Lindhard integral equation with binding energy and the light & charge yields of nuclear recoils in noble liquids

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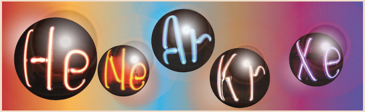

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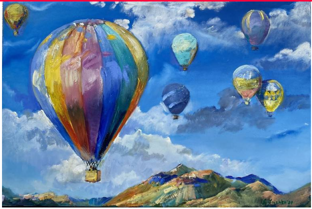
Overview



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Dual Phase TPC's

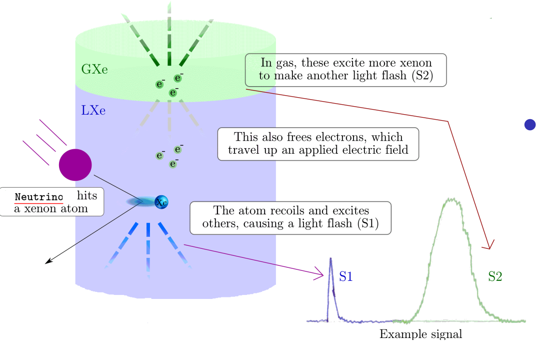
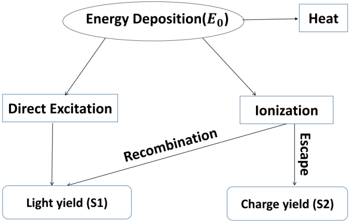


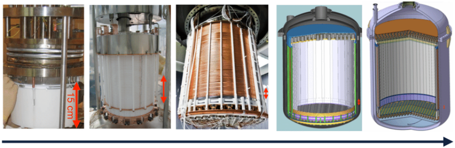
Image from: https://science.purdue.edu/xenon1t

- Total energy deposited in the detector is divided into::
  1. Direct Excitation.
  2. Ionization.
  3. Atomic movement.
- Ionization → light due to recombination.



J. Phys. G: Nucl. Part. Phys. 44 (2017) 055001

Dark Matter (DM) and CEνNS detections



XENON10	XENON100	XENON1T	XENONnT	DARWIN
2005 – 2007	2008 – 2016	2012 – 2018	2019 – 2023	2025 –
~15 kg	~62 kg	~2 t	~5.9 t	40 t
15 cm	30 cm	1 m	1.5 m	2.6 m
~10^{-43} cm^2	~10^{-45} cm^2	~10^{-47} cm^2	~10^{-48} cm^2	~10^{-49} cm^2

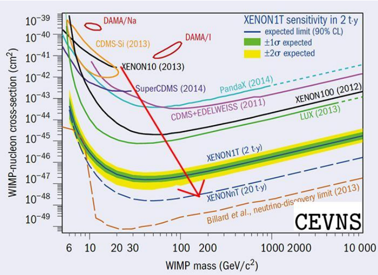


Image credit: XENON Collaboration.

- Massive detectors→ larger signals → lower limits.
- Systematic fluctuations for S_1 and S_2 signals will limit this tendency.
- Especially for large nT TPC's.
- Hence the importance of having a first principles theory for energy reconstruction.

Energy Dissipation in Noble Elements

- Energy E_R is deposited in the medium: $E_R = \bar{\mathcal{H}} + \bar{\mathcal{N}}$.
- Distributed between moving ions, $\bar{\mathcal{N}}$, and electrons $\bar{\mathcal{H}}$.
- Platzman equation states[2]: $\bar{\mathcal{H}} = N_i E_i + N_{ex} E_{ex} + N_i \tilde{E}$.
  - N_i: number of electron-ion pairs, $A + A \longrightarrow e + A^+ + A'$
  - N_{ex}: number of atoms excited, $A + A \longrightarrow A^* + A'$
  - $\tilde{E}$: average kinetic energy of sub-excitation electrons $\rightarrow$ Heat.
- The energy $\tilde{E}$ will be taken into account later in the binding energy.
- **Visible electrons and photons $\propto \bar{\mathcal{H}}$, ionization energy.**

[2] R. L. Platzman, Appl. Rad. Isot. 10, 116– 122 (1961)

