Magnetic Field Instabilities in Neutron Stars

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with

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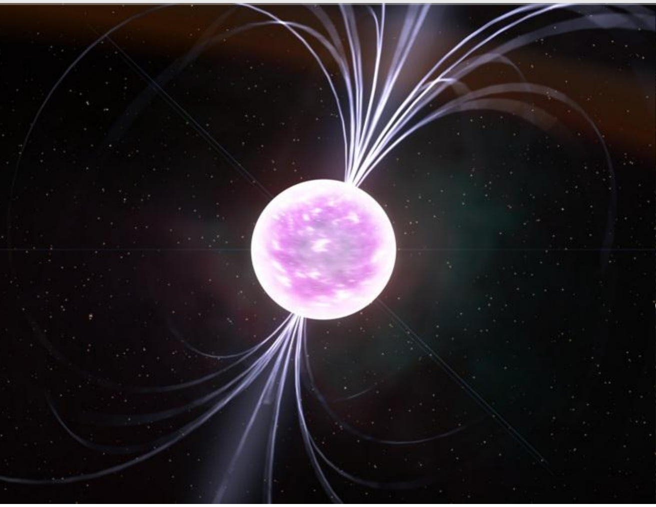
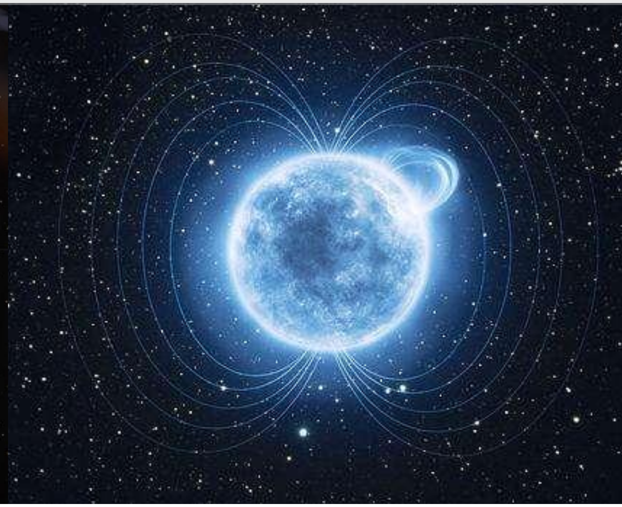
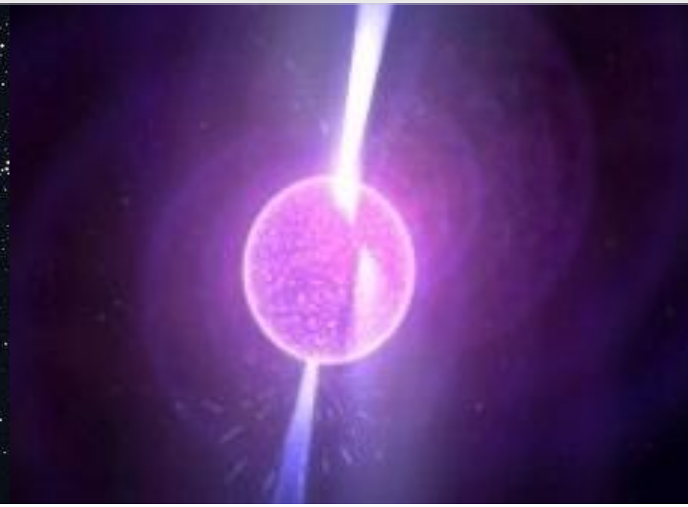
CAMK PAN

The 1st Conference of Young Researchers at CAMK PAN

13 June 2019

Neutron Stars

Credit:NASA

Neutron star | Magnetar | Pulsar

Neutron degeneracy pressure balances self-gravity

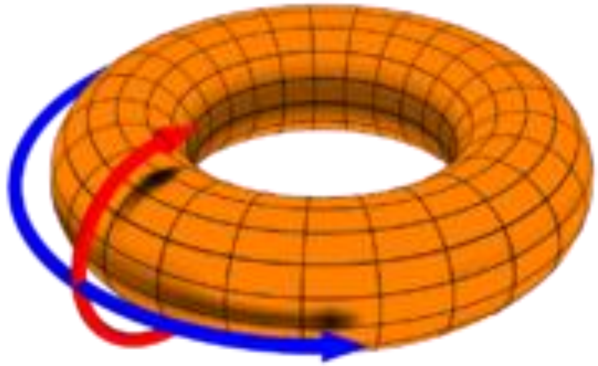
$\rho_c \geq 10^{15}$ gm cm^{-3} $\quad M \sim 1.4 M_\odot \quad R \sim 10$ km $\quad 10^8 G \leq B \leq 10^{16} G$

Extremely dense compact objects having the strongest magnetic field in the universe

Magnetic Field

- Magnetic field configuration inside the star is unknown.

$$\nabla . \mathrm{B} = 0 \rightarrow \mathrm{B} = \mathrm{P}\{r, \theta\} + \mathrm{T}\{\phi\}$$



