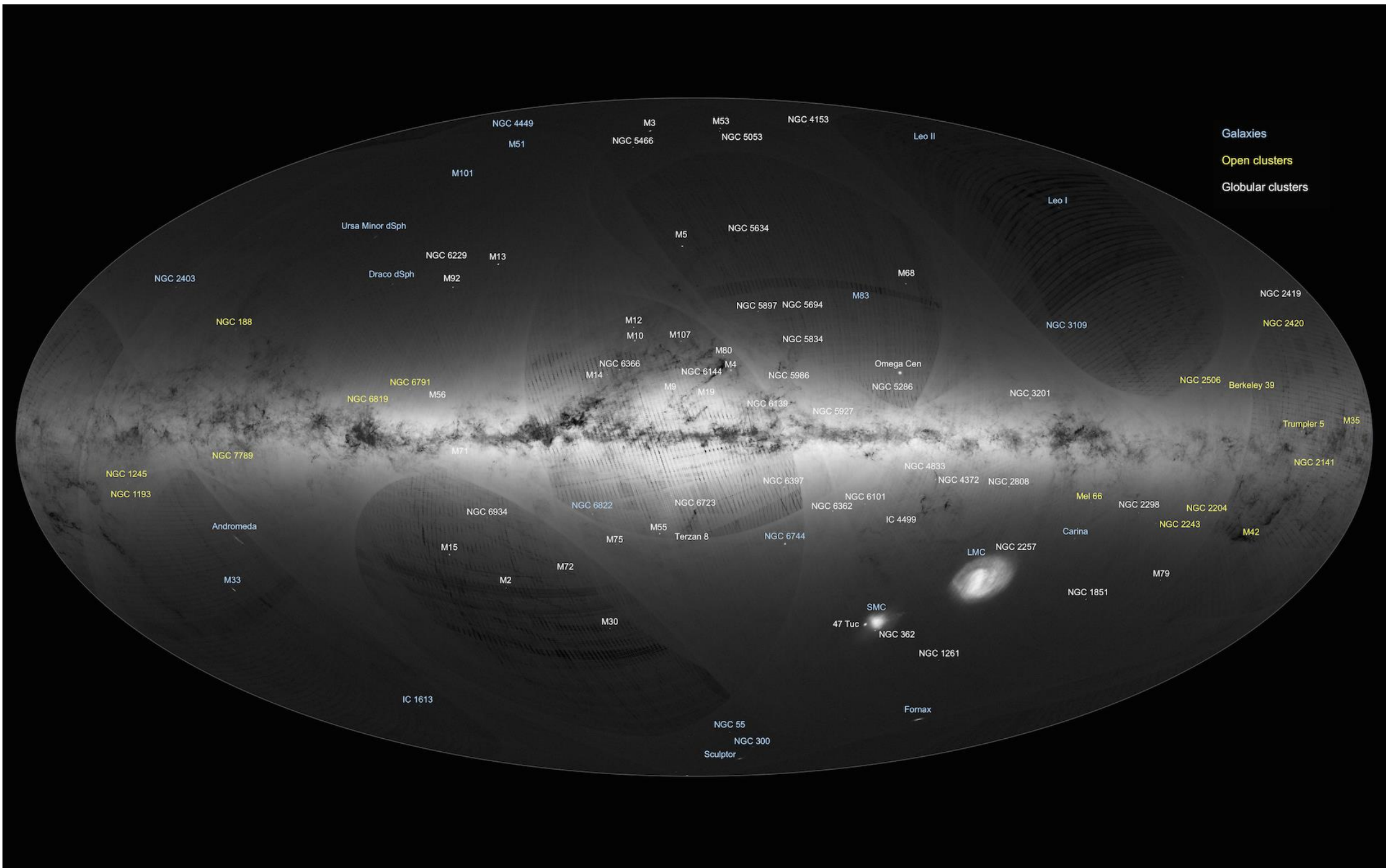
# LOCAL GROUP





Local Group and nearest galaxies



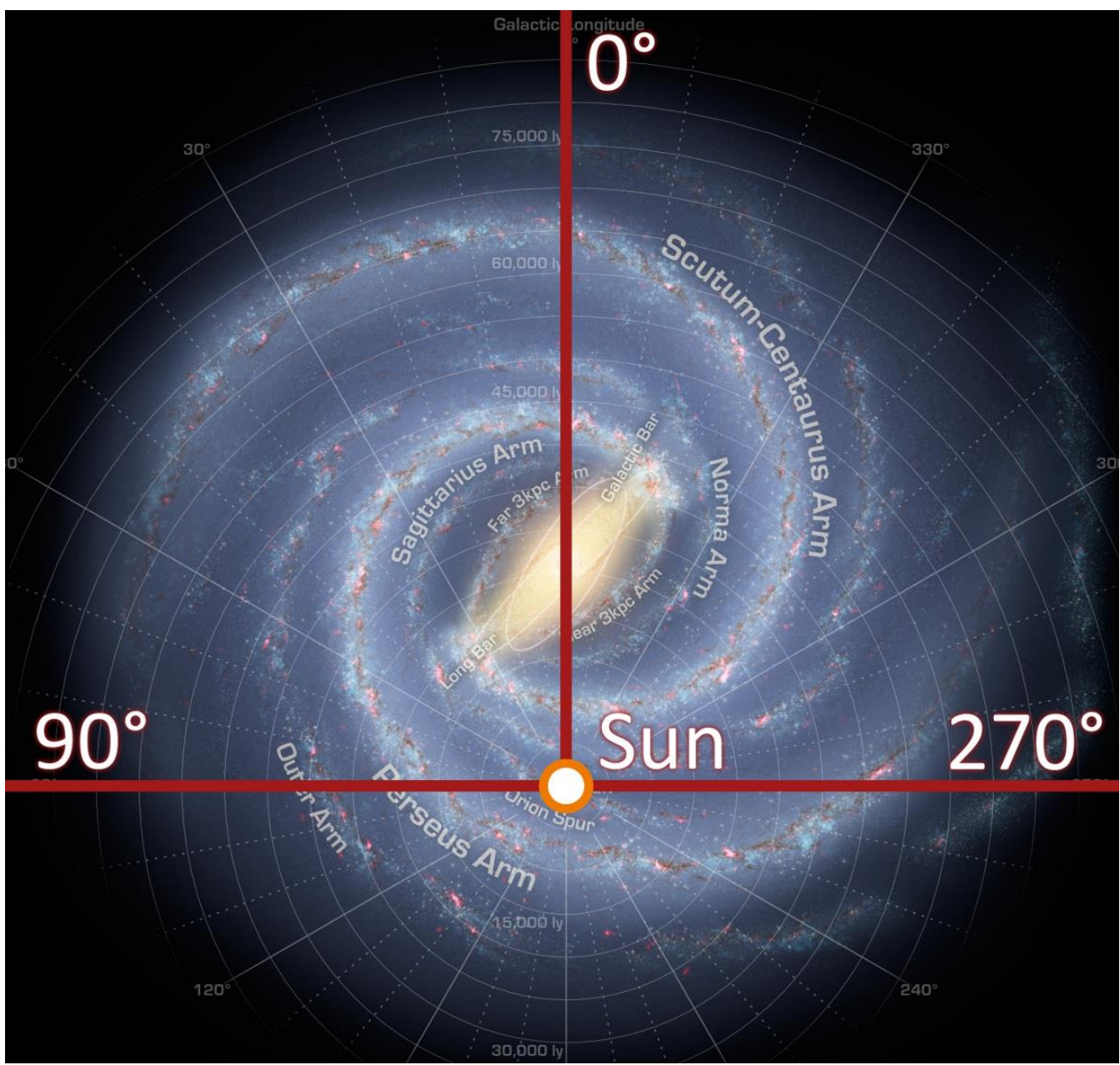
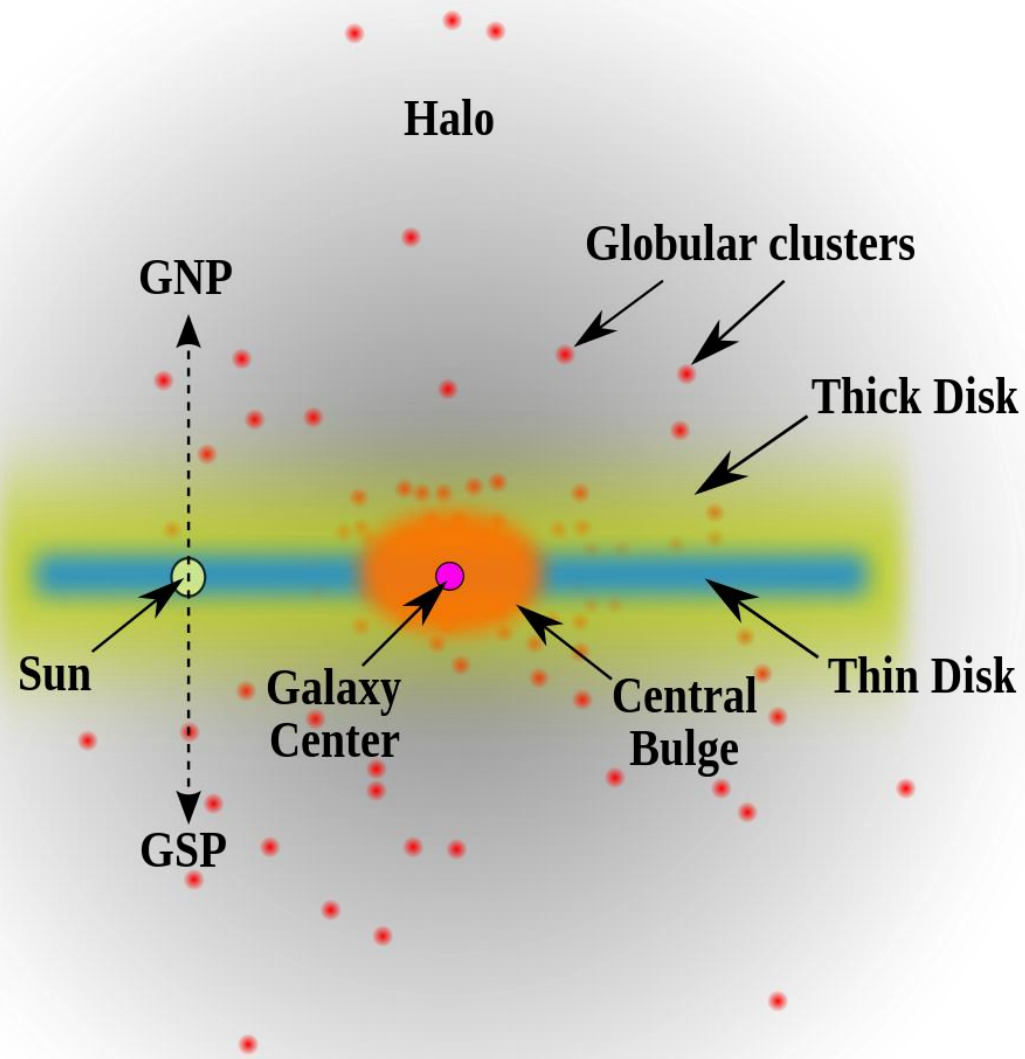


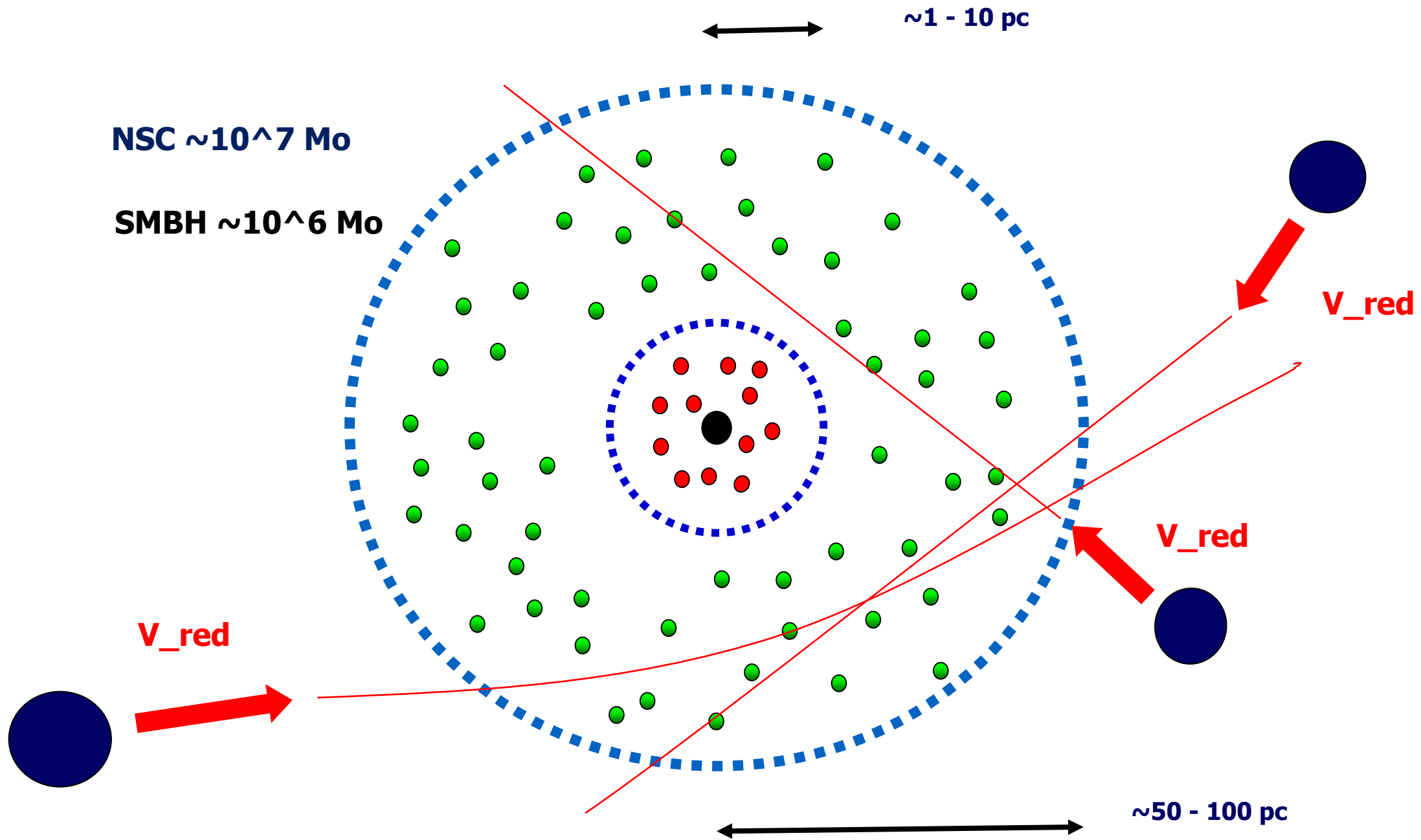
Galaxies

Open clusters

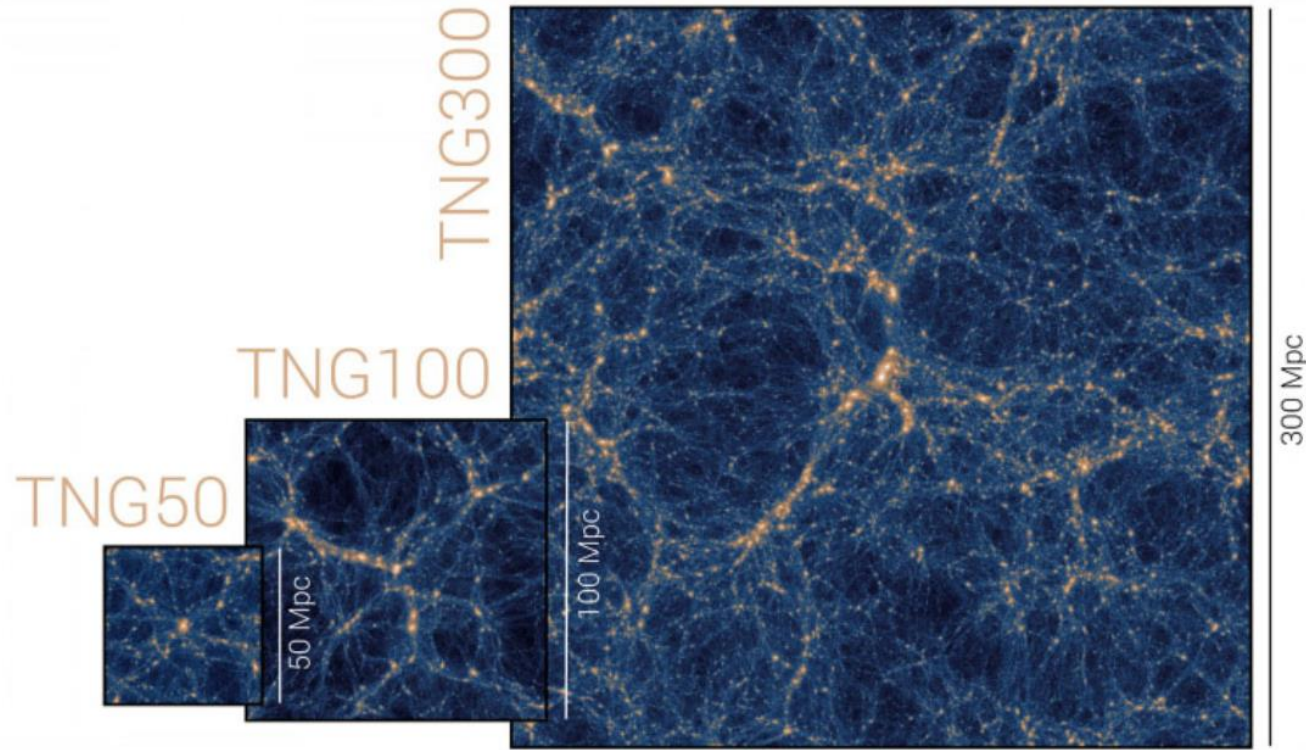
Globular clusters

NGC 4449 M3 M53 NGC 4153  
M51 NGC 5466 NGC 5053 Leo II  
M101  
Ursa Minor dSph M5 NGC 5634 Leo I  
NGC 6229 M13  
Draco dSph M92 M68  
NGC 2403 NGC 5897 NGC 5694 M83 NGC 2419  
NGC 188 M12 M10 M107 M80 NGC 5834 NGC 3109 NGC 2420  
NGC 6791 M14 NGC 6366 NGC 6144 NGC 5986 Omega Cen NGC 2506 Berkeley 39  
NGC 6819 M56 M9 M19 NGC 6139 NGC 5927 NGC 3201 Trumpler 5 M35  
NGC 1245 M71 NGC 4833 NGC 4372 NGC 2808 NGC 2141  
NGC 1193 NGC 6934 NGC 6822 NGC 6723 NGC 6101 NGC 6362 IC 4499 Mel 66 NGC 2298 NGC 2204  
Andromeda M15 M75 M55 Terzan 8 NGC 6744 LMC NGC 2257 Carina NGC 2243 M42  
M33 M2 M72 M79  
M30  
IC 1613  
NGC 55  
NGC 300  
Sculptor  
SMC  
47 Tuc NGC 362  
NGC 1261  
Fornax

