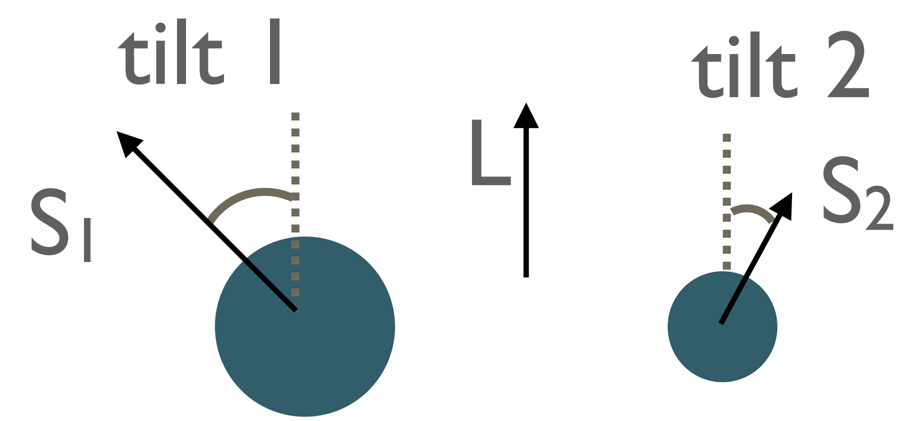
LIGO-G1901580-v1

COMPUTING THE SPIN TILT ANGLES AT FORMATION FROM GRAVITATIONAL WAVE OBSERVATIONS OF BINARY BLACK HOLES

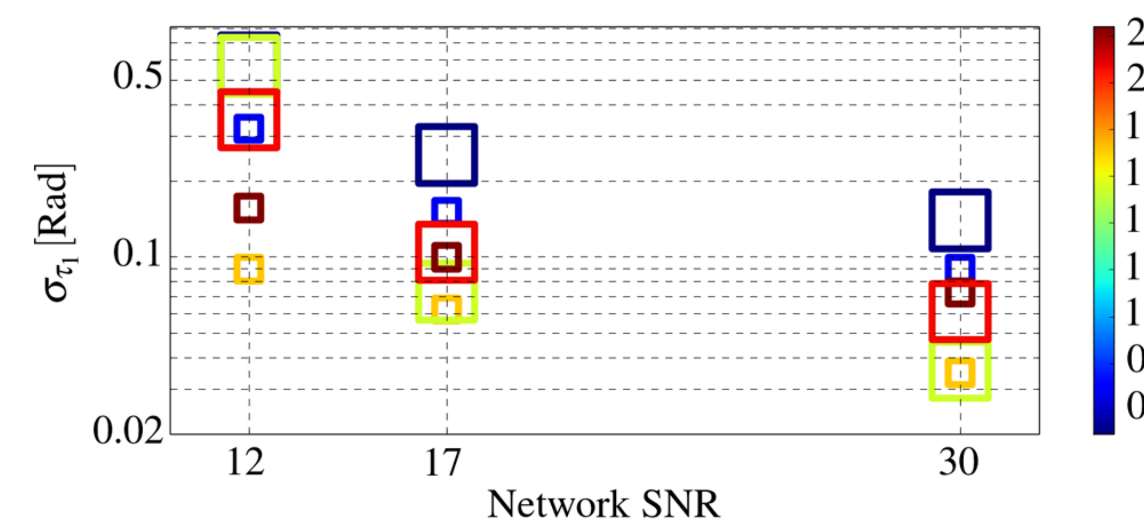
N. K. Johnson-McDaniel (in collaboration with Anuradha Gupta)
Astrophysics with GW detections, Warsaw
7.9.2019

MOTIVATION

- Spin misalignments in binary black holes can give insights into the formation of the binary, e.g., supernova kicks.
- LIGO/Virgo parameter estimation results currently give the tilt angles at a GW frequency close to merger (e.g., 20 Hz).
- To interpret formation channels, one wants to know the tilt angles when the binary was well separated (formally at infinite separation).
- One can compute these tilts at infinity efficiently using precession-averaged evolution [Kesden et al., PRL (2015); Gerosa et al., PRD (2015); Chatziioannou et al., PRD (2017)]



Example errors on tilt 1 for a (10, 1.4) Msun system



Vitale et al., PRL (2014)

MOTIVATION (CONT.)