Source: Wikipedia

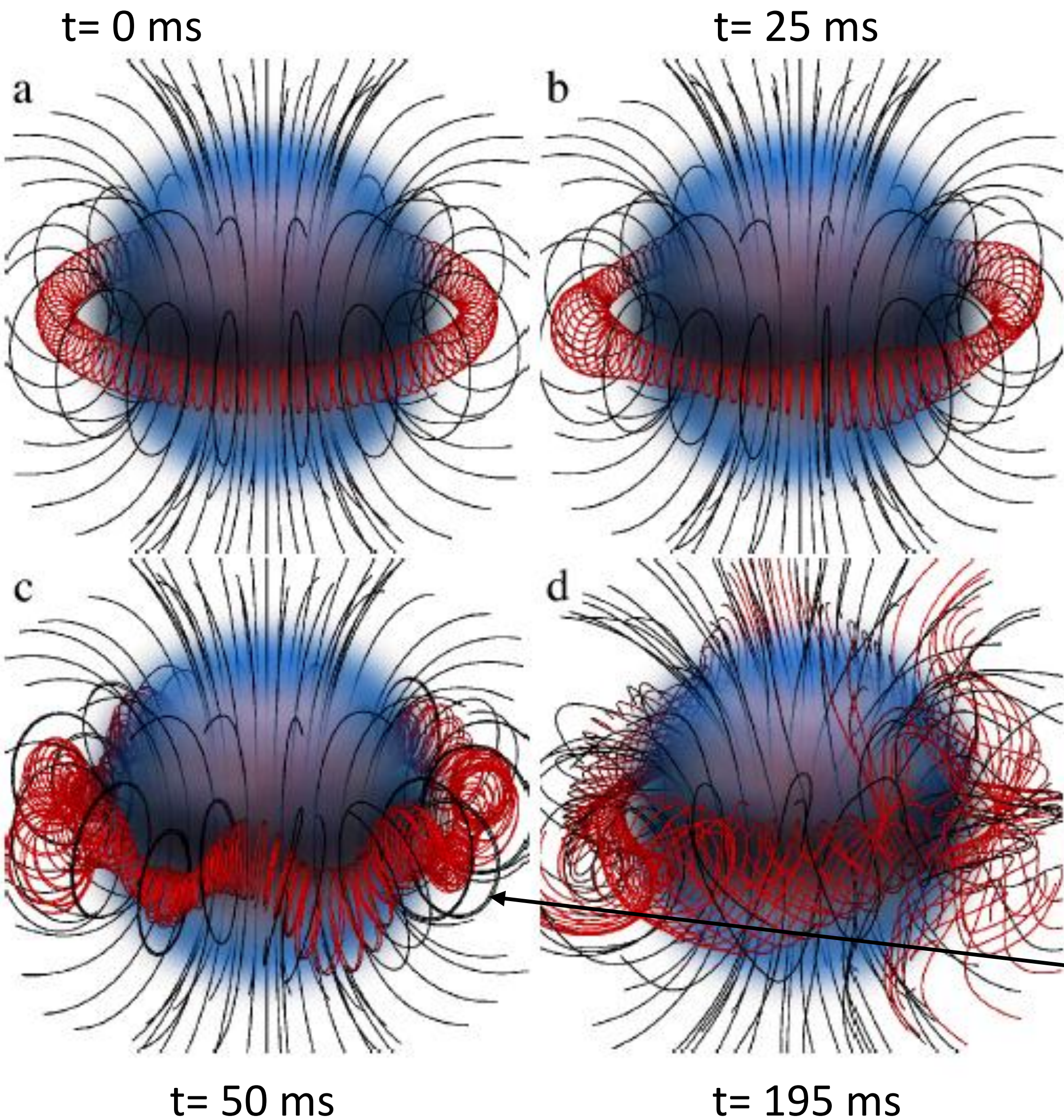
- Exterior field appears to be purely poloidal and dipolar.

- Initial random field evolution shows twisted-torus configuration. (Braithwaite and Nordlund 2006)

- Equilibrium models (Fujisawa et al 2012, Glampedakis et al. 2012; Lander & Jones 2009, 2012)
- Assessing the field stability is non trivial, simple geometries and non-rotating stars are considered.

Magnetic Field Instability

Lasky et al., ApJ, 2011



$$t_{Alfven} = \frac{2R\sqrt{4\pi<\rho>}}{<B>}$$

For a field of $< B >\sim 1.6 \times 10^{16}$ G,

$$t_{Alfven} \sim 5 \text{ ms}$$

Instability acts near the neutral line.

"Kink" instability visible

Magnetic Field Instability

Ciolfi, Lander, Manca, Rezzola, ApJ, 2011

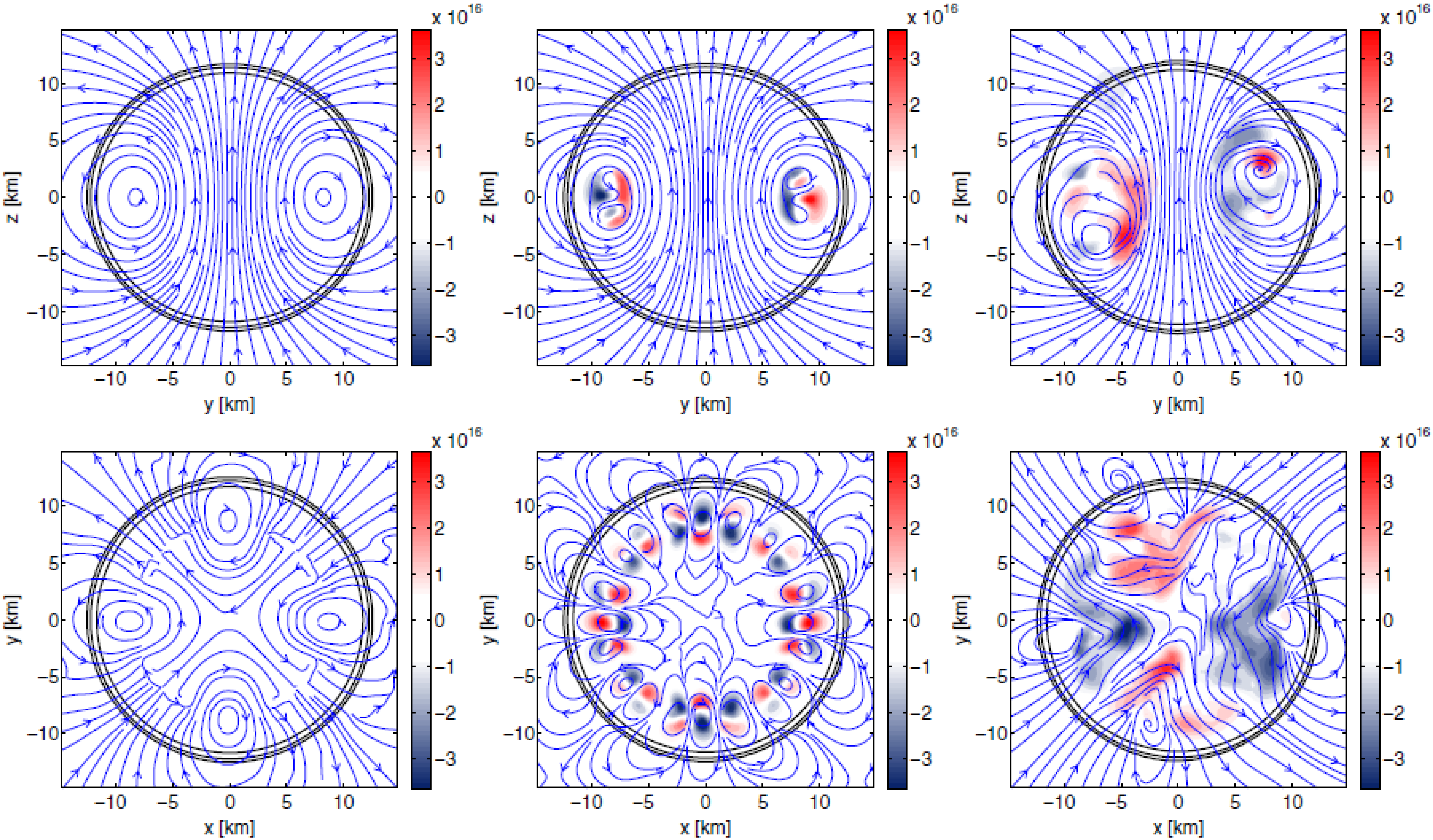


Figure 1. Snapshots of the development of the instability in our fiducial star, showing projections on the (x, z) plane (upper row) and (x, y) plane (lower row) of the simulation at times $t = 1, 3, 10$ ms (left to right), respectively. Shown with vector lines are the (global) magnetic-field lines, while the colors show the intensity of the toroidal magnetic field only; also reported are the isodensity contours of the rest-mass density near the stellar surface.