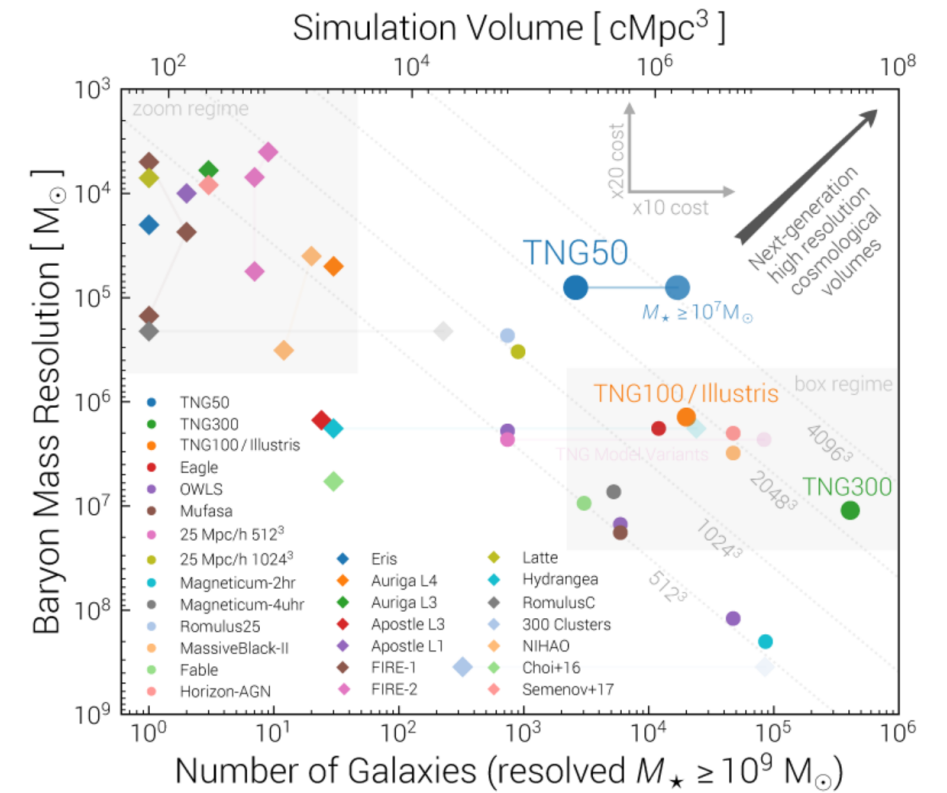




-The TNG Collaboration



	TNG50	TNG100	TNG300
Volume [Mpc <sup>3</sup> ]	51.7 <sup>3</sup>	110.7 <sup>3</sup>	302.6 <sup>3</sup>
$L_{\text{box}}$ [Mpc/h]	35	75	205
$N_{\text{GAS}}$	-	2160 <sup>3</sup>	2500 <sup>3</sup>
$N_{\text{DM}}$	-	2160 <sup>3</sup>	2500 <sup>3</sup>
$N_{\text{TR}}$	-	2160 <sup>3</sup>	2500 <sup>3</sup>
$m_{\text{baryon}}$ [M <sub>⊙</sub> ]	$8.5 \times 10^4$	$1.4 \times 10^6$	$1.1 \times 10^7$
$m_{\text{DM}}$ [M <sub>⊙</sub> ]	$4.5 \times 10^5$	$7.5 \times 10^6$	$5.9 \times 10^7$
$\epsilon_{\text{gas,min}}$ [pc]	74	185	370
$\epsilon_{\text{DM},\star}$ [pc]	288	740	1480



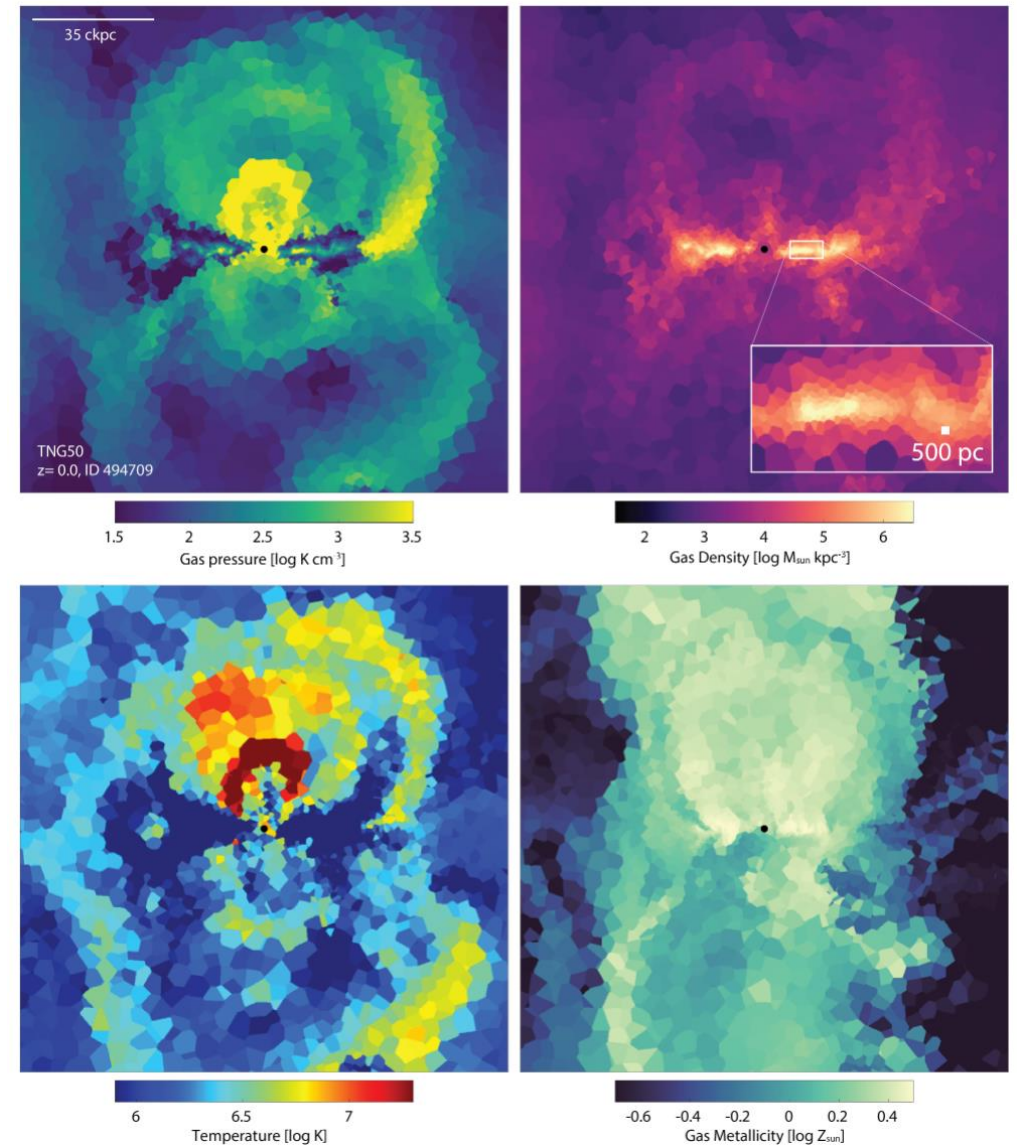
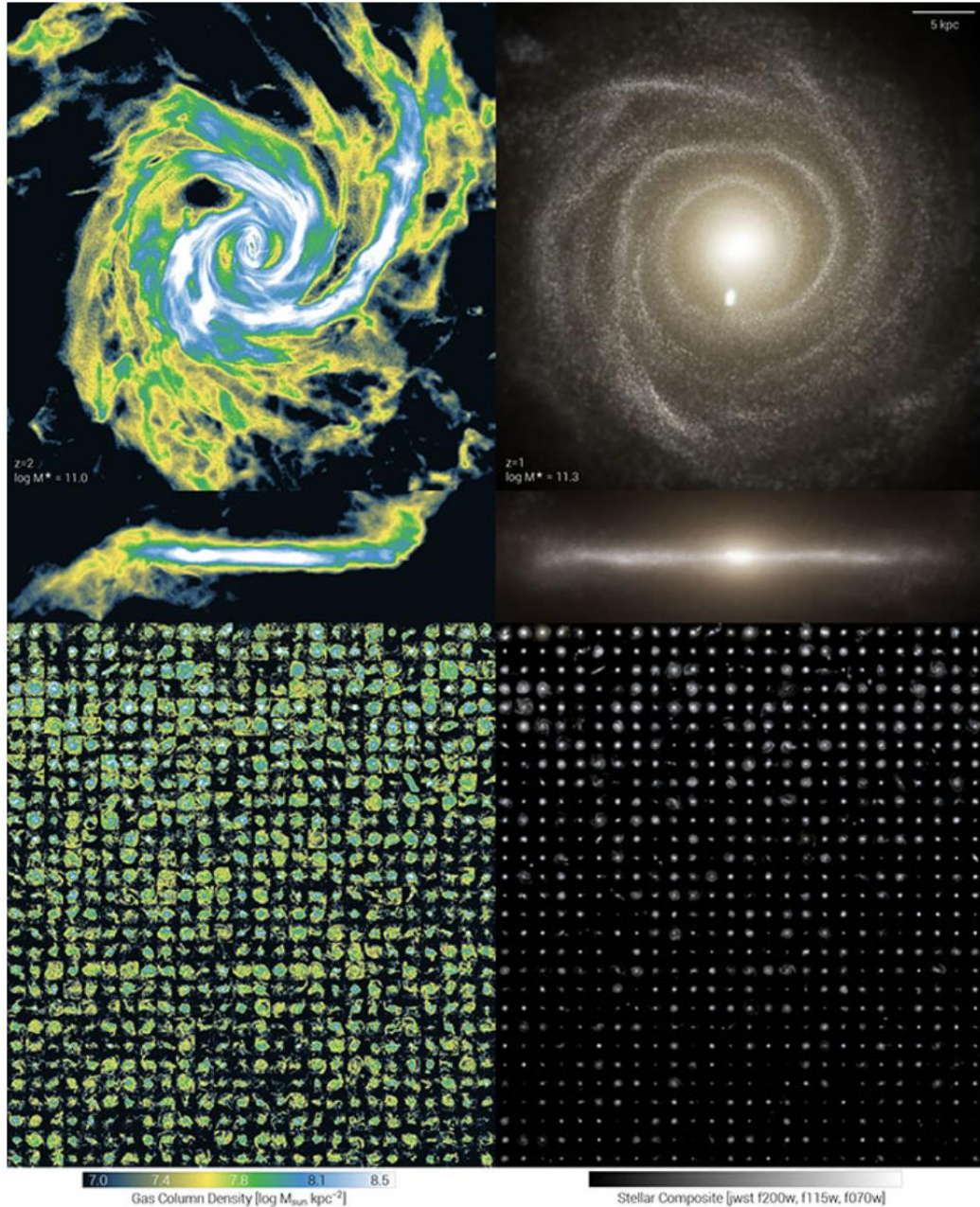
**Volker Springel**

Heidelberg Institute for Theoretical Studies → MPA  
PI: Overall TNG Project



**Lars Hernquist**

Harvard University

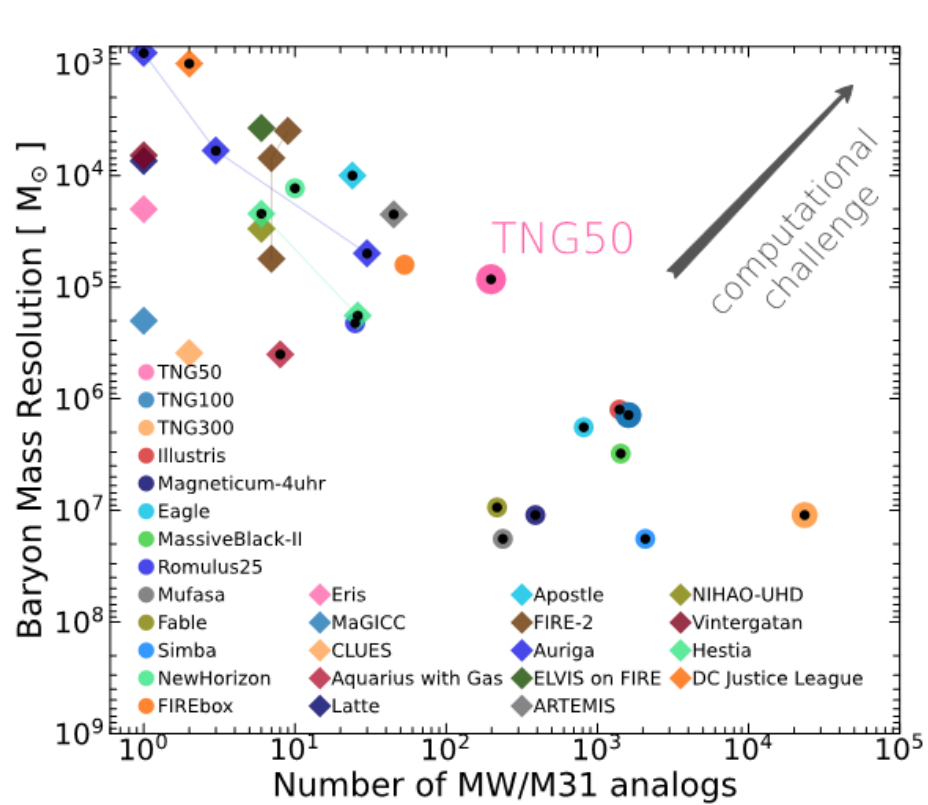


The gaseous (left) and stellar (right) structure of TNG50 galaxies at high redshift. The top shows a massive disk galaxy at  $z=2$  and a descendant at  $z=1$ , while the bottom montage reveals  $\sim 750$  central galaxies at redshift two, from large ellipticals at the center of galaxy groups to smaller, dwarf systems. [\[large\]](#) [\[ref\]](#)

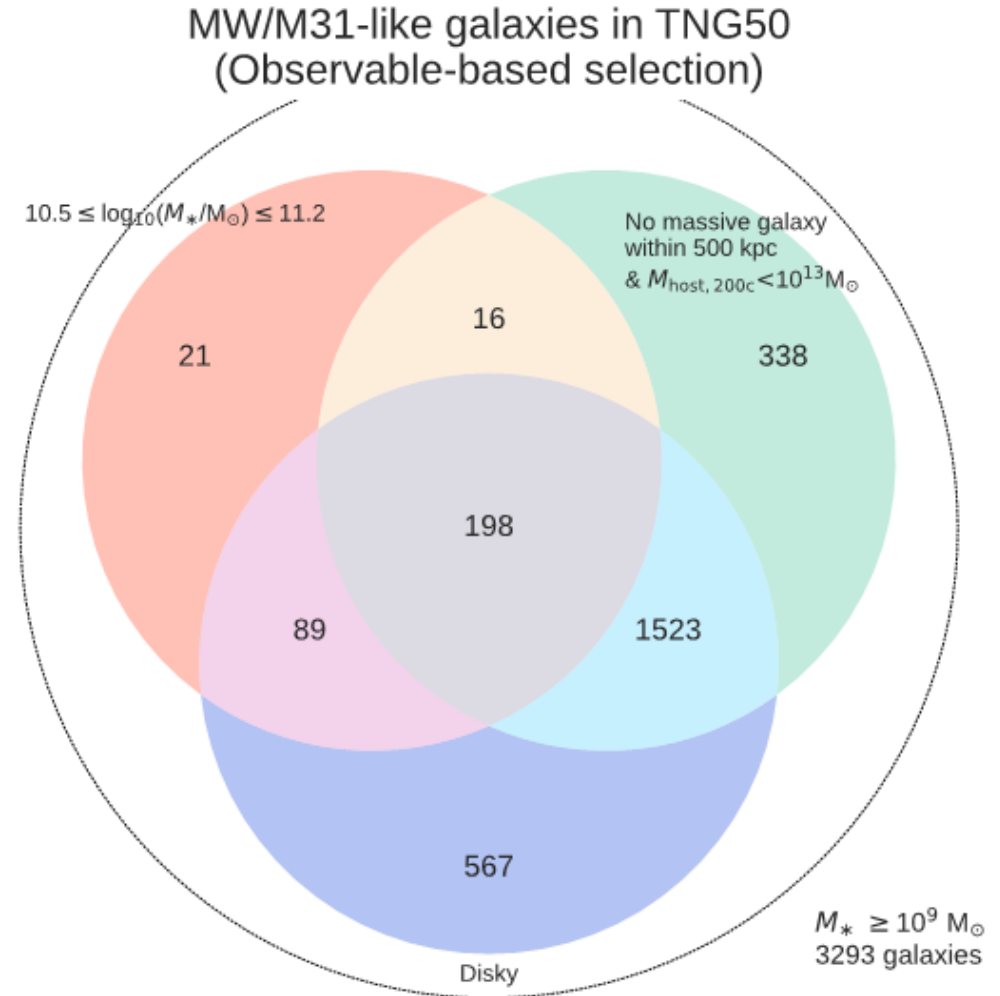
A outflow-driven "bubble" produced by supermassive black hole feedback, around a disk galaxy similar to our own Milky Way at redshift zero, found within the TNG50 simulation. This structure is similar to those observed in gamma-rays by the Fermi telescope, and in x-rays by the eROSITA mission, in our own Galaxy. [\[large\]](#) [\[ref\]](#)

# Milky Way and Andromeda analogs from the TNG50 simulation

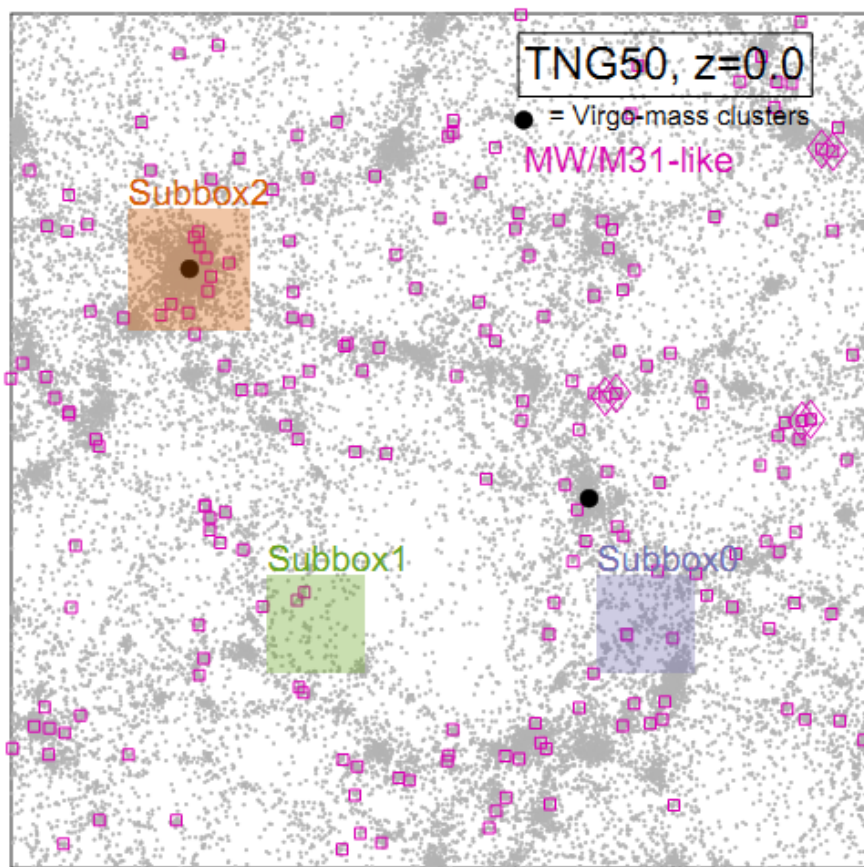
Annalisa Pillepich<sup>1,\*</sup>, Diego Sotillo-Ramos<sup>1</sup>, Rahul Ramesh<sup>2</sup>, Dylan Nelson<sup>2</sup>, Christoph Engler<sup>1</sup>  
 Vicente Rodriguez-Gomez<sup>3</sup>, Martin Fournier<sup>1</sup>, Martina Donnari<sup>1</sup>, Volker Springel<sup>4</sup>, and Lars Hernquist<sup>5</sup>



TNG100	2017	AREPO	unif.res.	yes	1606	$1.4 \times 10^6$
TNG50	2019	AREPO	unif.res.	yes	198 <sup>□</sup>	$8.5 \times 10^4$



**Figure 1. The landscape of  $\Lambda$ CDM cosmological simulations of MW/M31-like galaxies as of 2023.** We contrast numerical projects in terms of baryonic mass resolution vs. number of simulated MW/M31 analogs. The TNG50 simulation this paper focuses on (magenta circle) is compared to state-of-the-art cosmological zoom-in simulations (diamonds) and so-called box or uniform-resolution large-volume simulations (circles): this comparison is deliberately similar to that in Fig. 1 of Nelson et al. (2019b) but focuses on MW/M31-

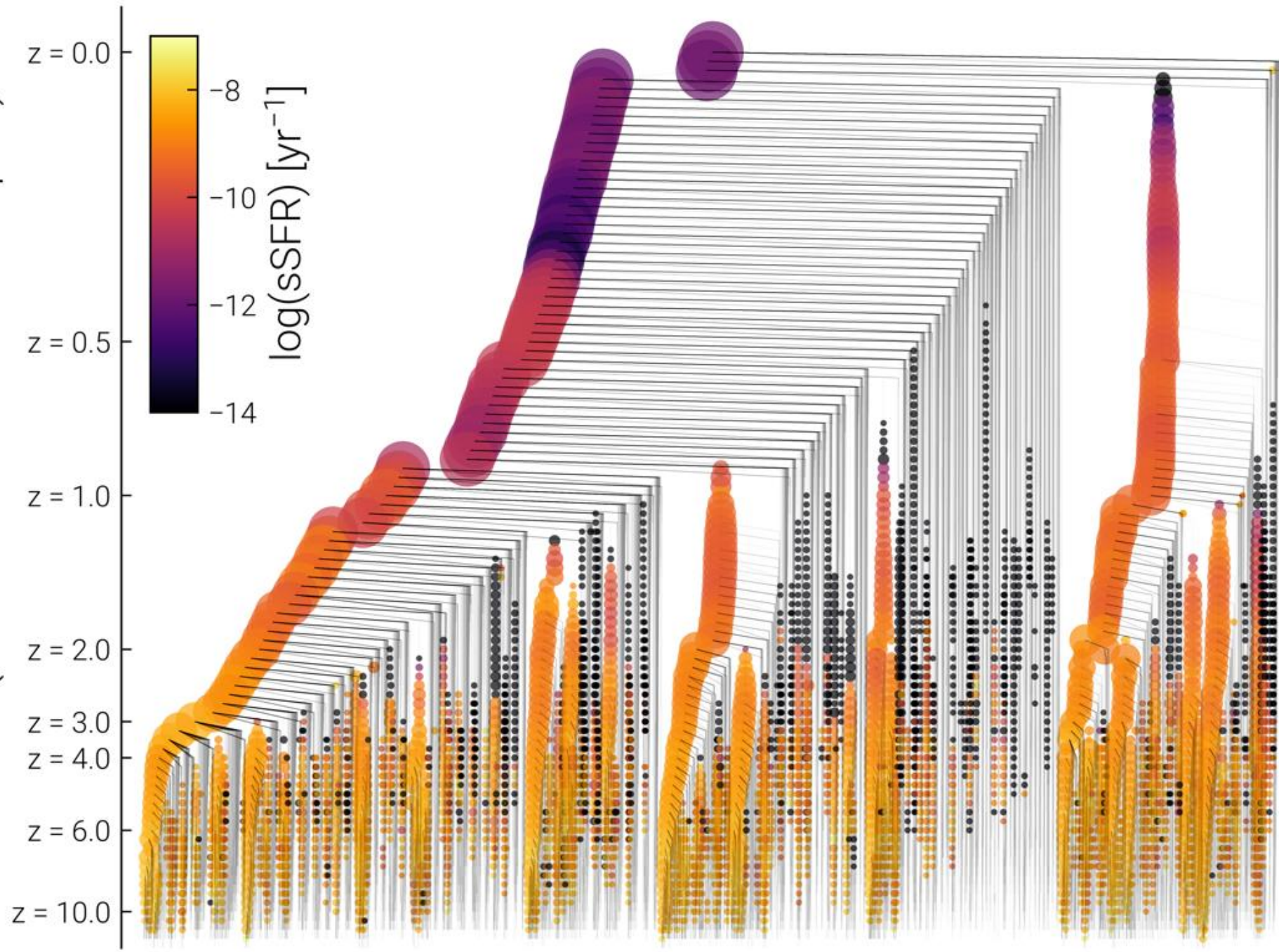


**Figure 2.** Example galaxies from the TNG50 simulation at  $z = 0$  with stellar mass and/or disk structure similar to the Milky Way or to Andromeda, among the 198 TNG50 MW/M31-like galaxies. These are shown in random projections, across 70 kpc a side, in a composite stellar light image of three HST/ACS bands (F435W, F606W, F775W; 0.049 arcsec/pixel) including radiative transfer in post-process with SKIRT (Camps & Baes 2015) and placing the galaxies at  $z \sim 0.08$ . A large gallery of all the TNG50 MW/M31-like galaxies can be found at <https://www.tng-project.org/explore/gallery/pillepich23a>.

