BLACK HOLES AT DIFFERENT FLAVOURS

WE-HERAEUS-EAS WORKSHOP FOR EARLY CAREER RESEARCHERS IN ASTRONOMY

Accretion and acceleration of matter around SMBH in GR regime

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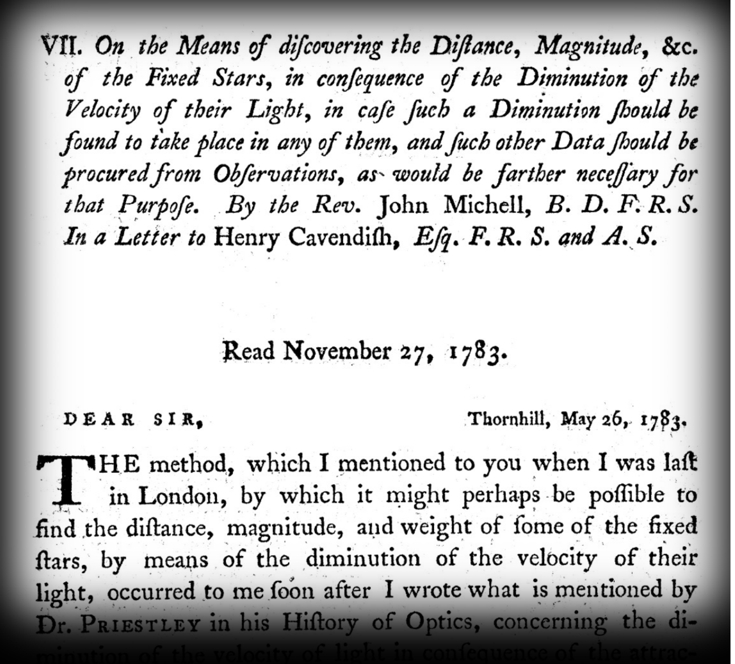
Nicolaus Copernicus Astronomical Center, Warsaw, 21-25 September 2026

Some historical remarks

John Michell (1784): Phil. Trans. Roy. Soc. Lond. LXXIV, 35

If there should really exist in nature any bodies whose density is not less than that of the sun, and whose diameters are more than 500 times the diameter of the sun, since their light could not arrive at us … we could have no information from sight; yet if any other luminous bodies should happen to revolve about them, we might still perhaps from the motions of these revolving bodies infer the existence of the central ones …



VII. *On the Means of diſcovering the Diſtance, Magnitude, &c. of the Fixed Stars, in conſequence of the Diminution of the Velocity of their Light, in caſe ſuch a Diminution ſhould be found to take place in any of them, and ſuch other Data ſhould be procured from Obſervations, as would be farther neceſſary for that Purpoſe. By the Rev.* John Michell, *B. D. F. R. S. In a Letter to* Henry Cavendiſh, *Eſq. F. R. S. and A. S.*

Read November 27, 1783.

DEAR SIR, Thornhill, May 26, 1783.

THE method, which I mentioned to you when I was laſt in London, by which it might perhaps be poſſible to find the diſtance, magnitude, and weight of ſome of the fixed ſtars, by means of the diminution of the velocity of their light, occurred to me ſoon after I wrote what is mentioned by Dr. PRIESTLEY in his Hiſtory of Optics, concerning the di-

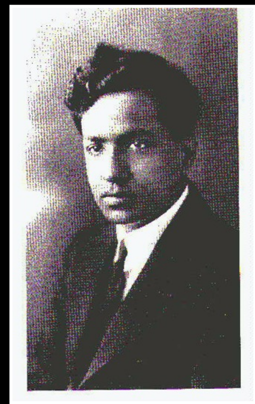
Pierre S. Laplace (1796): “Exposition du Systēme du Monde”

… the attractive force of a heavenly body could be so large that light could not flow out of it. (W. Israel, in “300 Years of Gravitation”)

On the maximum mass

S. Chandrasekhar (1931): "The maximum mass of ideal white dwarfs", Astrophysical Journal 74, 81

> *... there is no cause in the quantum theory that could prevent collapse of a body of the mass $M > M_0$ in a point …*



Limiting mass:

white dwarfs ≈ 1.4 M_S, neutron stars ≈ 2 M_S

S. Eddington (1935): "Relativistic degeneracy", Observatory 58, 37

W. Baade & F. Zwicky (1934): "On supernovae; Cosmic rays from supernovae", Proc. Nat. Acad. Sci. 20, 254

J. R. Oppenheimer & H. Snyder (1939): "On continued gravitational contraction", Phys. Rev. 56, 455

$$ds^2 = (1 - 2m/r)dt^2 - (1 - 2m/r)^{-1}dr^2 - r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)$$

