COSMIC MAGNETIC FIELDS

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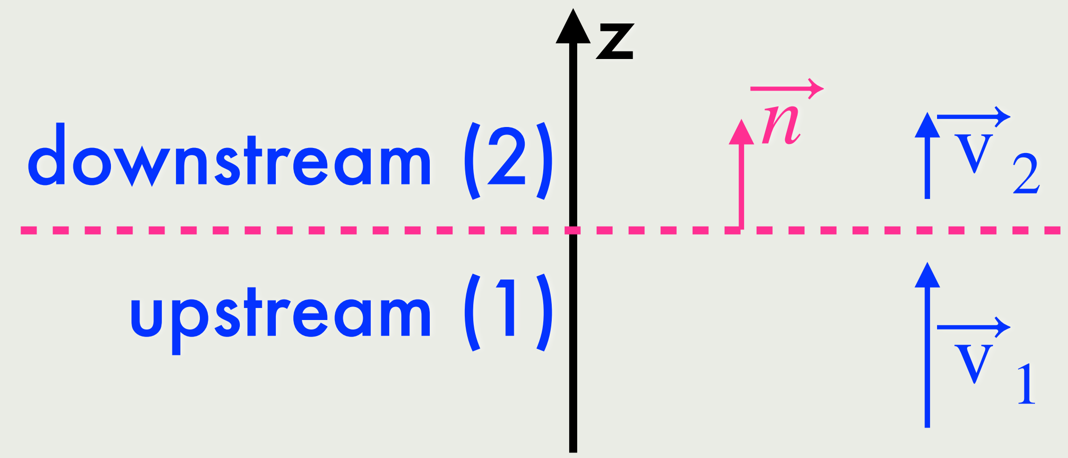
MHD shocks

THE SHOCK PROBLEM



- Consider a stationary discontinuity along the $z = 0$ plane (with a normal vector $\vec{n} = [0,0,1]$).

- A fluid flows from the upstream region 1 ($z < 0$) to the downstream region 2 ($z > 0$): $v_{1,z}, v_{2,z} > 0$.

- Given the upstream fluid parameters, what can be the downstream fluid parameters?

HYDRODYNAMIC SHOCK

- conservation of mass: $\Gamma_2 \mathrm{v}_{2,z} \rho_2 = \Gamma_1 \mathrm{v}_{1,z} \rho_1$ or $\left[\Gamma \mathrm{v}_z \rho\right] = 0$.
- conservation of energy-momentum: $\left[T_{\mathrm{fl}}^{\mu z}\right] = 0$, where $T_{\mathrm{fl}}^{\mu\nu} = w \dfrac{u^\mu u^\nu}{c^2} + P g^{\mu\nu}$ is the energy-momentum equation for a fluid.
- choosing the reference frame where $\mathrm{v}_{1,x} = \mathrm{v}_{1,y} = 0$, hence $\mathrm{v}_{2,x} = \mathrm{v}_{2,y} = 0$, leaves us with two non-trivial equations: $\left[T_{\mathrm{fl}}^{0z}\right] = 0$ and $\left[T_{\mathrm{fl}}^{zz}\right] = 0$.
- adiabatic equation of state: relativistic enthalpy density $w_2 = \rho_2 c^2 + [\kappa_2/(\kappa_2 - 1)] P_2$, pressure $P_2 = \Theta_2 \rho_2 c^2$, relativistic temperature $\Theta_2 = k_{\mathrm{B}} T_2 / m c^2$, and adiabatic index $4/3 < \kappa_2(\Theta_2) < 5/3$.
- 3 equations for 3 variables: $\mathrm{v}_{2,z}$, ρ_2, Θ_2.

NON-RELATIVISTIC HYDRODYNAMIC SHOCK

- The limit of non-relativistic velocities $\mathrm{v}_1, \mathrm{v}_2 \ll c$ and non-relativistic temperatures $\Theta_1, \Theta_2 \ll 1$, hence $\kappa_1, \kappa_2 = 5/3$.

- Shock jump equations (Rankine-Hugoniot conditions):
  $\rho_2 \mathrm{v}_2 = \rho_1 \mathrm{v}_1$ (mass conservation),
  $\rho_2 \mathrm{v}_2^2 + P_2 = \rho_1 \mathrm{v}_1^2 + P_1$ (momentum conservation),
  $\left(\frac{\rho_2 \mathrm{v}_2^2}{2} + \frac{5}{2} P_2 \right) \mathrm{v}_2 = \left(\frac{\rho_1 \mathrm{v}_1^2}{2} + \frac{5}{2} P_1 \right) \mathrm{v}_1$ (energy conservation).

- Introducing the shock velocity jump, equivalent to the compression ratio: $r = \mathrm{v}_1/\mathrm{v}_2 \equiv \rho_2/\rho_1$; and eliminating v_2, P_2 one obtains $(r-4)\rho_1 \mathrm{v}_1^2 + 5rP_1 = 0$.