Questions

What is the internal magnetic field strength ?

What is the structure of the field?

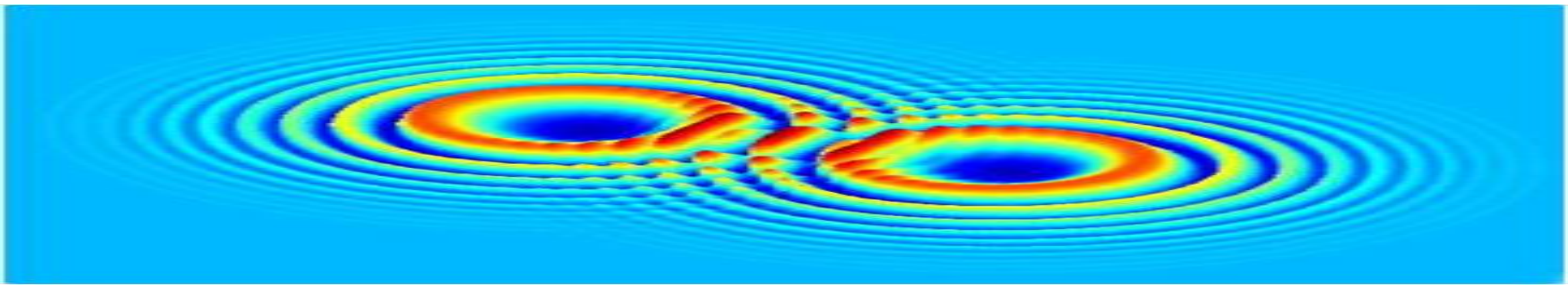
What are the modes of oscillations?

Magnetohydrodynamics

Hydro equations coupled with Maxwell's equations

Solve for $\{\rho, P, \vec{B}, \vec{V}\}$

$$
\begin{aligned}
\frac{\partial \rho}{\partial t} + \boldsymbol{v} \cdot \nabla \rho + \rho \nabla \cdot \boldsymbol{v} &= 0 \\
\frac{\partial \boldsymbol{v}}{\partial t} + \boldsymbol{v} \cdot \nabla \boldsymbol{v} + \frac{1}{\rho} \boldsymbol{B} \times (\nabla \times \boldsymbol{B}) + \frac{1}{\rho} \nabla p &= -\nabla \Phi + \boldsymbol{g} \\
\frac{\partial \boldsymbol{B}}{\partial t} + \boldsymbol{B}(\nabla \cdot \boldsymbol{v}) - (\boldsymbol{B} \cdot \nabla)\boldsymbol{v} + (\boldsymbol{v} \cdot \nabla)\boldsymbol{B} &= \boldsymbol{v}\,(\nabla \cdot \boldsymbol{B}) \\
\frac{\partial p}{\partial t} + \boldsymbol{v} \cdot \nabla p + \rho c_s^2 \nabla \cdot \boldsymbol{v} &= 0\,,
\end{aligned}
$$



Magnetohydrodynamics

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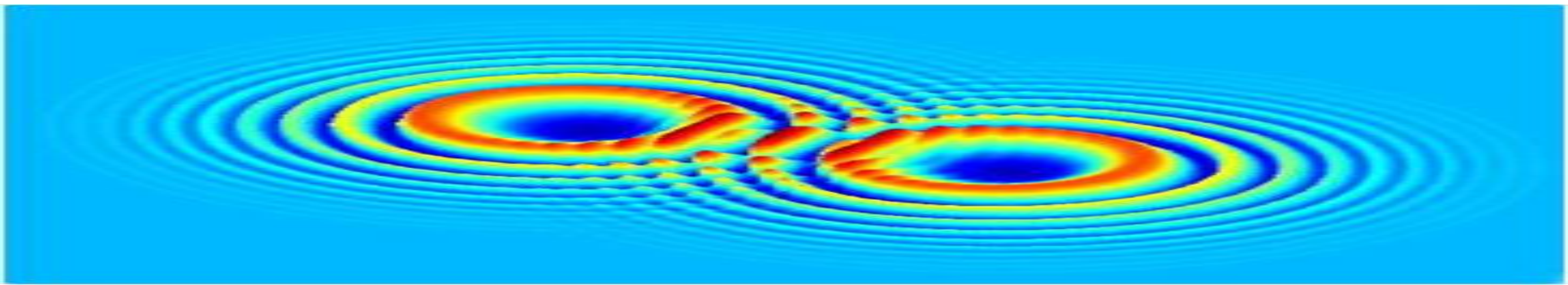
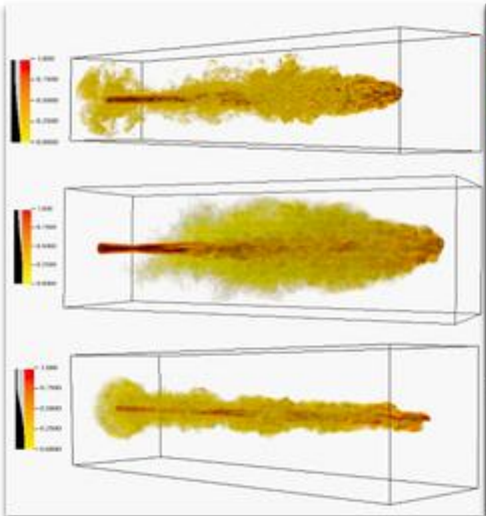
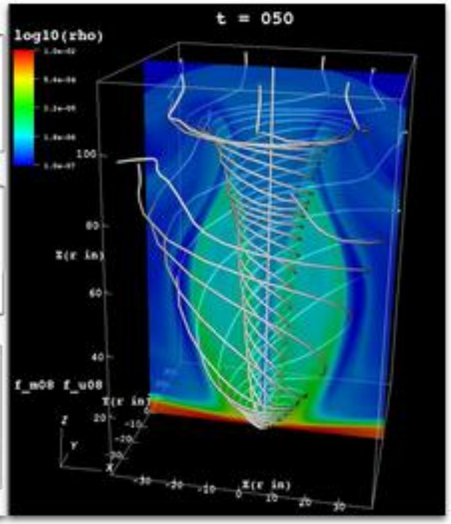
\end{aligned}
$$