- However, precession-averaged evolution is only valid when the precession timescale is sufficiently large, so not too close to merger.
- One thus has to evolve the spins backwards with orbit-averaged evolution before applying the precession-averaged evolution.
  [It might even be necessary to evolve backwards without any averaging if the reference frequency at which the spins are given is sufficiently close to merger. However, we have not yet tried this.]
- Additionally, the standard precession-averaged evolution (implemented in, e.g., the PRECESSION package [Gerosa and Kesden, PRD (2016)]) does not deal well with mass ratios close to unity (the tilts at infinity are not well-defined for exactly equal masses), and LIGO posterior samples have mass ratios of up to ~0.99999.

OUR WORK

- We have regularized the precession-averaged equations so that they are numerically stable for mass ratios close to unity.

- Additionally, we have noted that one can simplify the formalism (and make it numerically more stable) by linearizing in certain limits, and obtained rigorous bounds for the error incurred by this linearization.

- We are currently investigating the orbital velocity at which one needs to transition from orbit-averaged evolution to precession-averaged evolution to obtain the tilts at infinity with a given accuracy.

PRECESSION-AVERAGED EVOLUTION

- The idea of precession-averaged evolution is that there is a separation of timescales $t_{orb} << t_{prec} << t_{RR}$ when the binary is well separated—roughly, $t_{prec}/t_{orb} \sim r/M$ and $t_{RR}/t_{prec} \sim (r/M)^{3/2}$.
- Thus, one can evolve the spins on the radiation-reaction timescale, averaging over the precession timescale.
- One additionally notes that the 2PN spin evolution equations conserve the effective spin, so that the magnitudes of the total spin S, orbital angular momentum L, and total angular momentum J contain all the information about the spin evolution.

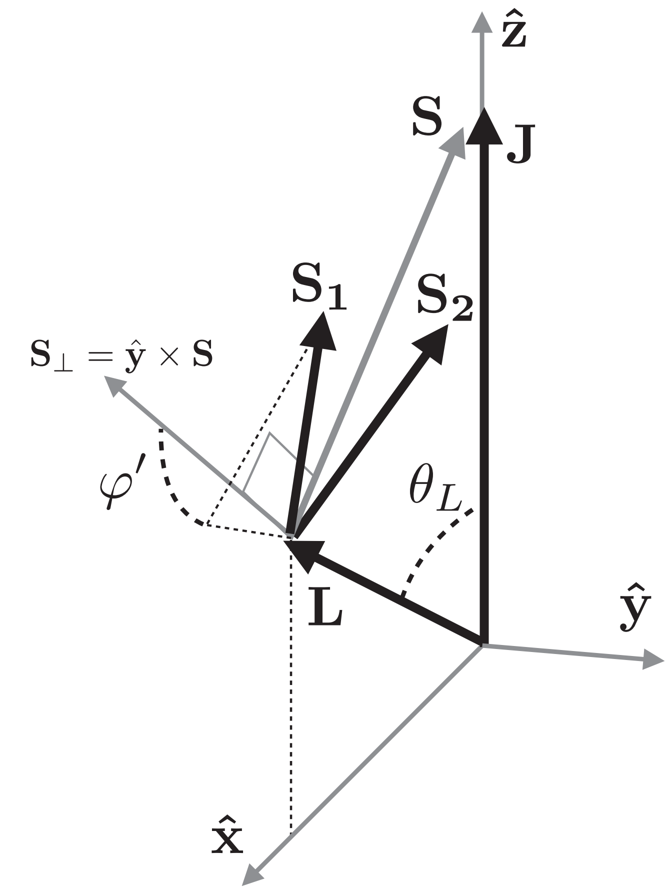


illustration from Gerosa et al., PRD (2015)