- Introducing the upstream sound speed $\mathrm{v}_{s,1} = \sqrt{5P_1/3\rho_1}$ and the Mach number $M_1 = \mathrm{v}_1/\mathrm{v}_{\mathrm{s},1}$, the result is $r = 4M_1^2/(M_1^2+3)$ and hence $P_2/P_1 = (4r-1)/(4-r)$.

- It can be shown that for $r > 1$ specific entropy satisfies $s_2 > s_1$, and also that $M_1 > 1$ and $M_2 < 1$: *the solution is physical when a supersonic upstream flow converts into a subsonic downstream flow.*

MAGNETIC FIELD JUMP

- Magnetic field jump is calculated from stationary source-free Maxwell's equations in ideal MHD.

- $\overrightarrow{\nabla} \cdot \overrightarrow{B} = 0 \rightarrow \partial_z B_z = 0 \rightarrow \left[B_z\right] = 0$
  magnetic field parallel to the shock normal is conserved

- $\overrightarrow{\nabla} \times \overrightarrow{E} = 0 \rightarrow \partial_z E_x = \partial_z E_y = 0 \rightarrow \left[B_x \mathrm{v}_z - B_z \mathrm{v}_x\right] = \left[B_y \mathrm{v}_z - B_z \mathrm{v}_y\right] = 0$
  magnetic field perpendicular to the shock normal is compressed

- The source Maxwell's equations allow for the presence of electric charge and/or electric current of surface densities $\Sigma_\mathrm{e} = [E_z]/4\pi$ and $\mathcal{J}_y = (c/4\pi)[B_x]$, etc.

MAGNETIZED SHOCKS

- Let $\vec{B}_1 = (B_{1,x}, 0, B_z)$, $\vec{B}_2 = (B_{2,x}, 0, B_z)$ and $\vec{\mathrm{v}}_1 = [0, 0, \mathrm{v}_{1,z}]$
- parallel shocks: $\vec{B}_1, \vec{B}_2 \parallel \vec{n}$, hence $B_{1,x} = B_{2,x} = 0$. Parallel magnetic field cancels out from the shock jump equations, which reduce to the hydrodynamic form.
- perpendicular shocks: $\vec{B}_1, \vec{B}_2 \perp \vec{n}$, hence $B_z = 0$. Perpendicular magnetic field contributes to the shock jump equations.
- oblique shocks: $B_z \neq 0$ and $B_{1,x} \neq 0$, hence $B_{2,x} \neq 0$.

MAGNETIC ENERGY-MOMENTUM TENSOR

- consider an upstream magnetic field $\overrightarrow{B}_1 = (B_{1,x}, 0, B_z)$
  recall the upstream velocity $\overrightarrow{\beta}_1 = (0, 0, \beta_1)$

- non-zero elements of the $T^{\mu z}_{\mathrm{EM},1}$ tensor:

  $T^{0z}_{\mathrm{EM},1} = \beta_1 \dfrac{B_{1,x}^2}{4\pi}$ (energy)

  $T^{xz}_{\mathrm{EM},1} = -\dfrac{B_{1,x} B_z}{4\pi}$ (perpendicular momentum, generates $\beta_{2,x}$ in oblique fields)

  $T^{zz}_{\mathrm{EM},1} = (1 + \beta_1^2)\dfrac{B_{1,x}^2}{8\pi} - \dfrac{B_z^2}{8\pi}$ (parallel momentum; B_z^2 and $\beta_1^2 B_{1,x}^2$ terms cancel out)

NON-RELATIVISTIC PERPENDICULAR SHOCK

- Shock jump equations:

$\rho_2 \mathrm{v}_2 = \rho_1 \mathrm{v}_1$ (continuity)