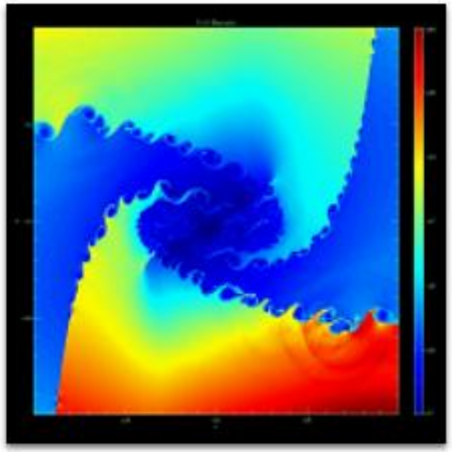
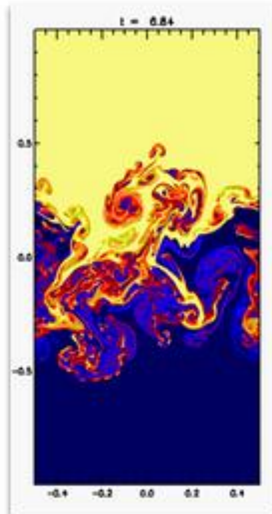
Equation of state



PLUTO code

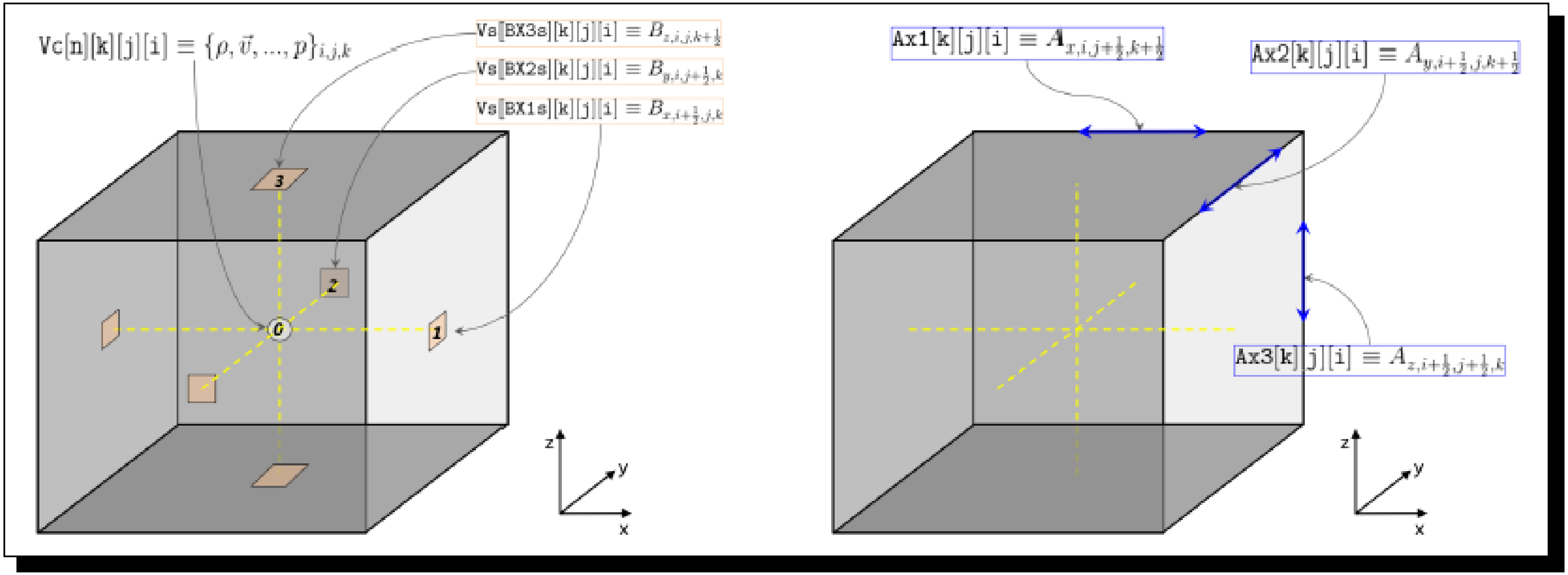
The PLUTO Code for Astrophysical GasDynamics