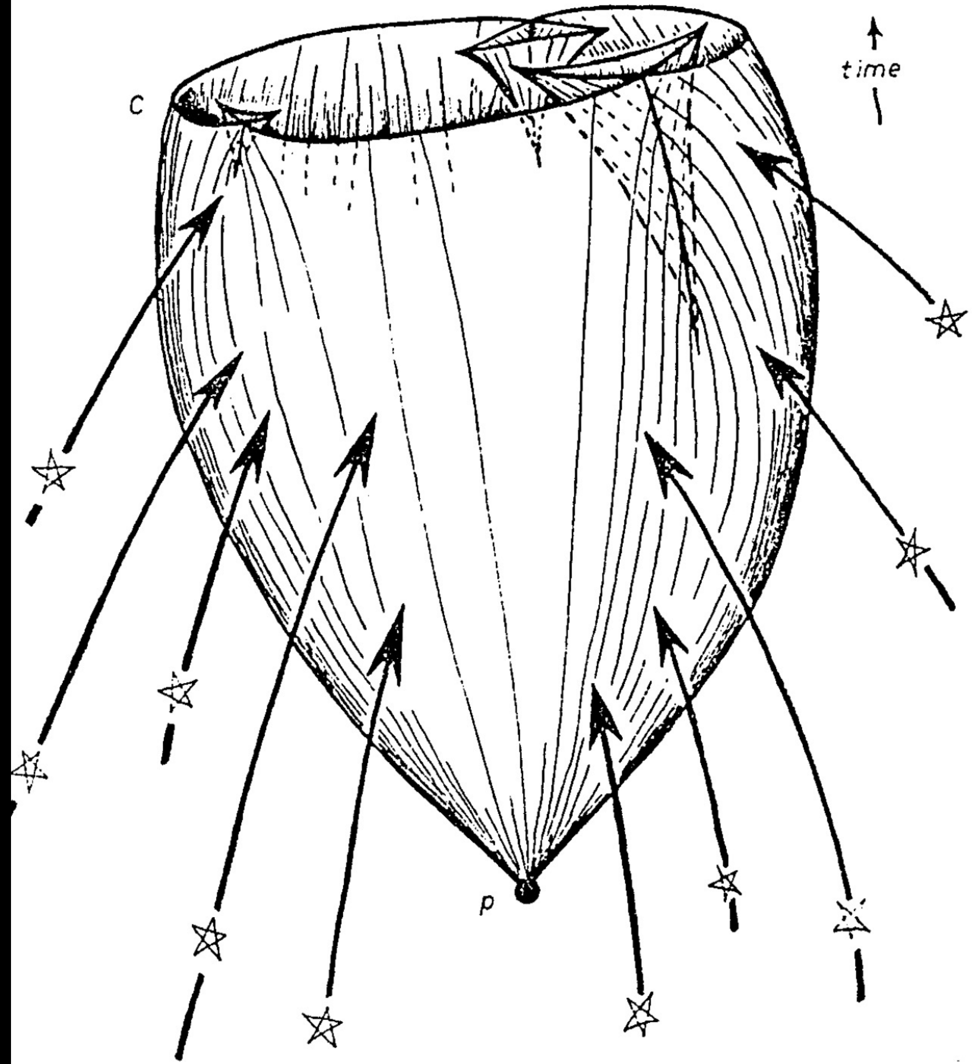


Figure 3. The future light cone of p is caused to reconverge by the falling stars.

	Non-rotating ($J = 0$)	Rotating ($J \neq 0$)
Uncharged ($Q = 0$)	Schwarzschild	Kerr
Charged ($Q \neq 0$)	Reissner–Nordström	Kerr–Newman

$$\boxed{\begin{aligned} R_g &= GM_{BH}/c^2 \\ &\cong 1.5 \times 10^5 \left(M_{BH}/M_S\right) \text{ cm} \end{aligned}}$$

- ✓ Super-massive black holes (~ $10^6 – 10^9$ M_S)
  - In cores of most galaxies (including the Milky Way)
  - Prime movers of Active Galactic Nuclei
- ✓ Intermediate black holes (~ 10^3 M_S)
  - Possibly in collapsed cores of stellar clusters

[1] rotating (Kerr) black hole

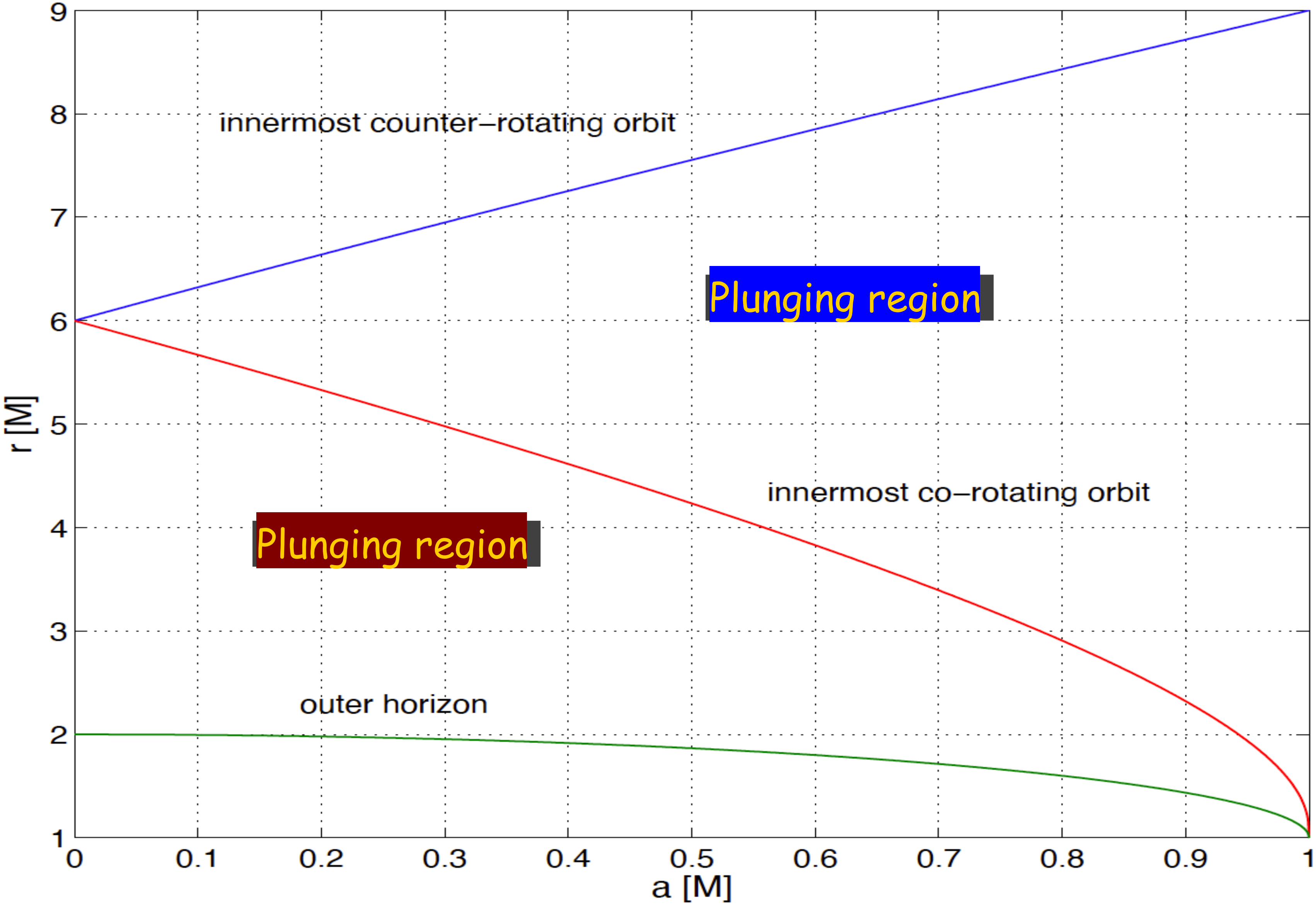
Kerr metric describing the geometry of the spacetime around the rotating black hole is expressed in Boyer-Lindquist coordinates $x^\mu = (t,\ r,\ \theta,\ \varphi)$ as follows (Misner et al. 1973):