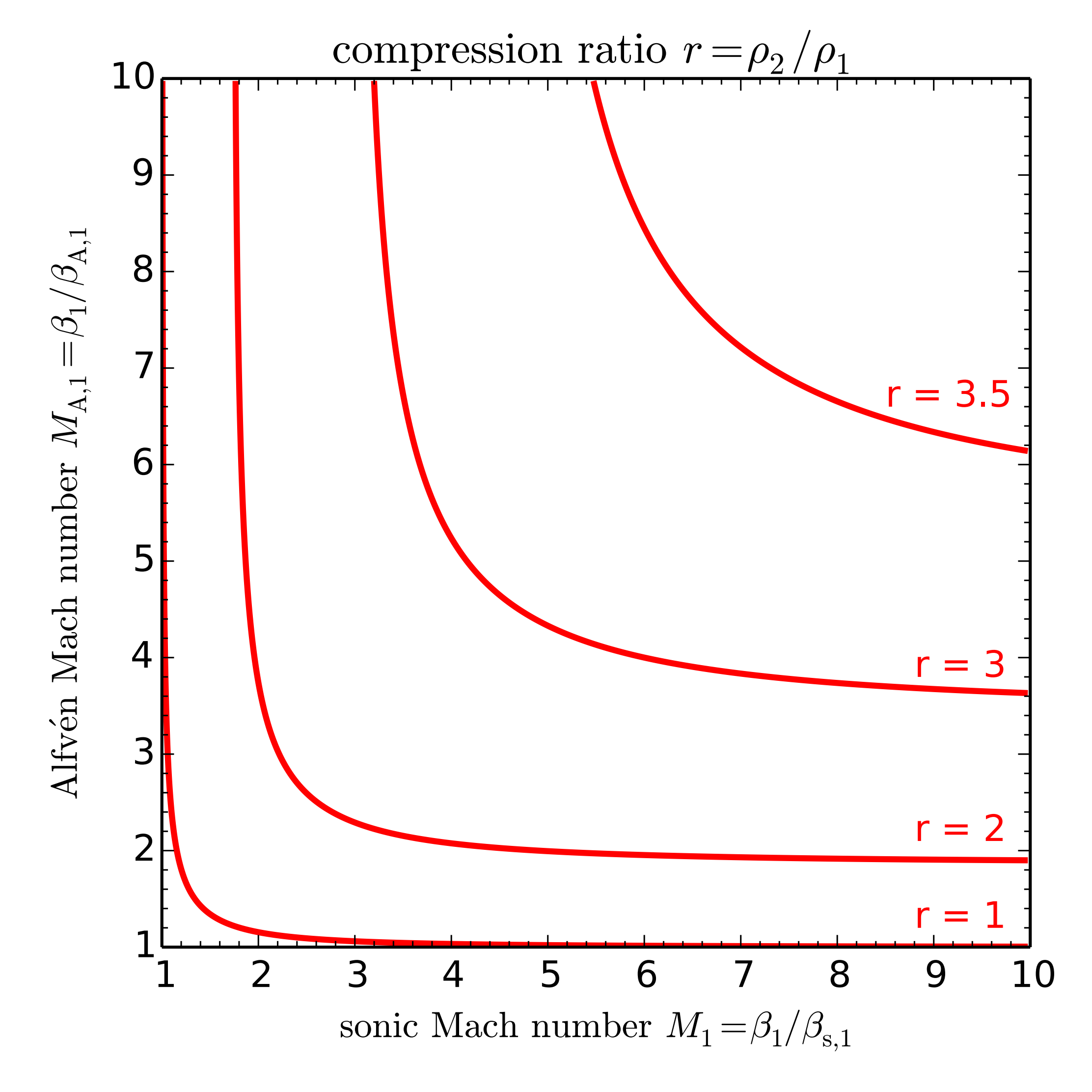
$\rho_2 \mathrm{v}_2^2 + P_2 + \frac{B_{2,x}^2}{8\pi} = \rho_1 \mathrm{v}_1^2 + P_1 + \frac{B_{1,x}^2}{8\pi}$ (momentum)

$\left(\frac{\rho_2 \mathrm{v}_2^2}{2} + \frac{5}{2}P_2 + \frac{B_{2,x}^2}{4\pi}\right)\mathrm{v}_2 = \left(\frac{\rho_1 \mathrm{v}_1^2}{2} + \frac{5}{2}P_1 + \frac{B_{1,x}^2}{4\pi}\right)\mathrm{v}_1$ (energy)

$B_{2,x}\mathrm{v}_2 = B_{1,x}\mathrm{v}_1$ (electric field)

- Using $r = \mathrm{v}_1/\mathrm{v}_2$, they reduce to:

$(r-4)\rho_1 \mathrm{v}_1^2 + 5rP_1 + r(r+5)\frac{B_{1,x}^2}{8\pi} = 0$



- Introducing the upstream Alfvén speed $\mathrm{v}_{A,1} = B_{1,x}/\sqrt{4\pi\rho_1}$ and the Alfvén Mach number $M_{A,1} = \mathrm{v}_1/\mathrm{v}_{A,1}$ results in a quadratic equation $r - 4 + \frac{3r}{M_1^2} + \frac{r(r+5)}{2M_{A,1}^2} = 0$ that has only one physical solution for $\mathrm{v}_1^2 > \mathrm{v}_{A,1}^2 + \mathrm{v}_{s,1}^2 \equiv \mathrm{v}_{FM,1}^2$, i.e., *super-fast-magnetosonic upstream flow.*

RELATIVISTIC PERPENDICULAR SHOCK

- $\Gamma_2 \rho_2 \beta_2 = \Gamma_1 \rho_1 \beta_1$ (continuity)