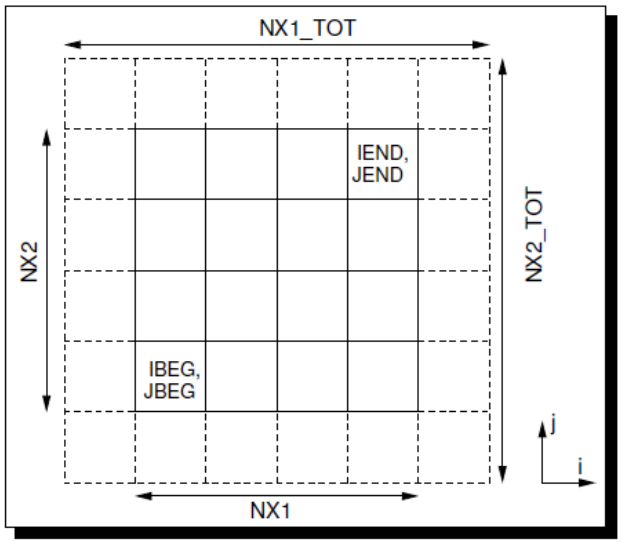



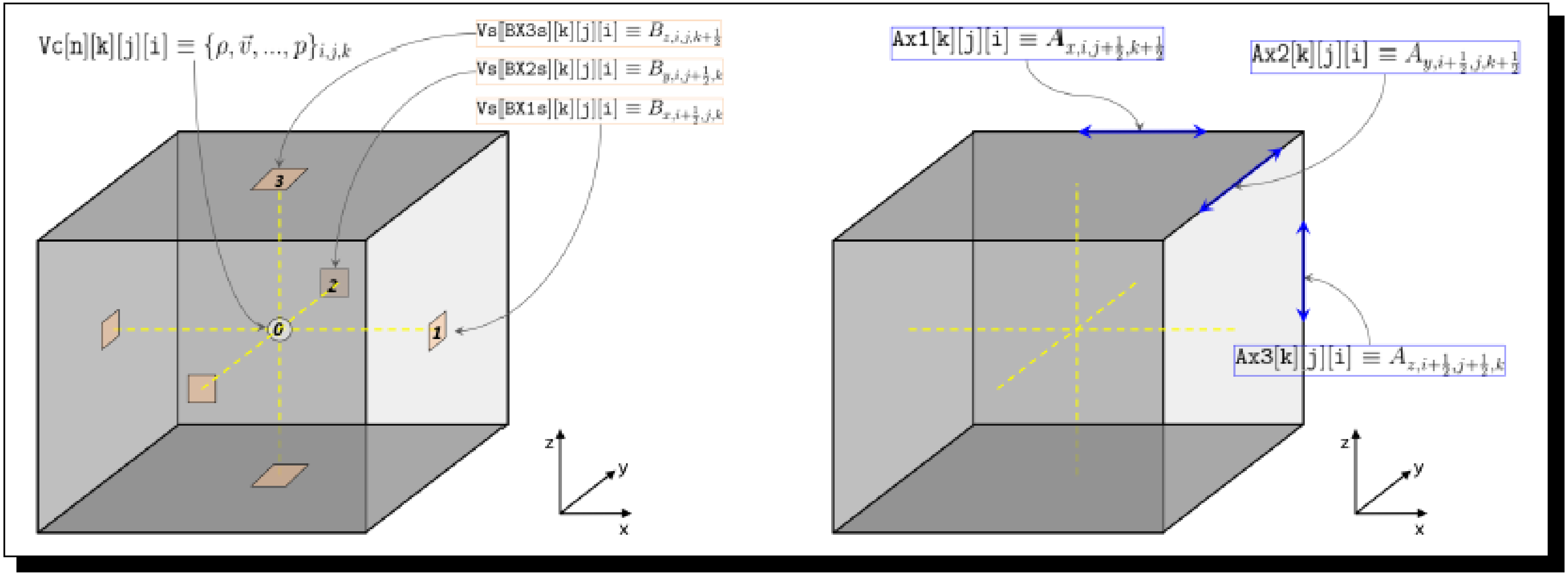
- Solves different modules using finite difference/finite volume method.
- Equations are discretized and solved in a structured mesh.
- Code written in C and Fortran.
- Works in normalized units.

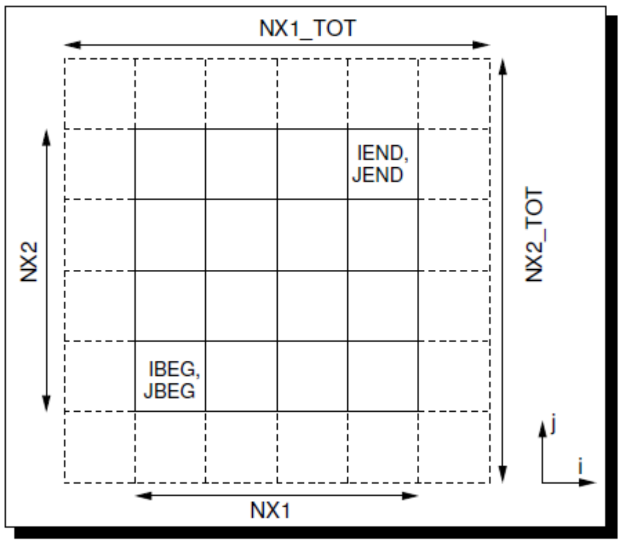
Numerical Computations





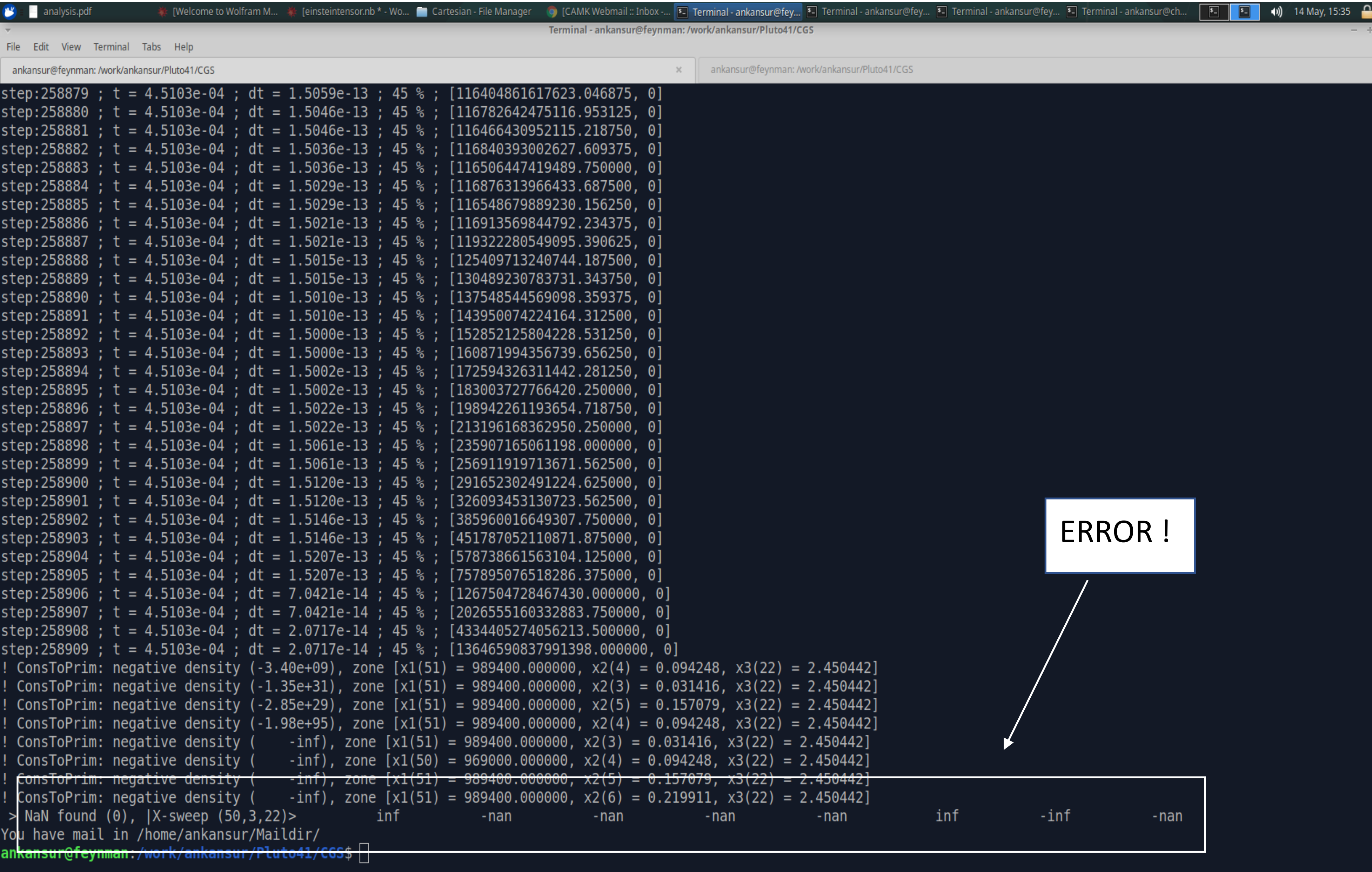
Numerical Computations





Boundary conditions are very challenging in numerics!