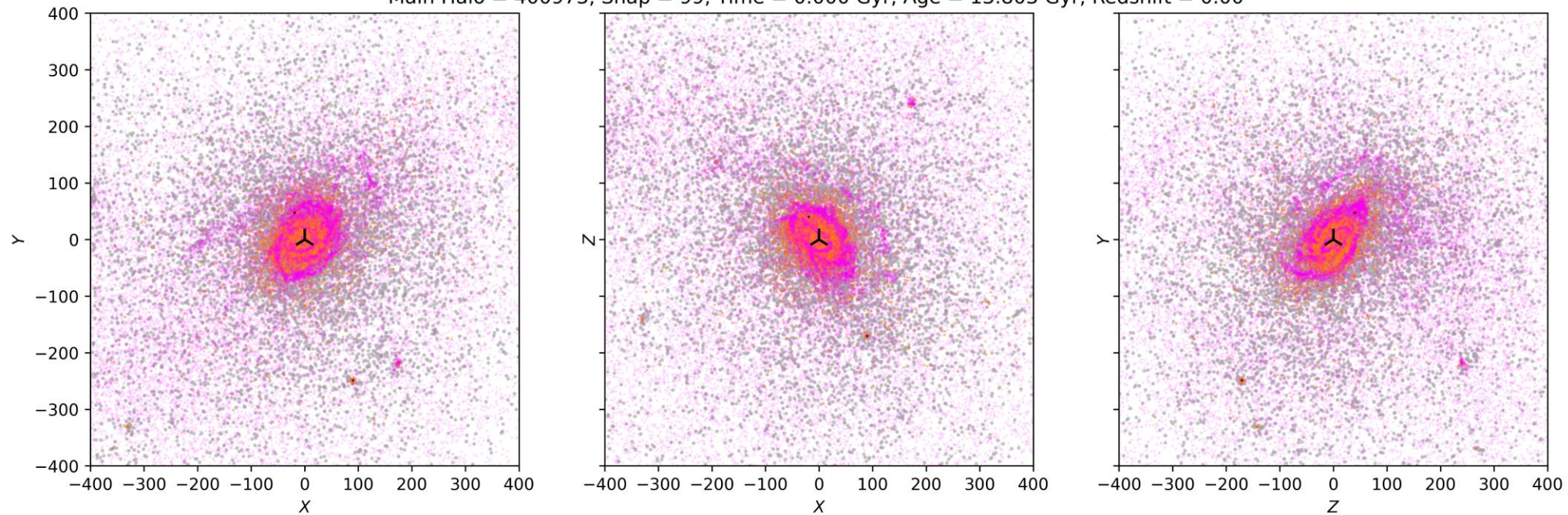
**Figure 6.** Location and large-scale environment of the 198 MW/M31-like galaxies (magenta) within the whole TNG50 simulated volume. The large-scale structure throughout the comoving volume of TNG50 (which spans about 52 comoving Mpc a side) is here depicted at  $z = 0$  through the position of TNG50 galaxies more massive than  $10^6 M_\odot$  in stars (gray circles). This is a random projection, along the  $z$ -axis of the box, and also shows the location of the two Virgo-mass clusters in the simulation (large black circles). A handful of the selected objects are within the so-called Subboxes (shaded comoving cubes), whereby data is available every few million years (see Section A and Table A1). Three pairs compose Local-Group analogs (magenta diamonds).



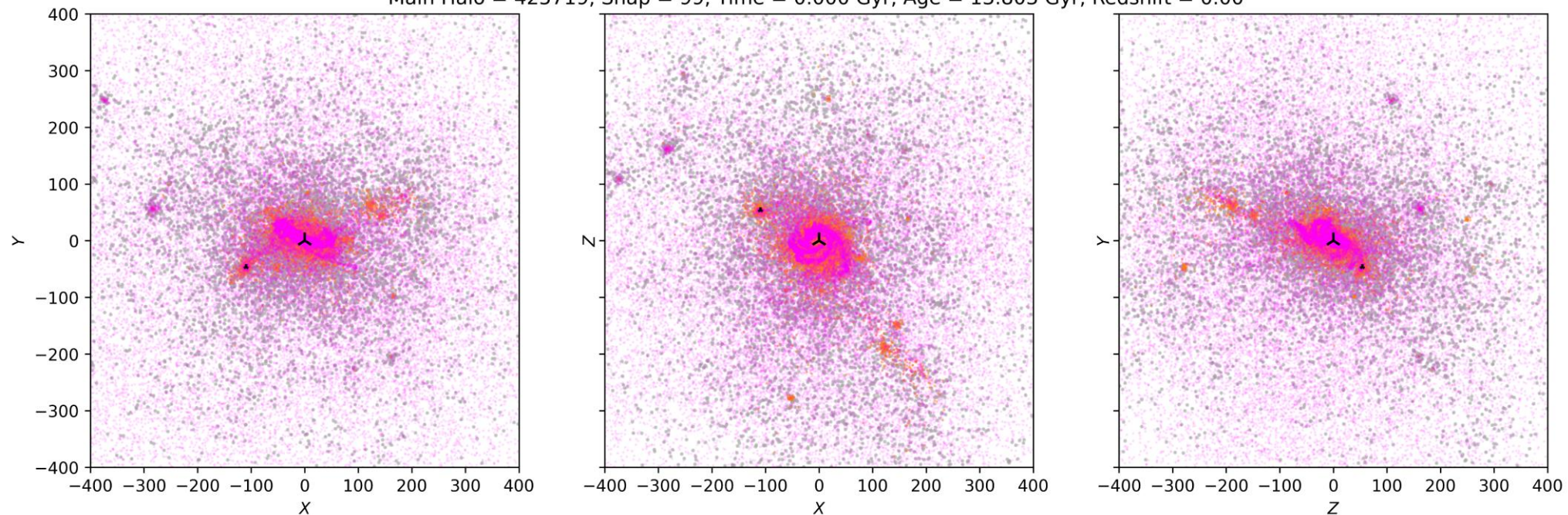
TNG50-1 (subhaloID = 400973 at snap 99)



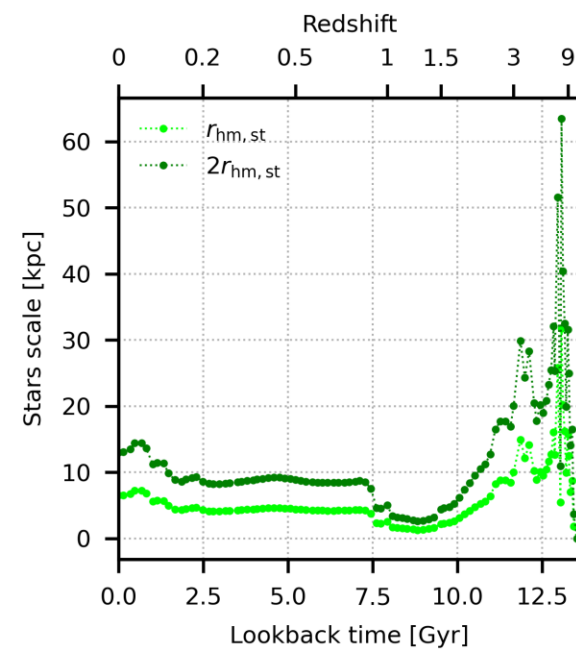
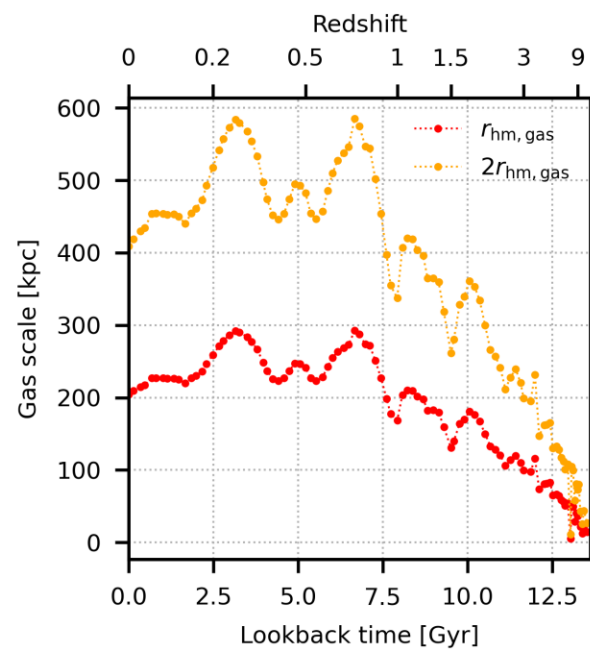
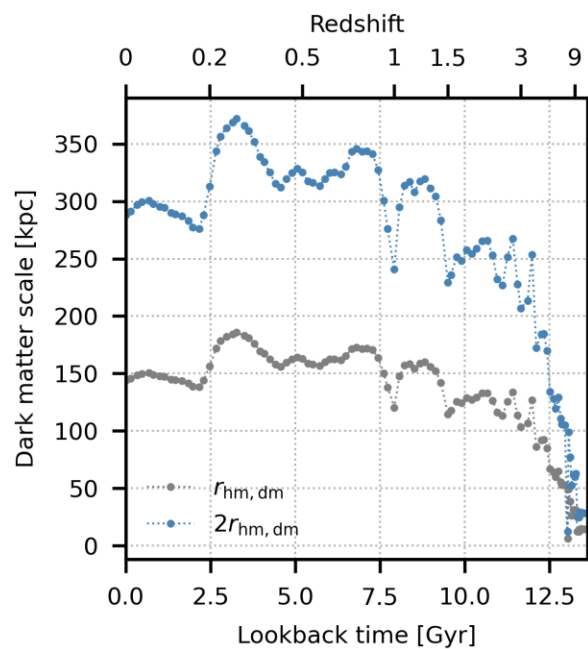
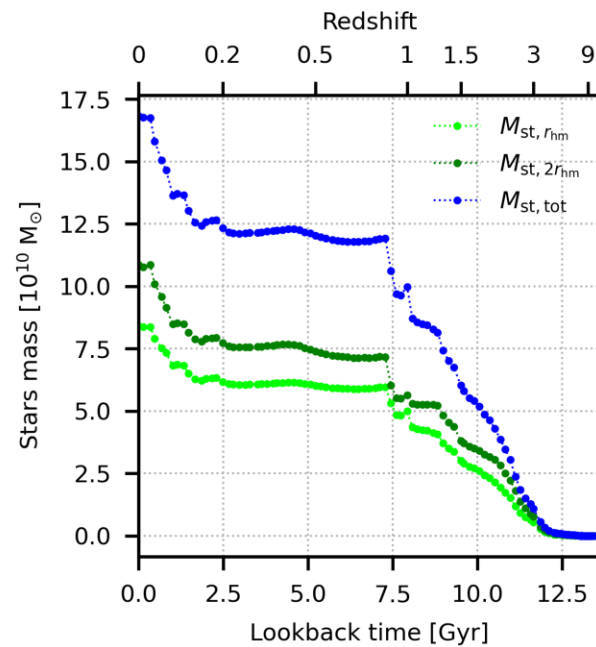
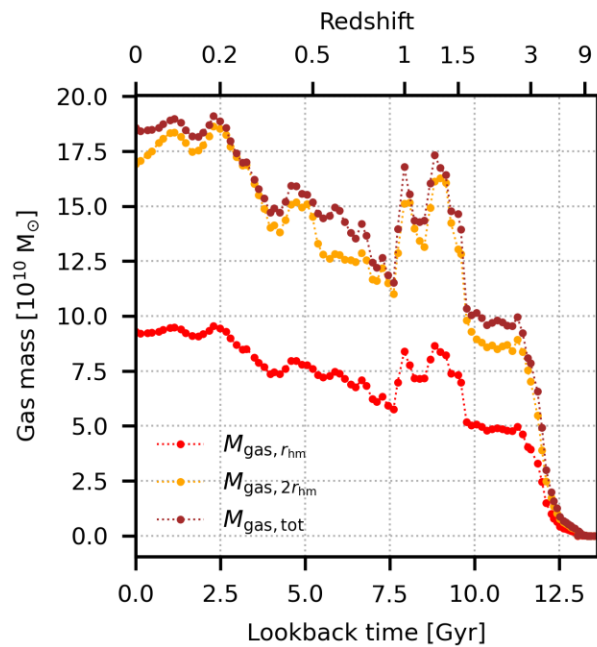
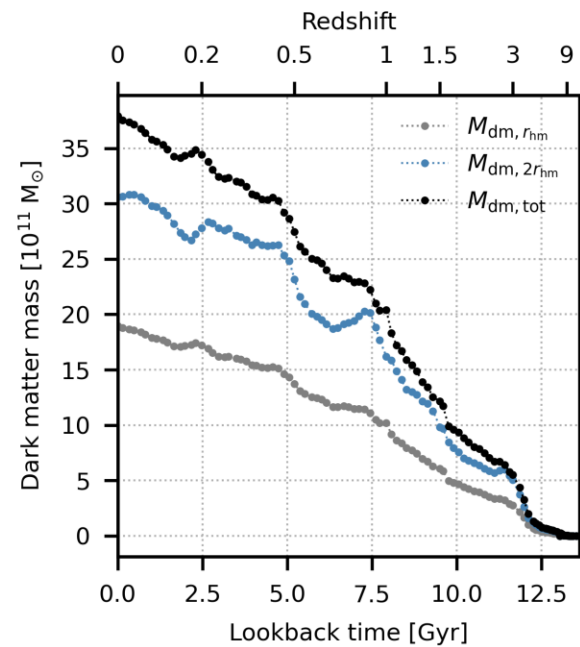
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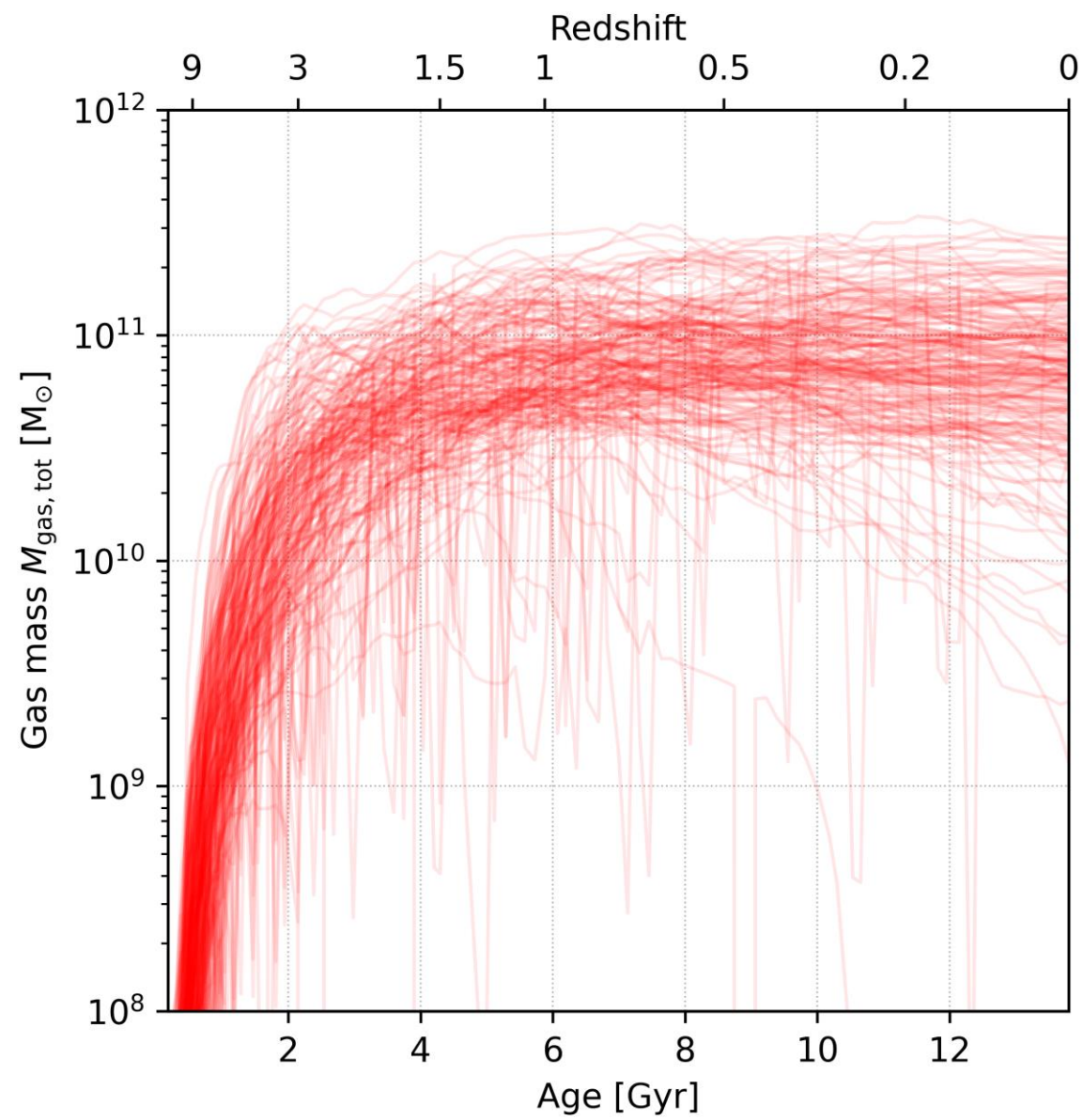
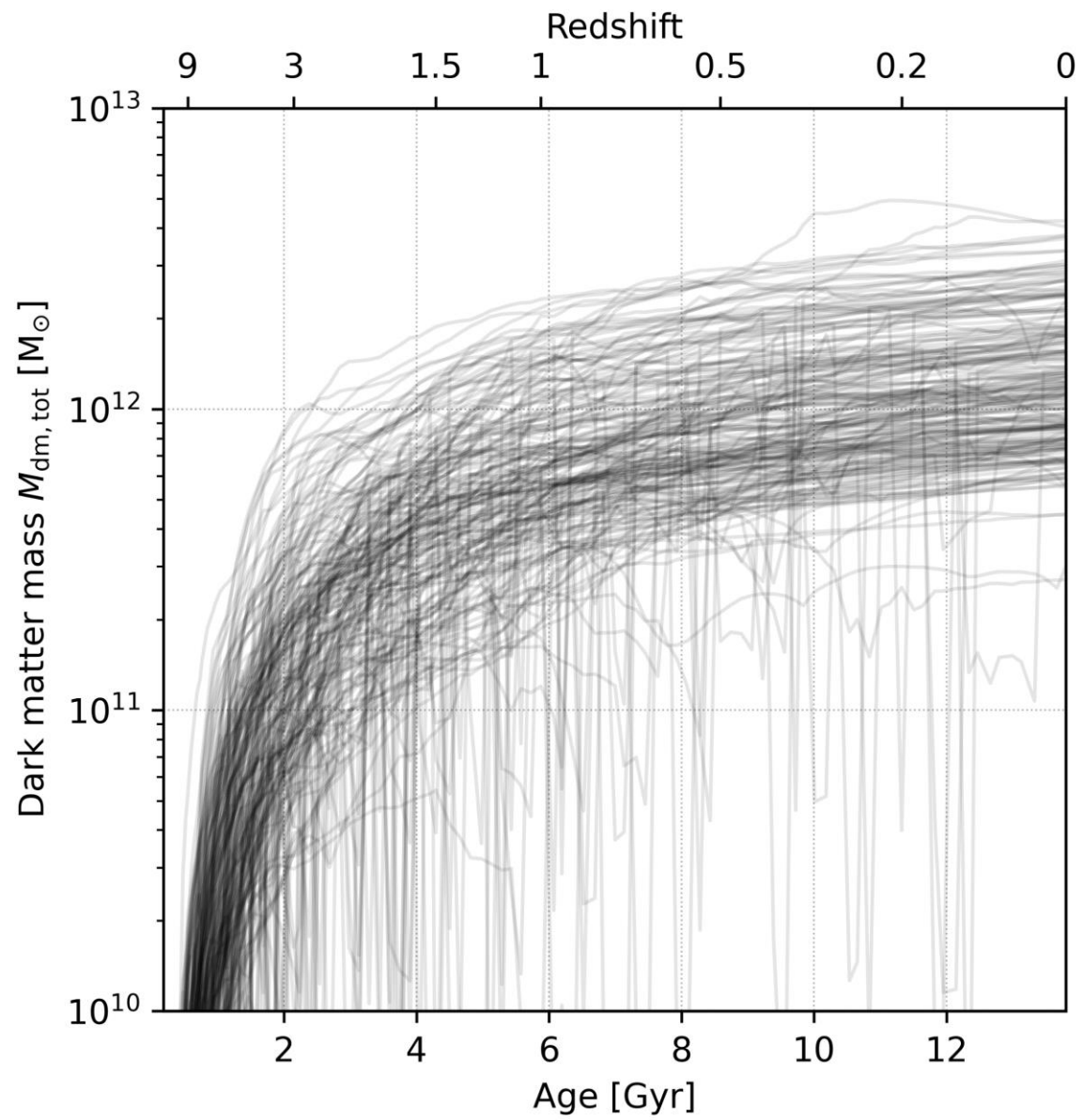