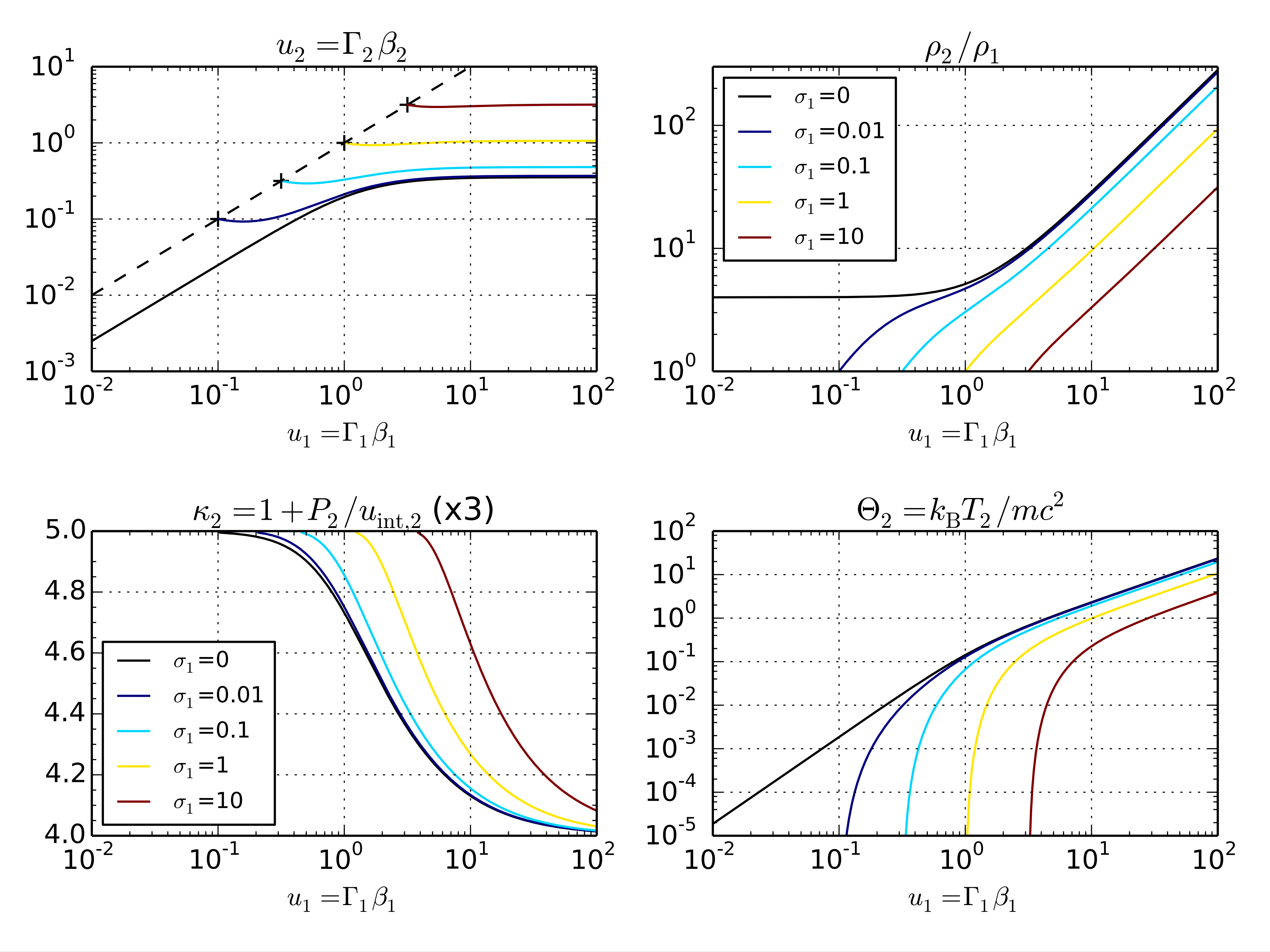
- $\Gamma_2^2 w_2 \beta_2^2 + P_2 + \frac{B_{2,x}^2}{8\pi} = \Gamma_1^2 w_1 \beta_1^2 + P_1 + \frac{B_{1,x}^2}{8\pi}$ (momentum)

- $\Gamma_2^2 w_2 \beta_2 + \frac{B_{2,x}^2}{4\pi} \beta_2 = \Gamma_1^2 w_1 \beta_1 + \frac{B_{1,x}^2}{4\pi} \beta_1$ (energy)

- $B_{2,x} \beta_2 = B_{1,x} \beta_1$ (electric field)

RELATIVISTIC PERPENDICULAR SHOCK

- numerical solutions in the limit of cold upstream gas ($T_1 = 0$).