Simulations with PLUTO



Simulations with PLUTO

The PLUTO Code for Astrophysical GasDynamics

PLUTO 4.3: DID NOT WORK! :(

- Solves different modules using finite difference/finite volume method.
- Equations are discretized and solved in a structured mesh.
- Code written in C++ and Fortran.

Simulations with PLUTO

The PLUTO Code for Astrophysical GasDynamics

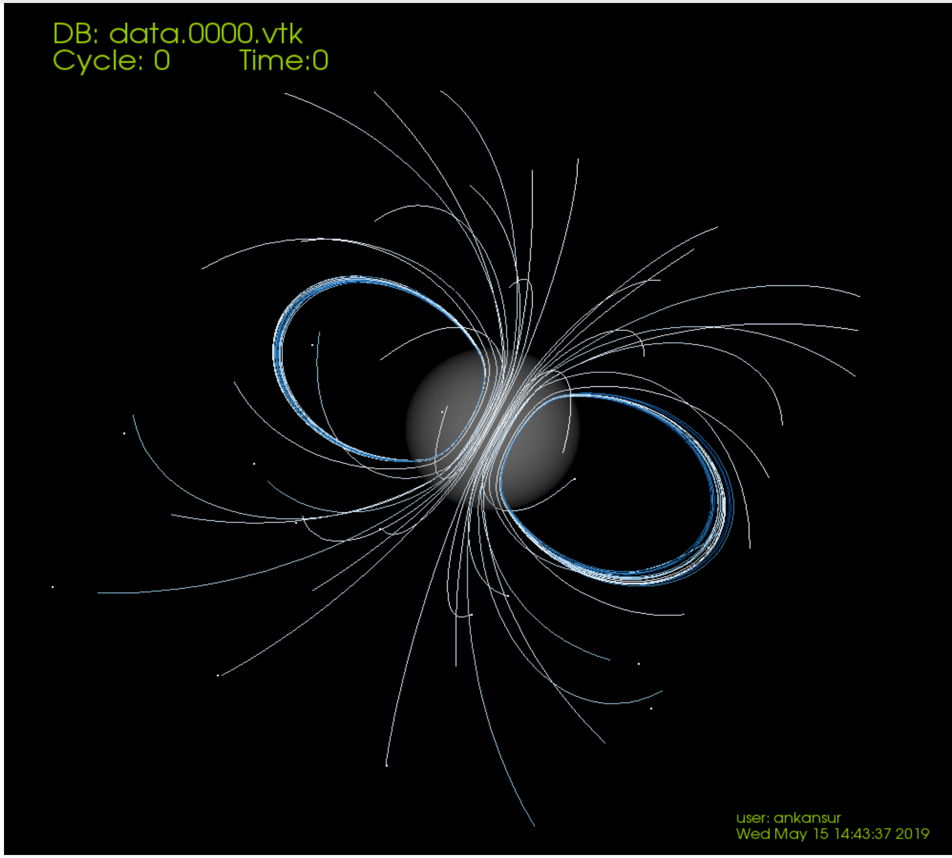
PLUTO 4.1: Worked ! :)

- Solves different modules using finite difference/finite volume method.
- Equations are discretized and solved in a structured mesh.
- Code written in C++ and Fortran.

Problem Setup

We consider :

- A nonrotating star
- $M \sim 1.4 M_\odot$
- Radius = 10 km
- Atmosphere with low density
- Field strength = B_s = 1-3 x 10^{17} G
- Evolution time = 5-10 Alfven timescales