$$ds^2 = -\frac{\Delta}{\Sigma}\left[dt - a\sin\theta\, d\varphi\right]^2 + \frac{\sin^2\theta}{\Sigma}\left[(r^2 + a^2)d\varphi - a\, dt\right]^2 + \frac{\Sigma}{\Delta}\, dr^2 + \Sigma d\theta^2, \tag{1}$$

where

$$\Delta \equiv r^2 - 2Mr + a^2, \quad \Sigma \equiv r^2 + a^2\cos^2\theta. \tag{2}$$

Coordinate singularity at $\Delta = 0$ corresponds to outer/inner horizon of the black hole $r_\pm = M \pm \sqrt{M^2 - a^2}$. Rotation of the black hole is measured by the spin parameter $a \in \langle -M, M\rangle$. Here we only consider $a \geq 0$ without the loss of generality.



innermost counter–rotating orbit
Plunging region
innermost co–rotating orbit
Plunging region
outer horizon
r [M]
a [M]
9
8
7
6
5
4
3
2
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

“Magnetized” black holes

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Usp. Fiz. Nauk **157**, 129–162 (January 1989)

Physical aspects of the theory of black holes in an external electromagnetic field are reviewed. The “magnetized” black hole model is currently widely discussed in astrophysics because it provides a basis for the explanation of the high energy activity of galactic cores and quasars. The particular feature of this model is that it predicts unusual “gravimagnetic” phenomena that arise as a result of a natural combination of effects in electrodynamics and gravitation, namely, the appearance of an inductive potential difference during the rotation of a black hole in a magnetic field, the drift of a black hole in an external electromagnetic field, the change in the chemical potential of the event horizon, the creation of an effective ergosphere of a black hole in a magnetic field, and so on. Questions relating to the description of electromagnetic fields in Kerr space-time are examined, including their influence on the space-time metric, the interaction between a rotating charged black hole and an external electromagnetic field, the motion of charged particles near “magnetized” black holes, including their spontaneous and stimulated emission, and the influence of magnetic fields on quantum-mechanical processes in black holes.

The spacetime metric

$$\mathrm{d}s^2 = f^{-1}\left[e^{2\gamma}\left(\mathrm{d}z^2 + \mathrm{d}\rho^2\right) + \rho^2\,\mathrm{d}\phi^2\right] - f\left(\mathrm{d}t - \omega\,\mathrm{d}\phi\right)^2,$$

with f, ω, and γ functions of z and ρ only.

We consider coupled Einstein-Maxwell equations:

electrovacuum case with a black hole,

axial symmetry, stationarity,

not necessarilly asymptotically flat.

- Kramer D., Stephani H., MacCallum M., and Herlt E., *Exact Solutions of the Einstein's Field Equations* (Deutscher Verlag der Wissenschaften, Berlin 1980)
- Alekseev G. A., Garcia A. A., Phys. Rev. D **53**, 1853 (1996)
- Ernst F. J., and Wild W. J., J. Math. Phys. **12**, 1845 (1976)
- Hiscock W. A., J. Math. Phys. **22**, 1828 (1988)
- Karas V., Vokrouhlický D., J. Math. Phys. **32**, 714 (1990)

Vacuum field equations (without electromagnetic field):

$$\boxed{\begin{aligned} &f\boldsymbol{\nabla}^2 f = \boldsymbol{\nabla} f\cdot\boldsymbol{\nabla} f - \rho^{-2} f^4 \boldsymbol{\nabla}\omega\cdot\boldsymbol{\nabla}\omega,\\ &\boldsymbol{\nabla}\cdot\left(\rho^{-2} f^2 \boldsymbol{\nabla}\omega\right) = 0.\end{aligned}}$$

Strongly Magnetized Kerr-Newman (MKN) black hole:

Magnetic generalizations of the Carter metrics

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(Received 20 February 1987; accepted for publication 14 December 1988)