Relation with ionization efficiency f_n

- We define the *ionization Efficiency*: $\boxed{\frac{\bar{\mathcal{H}}}{E_R} = \mathbf{f_n}}$.
- The total quanta is: $N = N_i + N_{ex} = N_i(1 + \frac{N_{ex}}{N_i})$,
- where $\frac{N_{ex}}{N_i} = \beta$ will be a constant, $N = N_i(1 + \beta)$.
- $W_i = \frac{\bar{\mathcal{H}}}{N_i}$ is the energy to create a e-h pair.
- Then the average energy to create an electron or exciton[3] is $W_{sc} = W_i/(1 + \beta)$.
  - **Finally** $\bar{\mathcal{H}} = W_i N_i = f_n E_R = W_{sc}(N_i + N_{ex}) = W_{sc}\left(\frac{S_2}{g_2} + \frac{S_1}{g_1}\right)$**.**

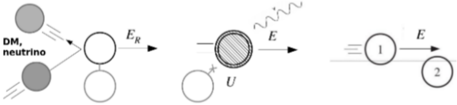
$$\boxed{E_R = W_{sc}\left(\frac{S_2}{g_2} + \frac{S_1}{g_1}\right)/\mathbf{f_n}}$$

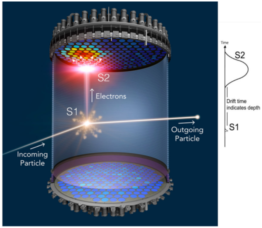
[3] For zero electric field all quanta goes to scintillation, $N = N_{sc}$.

Ionization Efficiency

To start the cascade of recoiling atoms the particle,e.g WIMP, have to deposit the necessary energy to free the ion.

$\bar{\mathcal{N}}$: Nuclear collisions[4]. ($\bar{\nu}$)
$\bar{\mathcal{H}}$: Ionization (visible) energy $[\mathrm{keV_{ee}}]$ ($\bar{\eta}$).





Recent Results from LUX and

Prospects for DM Searches with LZ

- $\frac{\text{total ionization energy}}{\text{total deposited energy}} = \boxed{f_n = \frac{\bar{\eta}}{\varepsilon_R}}$.
- $\varepsilon_R = \bar{\eta} + \bar{\nu}$, where ε_R is the recoil energy.
- Energy u is lost to some disruption of the atomic bonding: $\varepsilon_R = \varepsilon + u$.
- This sets a dissipative cascade of slowing-down processes.

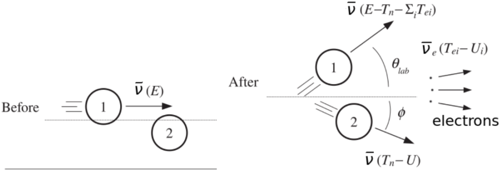
[4]Using dimensionless units ($\varepsilon = 11.5E(\mathrm{keV})/\mathrm{Z}^{7/3}$)

(T_n : Nuclear kinetic energy and T_{ei} electron kinetic energy.)

$$\underbrace{\int d\sigma_{n,e}}_{\textit{total cross section}} \left[\underbrace{\bar{\nu}\left(E - T_n - \sum_i T_{ei}\right)}_{\text{A}} + \underbrace{\bar{\nu}\,(T_n - U)}_{\text{B}} - \underbrace{\bar{\nu}(E)}_{\text{C}} + \underbrace{\sum_i \bar{\nu}_e\,(T_{ei} - U_{ei})}_{\text{D}} \right] = 0 \qquad (1)$$

Lindhard's (five) approximations

- I Neglect contribution to atomic motion coming from electrons.
- II **Neglect the binding energy, U = 0.**
- III Energy transferred to electrons is small compared to that transferred to ions.
- IV Effects of electronic and atomic collisions can be treated separately.
- V T_n is small compared to the energy E.