ULTRA-RELATIVISTIC PERPENDICULAR SHOCK

- Consider the limit of $\Gamma_1 \gg 1$, hence $\beta_1 \simeq 1$.
  Let $r = \beta_1/\beta_2 \simeq 1/\beta_2$, and $B_{2,x} = rB_{1,x}$ like before.
  Introduce upstream magnetization $\sigma_1 = B_{1,x}^2/(4\pi\Gamma_1^2 w_1)$.
  Anticipating $\Theta_2 \gg 1$, adopt $\kappa_2 \simeq 4/3$ and $w_2 = 4P_2$.

- $$\Gamma_2^2 w_2 \beta_2^2 + P_2 + \frac{B_{2,x}^2}{8\pi} = \Gamma_1^2 w_1 \beta_1^2 + P_1 + \frac{B_{1,x}^2}{8\pi} \quad \text{(momentum)}$$
  $$\frac{r^2+3}{r^2-1} P_2 \simeq \left[1 - (r^2-1)\frac{\sigma_1}{2}\right] \Gamma_1^2 w_1$$

- $$\Gamma_2^2 w_2 \beta_2 + \frac{B_{2,x}^2}{4\pi}\beta_2 = \Gamma_1^2 w_1 \beta_1 + \frac{B_{1,x}^2}{4\pi}\beta_1 \quad \text{(energy)}$$
  $$\frac{4r}{r^2-1} P_2 \simeq \left[1 - (r-1)\sigma_1\right] \Gamma_1^2 w_1$$

- $$\frac{P_2}{(r^2-1)\Gamma_1^2 w_1} \simeq \frac{2-(r^2-1)\sigma_1}{2(r^2+3)} \simeq \frac{1-(r-1)\sigma_1}{4r}$$

- $$\sigma_1 \simeq -\frac{(r-3)}{(r-1)(r+3)}$$

ULTRA-RELATIVISTIC PERPENDICULAR SHOCK

- $\sigma_1 \simeq -\dfrac{(r-3)}{(r-1)(r+3)}$

- Solution: $r \simeq \dfrac{1+2\sigma_1}{2\sigma_1}\left[2\sqrt{1-\dfrac{3}{4(1+2\sigma_1)^2}}-1\right]$

- Hydrodynamic limit ($\sigma_1 = 0$): $r \simeq 3$, velocity $\beta_2 \simeq 1/3$, density $\rho_2 \simeq \sqrt{8}\,\Gamma_1\rho_1$, pressure $P_2 \simeq (2/3)\Gamma_1^2 w_1$, temperature $\Theta_2 \simeq (1/3\sqrt{2})(\Gamma_1 w_1/\rho_1 c^2)$.

- Relativistic magnetization ($\Gamma_1 > \sigma_1 \gg 1$): $r \simeq 1 + 1/(2\sigma_1)$, velocity $\beta_2 \simeq 1 - 1/(2\sigma_1)$, density $\rho_2 \simeq \Gamma_1\rho_1/\sqrt{\sigma_1}$, pressure $P_2 \simeq \Gamma_1^2 w_1/(8\sigma_1)$, temperature $\Theta_2 = P_2/\rho_2 c^2 = (1/8\sqrt{\sigma_1})(\Gamma_1 w_1/\rho_1 c^2)$.

FERMI PROCESS IN COLLISIONLESS SHOCKS

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PARTICLE ACCELERATION IN RELATIVISTIC COLLISIONLESS SHOCKS: FERMI PROCESS AT LAST?