TILTS AT INFINITY

- Following Gerosa et al., PRD (2015), one computes the tilts at infinity using

$$\cos\theta_{1\infty} = \frac{-\xi + \kappa_\infty(1+q^{-1})}{S_1(q^{-1}-q)},$$

$$\cos\theta_{2\infty} = \frac{\xi - \kappa_\infty(1+q)}{S_2(q^{-1}-q)}$$

← numerically ill-conditioned for q → 1

with

$$\xi := [(1+q)\mathbf{S}_1 + (1+q^{-1})\mathbf{S}_2]\cdot\hat{\mathbf{L}}$$

← effective spin

and

$$\kappa := \frac{J^2 - L^2}{2L} \qquad \frac{d\kappa}{du} = \langle S^2\rangle_{\rm pr} \qquad u := 1/(2L)$$

COMPUTING $\langle S^2 \rangle_{PR}$

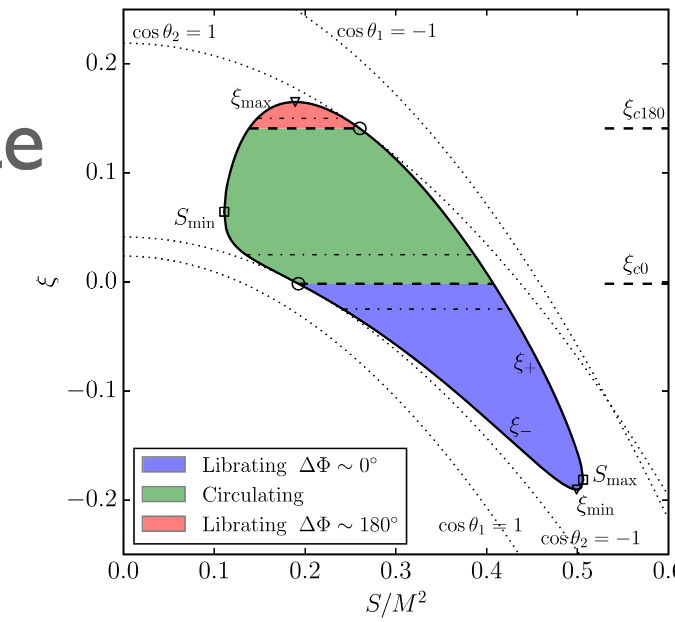
- We follow Chatziioannou et al. PRD (2017) and write

$$\langle S^2 \rangle_{\rm pr} = S_+^2 + (S_+^2 - S_3^2)\left[\frac{E(m)}{K(m)} - 1\right],$$

where $S_+^2 > S_-^2 > S_3^2$ are the roots of

$$S^6 + BS^4 + CS^2 + D = 0,$$

and B, C, and D are complicated functions of the mass ratio, spin magnitudes, L, J, and ξ.



Kesden et al., PRL (2015)

elliptic functions

$$m := \frac{S_+^2 - S_-^2}{S_+^2 - S_3^2} \qquad E(m) := \int_0^{\pi/2} \sqrt{1 - m\sin^2\theta}\, d\theta, \qquad K(m) := \int_0^{\pi/2} \frac{d\theta}{\sqrt{1 - m\sin^2\theta}}$$

SIMPLIFYING THE EXPRESSION FOR <S^2>$_{PR}$

- When S_3^2 is large, e.g., when L is large, then it is often not obtained numerically very accurately.

- Fortunately, in that case m is small, so one can linearize <S^2>$_{pr}$ in m, giving a result independent of S_3^2:

$$\langle S^2\rangle_{\text{pr}} = \frac{1}{2}(S_+^2 + S_-^2) + O(m^2).$$

- Using Taylor's theorem with remainder and standard inequalities, one can obtain the following bound on m for the linearization to be accurate to δ_{lin}. (This is a simplified version that is slightly weaker than the full bound, but is easier to work with.)

$$m \leq \min\left[0.01, 15.6\delta_{\text{lin}} \max\left(1, \frac{1}{S_+^2 - S_-^2}\right)\right]$$

REGULARIZING THE EXPRESSIONS FOR THE TILTS AT INFINITY

- In order to obtain an expression for the tilts at infinity that is well behaved for q close to unity, we note that S is also conserved for q = 1.