Lindhard Simplified Equation

Lindhard deduced a simplified integro-differential equation,

$$\underbrace{\left(k\varepsilon^{1/2}\right)}_{S_e}\bar{\nu}'(\varepsilon) = \int_0^{\varepsilon^2} \underbrace{dt\frac{f\left(t^{1/2}\right)}{2t^{3/2}}}_{d\sigma_n}\left[\bar{\nu}(\varepsilon - t/\varepsilon) + \bar{\nu}(t/\varepsilon) - \bar{\nu}(\varepsilon)\right], \quad (2)$$

L.H.S R.H.S

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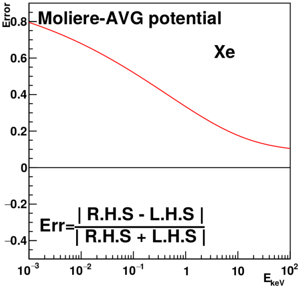
but since binding energy was neglected it is only valid at high energies, since $\bar{\nu}(\varepsilon \to 0) \to \varepsilon$, by the above equation we get $\bar{\nu}'(0) = 0!$.

- Lindhard found a good parametrization for the solution.
- Valid at high energies (U=0).
- $\bar{\nu}_L(\varepsilon) = \frac{\varepsilon}{1+kg(\varepsilon)}$, $g(\varepsilon) = 3\varepsilon^{0.15} + 0.7\varepsilon^{0.6} + \varepsilon$.
- **For low energies and a realistic potential Lindhard formula is not an accurate solution.**



Integro-Differential Equation With Binding Energy

- To include the binding energy, we recover the term **B** in Eq.(1): $\bar{\nu}(T_n) \rightarrow \bar{\nu}(T_n - U)$.
- It's also important to expand the term **A** in Eq.(1) up to second order.

$$\bar{\nu}\left(E - T_n - \Sigma_i T_{ei}\right) \approx \bar{\nu}\left(E - T_n\right) - \bar{\nu}'(E)\left(\Sigma_i T_{ei}\right) + \bar{\nu}''(E) T_n \left(\Sigma_i T_{ei}\right). \quad (3)$$

This leads to the appearance of the electronic stopping power in the first derivative term

$$\int d\sigma_{n,e} \bar{\nu}'(E)\left(\Sigma_i T_{ei}\right) = \bar{\nu}'(E) S_e(E), \ \int d\sigma_e \left(\Sigma_i T_{ei}\right) = S_e(E). \quad (4)$$

For the second derivative term, we can apply the integral mean value theorem

$$\int d\sigma_{n,e} \bar{\nu}''(E) T_n \left(\Sigma_i T_{ei}\right) = < T_n > \bar{\nu}''(E) S_e(E) = (\frac{1}{2}E) \bar{\nu}''(E) S_e(E). \quad (5)$$

Simplified equation with binding energy

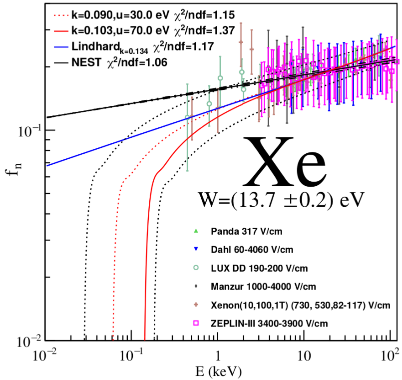
- Other works [5] didn't notice the necessity to change the lower limit of integration in order to be consistent with the term $\bar{\nu}(t/\varepsilon - u)$.
- In our work, we take this into account, so Eq.1 becomes:

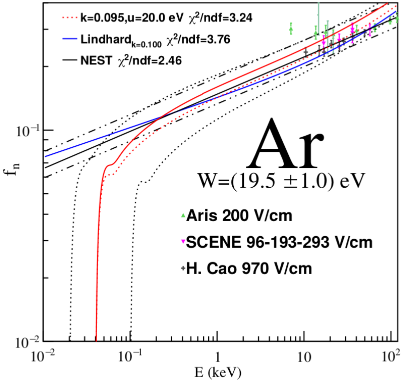
$$\boxed{-\frac{1}{2}k\varepsilon^{3/2}\bar{\nu}''(\varepsilon)} + \underbrace{k\varepsilon^{1/2}}_{S_e}\bar{\nu}'(\varepsilon) = \int_{\boxed{\varepsilon u}}^{\varepsilon^2} \underbrace{dt\frac{f\left(t^{1/2}\right)}{2t^{3/2}}}_{d\sigma_n}[\bar{\nu}(\varepsilon - t/\varepsilon) + \bar{\nu}(t/\varepsilon - \boxed{u}) - \bar{\nu}(\varepsilon)] \tag{6}$$

- This equation can be solved numerically from $\varepsilon \geqslant u$.
- The equation predicts a threshold energy at, $\varepsilon_R^{threshold} = 2u$.
- As a first approach, we assume a constant binding energy $u = u_0$.
- For more details, **Y. Sarkis et al, Phys.Rev.D 101,102001 (2020)** .

