A homework assigned to me by Roger

the spin is parallel-transported along the pulsar's geodesic,
that this implies

$$\frac{d^2 S_\alpha}{d\tau^2} = \Gamma^\delta_{\ \alpha\gamma} \Gamma^\beta_{\ \delta\epsilon} u^\gamma u^\epsilon S_\beta.$$

Next use Boyer-Lindquist coordinates and the results from Ex
puter algebra to show that the spatial part of Eq. (26.96a) tal
simple form

$$\frac{d^2 S_i}{d\tau^2} = -\frac{M S_i}{r^3},$$

Use computer algebra to evaluate Ω_p, and show that a Taylor expansion for $r \gg M$ gives

$$\Omega_p = \frac{3M^{3/2}}{2r^{5/2}} - \frac{aM}{r^3} + \ldots. \tag{26.96d}$$

The first term on the right-hand side is known as *geodetic* precession; the second as *Lense-Thirring* precession.

Solve the MTW exercise 26.19 using your Killing vectors trick.

My first homework (1951)

Abramowicz, Nurowski & Wex (1995)

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Optical reference geometry for stationary and axially symmetric spacetimes

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Abstract. We discuss the acceleration and Fermi–Walker transport for the circular motion of particles, photons and gyroscopes in stationary, axially symmetric spacetimes in terms of the optical reference geometry.

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4.2. Fermi–Walker transport and gyroscope precession

The Fermi–Walker derivative of τ^i with respect to u^i equals $\delta\tau_k/\delta s = u^i\nabla_i\tau_k - (a_k u_i - a_i u_k)\tau^i$, and the vector Ω^j_G,

$$\Omega^j_G = \left(\frac{\delta\tau^k}{\delta s}\Lambda_k\right)\lambda^j - \left(\frac{\delta\tau^k}{\delta s}\lambda_k\right)\Lambda^j \tag{4.6}$$

gives the rate of precession (with respect to τ^i) of a gyroscope which moves with the 4-velocity u^i, see, for example, [9].

For a steady circular motion,

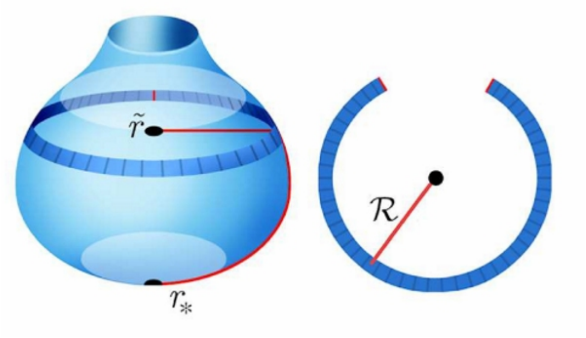
$$-mv\gamma^{-1}\frac{\delta\tau_k}{\delta s} = \tfrac{1}{2}(1+v^2)C_k + Z_k\,. \tag{4.7}$$

One sees that the *precession of the gyroscope is not influenced by the gravitational force*. On the equatorial plane,

$$\Omega^j_G = \gamma^3\left[\tfrac{1}{2}(1+v^2)C(\tilde{r}) + \varepsilon e^{\Phi}\tilde{\Omega}\frac{\tilde{R}}{\mathcal{R}}\right]E^j\,. \tag{4.8}$$

In this formula one easily recognizes different types of precession (Thomas, geodesic, Lense–Thirring). Its generalization will be discussed in [10].

Final formula



M. Abramowicz, G.F.R. Ellis, J. Horak &
M. Wielgus (2013)

$$\Omega_G = \text{MA-Nurowski-Wex} = (\Omega_K - \omega)\left(\frac{r}{\mathcal{R}}\right)$$

$$\mathcal{R} = r\,(1 - 3M/r)^{-1/2}$$

$$\Omega_G \approx (\Omega_K - \omega)\,(1 + (3/2)M/r)$$

$$\Omega_P = \Omega_G - \Omega_K \approx \frac{3}{2}\Omega_K \frac{M}{r} - \omega$$

$$\approx \frac{3}{2}\frac{M^{3/2}}{r^{5/2}} - \frac{Ma}{r^3}$$

Stanford Gravity Probe B