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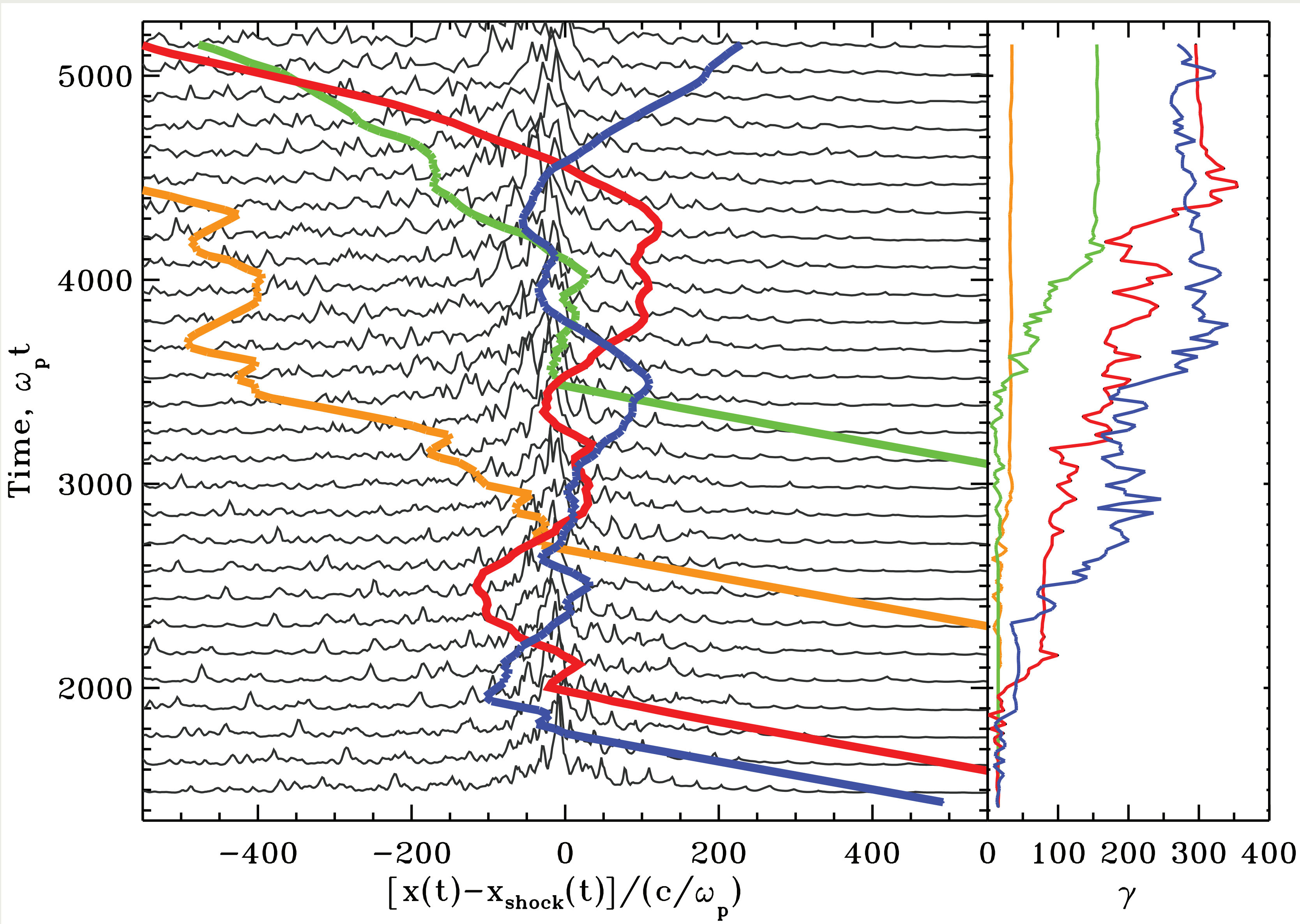
