PROBLEM 6: SUPERLUMINAL SHOCK

- Consider a relativistic shock in reference frame $\mathcal{O}$ with normal upstream velocity $\overrightarrow{\mathrm{v}}_1 = [0,0,\mathrm{v}_1]$ in coordinates (x, y, z), and oblique upstream magnetic field $\overrightarrow{B}_1 = B_1[\sin\theta_1, 0, \cos\theta_1]$. Ideal MHD is satisfied both upstream and downstream.

- Consider another reference frame $\mathcal{O}'$ moving in $\mathcal{O}$ with boost velocity $\overrightarrow{\mathrm{v}}_\mathrm{b} = [\mathrm{v}_b,0,0]$. Using the Lorentz transformation, find what are the conditions to have:
  (1) $B'_z = 0$,
  (2) $\overrightarrow{E}'_1 = 0$.

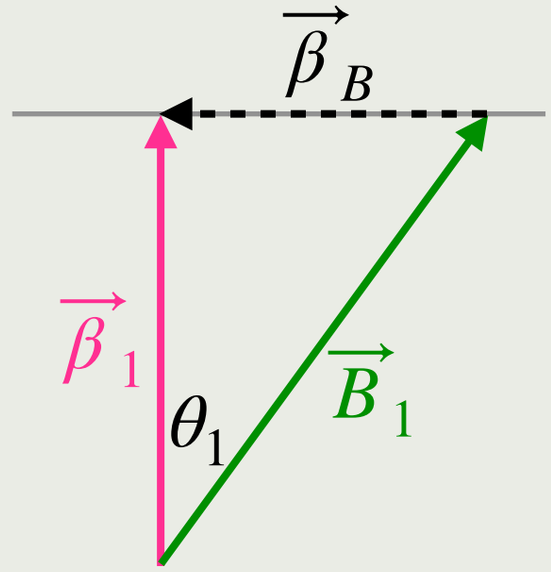
- Consider a particle that can only move along the local magnetic field. Such a particle can easily pass from the upstream region to the downstream region. In which case is it possible for this particle to return to the upstream region?

This problem is worth 5 points. Solutions should be sent as 1-page PDF files to knalew@camk.edu.pl before the next lecture.

LORENTZ TRANSFORMATIONS

- Let $\vec{\beta}_1 = \vec{\mathrm{v}}/c$ and $\vec{\beta}_\mathrm{b} = \vec{\mathrm{v}}_\mathrm{b}/c$. Introduce the boost Lorentz factor $\Gamma_\mathrm{b} = (1-\beta_\mathrm{b}^2)^{-1/2}$.

- The upstream electric field in $\mathcal{O}$:
  $\vec{E}_1 = \vec{B}_1 \times \vec{\beta}_1 = -B_1\beta_1 \sin\theta_1 \hat{y}$
  $\vec{\beta}_\mathrm{b} \times \vec{E}_1 = -\beta_\mathrm{b}\beta_1 B_1 \sin\theta_1 \hat{z}$
  $\vec{\beta}_\mathrm{b} \times \vec{B}_1 = -\beta_\mathrm{b} B_1 \cos\theta_1 \hat{y}$



- (1) Transformation of $B_{1,z}$:
  $B'_{1,z} = \Gamma_\mathrm{b}\left[B_{1,z} - \left(\vec{\beta}_\mathrm{b} \times \vec{E}_1\right)_z\right] = \Gamma_\mathrm{b} B_1 \left(\cos\theta_1 + \beta_\mathrm{b}\beta_1 \sin\theta_1\right)$
  $B'_{1,z} = 0$ implies that $\beta_\mathrm{b} = -\dfrac{1}{\beta_1 \tan\theta_1} \equiv -\dfrac{1}{\beta_B}$
  β_B is the speed of magnetic field line along the shock front

- (2) Transformation of $E_{1,y}$:
  $E'_{1,y} = \Gamma_\mathrm{b}\left[E_{1,y} + \left(\vec{\beta}_\mathrm{b} \times \vec{B}_1\right)_y\right] = -\Gamma_\mathrm{b} B_1 \left(\beta_1 \sin\theta_1 + \beta_\mathrm{b}\cos\theta_1\right)$
  $E'_{1,y} = 0$ implies that $\beta_\mathrm{b} = -\beta_1 \tan\theta_1 \equiv -\beta_B$

SUBLUMINAL
SUPERLUMINAL

- Transformation is possible if $|\beta_{\rm b}| < 1$.

- $E'_{1,y} = 0$ is possible for $|\beta_B| < 1$ (subluminal)
  or $|\tan\theta_1| < 1/\beta_1$
  (including $\theta_1 = 0$: parallel shock).
  $\overrightarrow{E}'_1 = 0$, hence $\overrightarrow{B}'_1 \parallel \overrightarrow{\beta}'_1$, particles can return upstream.

- $B'_{1,z} = 0$ is possible for $|\beta_B| > 1$ (superluminal)
  or $|\tan\theta_1| > 1/\beta_1$
  (including $\theta_1 = \pi/2$: perpendicular shock).
  $\overrightarrow{B}'_1 \perp \overrightarrow{n}'$, particles cannot return upstream.