[5] Phys. Rev. D91, 0835098 (2015)

$(N_R)_s$ ionization efficiency for LAr and LXe





Best Fit by using Eq.(6) for k and u (10×10 Grid) to data.

Charge exchange effects like Bohr stripping, will damp f_n at high energies.

Recombination Model for Charge and Light Yields. Results

Thomas Imel Box Model

- Diffusion equation for ions-electrons recombination model[6].

$$\frac{\partial N_+}{\partial t} = -\alpha N_- N_+, \qquad \frac{\partial N_-}{\partial t} = u_- E \frac{\partial N_-}{\partial z} - \alpha N_+ N_-. \tag{7}$$

- Each excited or ionized atom leads to one photon or electron.
- $\Rightarrow N_i + N_{ex} = n_\gamma + n_e$, $n_e = (1-r)N_i$ & $n_\gamma = N_{ex} + rN_i$.
- Hence, the fraction of ionizations predicted is

$$\frac{n_e}{N_i} = \frac{1}{\xi}\ln(1+\xi), \quad 1 - r = \frac{1}{\xi}\ln\left(1+\xi\right), \quad \xi = \frac{N_i \alpha}{4a^2 v}.$$

$N_i = \frac{E_R \mathbf{f_n}}{W_{sc}(1+\beta)}$, Where β and $\gamma \equiv \frac{\alpha}{4a^2 v}$ are free parameters.

[6] Ann.Phys.IV, V42, pp.303–344, (1913). PRA 36, 614 (1987)