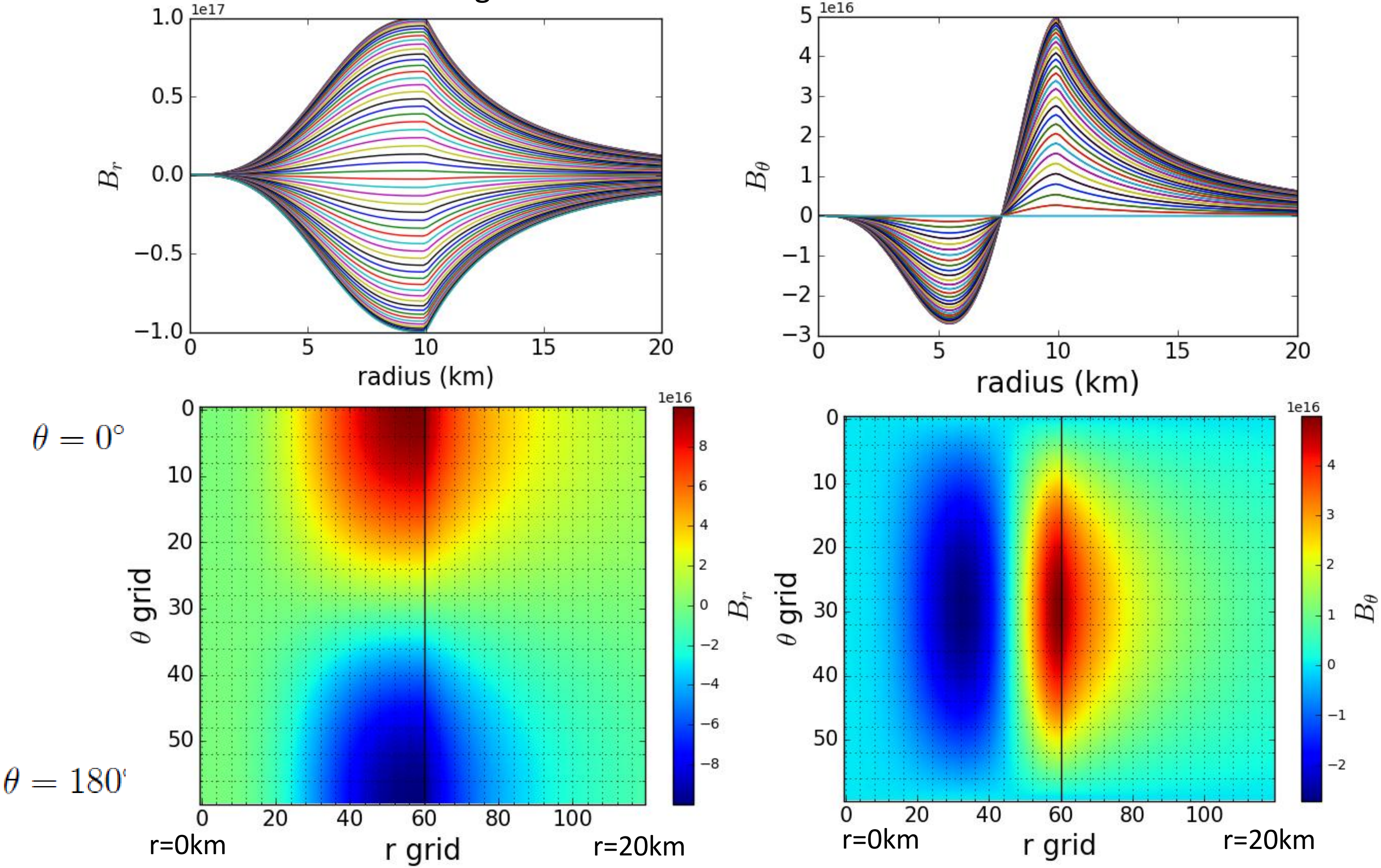


Simulation setup:
Visualization using Visit

Problem Setup

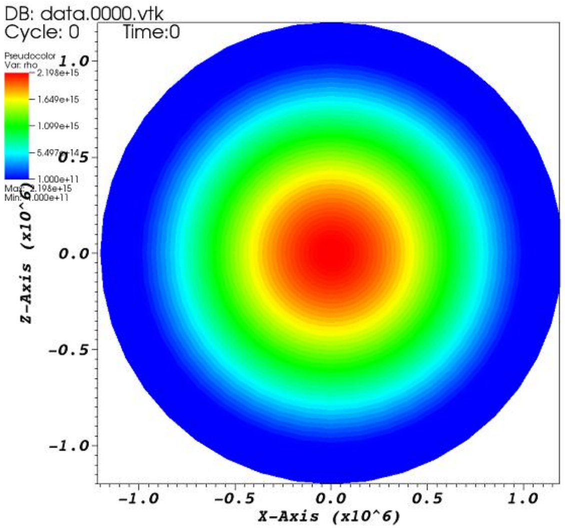
We use n=1 polytrope model ⟶ $\rho = \rho_c R \frac{sin(\pi r/R)}{\pi r}$ $P = K\rho^2$