Magnetic generalizations of all Carter metrics—Carter A, $B(+)$, $B(-)$, and D solutions—are obtained by applying a Harrison transformation to the Ernst potentials determined from a linear combination of two Killing vectors. The most general magnetized Carter-A metric contains, among other solutions, the magnetized Kerr–Newman metrics and, modulo limiting transitions, the magnetized branches $B(+)$, $B(-)$, and D.

$$ds^2 = -e^{2\nu}\,dt^2 + e^{2\psi}(d\varphi - \omega\, dt)^2 + e^{2\lambda}\,dr^2 + e^{2\mu}\,d\theta^2$$

$$e^{2\nu} = |\Lambda|^2\,\Sigma\,\Delta A^{-1}, \qquad e^{2\psi} = |\Lambda|^{-2} A\Sigma^{-1}\sin^2\theta,$$

$$e^{2\lambda} = |\Lambda|^2\,\Sigma\,\Delta^{-1}, \qquad e^{2\mu} = |\Lambda|^2\,\Sigma,$$

$$\Sigma = r^2 + a^2\cos^2\theta, \qquad \Delta = r^2 - 2Mr + a^2 + e^2,$$

$$A = (r^2 + a^2) - \Delta a^2\sin^2\theta, \qquad \Lambda = 1 + B_0\Phi - \frac{1}{4}B_0^2\mathcal{E}.$$

Strongly Magnetized Kerr-Newman (MKN) black hole:

Now we turn to Komar's integral relations for the angular momentum and mass contained within a black hole:

$$J_H = -\frac{1}{16\pi}\int_{\mathscr{H}} m^{\mu;\nu}\, d\Sigma_{\mu\nu}$$

$$= -\tfrac{1}{8}|\Lambda_0|^2(r_+^2 + a^2)^4 \int_0^\pi \Sigma^{-2}|\Lambda|^{-4}\,\partial_r\omega' \sin^3\theta\, d\theta, \qquad (15)$$

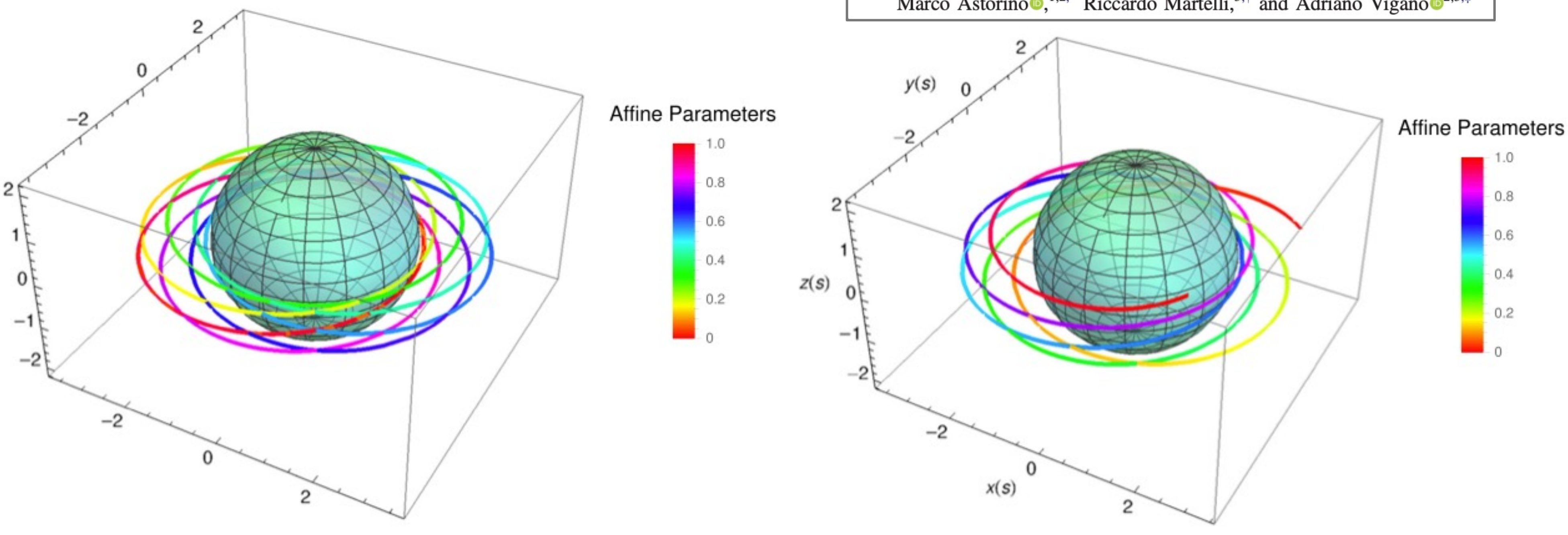
$$M_H = \frac{1}{8\pi}\int_{\mathscr{H}} k^{\mu;\nu}\, d\Sigma_{\mu\nu} = 2\omega'_H J_H + \frac{1}{4\pi}\kappa_H \mathscr{A}, \qquad (16)$$

where $\mathscr{A} = 4\pi(r_+^2 + a^2)|\Lambda_0|^2$ is the surface of the horizon and $\omega'_H = \omega'(r_+)$. Equation (16) acquires the form of the well-known Smarr formula, which has been, however, usually considered only for asymptotically flat space-times.