- The q → 1 limit is singular, but we can still use the q = 1 results to obtain our regularized expressions. We have

$$\kappa = \frac{S_0^2}{2L} + \frac{\xi}{2} \qquad (q = 1),$$

so with ε := 1 - q, we take

initial value of S^2

$$\kappa = \frac{S_0^2}{2L} + \frac{\xi}{2} + \epsilon\kappa^{(\epsilon)} + O(\epsilon^2),$$

and then have

$$\cos\theta_{1\infty} = \frac{-\xi + \kappa_\infty(2-\epsilon)}{2S_1\epsilon} + O(\epsilon) = \frac{\kappa_\infty^{(\epsilon)} - \xi/4}{S_1} + O(\epsilon), \quad \cos\theta_{2\infty} = \frac{\xi - \kappa_\infty(2-\epsilon)}{2S_2\epsilon} + O(\epsilon) = \frac{-\kappa_\infty^{(\epsilon)} + \xi/4}{S_2} + O(\epsilon),$$

REGULARIZING THE EXPRESSIONS FOR THE TILTS AT INFINITY

- In fact, we can write

$$S_2 \cos\theta_{2\infty} = \frac{\xi - (1+q)S_1 \cos\theta_{1\infty}}{1+q^{-1}}.$$

so we take

$$\kappa_{\xi q} := \frac{1}{1-q}\left(\kappa - \frac{S_0^2}{2L} - \frac{q\xi}{1+q}\right) = S_1 \cos\theta_1 + \frac{S^2 - S_0^2}{2L(1-q)},$$

and have

$$S_1 \cos\theta_{1\infty} = \kappa_{\xi q,\infty}$$

EQUATION FOR $\kappa_{\xi q}$

- We find that

$$\frac{d\kappa_{\xi q}}{du} = \langle \bar{S}^2 \rangle_{\text{pr}}$$

where

$$\bar{S}_\star^2 := \frac{S_\star^2 - S_0^2}{1-q},$$

and

$$q(1-q^2)u^2\bar{S}^6 + \bar{B}\bar{S}^4 + \bar{C}\bar{S}^2 + \bar{D} = 0,$$

so we can apply all our previous results about linearizing in m.

DISCUSSION

- With this regularization, we obtain numerically stable results for almost all cases even with mass ratios up to ~0.99999.
- However, there are still some fairly innocuous-seeming (though special) cases where the evolution fails, e.g., masses of 35 and 34.9 Msun, and parallel spins of magnitude 0.95 in the orbital plane. (Such cases also fail with the PRECESSION code.)
- It is possible that this issue might be fixed if we were able to solve a quadratic instead of a cubic in cases where the leading coefficient of the cubic is small.

$$q(1-q^2)u^2\bar{S}^6 + \bar{B}\bar{S}^4 + \bar{C}\bar{S}^2 + \bar{D} = 0,$$

  However, I have yet to derive a rigorous bound for this case, so I have not tried it in the code.
- Nevertheless, these cases are special enough that they are unlikely to be a problem for the application to GW posterior samples.

ORBIT-AVERAGED SPIN EVOLUTION

- We are currently experimenting with the LALSimulation orbit-averaged spin evolution (from SimInspiralSpinTaylorPNEvolveOrbit). This is reasonably fast, but not optimized for this sort of evolution.