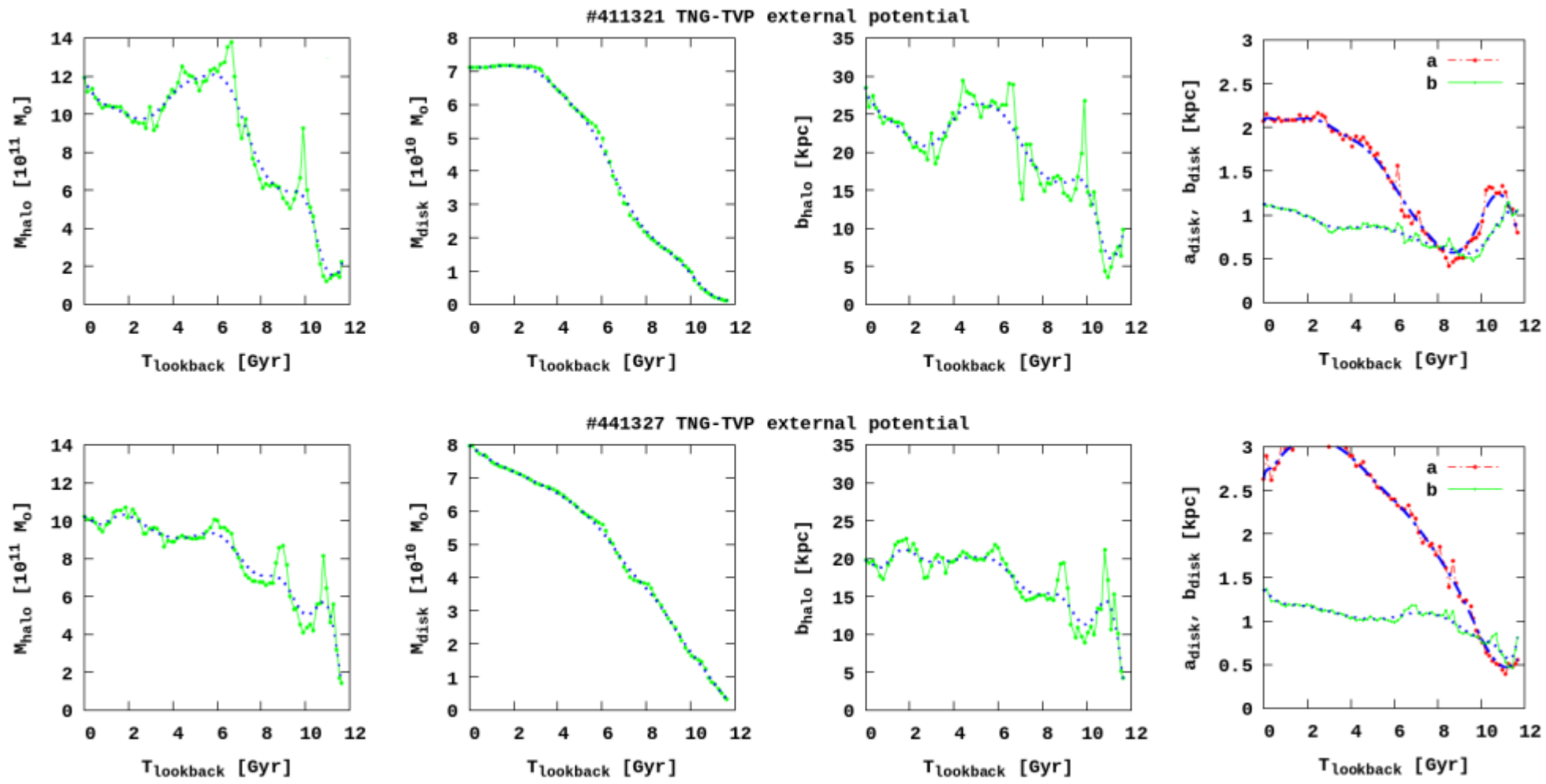
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## Main Halo = 400973







Mardini et al. 2020

<https://github.com/Mohammad-Mardini/The-ORIENT>

$$\begin{aligned}
 \Phi_{\text{tot}} &= \Phi_{\text{d}}(R, z) + \Phi_{\text{h}}(R, z) = \\
 &= -\frac{GM_{\text{d}}}{\sqrt{R^2 + (a_{\text{d}} + \sqrt{z^2 + b_{\text{d}}^2})^2}} - \frac{GM_{\text{h}} \cdot \ln\left(1 + \frac{\sqrt{R^2 + z^2}}{b_{\text{h}}}\right)}{\sqrt{R^2 + z^2}}, \quad (1)
 \end{aligned}$$

**Table 1.** Parameters of the time-varying potentials selected from the IllustrisTNG-100 simulation at redshift zero. The last column shows the parameters of the corresponding MW components according to [Bennett et al. \(2022\)](#) at present.

Parameter	Unit	#411321	#441327	#451323	#462077	#474170	Milky Way
Disk mass, $M_d$	$10^{10} M_\odot$	7.110	7.970	7.670	7.758	5.825	6.788
Halo mass, $M_h$	$10^{12} M_\odot$	1.190	1.020	1.024	1.028	0.898	1.000
Disk scale length, $a_d$	1 kpc	2.073	2.630	2.630	1.859	1.738	3.410
Disk scale height, $b_d$	1 kpc	1.126	1.356	1.258	1.359	1.359	0.320
Halo scale height, $b_h$	10 kpc	2.848	1.981	2.035	2.356	1.858	2.770

[Massari et al. 2019; Malhan et al. 2022](#)

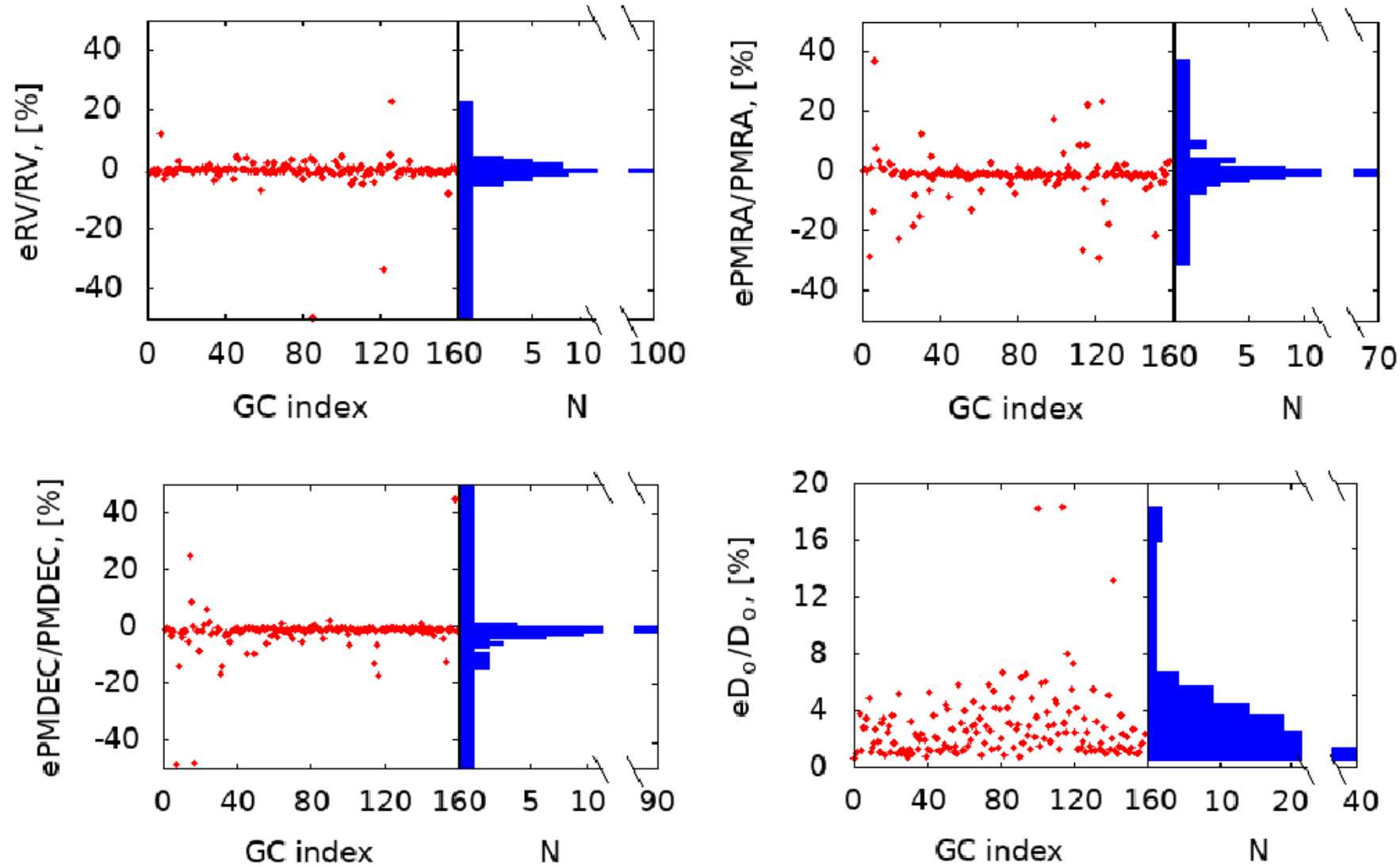
[Armstrong et al. 2021; Haghi et al. 2015](#)

[Bajkova et al. 2021](#)

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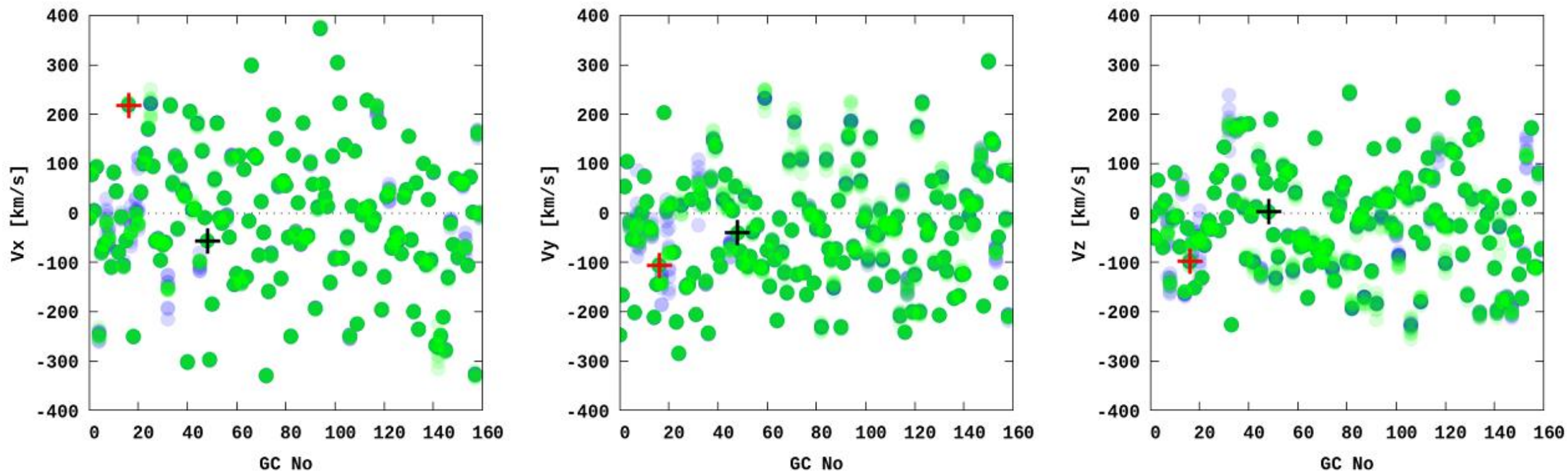
To transform the positions and velocities into the Cartesian galactocentric rest frame (see [Johnson & Soderblom 1987; Bovy 2011](#), for the coordinate transformation equations), we assumed a galactocentric distance of the Sun  $R_\odot = 8.178$  kpc ([Gravity Collaboration et al. 2019; Reid & Brunthaler 2004](#)), a height above the Galactic plane  $Z_\odot = 20.8$  pc ([Bennett & Bovy 2019](#)), and the velocity of the local standard of rest (LSR)  $V_{\text{LSR}} = 234.737$  km s<sup>-1</sup> ([Bovy et al. 2012; Drimmel & Poggio 2018](#)). Accordingly, the Sun is located at  $X_\odot = -8178$  pc,  $Y_\odot = 0$  pc and  $Z_\odot = 20.8$  pc in our Cartesian galactocentric coordinate system. For the peculiar velocity of the Sun, we used the following values with respect to the LSR:  $U_\odot = 11.1$  km s<sup>-1</sup>,  $V_\odot = 12.24$  km s<sup>-1</sup>, and  $W_\odot = 7.25$  km s<sup>-1</sup> ([Schönrich et al. 2010](#)).

[https://people.smp.uq.edu.au/HolgerBaumgardt/globular/orbits\\_table.txt](https://people.smp.uq.edu.au/HolgerBaumgardt/globular/orbits_table.txt)



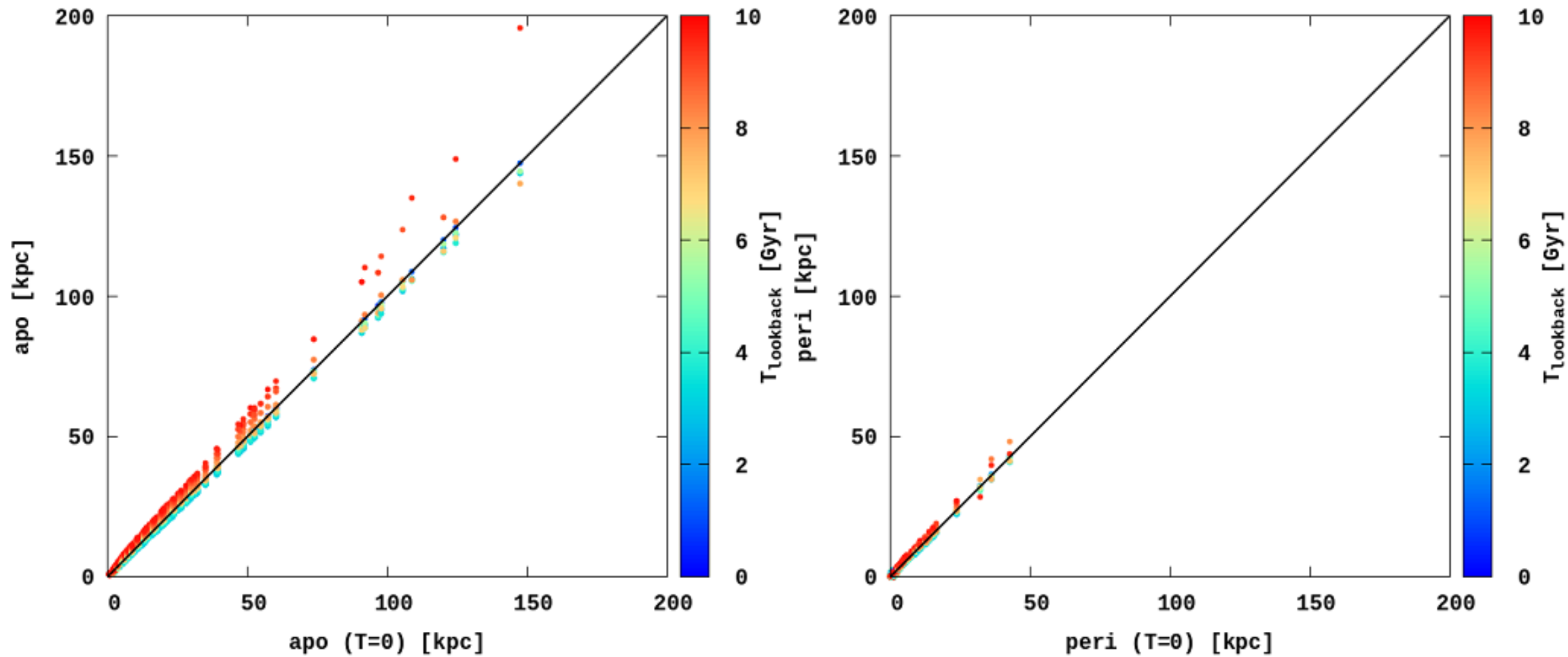
Baumgardt & Vasiliev (2021)

**Fig. 1.** Distribution of the relative errors (*from left to right*): Radial velocity ( $eRV$ ), proper motions in right ascension ( $ePMRA$ ), and declination ( $ePMDEC$ ), and in heliocentric distance  $eD_0$ . Blue histograms represent the error distributions. The following GCs are not shown because the error values are too high: Crater, Lae 3, NGC 6760, BH 261, NGC 6553, and Pal 13, except in the graph in the bottom right panel.