Magnetic field from Haskell et al 2008

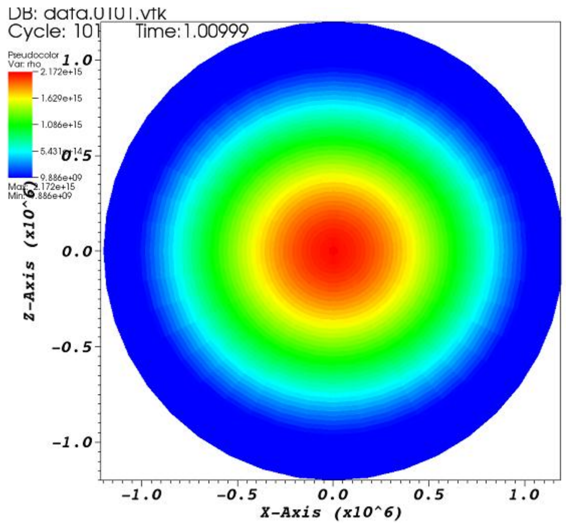


Simulation results

ρ

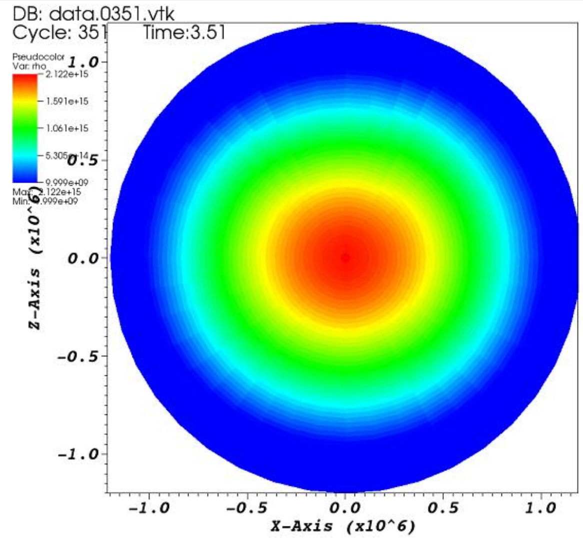


t = 0 ms



t = 1.0 ms

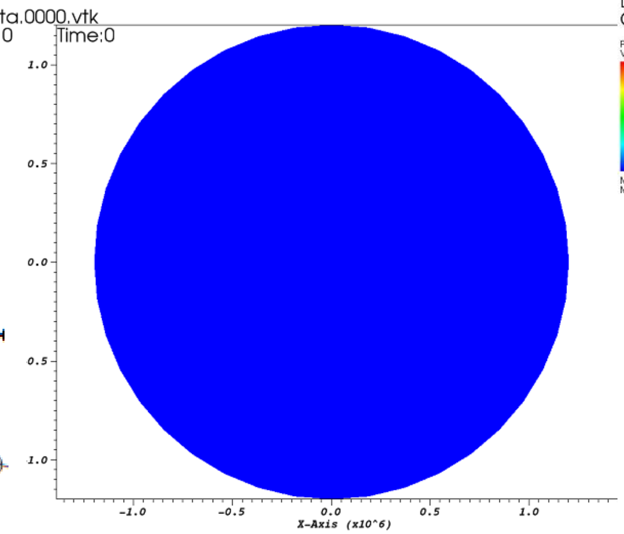
= 1 Alfven time

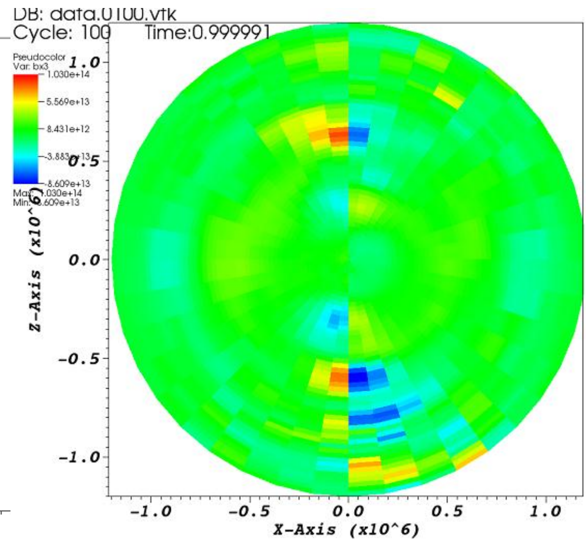


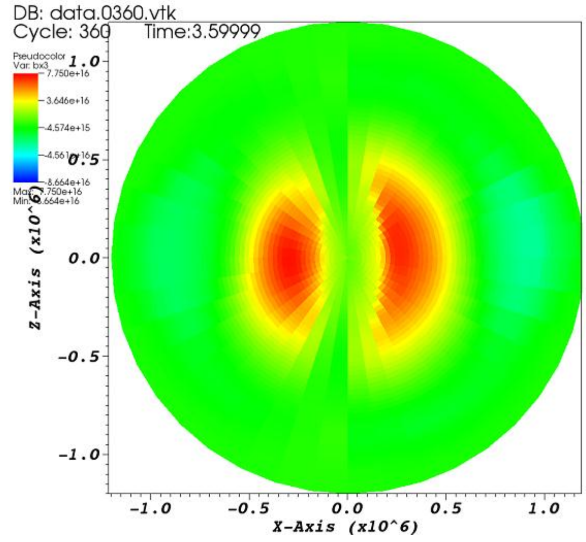
t = 3.2 ms

= 3 Alfven times

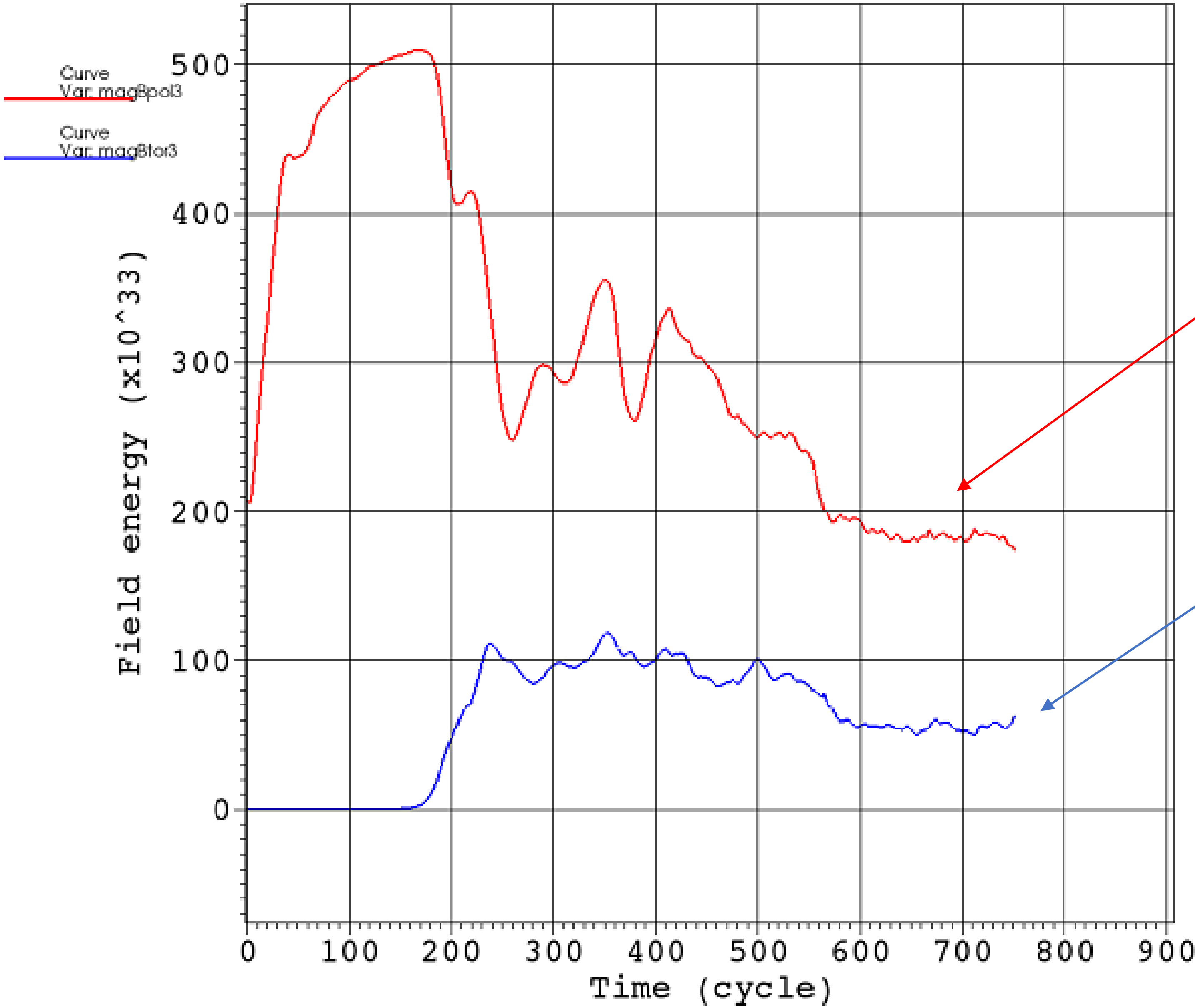
B_ϕ component







Magnetic field energies



Astrophysical importance

- The deformation of the star with misaligned spin and magnetic moment axes produces gravitational waves.

$$\epsilon = \frac{I_{zz} - I_{xx}}{I_0} \longrightarrow h = \frac{16\pi^2 G}{c^4}\frac{\epsilon I_0 f^2}{r}$$

- The internal rearrangement of the field are strong candidates for magnetic flares.

- The field configuration provide initial conditions for magneto-thermal evolution studies.

What else?

- We have similar setup in Cartesian coordinates.
- Compare results : Different coordinate systems? Different resolutions?
- Calculate instability growth times.
- How about changing initial conditions?

Acknowledgement

I am grateful to :

- Dr. Milijenko Cemeljic
- Dr. Varadaranjan Parthasarathy
- Dr. Samuel Lander
- Dr. Dipanjan Mukherjee

(others: Deepika, David,…)

And of course, my supervisor: Prof. Bryn Haskell