PHYSICAL REVIEW D **106,** 064014 (2022)

Black holes in a swirling universe

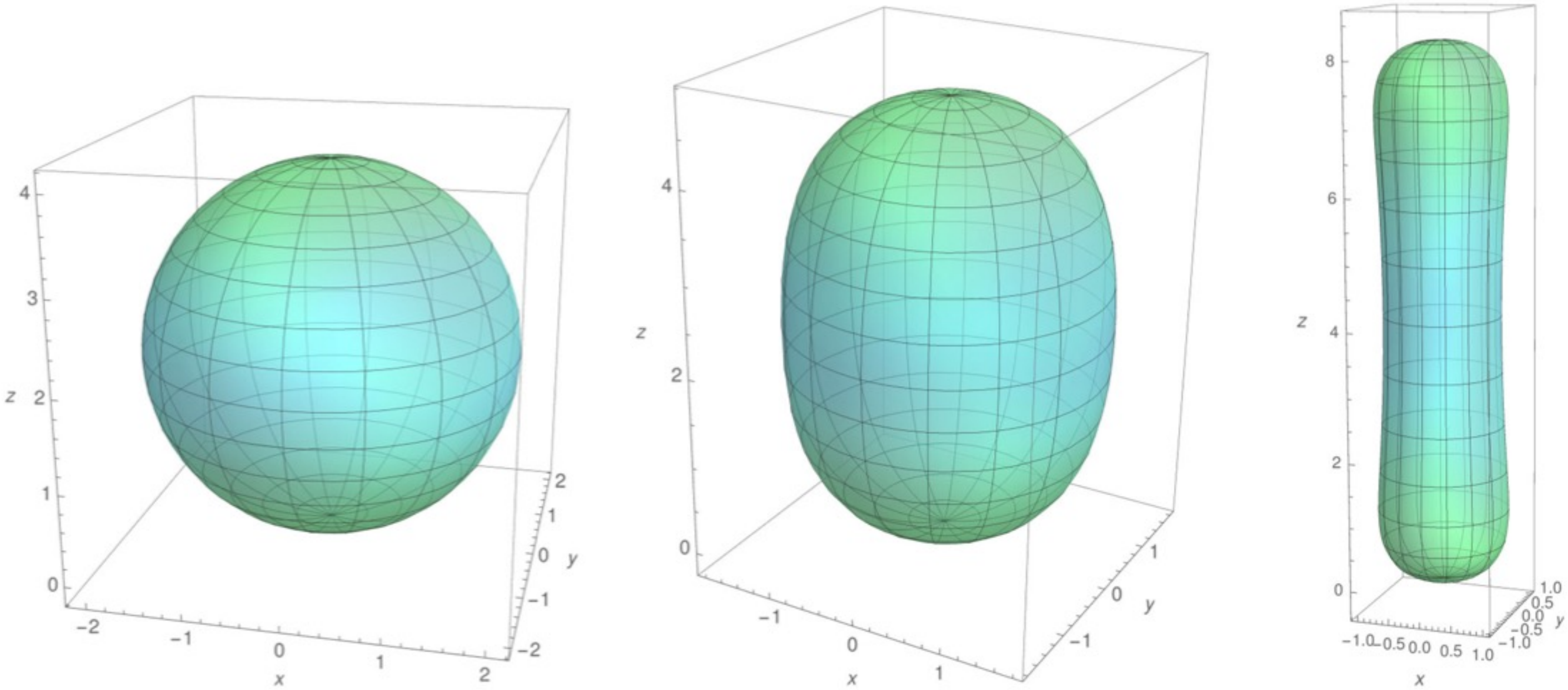
Marco Astorino,[1,2,*] Riccardo Martelli,[3,†] and Adriano Viganò[2,3,‡]



$m = 1,\ \jmath = 0$ $\qquad$ $m = 1,\ \jmath = 10^{-8}$

FIG. 4. Geodesic motion around the black hole. The left panel shows the Schwarzschild spacetime, while the right panel shows the new black hole solution (3.13). The plots share the same initial conditions with $E = 1$, $L = 12$.

Melvin Universe : Harrison transformation = Rotating Universe : Ehlers transformation



$m = 1,\ \jmath = 0.05$ $m = 1,\ \jmath = 0.1$ $m = 1,\ \jmath = 0.3$

FIG. 2. Embedding in Euclidean three-dimensional space $\mathbb{E}^3$ of the event horizon of the black hole distorted by the rotating background, for three different values of the background rotational parameter $\jmath$.