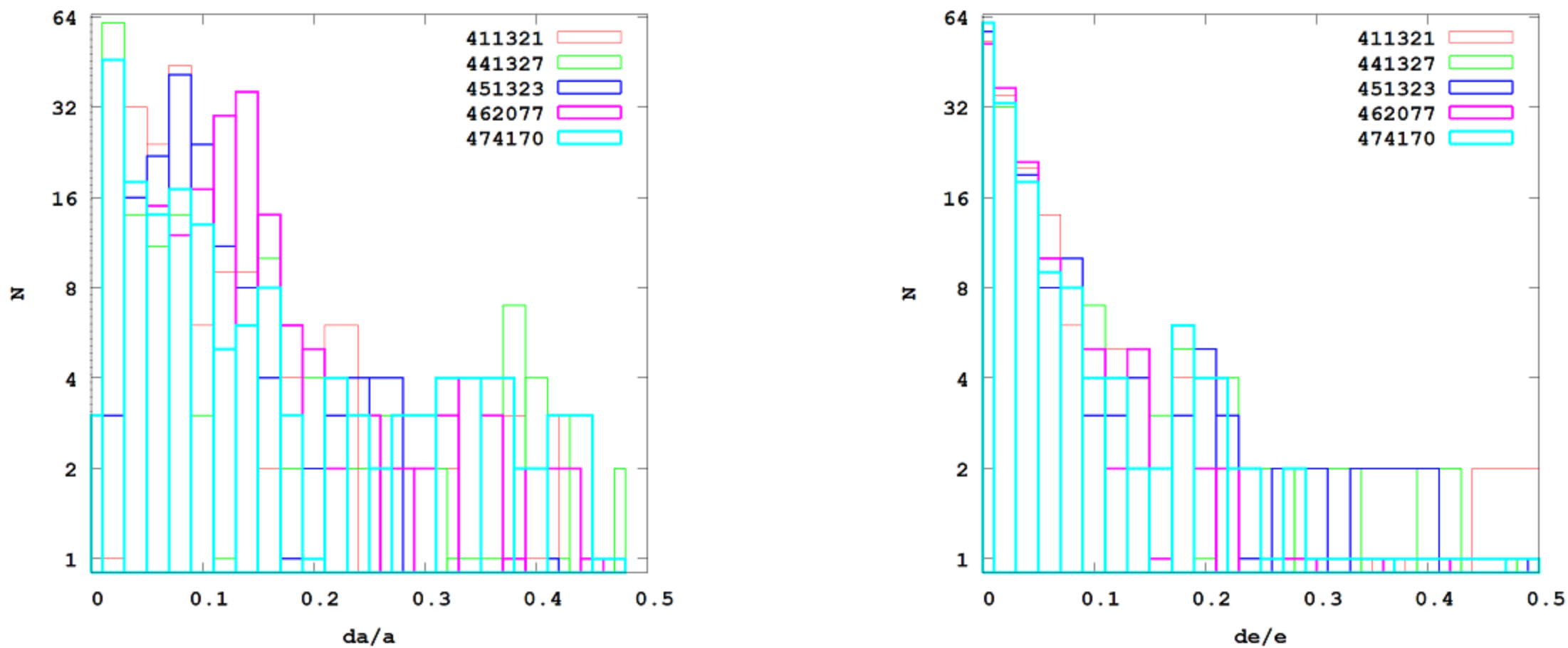


**Fig. 3.** Influence of the error randomisation on the initial velocity components in the galactocentric Cartesian reference frame. The distance determination is shown by green circles and the velocity determination by blue circles. The red cross marks Crater, and the black cross indicates NGC 6121.

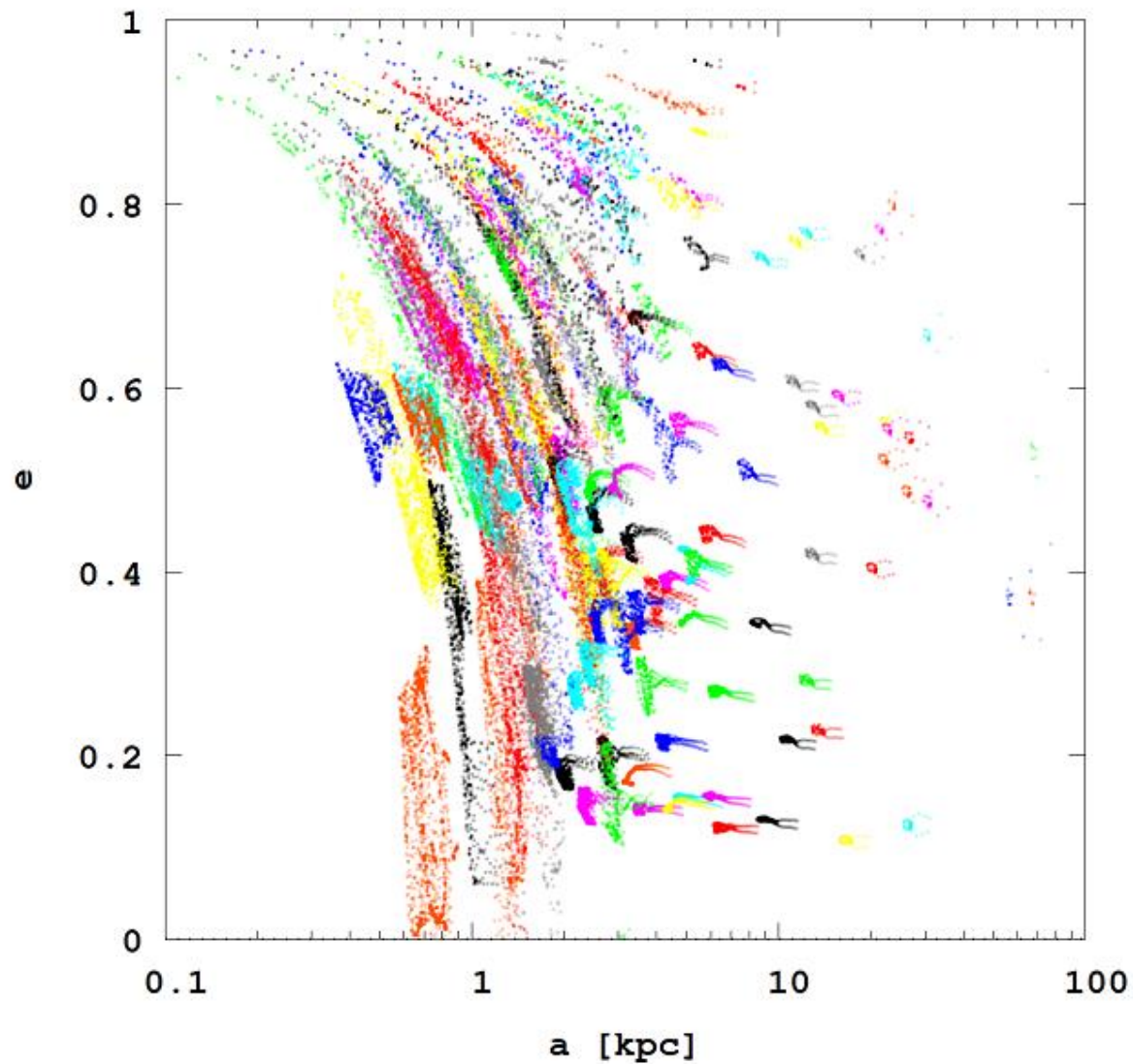




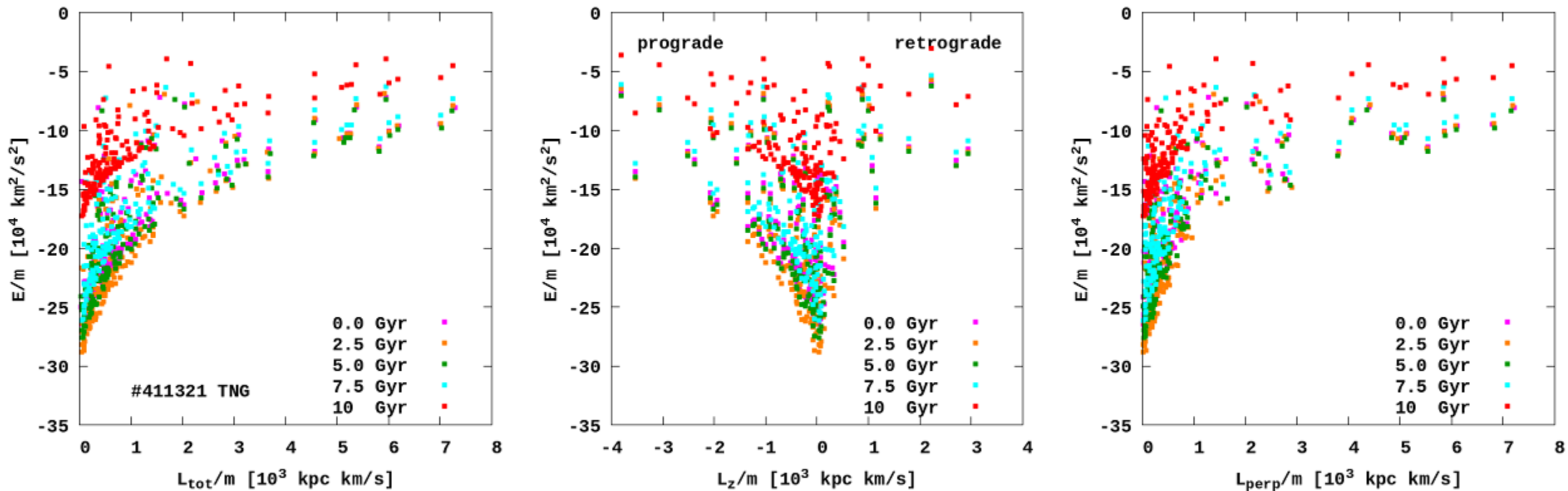
**Fig. 7.** Evolution of the apocenters (*left panel*) and pericenters (*right panel*) for the 159 GCs in the 411321 TNG-TVP. The colour-code corresponds to the lookback time.



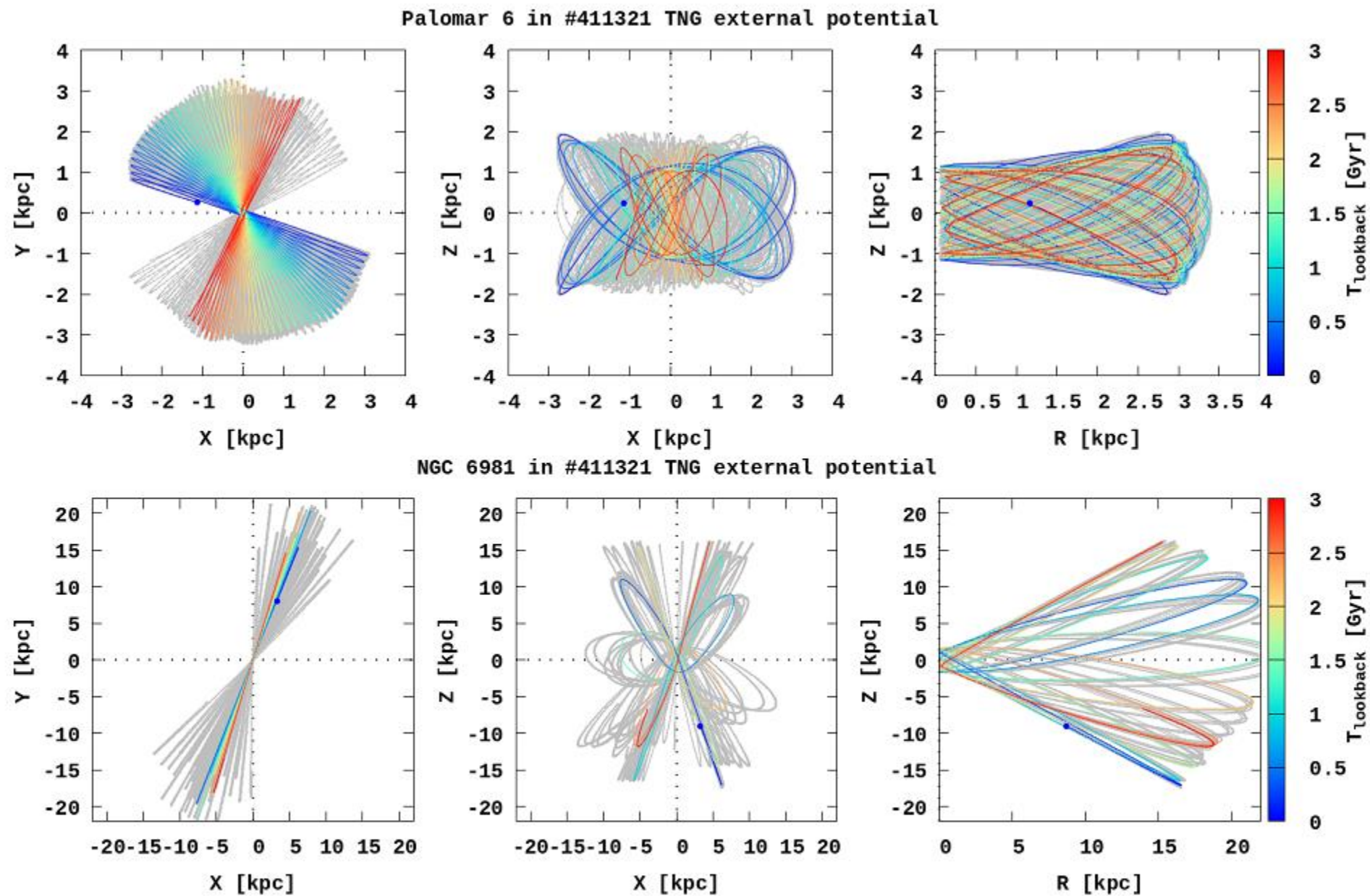
**Fig. 5.** Relative orbital semi-major  $\Delta a/a$  (left) and eccentricity  $\Delta e/e$  (right) changes during the GC orbital evolution for all five TNG-TVPs.



**Fig. 6.** Evolution of the GC orbital semi-major and eccentricity during the whole backward integration time in the case of 411321 TNG-TVP. The inner GCs ( $a \leq 3$  kpc) have more regular and larger eccentricity changes during the evolution. The outer GCs ( $a > 3$  kpc) have much smaller eccentricity changes during the whole backward integration time.



**Fig. 14.** Evolution of the total angular momentum vs energy phase space ( $L_{\text{tot}}, E$ ) (left panels, top to bottom),  $z$ th component of the angular momentum vs energy ( $L_z, E$ ) (middle, top to bottom), and the perpendicular component of the angular momentum vs energy ( $L_{\text{perp}}, E$ ) (right, top to bottom) for 411321, 441327, and 451323 TNG-TVPs. The colours represent data at different times in Gyr lookback time: magenta for  $T = 0$ , orange for  $T = 2.5$ , green for  $T = 5$ , cyan for  $T = 7.5$ , and red for  $T = 10$ . The GCs Pal 3, Crater, and Sagittarius II are not shown in the ( $L_{\text{tot}}, E$ ) and ( $L_{\text{perp}}, E$ ) phase spaces, and Sagittarius II is not shown in ( $L_z, E$ ) phase space because their values are extremely high.



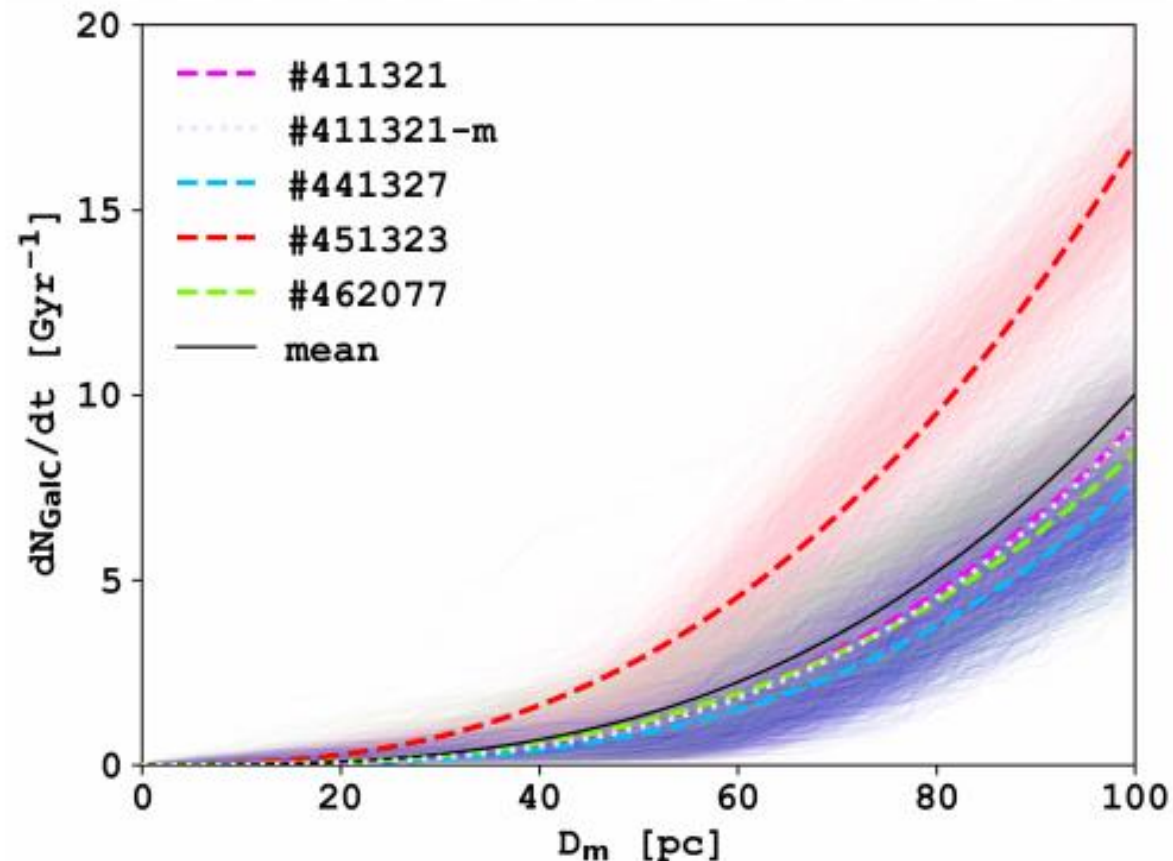
**Fig. 1.** Orbital evolution of two selected clusters: Pal 6 and NGC 6981. On the plot we show only the first 3 Gyr evolution in #411321 TNG-TVP external potential. The colour-coded line presents the orbit based on catalogue initial positions and velocities. The grey colour represents the orbits for ten different random realisations of initial data.

$$\frac{dN_{\text{GalC}}(D_m)}{dt} = 10^{a \cdot \lg(D_m) + b},$$

**Table 2.** Fitting parameters for the interaction rate GC with GalC as a function of the relative distance from the centre for four TNG-TVP external potentials.

Potential	a	b
#411321	$3.073 \pm 0.588$	$-5.181 \pm 1.205$
#411321-m*	$3.183 \pm 0.609$	$-5.404 \pm 1.247$
#441327	$3.202 \pm 0.584$	$-5.519 \pm 1.187$
#451323	$2.563 \pm 0.328$	$-3.900 \pm 0.670$
#462077	$2.897 \pm 0.594$	$-4.867 \pm 1.218$
Mean	$2.934 \pm 0.524$	$-4.867 \pm 1.070$

**Notes.** \* simulations for which additional SMBH mass has been taken into account. This value was not used for the mean calculation.



**Fig. 2.** Interaction rate of the GCs with GalC as a function of the relative distance from the centre for the five TNG-TVP external potentials and 1000 random realisations (thin solid coloured lines). Thick dashed lines are a power-law fit in TNG potentials. The solid black line is a mean fitting line (see Table 2). The pale violet dotted line is a power-law fit for potential #411321-m for which the SMBH mass has been taken into account.

**Table 3.** Percent of probability of the GCs' interaction with the GalC in all 1000 sets of randomisation for four TNG TVPs.

GC	#411321	#411321-m	#441327	#451323	#462077	Mean
(1)	(2)	(3)	(4)	(5)	(6)	(7)
NGC 6401	100.0	99.9	100.0	100.0	100.0	100.0
Pal 6	100.0	99.6	99.9	100.0	100.0	99.9 ± 0.1
NGC 6681	99.9	99.9	100.0	100.0	100.0	99.9 ± 0.1
NGC 6712	99.9	100.0	99.9	99.8	100.0	99.9 ± 0.1
NGC 6287	100.0	100.0	100.0	97.0	92.1	97.3 ± 3.7
NGC 6642	99.8	99.8	99.3	100.0	99.5	99.7 ± 0.3
NGC 6981	83.9	84.9	90.2	93.5	87.8	88.9 ± 4.0
HP 1	98.7	99.0	70.7	99.5	83.0	89.9 ± 13.8
NGC 1904	72.4	73.0	73.6	83.2	76.7	76.5 ± 4.8
NGC 362	24.4	27.9	30.7	41.2	12.9	27.3 ± 11.8

**Notes.** Column (1) – name of a GC. Columns (2)–(6) – interaction probabilities for GCs with the GalC for each TNG-TVP external potential in percent. #411321-m – the special potential with the MW SMBH mass. Column (7) – the average probability value with error over all potentials, excluding the #411321-m potential.

**Table 4.** Statistics of the characteristics of the GCs' interaction with the GalC in all four TNG-TVP external potentials.