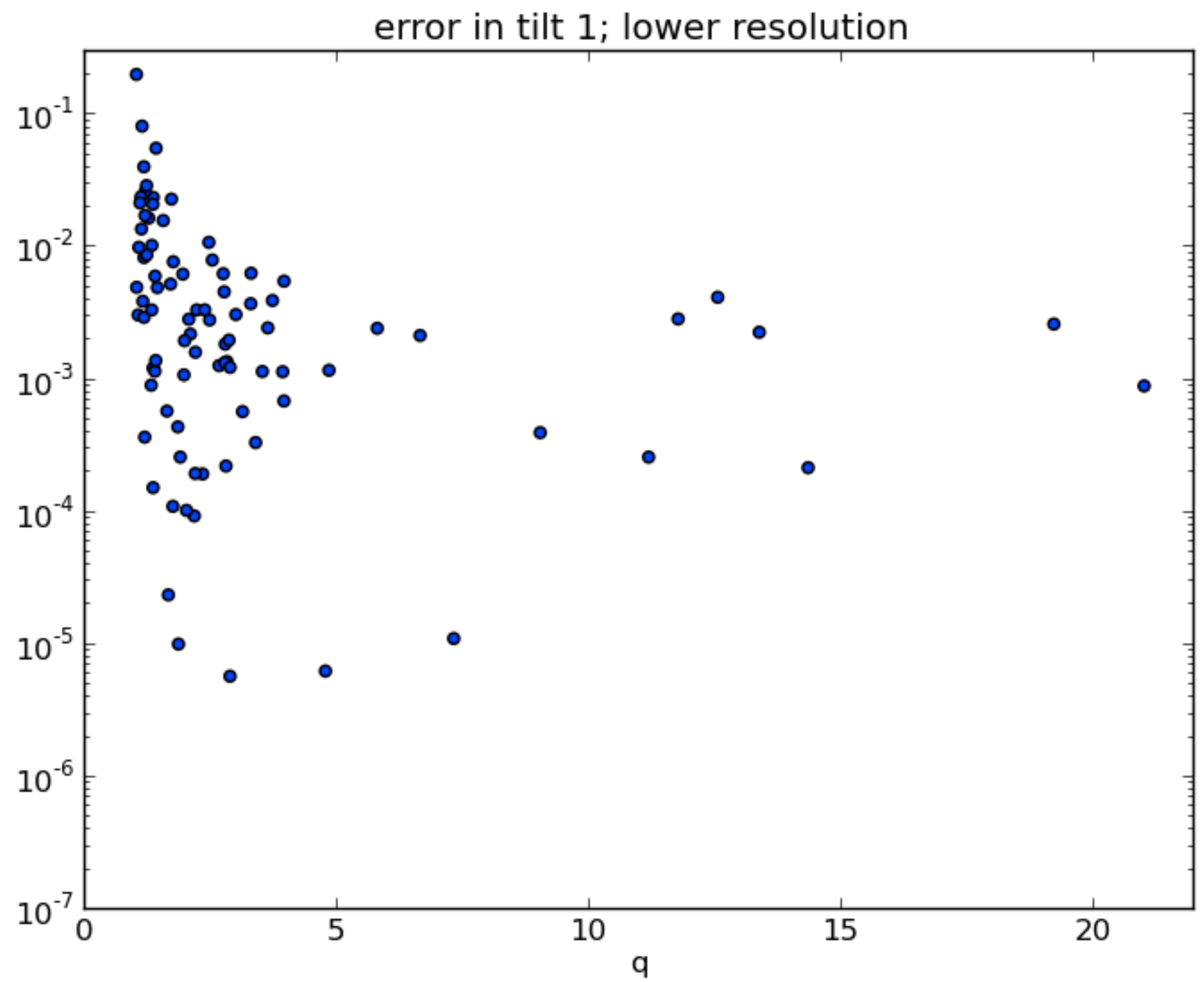
- We have found that one needs quite low transition frequencies (< 0.1 Hz for a total detector frame mass of 80 Msun, or an orbital velocity of < 0.05c) to obtain good accuracy for the tilts at infinity.

- So far, we do not find as nice convergence with the transition frequency as we might like, so we likely need to consider even lower transition frequencies, for which optimization of SimInspiralSpinTaylorPNEvolveOrbit may be necessary.