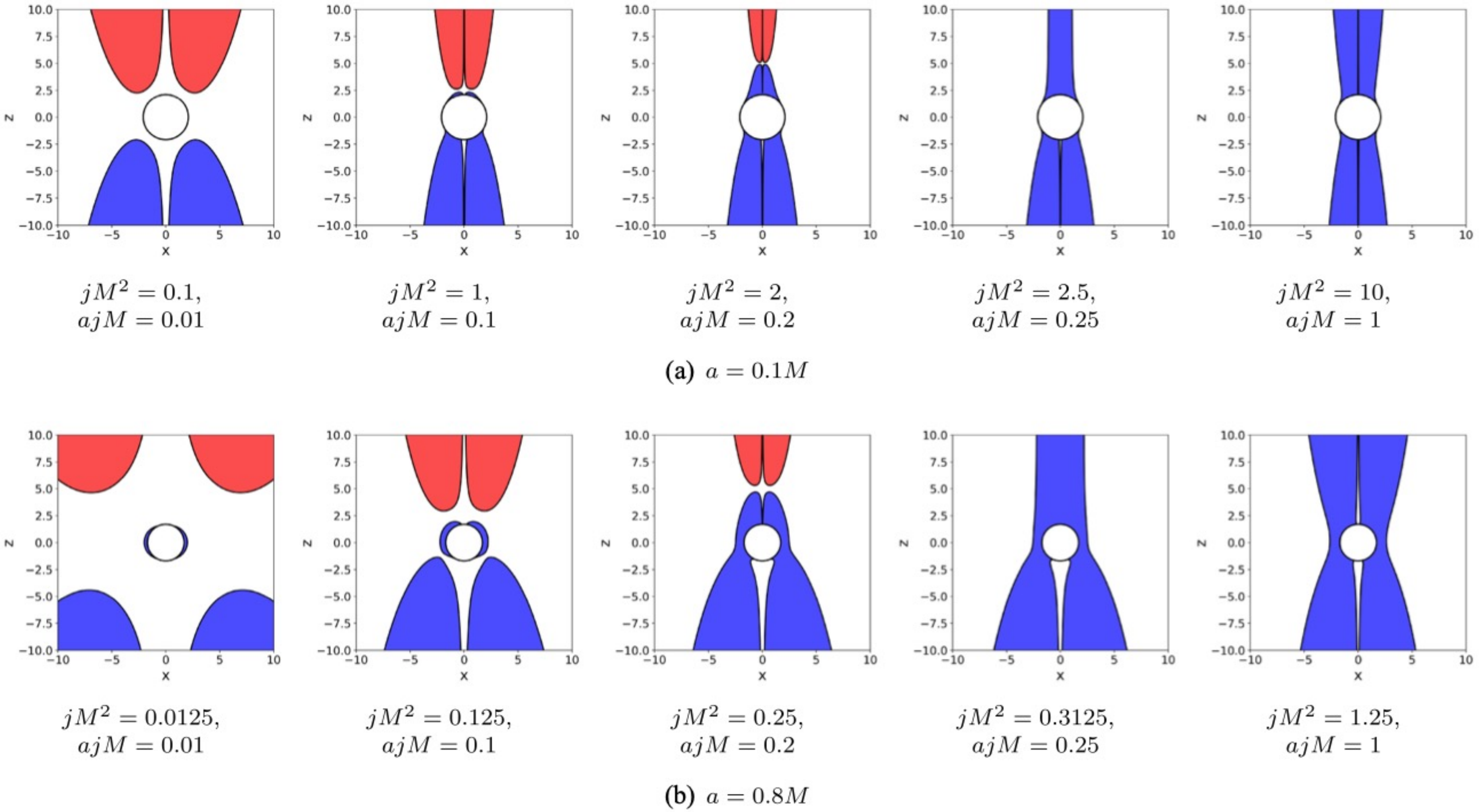


FIG. 1. Cross section ($x = r\sin\theta\cos\varphi$, $y = r\sin\theta\sin\varphi$, $z = r\cos\theta$, with $\varphi = 0, \pi$) of the ergoregions (colored) with different values of jM^2 and (a) $a = 0.1M$ and (b) $a = 0.8M$. The angular velocity of frame dragging $\Omega = \omega$ in these regions is positive when colored blue and negative when colored red. The region behind the event horizon $r < r_h^{(+)}$ has been excluded from these plots.

[2] gaseous accretion flow

- ✓ **Standard accretion disk?**

  … succesful, but cannot account for rich phenomenology

- ✓ **Variability on different time-scales?**

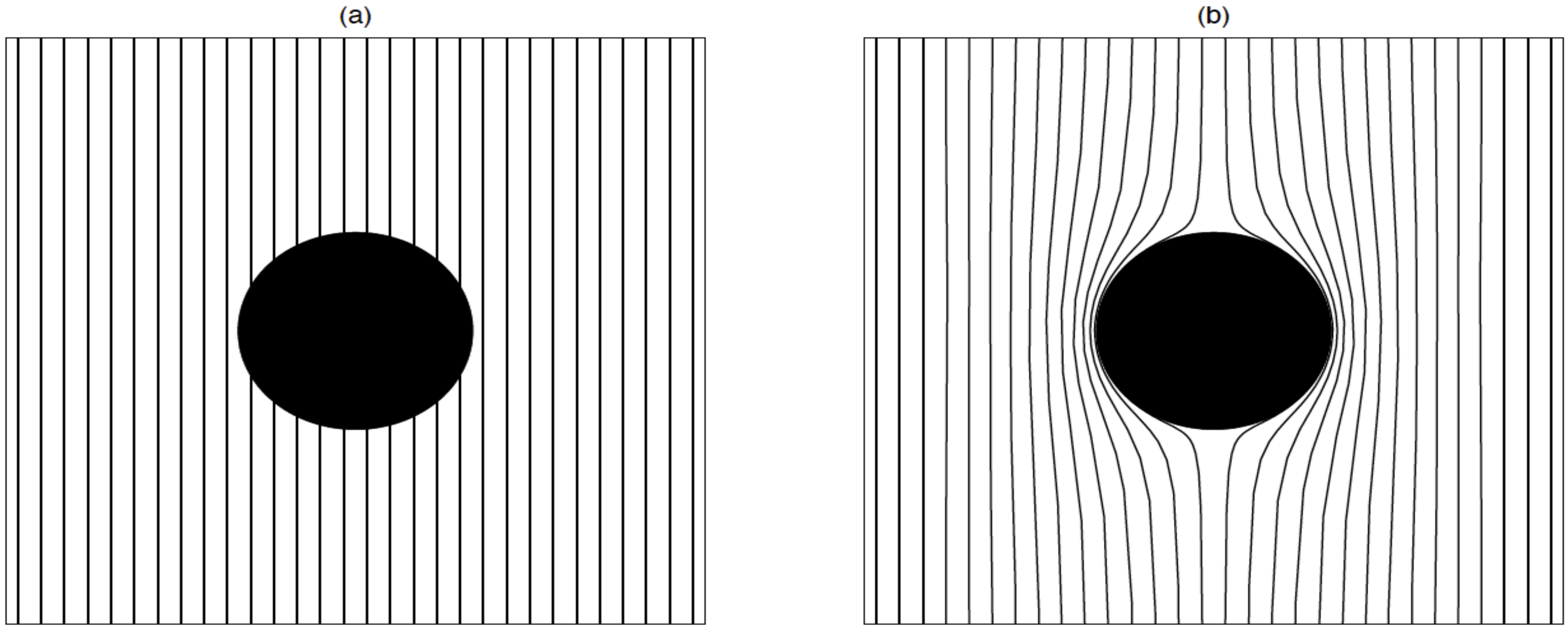
  … QPOs, QPEs, …

- ✓ **Hard X-ray spectra?**

  … hot optically thin corona

- ✓ **Jets and outflows?**
- ✓ *… B-field vs. spin direction*

[3] the initial magnetic field (weak)