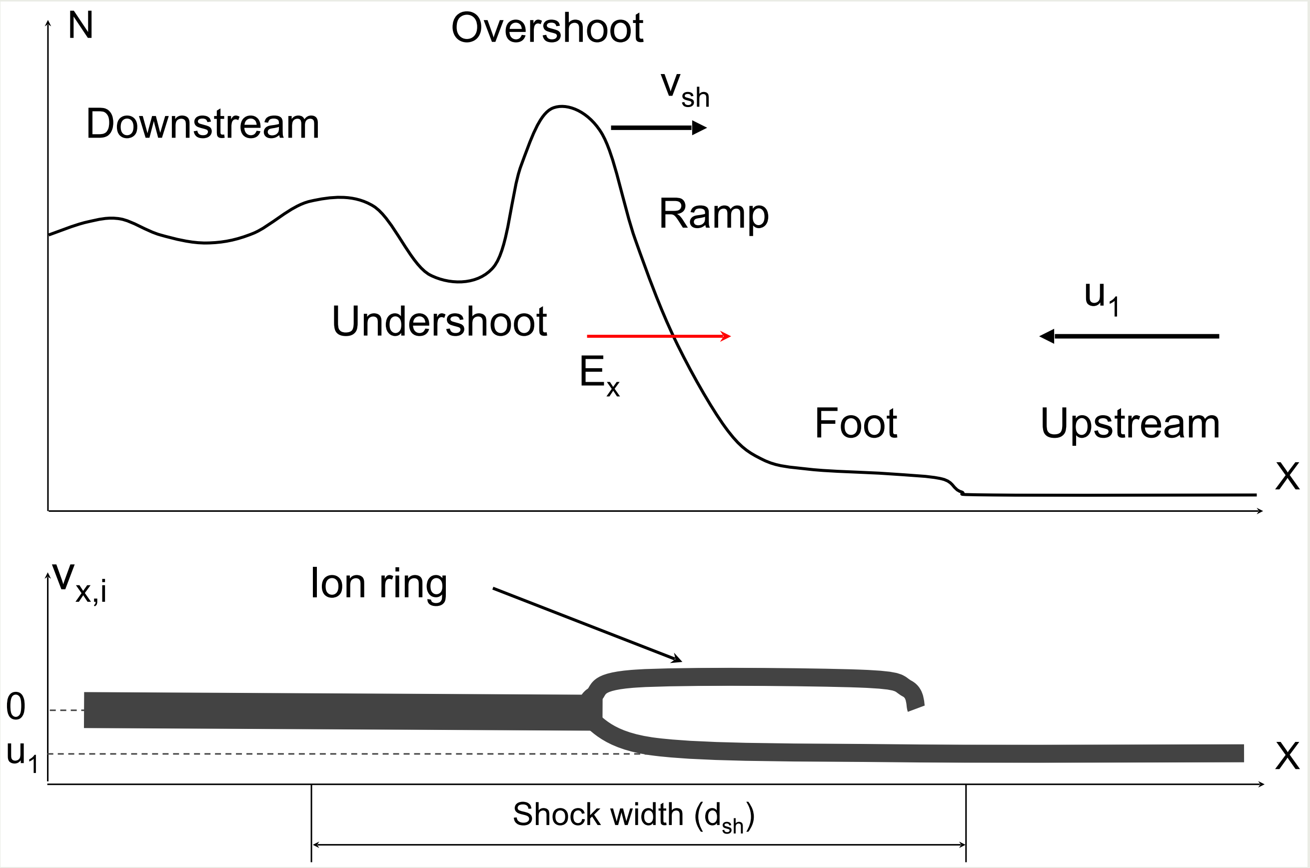
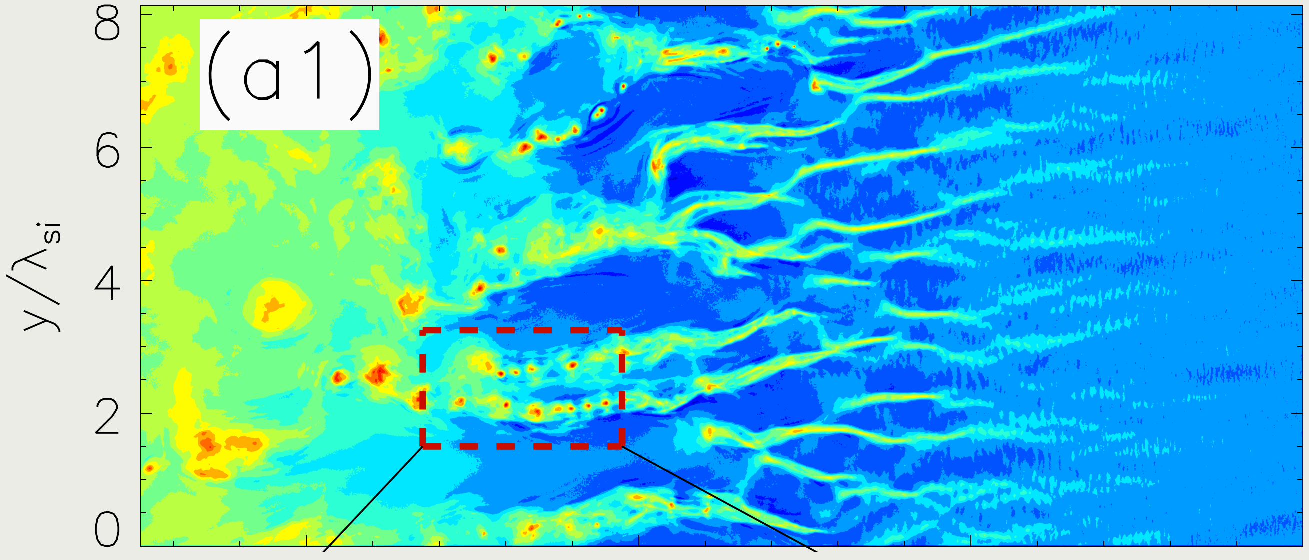