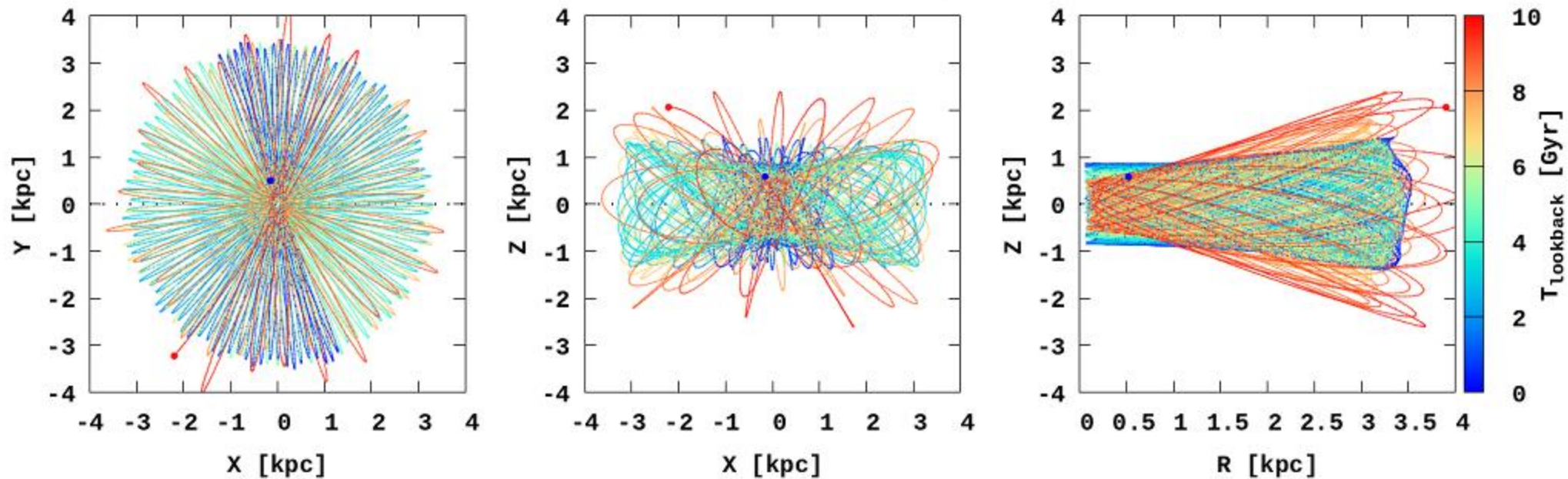
Pot \ GC	#411321			#411321-m					
	$\langle D_m \rangle$ pc	$\langle dV \rangle$ km s <sup>-1</sup>	$\langle N_{int} \rangle$	$\langle D_m \rangle$ pc	$\langle dV \rangle$ km s <sup>-1</sup>	$\langle N_{int} \rangle$			
NGC 6401	56 ± 10	331 ± 18	19 ± 2	58 ± 10	333 ± 17	18 ± 4			
Pal 6	58 ± 12	340 ± 10	11 ± 6	58 ± 13	341 ± 10	10 ± 5			
NGC 6681	42 ± 16	410 ± 4	3 ± 1	45 ± 17	411 ± 1	3 ± 1			
NGC 6712	52 ± 8	379 ± 10	8 ± 2	53 ± 8	380 ± 10	8 ± 2			
NGC 6287	82 ± 5	405 ± 10	4 ± 1	81 ± 4	398 ± 11	4 ± 1			
NGC 6642	62 ± 16	262 ± 19	27 ± 10	63 ± 14	260 ± 19	28 ± 9			
NGC 6981	57 ± 24	545 ± 8	2 ± 1	57 ± 24	547 ± 8	2 ± 1			
HP 1	50 ± 21	304 ± 10	11 ± 4	50 ± 22	306 ± 11	16 ± 4			
NGC 1904	53 ± 25	517 ± 17	3 ± 2	54 ± 24	518 ± 18	3 ± 1			
NGC 362	88 ± 10	484 ± 6	1 ± 1	87 ± 10	481 ± 5	1 ± 1			

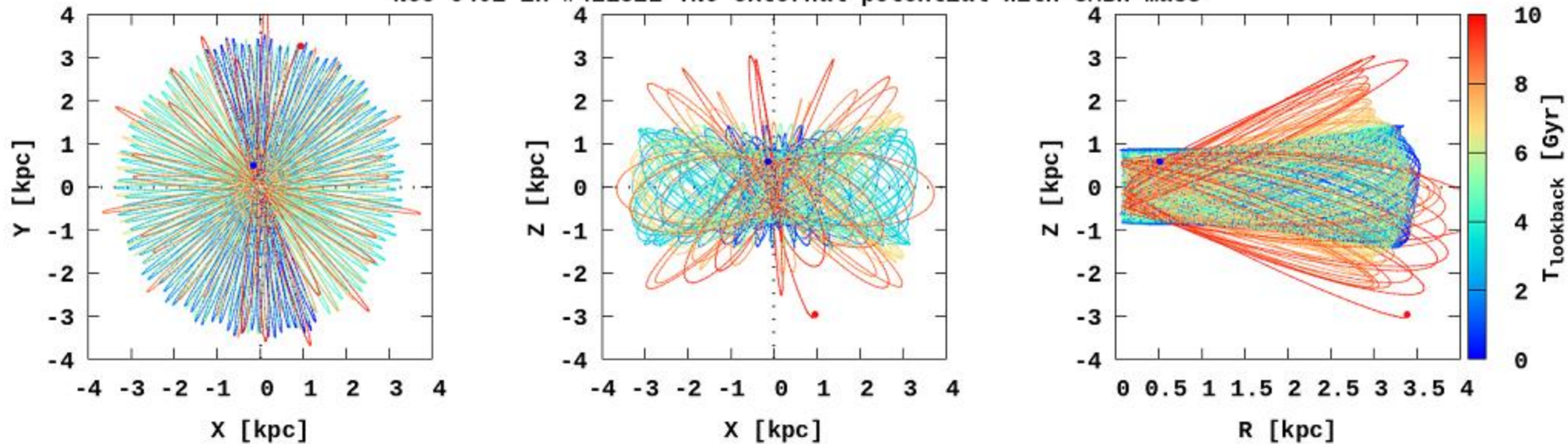
Pot \ GC	#441327			#451323			#462077		
	$\langle D_m \rangle$ pc	$\langle dV \rangle$ km s <sup>-1</sup>	$\langle N_{int} \rangle$	$\langle D_m \rangle$ pc	$\langle dV \rangle$ km s <sup>-1</sup>	$\langle N_{int} \rangle$	$\langle D_m \rangle$ pc	$\langle dV \rangle$ km s <sup>-1</sup>	$\langle N_{int} \rangle$
NGC 6401	48 ± 7	352 ± 12	38 ± 6	50 ± 7	366 ± 19	20 ± 3	54 ± 10	305 ± 12	14 ± 3
Pal 6	58 ± 9	359 ± 15	22 ± 4	70 ± 8	351 ± 29	10 ± 4	65 ± 9	299 ± 7	10 ± 3
NGC 6681	25 ± 10	440 ± 14	13 ± 2	34 ± 13	439 ± 14	7 ± 1	21 ± 11	368 ± 5	7 ± 2
NGC 6712	49 ± 10	427 ± 16	13 ± 3	54 ± 6	382 ± 25	8 ± 3	59 ± 10	342 ± 6	7 ± 2
NGC 6287	94 ± 4	409 ± 11	2 ± 1	84 ± 7	418 ± 20	2 ± 1	77 ± 3	375 ± 4	4 ± 2
NGC 6642	51 ± 13	300 ± 10	44 ± 7	59 ± 22	276 ± 27	25 ± 13	66 ± 13	226 ± 13	20 ± 6
NGC 6981	46 ± 24	590 ± 14	3 ± 1	51 ± 24	540 ± 28	3 ± 1	48 ± 24	511 ± 12	3 ± 1
HP 1	46 ± 21	327 ± 14	16 ± 6	65 ± 23	290 ± 2	4 ± 2	61 ± 25	282 ± 5	7 ± 4
NGC 1904	51 ± 25	588 ± 19	4 ± 2	54 ± 25	539 ± 47	3 ± 1	57 ± 25	500 ± 9	3 ± 1
NGC 362	85 ± 11	530 ± 12	2 ± 1	88 ± 10	482 ± 10	1 ± 1	87 ± 10	457 ± 6	1 ± 1

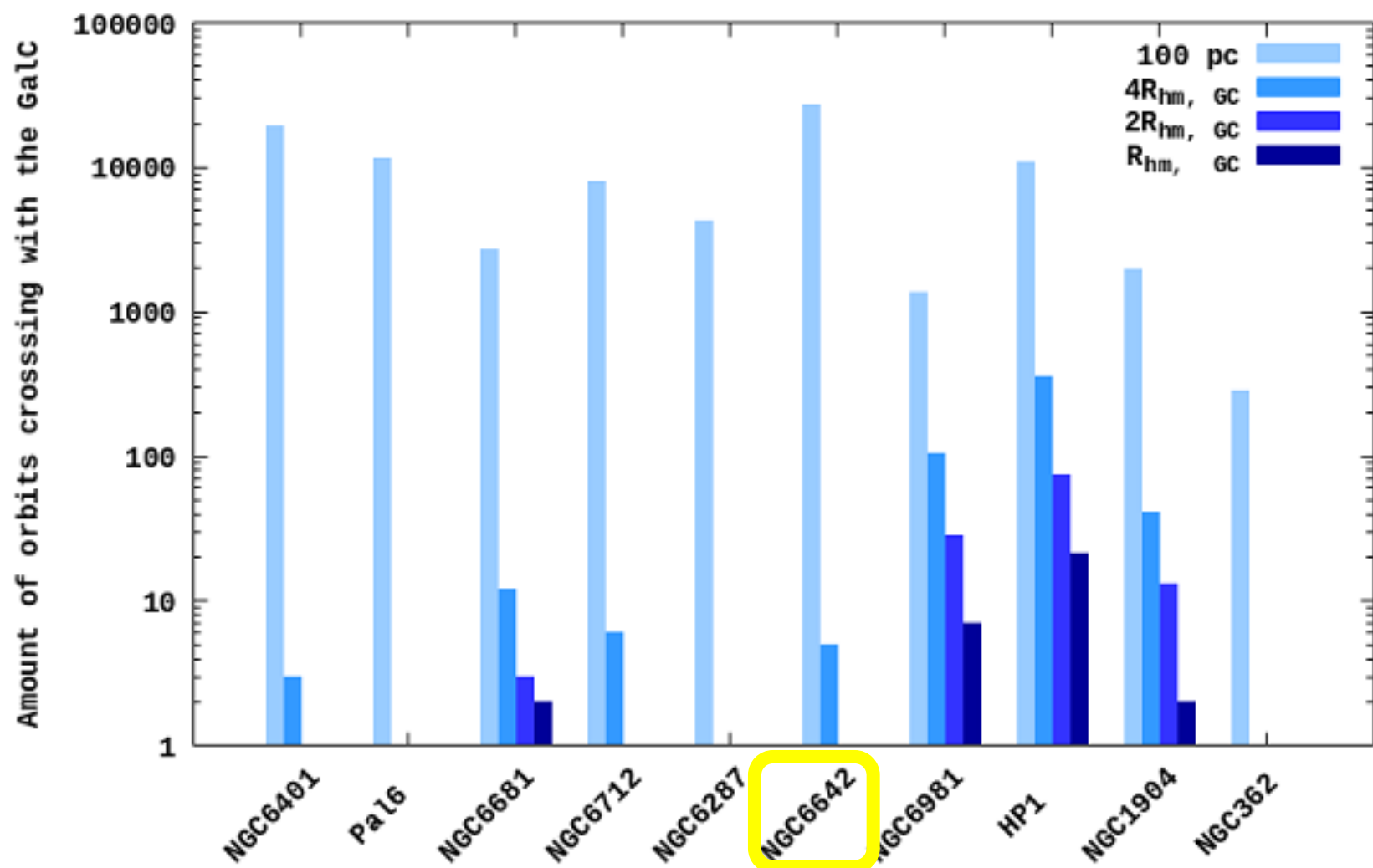


NGC 6401 in #411321 TNG external potential



NGC 6401 in #411321 TNG external potential with SMBH mass



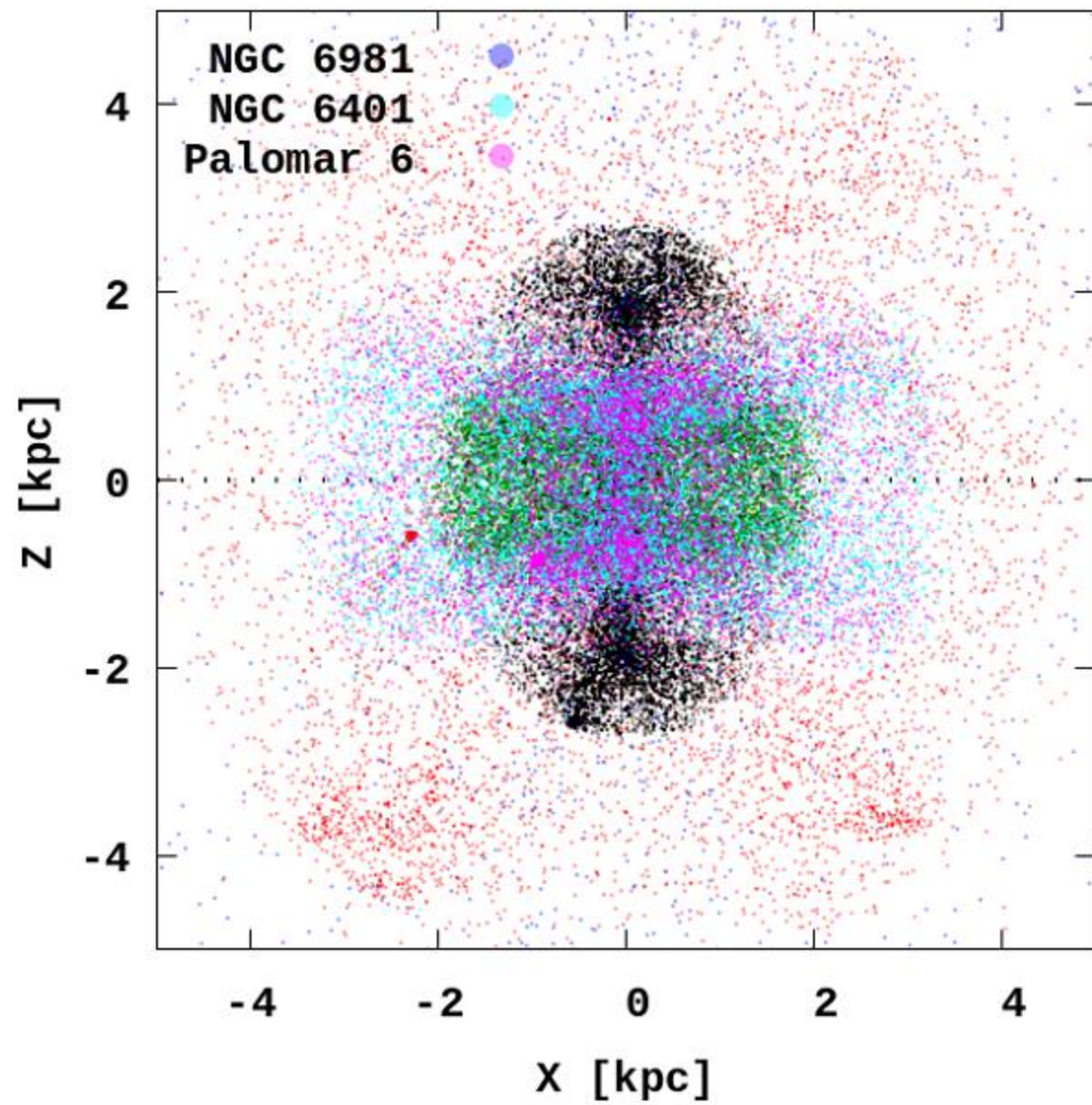
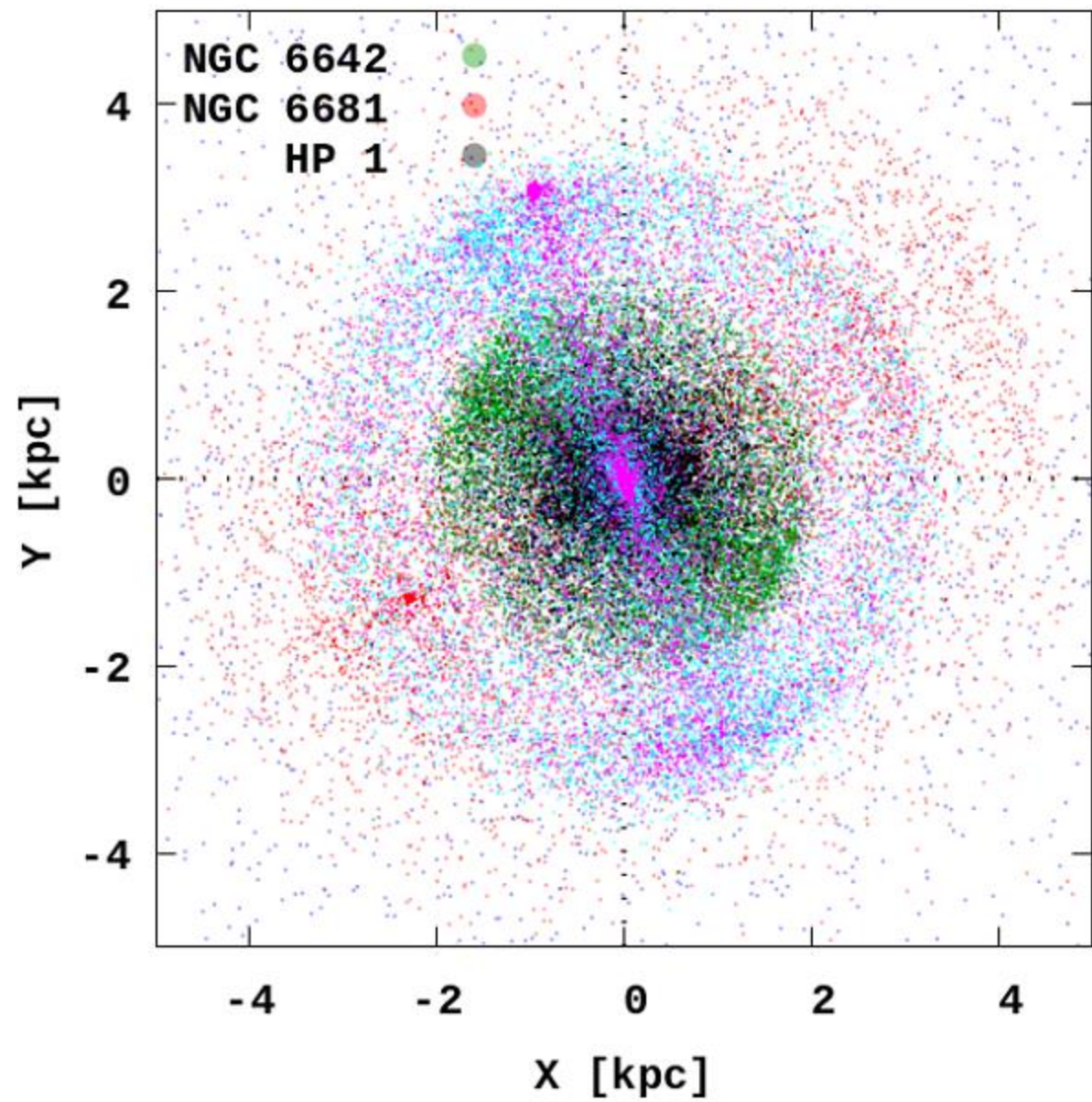


**Fig. 1.** Amount of orbits crossings with the GalC for selected GCs for four different distance criteria in 411321 TNG-TVP external potential.