An axisymmetric case: (a) $a = 0$ (a static black hole), and (b) $a = M$ (a maximally rotating black hole).

(Wald 1974)

PHYSICAL REVIEW D VOLUME 10, NUMBER 6 15 SEPTEMBER 1974

Black hole in a uniform magnetic field*

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(Received 22 April 1974)

Using the fact that a Killing vector in a vacuum spacetime serves as a vector potential for a Maxwell test field, we derive the solution for the electromagnetic field occurring when a stationary, axisymmetric black hole is placed in an originally uniform magnetic field aligned along the symmetry axis of the black hole. It is shown that a black hole in a magnetic field will selectively accrete charges until its charge becomes $Q = 2B_0 J$, where B_0 is the strength of the magnetic field and J is the angular momentum of the black hole. As a by-product of the analysis given here, we prove that the gyromagnetic ratio of a slightly charged, stationary, axisymmetric black hole (not assumed to be Kerr) must have the value $g = 2$.

I. INTRODUCTION

From the results on black-hole uniqueness which have been proved during the last several years (particularly the theorems of Israel,[1] Carter,[2] Hawking,[3] and Robinson[4]), it is now well established that an isolated black hole cannot have an electromagnetic field unless it is endowed with a net electric charge. Thus, an isolated black hole can participate in electromagnetic effects only if there is a mechanism for charging it up. However, if the black hole is not isolated, electromagnetic fields produced by external sources (e.g., plasma accreting onto the black hole) may be present.

We employ the test-field solution of Maxwell's equations for a weakly magnetized Kerr black hole immersed in an asymptotically uniform magnetic field specified by the component B_z parallel to the spin axis and the perpendicular component B_x. The electromagnetic vector potential A_μ is given as follows (Bičák & Janiš 1985):

$$A_t = \frac{B_z a M r}{\Sigma}\left(1+\cos^2\theta\right) - B_z a + \frac{B_x a M \sin\theta\cos\theta}{\Sigma}\left(r\cos\psi - a\sin\psi\right), \tag{3}$$

$$A_r = -B_x (r-M)\cos\theta\sin\theta\sin\psi, \tag{4}$$

$$A_\theta = -B_x a (r\sin^2\theta + M\cos^2\theta)\cos\psi - B_x (r^2\cos^2\theta - Mr\cos 2\theta + a^2\cos 2\theta)\sin\psi, \tag{5}$$

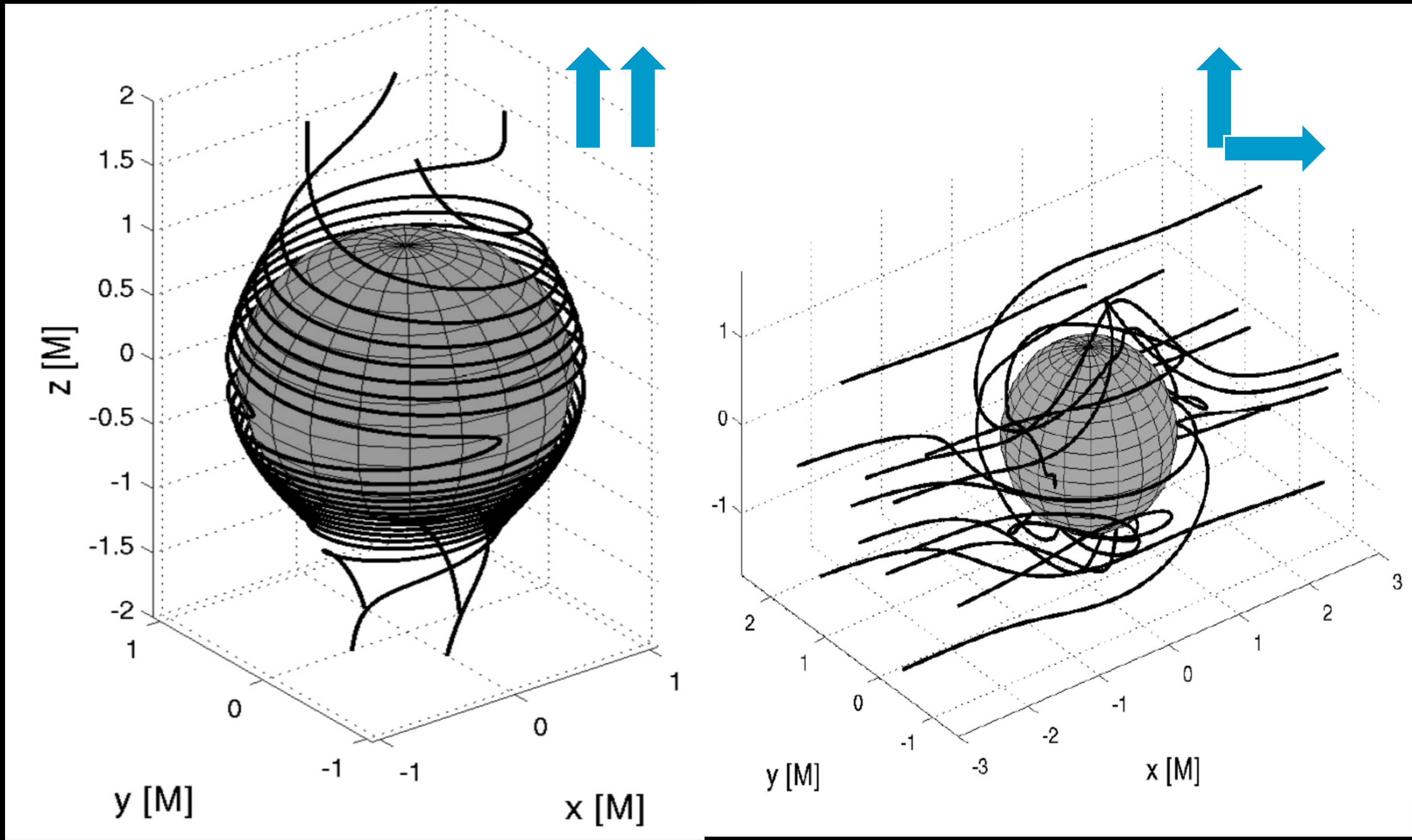
$$\begin{aligned} A_\varphi = {} & B_z \sin^2\theta\left[\frac{1}{2}(r^2+a^2) - \frac{a^2 M r}{\Sigma}(1+\cos^2\theta)\right] \\ & - B_x \sin\theta\cos\theta\left[\Delta\cos\psi + \frac{(r^2+a^2)M}{\Sigma}\left(r\cos\psi - a\sin\psi\right)\right], \end{aligned} \tag{6}$$