Xenon Charge Yield

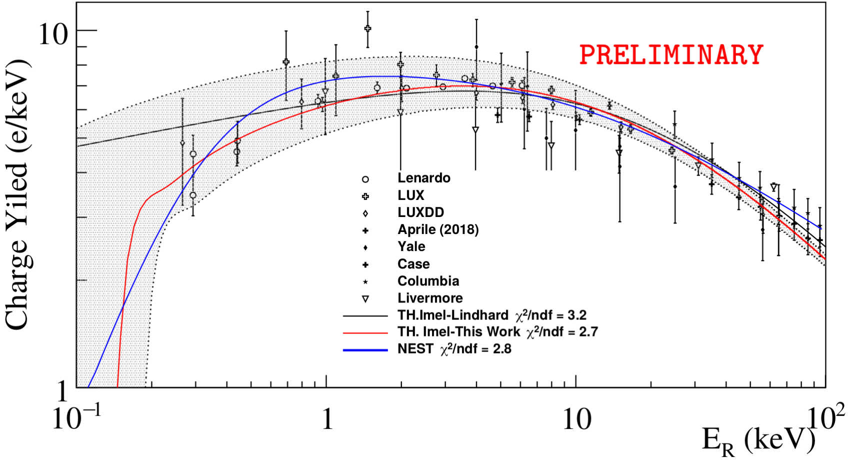


Figure: Charge Yield for Xe; $N_{ext}/N_i = 0.315$ and $\beta = 0.0127$

Xenon Light Yield

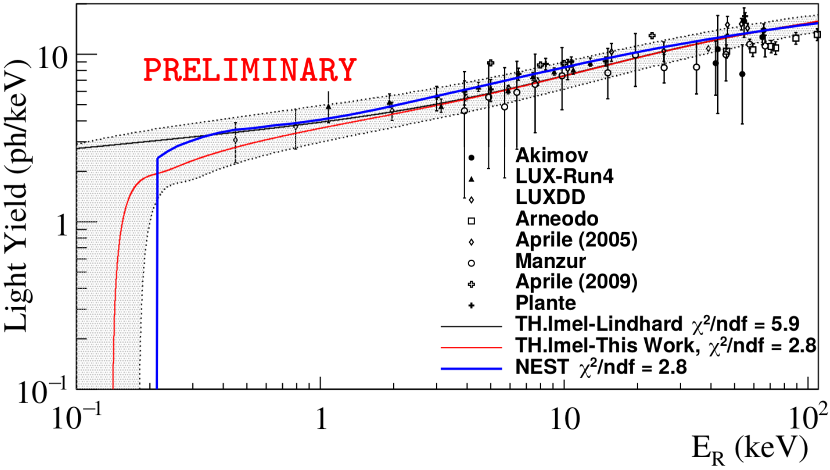


Figure: Light Yield for Xe; $N_{ext}/N_i = 0.701$ and $\beta = 0.0127$

Argon Charge Yield

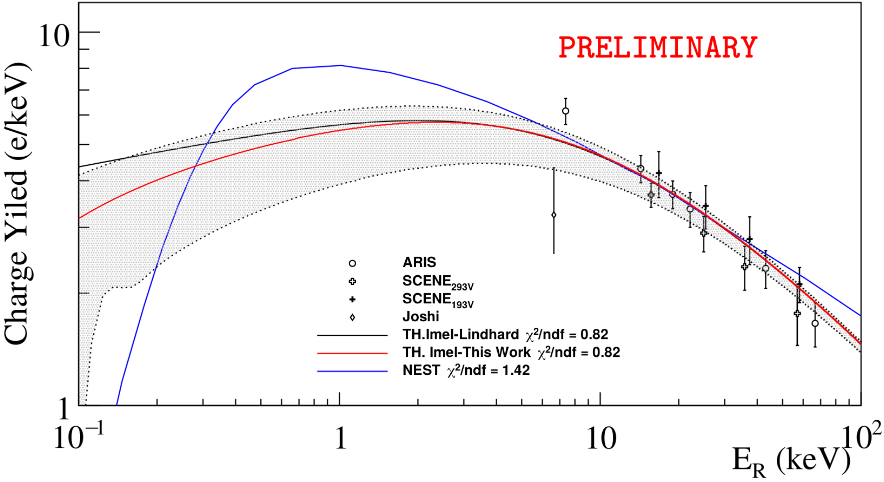


Figure: Charge Yield for Xe; $N_{ext}/N_i = 0.404$ and $\beta = 0.025$

Argon Light Yield

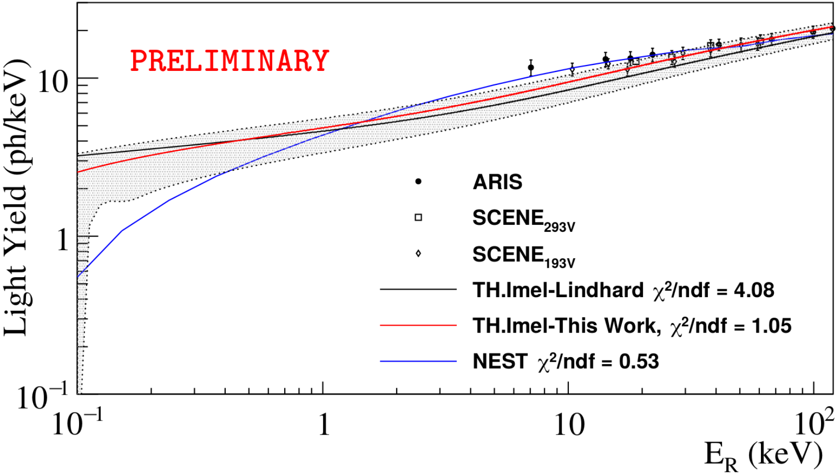


Figure: Light Yield for Xe; $N_{ext}/N_i = 1.168$ and $\beta = 0.025$

Conclusions

1. We present a first principles study based on Lindhard integral equation for nuclear recoil ionization efficiency $\mathbf{f_n}$, in LXe and LAr.
2. We show the implicit dependence of charge and light yield on $\mathbf{f_n}$.
3. The model predicts the turnover of f_n at low energies, already observed in Xe for $E_R < 1$ keV.
4. A similar behavior is expected for Ar. Lindhard's model fails to predict this effect.
5. At higher energies the model for f_n can be improved by considering Bohr stripping effects for electronic stopping power.
6. Lindhard integral equation with binding energy is a promising first-principles approach to study signal production in noble elements.

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