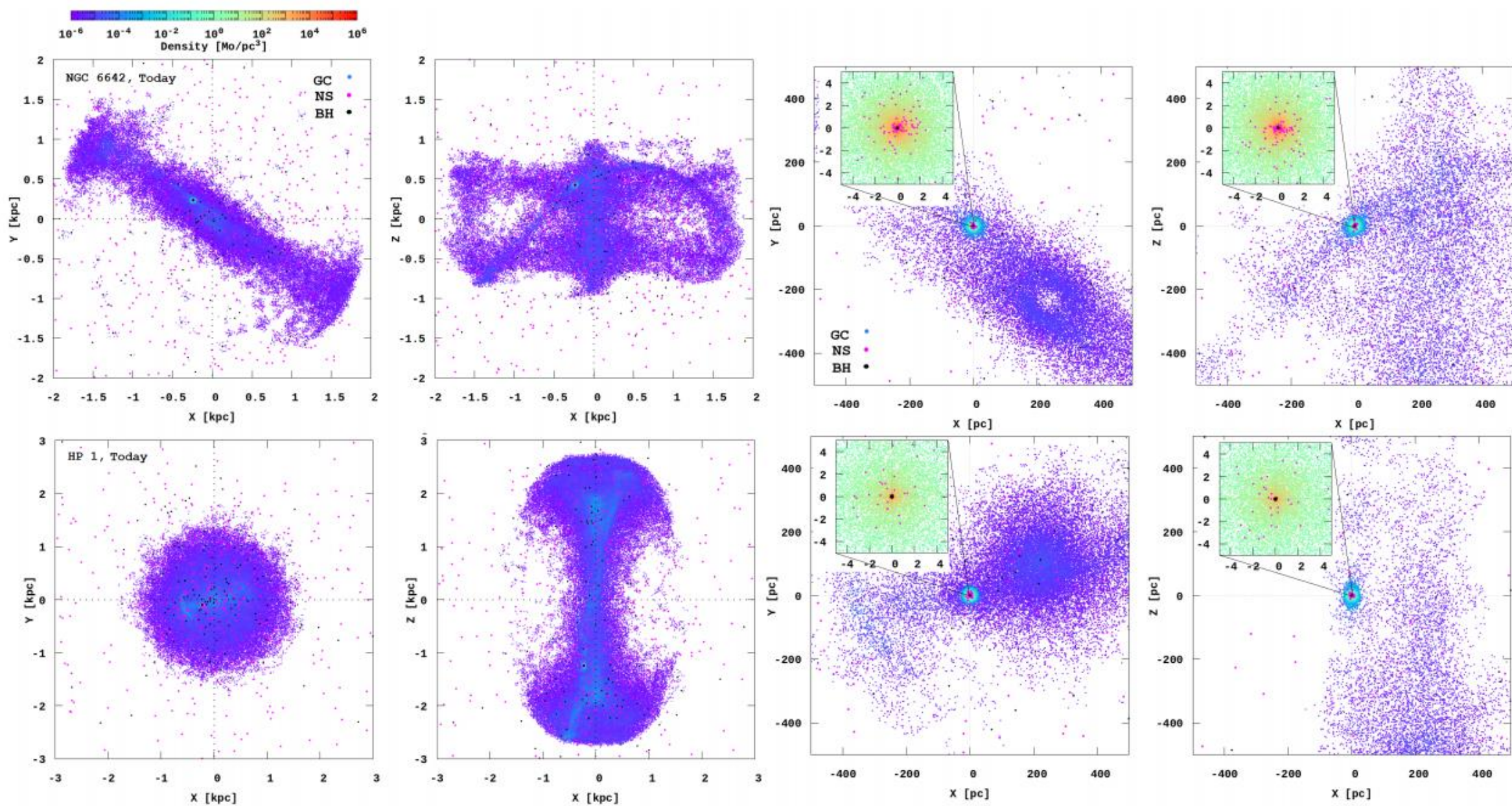
**Table 2.** Initial kinematics and physical characteristics at eight billion years lookback time for GCs.

GC	$X$	$Y$	$Z$	$V_x$	$V_y$	$V_z$	$E/m$	$L_{\text{tot}}/M$	$M$	$N$	$r_{\text{hm}}$	$W_0$
	pc	pc	pc	km s <sup>-1</sup>	km s <sup>-1</sup>	km s <sup>-1</sup>	10 <sup>4</sup> km <sup>2</sup> s <sup>-2</sup>	10 <sup>2</sup> kpc km s <sup>-1</sup>	10 <sup>6</sup> M <sub>⊙</sub>		pc	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
NGC 6401	-1493	-3213	398	5	-3	-57	-18.8	2.03	1.2	2 249640	6.5	9
Palomar 6	1499	465	-1655	140	51	-81	-18.8	1.14	1.0	1 751 970	3.5	9
NGC 6681	1392	-4638	4243	-0.7	-1	61	-16.6	2.9	1.3	2 265 380	3.0	8
NGC 6642	983	-897	545	-47	54	-117	-21.2	1.1	1.5	2 613 856	4.0	9
NGC 6981	1774	4001	-4943	70	159	-270	-11.5	3.7	1.0	1 742 560	7.0	9
HP 1	-580	242	1859	71	-36	128	-19.5	2.3	1.3	2 265 340	6.0	8

**Notes.** Column (1) – the GCs names; columns (2) – (4) – initial position in Cartesian Galactocentric frame; columns (5) – (7) – initial velocities in Cartesian Galactocentric frame; column (8) – total specific energy; column(9) – total specific angular momentum; column (10) – initial mass; column (11) – initial number of stars; column (12) – initial radius of half-mass; column (13) – initial concentration of King profile.



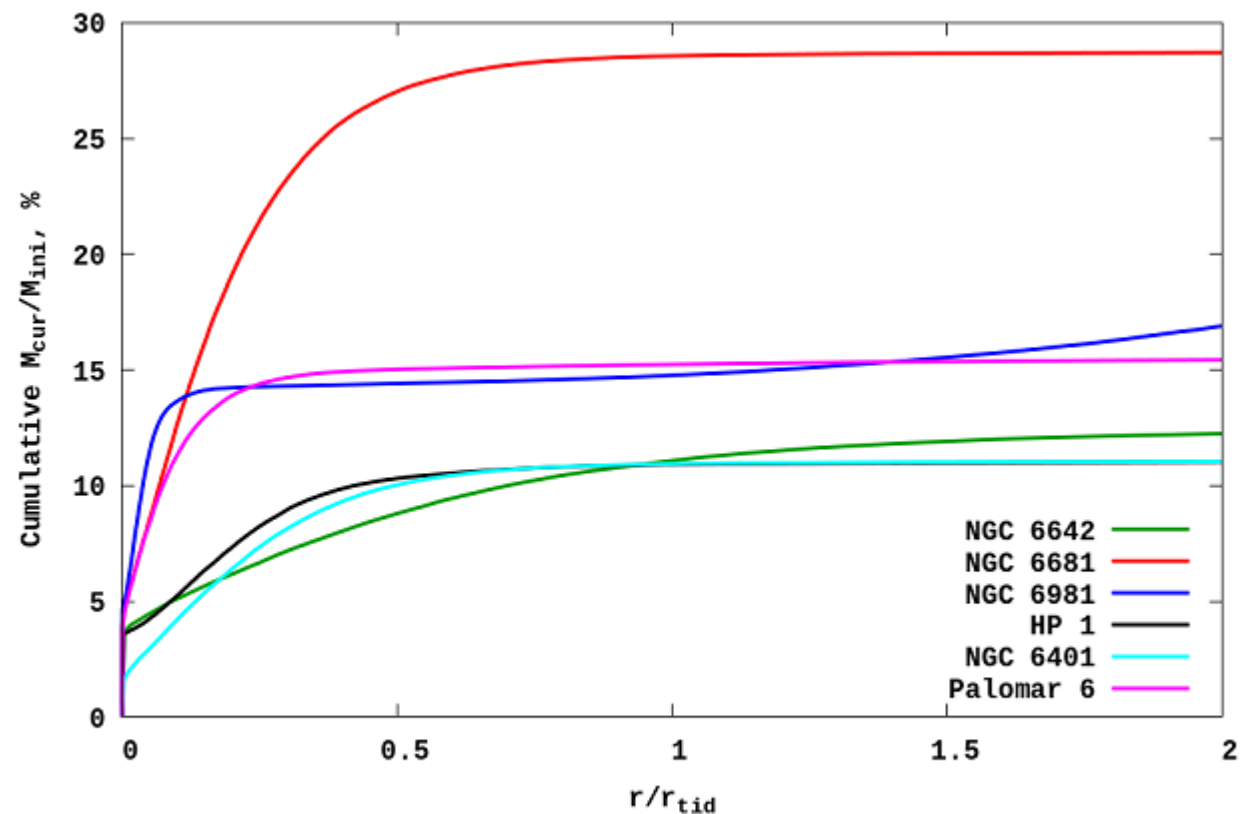
**Fig. 8.** Positions of the stars for six GCs in the central region of the Galaxy at present time.



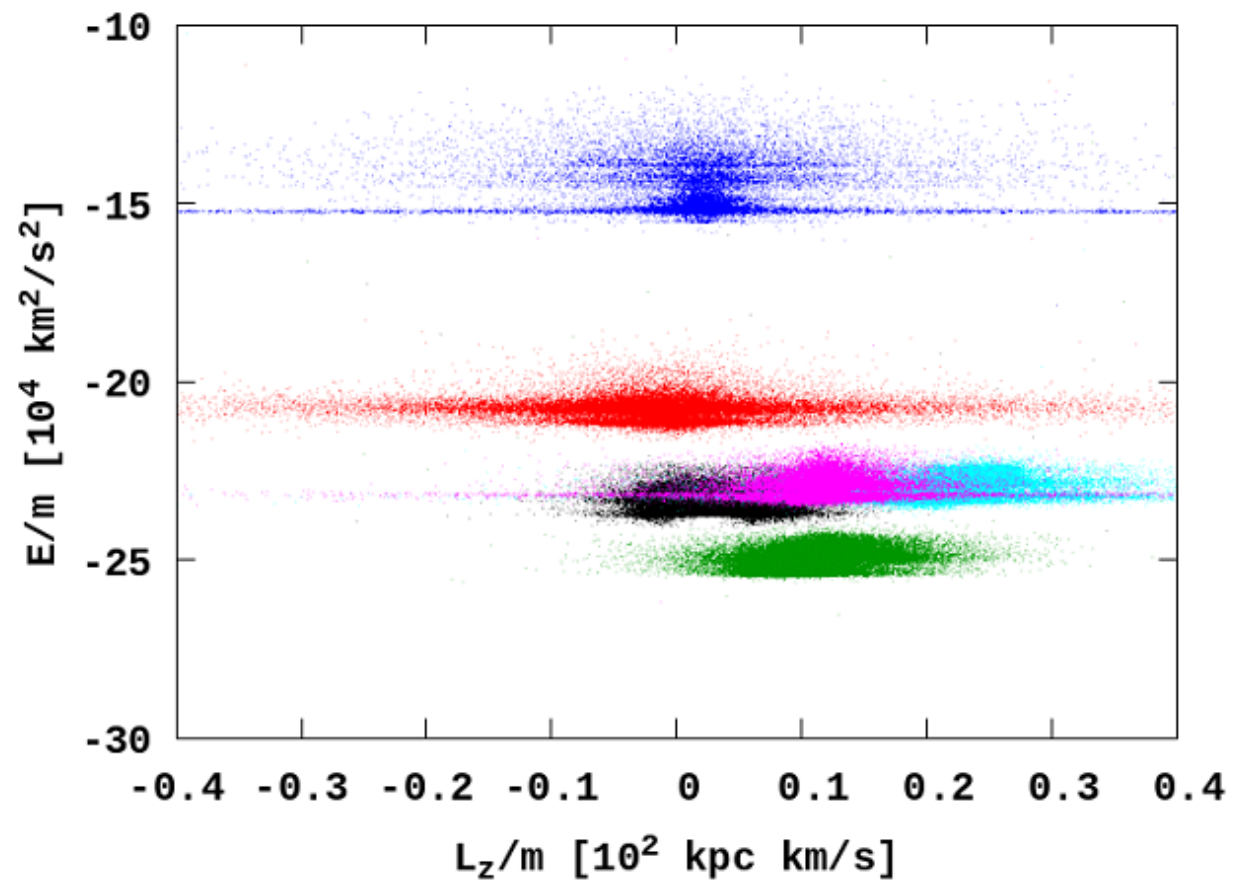
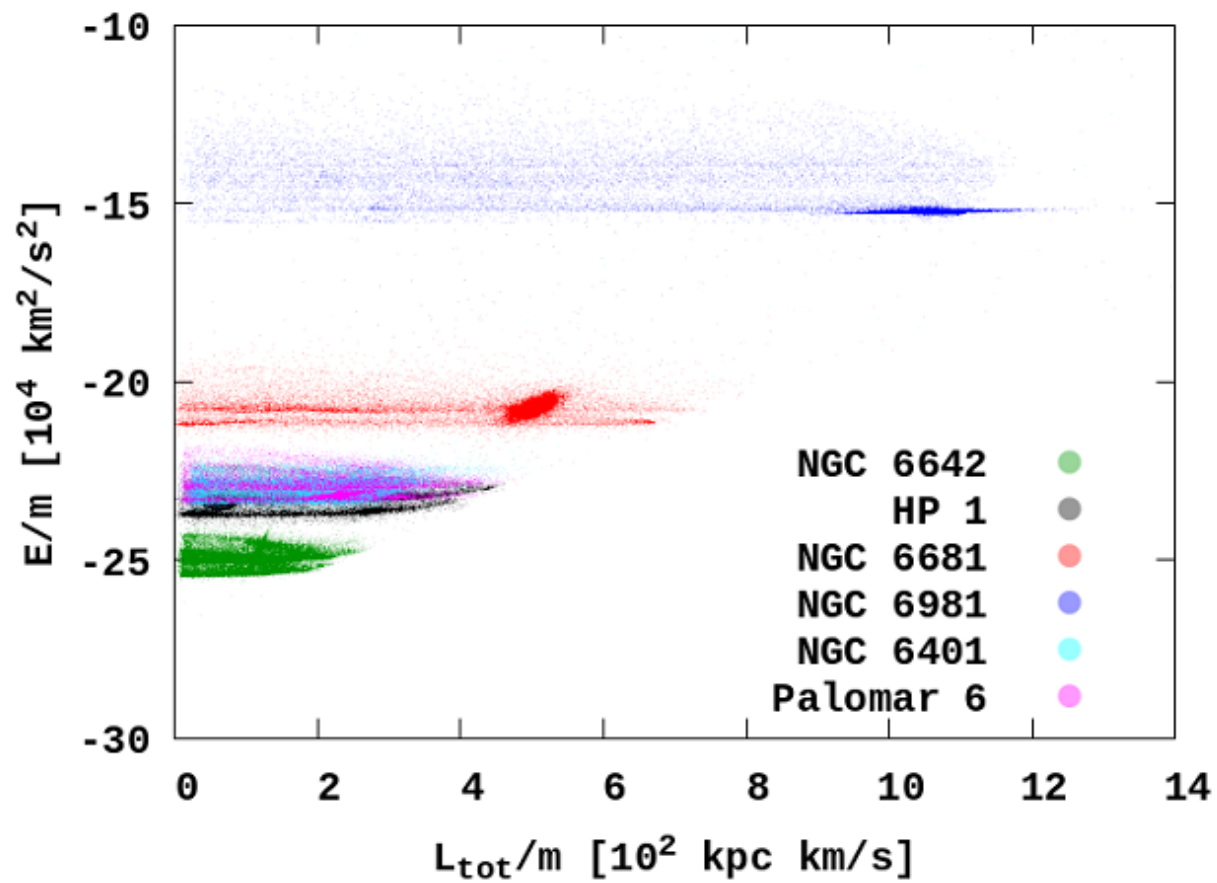
**Fig. 9.** NGC 6642 and HP 1 clusters today density distributions in 411321 TNG-TVP external potential. The orbital global evolution is present in two planes ( $X, Y$ ) and ( $X, Z$ ), *left panels*. GC central part in local frame with the BHs (black dots), NSs (magenta dots) and with detail central area (with box size 10 pc) we present in *two right panels*.

**Table 3.** Masses inside  $r_{\text{hm}}$  and  $r_{\text{tid}}$  for today, according to the numerical modelling.

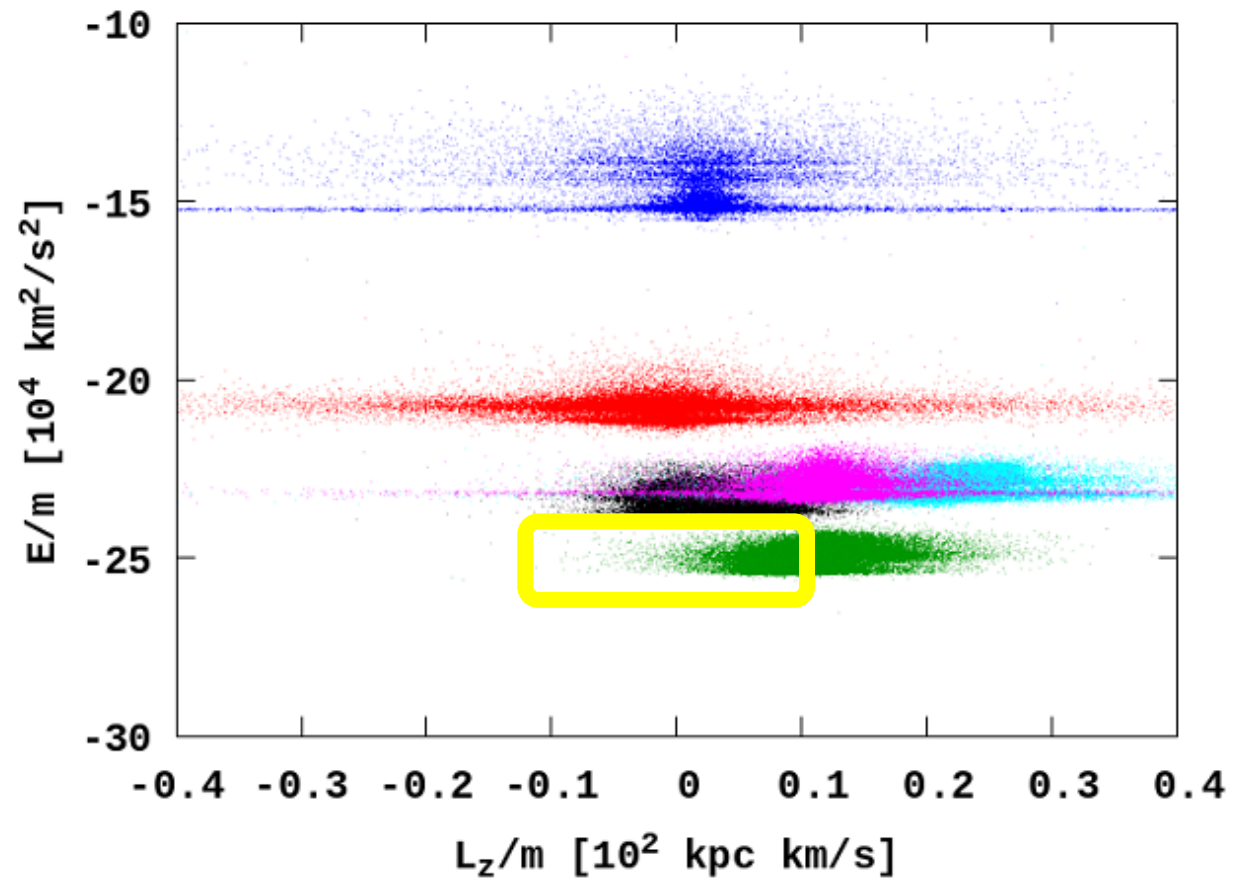
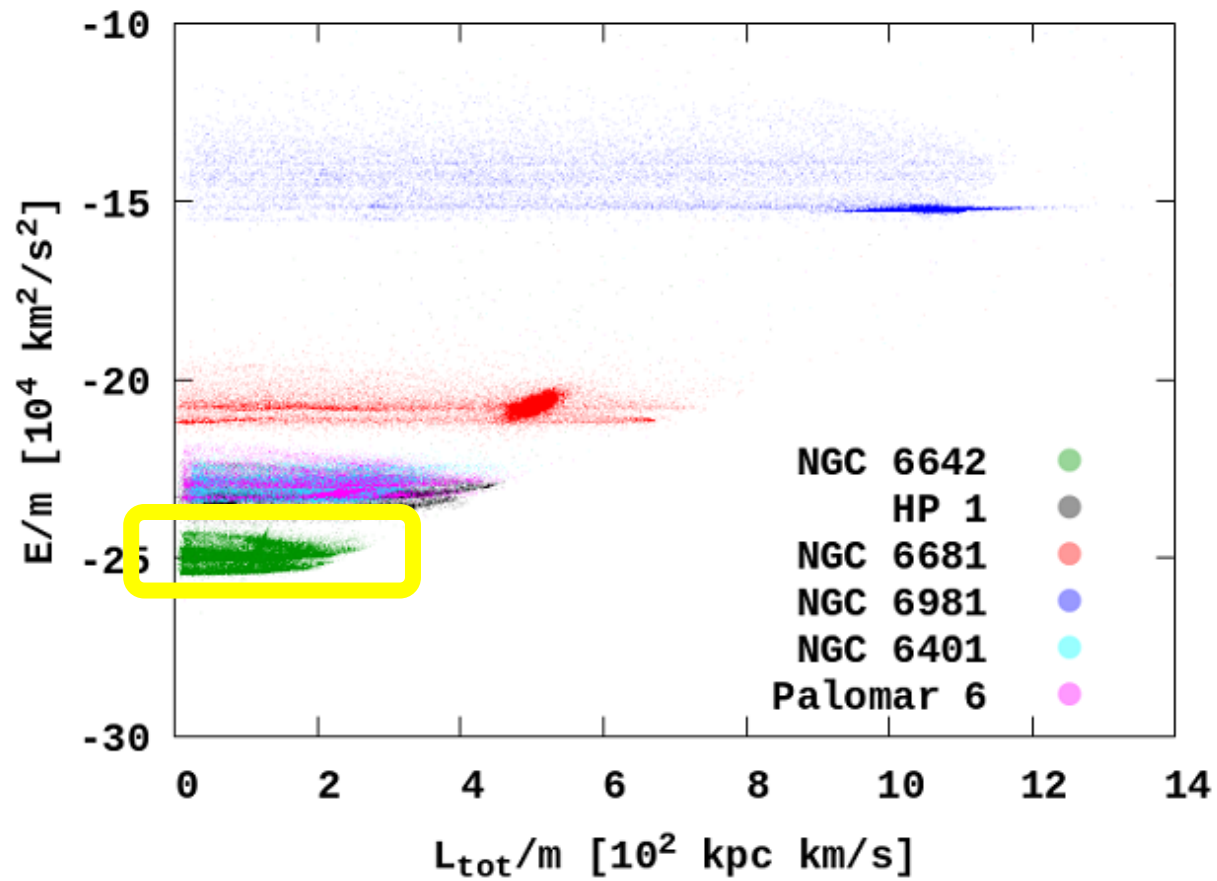
GC	$r_{\text{hm}}$ pc	$r_{\text{tid}}$ pc	$M(r_{\text{hm}})$ $10^4 M_{\odot}$	$M(r_{\text{tid}})$ $10^4 M_{\odot}$
NGC 6401	4.3	28	7.1	14.2
Palomar 6	2.7	70	7.6	15.2
NGC 6681	4.5	37	18.5	37.0
NGC 6642	1.5	11	8.2	16.6
NGC 6981	8.9	215	10.4	15.9
HP 1	4.0	37	7.0	14.2



**Fig. 4.** Internal mass distribution of the GCs for today.



**Fig. 10.** Total energy ( $E/m$ ) versus total angular momentum ( $L_{\text{tot}}/m$ ) and total energy versus  $z$ -th component of the angular momentum ( $L_z/m$ ) for GCs for today.