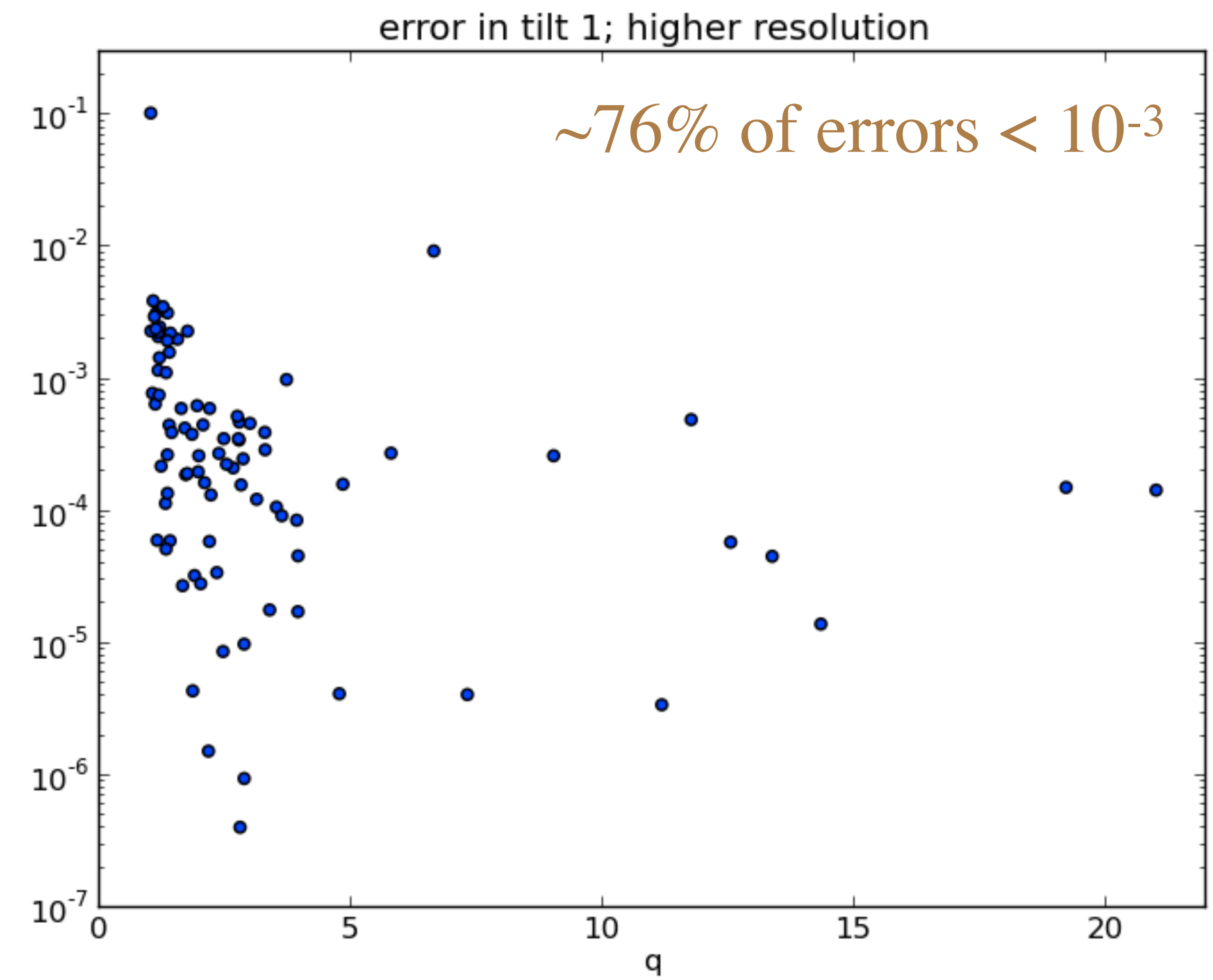
  However, we do find that the errors mostly decrease with decreasing transition frequency.