- Selected particles gain energy at each shock crossing.
- Multiple crossing allowed by scattering off turbulent fluctuations.

PARTICLE ACCELERATION AT SHOCKS

- MHD jump conditions apply to shocks in strongly collisional plasmas.
- Weakly collisional or collisionless plasmas develop shocks with complex structure.
- Several acceleration mechanisms have been identified, supported by various plasma instabilities.
- Shocks in completely unmagnetized plasmas generate magnetic fields (Weibel instability).



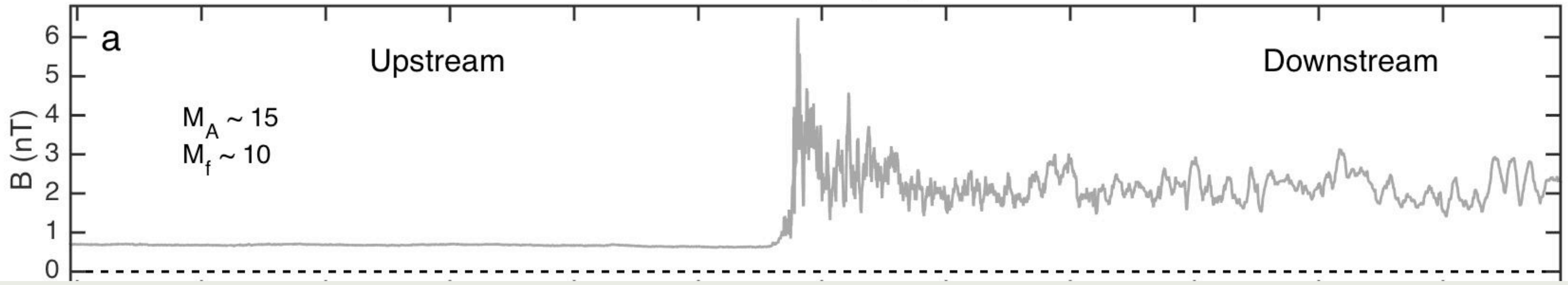


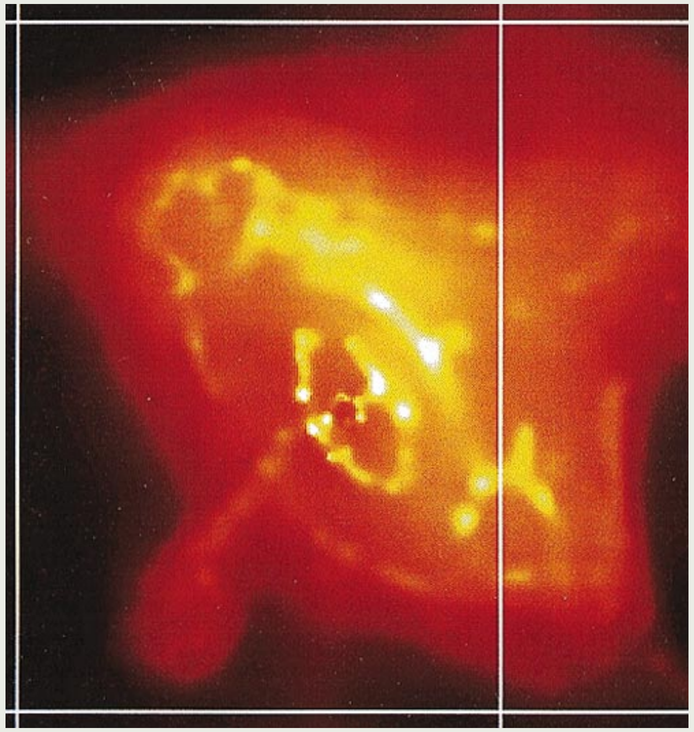
Bohdan et al. (2020)

PARTICLE ACCELERATION AT SHOCKS

- Planetary bow shocks (non-relativistic, magnetized)
- Supernova remnants (non-relativistic, magnetized)
- Pulsar wind nebulae (PWNe) (relativistic, magnetized)
- AGN jets (relativistic, magnetized)
- Gamma-ray bursts (GRBs) (ultra-relativistic, unmagnetized)

Saturn/Cassini (Masters et al. 2017)





Crab PWN (Chandra)
Weisskopf et al. (2000)

SNR 0509-67.5 (HST/Chandra)

PROBLEM 6: SUPERLUMINAL SHOCK

- Consider a relativistic shock in reference frame $\mathcal{O}$ with normal upstream velocity $\vec{\mathrm{v}}_1 = [0,0,\mathrm{v}_1]$ in coordinates (x, y, z), and oblique upstream magnetic field $\vec{B}_1 = B_1[\sin\theta_1, 0, \cos\theta_1]$. Ideal MHD is satisfied both upstream and downstream.

- Consider another reference frame $\mathcal{O}'$ moving in $\mathcal{O}$ with boost velocity $\vec{\mathrm{v}}_\mathrm{b} = [\mathrm{v}_b, 0, 0]$. Using the Lorentz transformation, find what are the conditions to have:
  (1) $B'_z = 0$,
  (2) $\vec{E}'_1 = 0$.

- Consider a particle that can only move along the local magnetic field. Such a particle can easily pass from the upstream region to the downstream region. In which case is it possible for this particle to return to the upstream region?

This problem is worth 5 points. Solutions should be sent as 1-page PDF files to knalew@camk.edu.pl before the next lecture.

SUMMARY

- Shock waves are discontinuities in flows that dissipate kinetic energy.
- Shock jump equations use conservation laws to related the upstream and downstream parameters.
- Magnetic field component parallel to the shock normal is conserved, the perpendicular component is compressed.
- Shock waves can be sites of non-thermal particle acceleration. Selected particles gain energy by multiple shock crossings.