**Fig. 10.** Total energy ( $E/m$ ) versus total angular momentum ( $L_{\text{tot}}/m$ ) and total energy versus  $z$ -th component of the angular momentum ( $L_z/m$ ) for GCs for today.

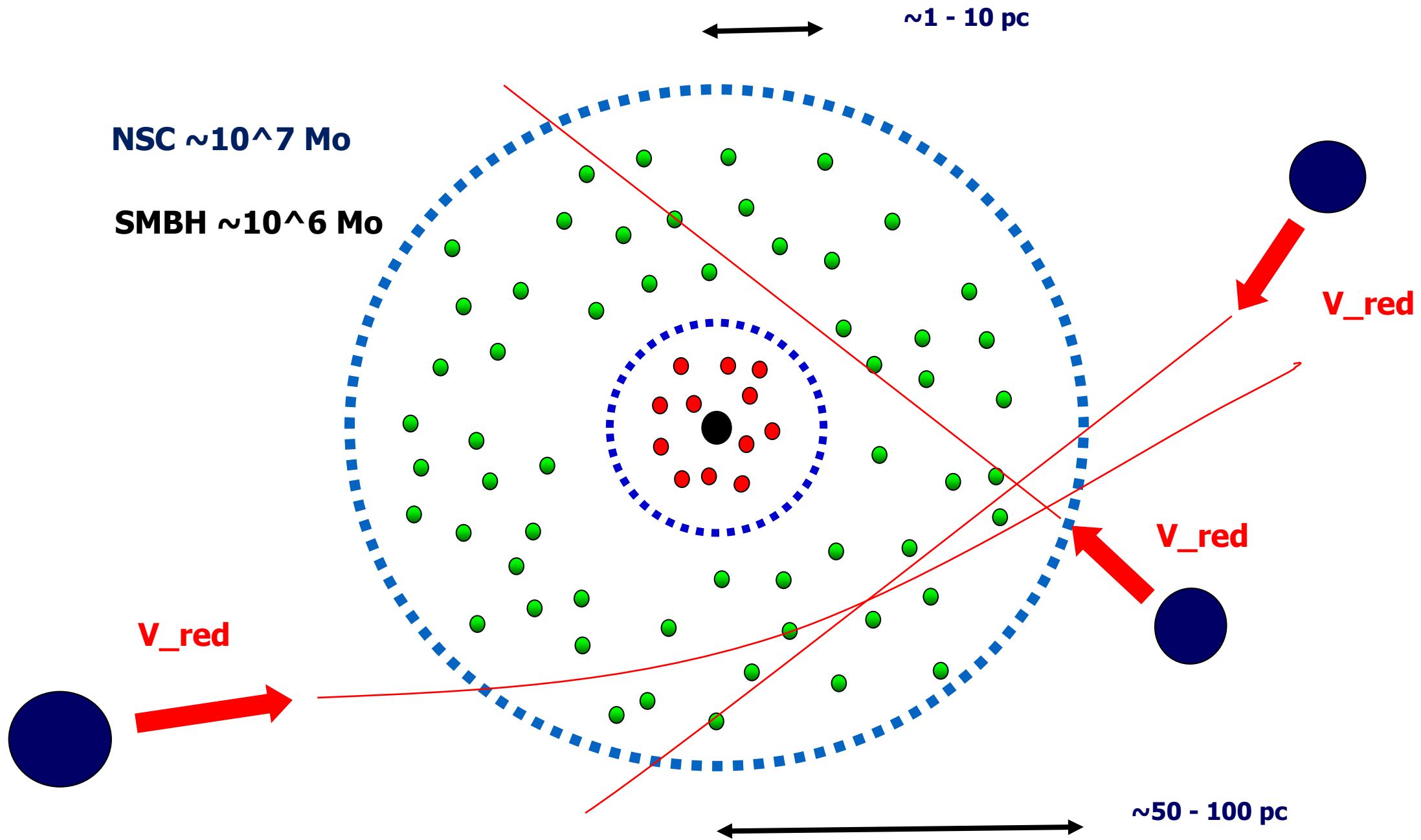


$$E_{\text{bound}}^* = -(M_{\text{NSC}} + m_*)/R_* + \underline{V_{\text{red}} \cdot 0.5 \cdot V_*^2},$$

**Table 5.** Total amount of the bound particles with the NSC and their masses with different velocity redaction during 8 billion years evolution in units of  $10^3$ .

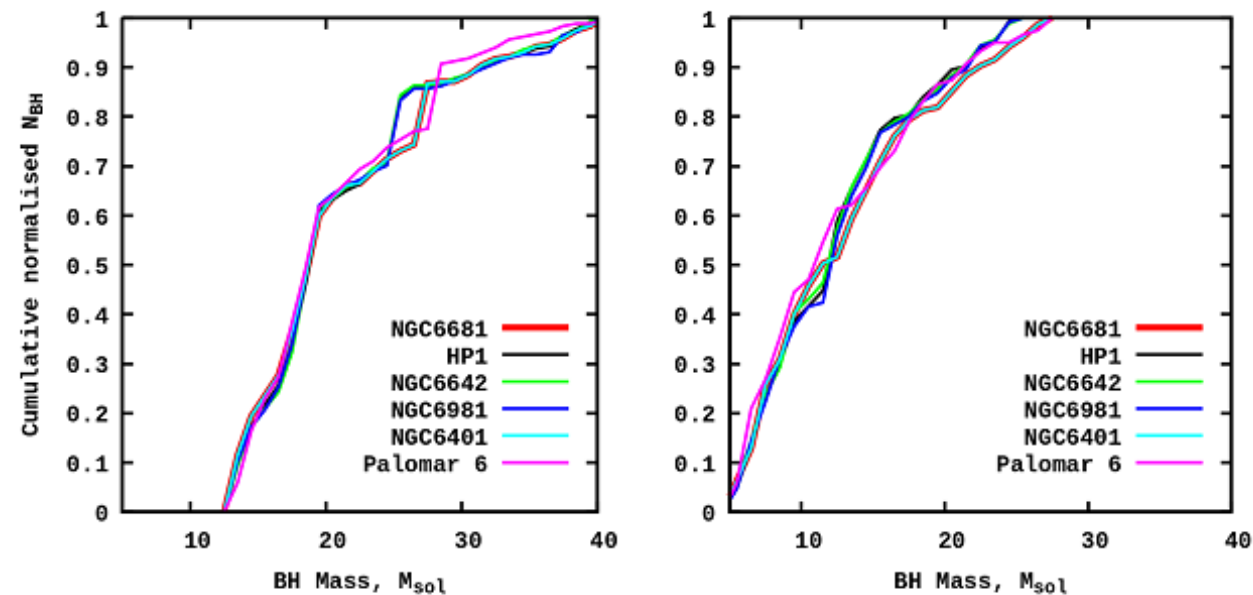
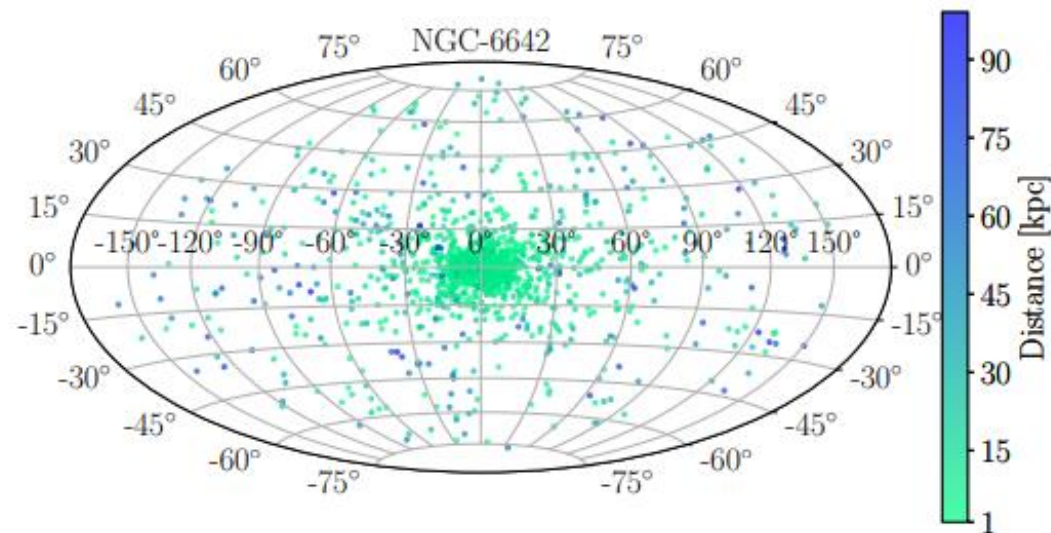
Vel <sub>red</sub>	NGC 6642			NGC 6401			NGC 6681			HP 1		
	Tot.	Unq. / in %	M <sub>⊙</sub>	Tot.	Unq. / in %	M <sub>⊙</sub>	Tot.	Unq. / in %	M <sub>⊙</sub>	Tot.	Unq. / in %	M <sub>⊙</sub>
V <sub>red</sub> =1.00	9.8	<u>2.6 / 0.1</u>	0.1	0.2	0.1 / 0.01	0.01	0	0	0	0	0	0
V <sub>red</sub> =0.80	22	8 / 0.3	0.4	1.2	0.1 / 0.01	0.01	0	0	0	0	0	0
V <sub>red</sub> =0.75	72	<u>29 / 1.1</u>	1.2	1.7	0.1 / 0.01	0.01	0	0	0	0	0	0
V <sub>red</sub> =0.60	4 550	1 066 / 41	40.7	2.6	0.1 / 0.01	0.01	0	0	0	0	0	0
V <sub>red</sub> =0.50	14 534	1 984 / 76	73.5	3.9	0.2 / 0.7	0.03	0	0	0	0	0	0
V <sub>red</sub> =0.40	–	–	–	20	1.4 / 0.6	0.7	0	0	0	0	0	0
V <sub>red</sub> =0.35	–	–	–	–	–	–	0	0	0	2 760	813 / 35	28.5
V <sub>red</sub> =0.30	–	–	–	–	–	–	0.37	0.17 / 0.01	0.01	5 127	1 504 / 66	56.5
V <sub>red</sub> =0.25	–	–	–	–	–	–	6.73	4.7 / 0.2	0.02	5 128	1 518 / 67	56.6
V <sub>red</sub> =0.20	–	–	–	–	–	–	5 820	199 / 88	7.1	–	–	–

**Notes.** – indicates, that no calculations were carried out for these V<sub>red</sub>.

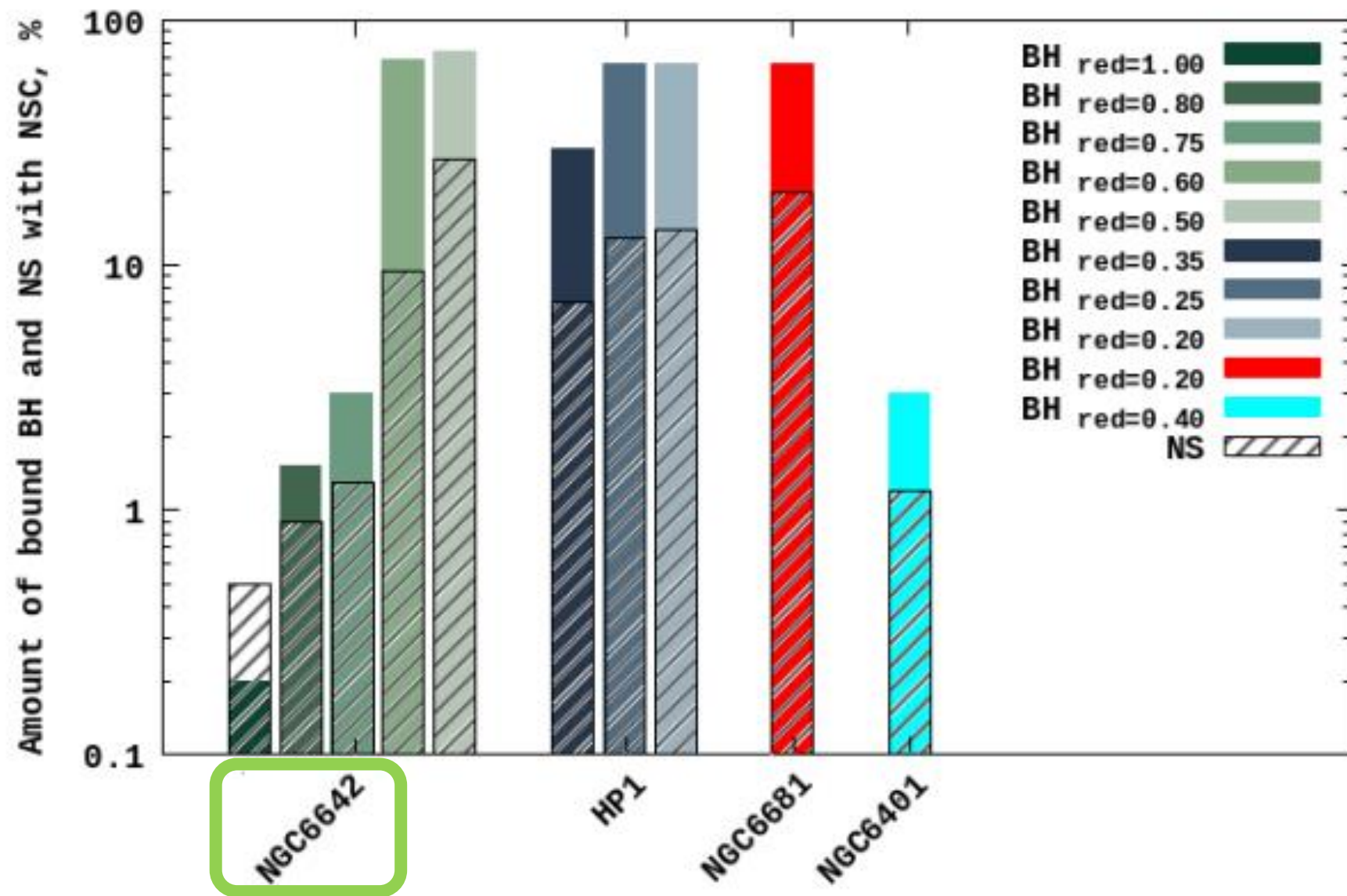


**Table 4.** Total amount of NSs, BHs and BH systems formed in GCs.

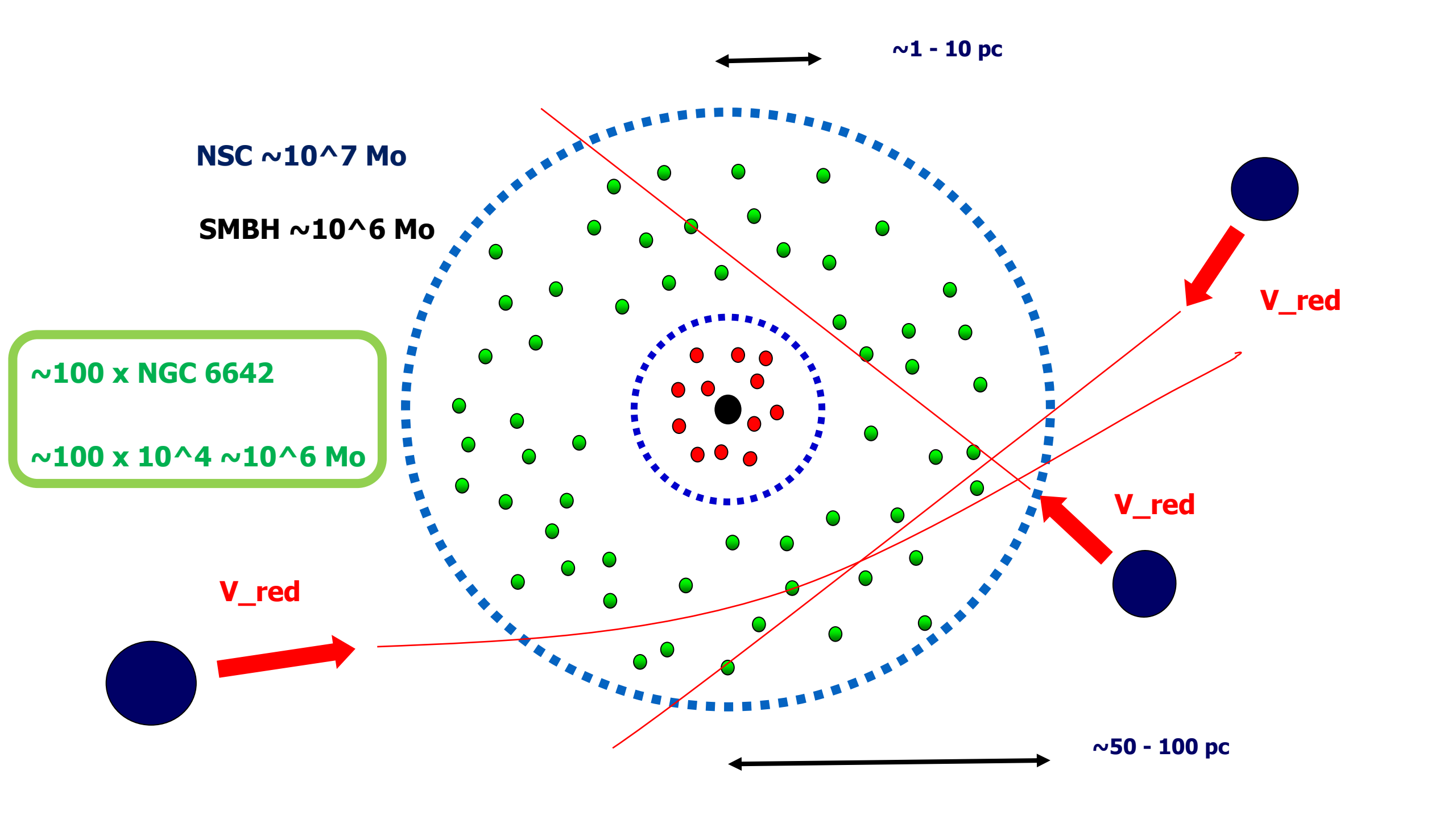
GC	NS	BH	$N_{\text{sub-sys,BH}}$	$M_{\text{sub-sys,BH}}, M_{\odot}$
NGC 6401	12 480	3 570	1340	$1.99 \times 10^4$
Palomar 6	13 540	3 020	1870	$3.99 \times 10^4$
NGC 6681	15 910	3 970	2610	$5.61 \times 10^4$
NGC 6642	17 730	4 440	2740	$5.82 \times 10^4$
NGC 6981	11 710	2 990	1720	$3.66 \times 10^4$
HP 1	15 140	3 890	2260	$4.76 \times 10^4$



**Fig. 7.** Statistics of the natal kick dependency from the BH remnant final masses. For  $V_{\text{kick}} = 0 \text{ km s}^{-1}$  – *left panel*, for  $V_{\text{kick}} > 0 \text{ km s}^{-1}$  – *right panel*.



**Fig. 16.** Amount of compact stellar remnants bound with the NSC for NGC 6642, NGC 6401, HP 1 and NGC 6681.



**Thank you for your attention...**