ERRORS WITH ORBIT-AVERAGED EVOLUTION—RANDOM BINARY PARAMETERS

v transition: 0.2 and 0.1



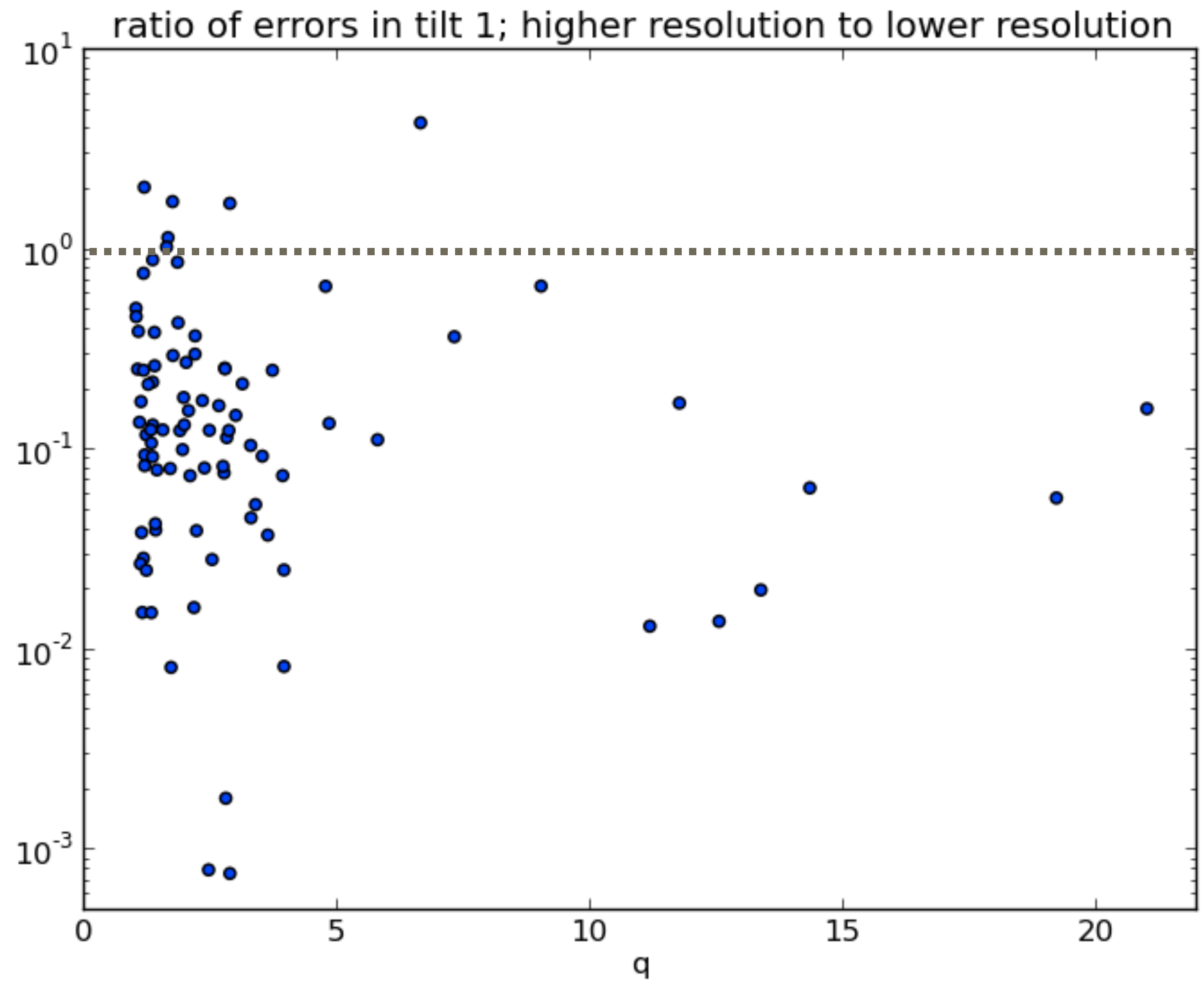
v transition: 0.1 and 0.05



tilt 2 errors are similar

ERRORS WITH ORBIT-AVERAGED EVOLUTION —RANDOM BINARY PARAMETERS

ratio of 0.1 and 0.05 errors to 0.2 and 0.1 errors



tilt 2 errors are similar

CONCLUSIONS

- Precession-averaged evolution is a powerful tool for obtaining the tilt angles at infinity, which are useful in comparing with binary evolution, e.g., supernova kicks.
- We have derived regularized equations that allow us to obtain the tilts at infinity for mass ratios close to unity (e.g., 0.99999) and a rigorous error bound on the linearization needed to help the numerical stability of the equations in certain regimes.
- We are currently checking the transition velocity from orbit-averaged to precession-averaged evolution needed to obtain a given accuracy for the tilts at infinity.
- The regularized precession-averaged evolution code will be released as part of LALSuite when we have finalized the interface with the orbit-averaged evolution.