where ψ denotes the azimuthal coordinate of Kerr ingoing coordinates, which is expressed in Boyer–Lindquist coordinates as follows:

$$\psi = \varphi + \frac{a}{r_+ - r_-}\ln\frac{r - r_+}{r - r_-}. \tag{7}$$

(Bičák & Janiš 1985)

The initial magnetic field



(Karas & Kopáček 2009)

The initial magnetic field

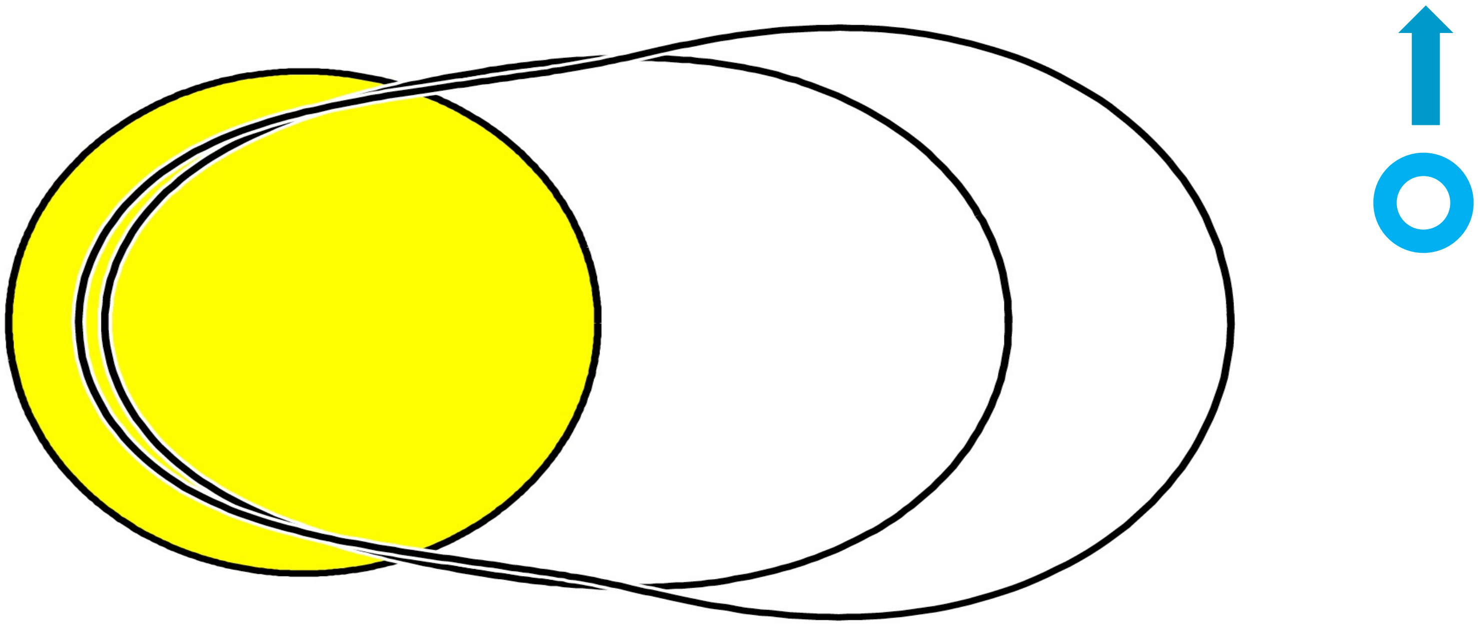


Figure 5. Cross-sectional area for the capture of magnetic field lines (asymptotically uniform magnetic field perpendicular to the rotation axis). The three curves correspond to different values of the black-hole angular momentum: $a = 0$ (cross section is a perfect circle; its projection coincides with the black-hole horizon, indicated here by shading), $a = 0.95M$, and $a = M$ (the most deformed shape refers to the maximally rotating case).

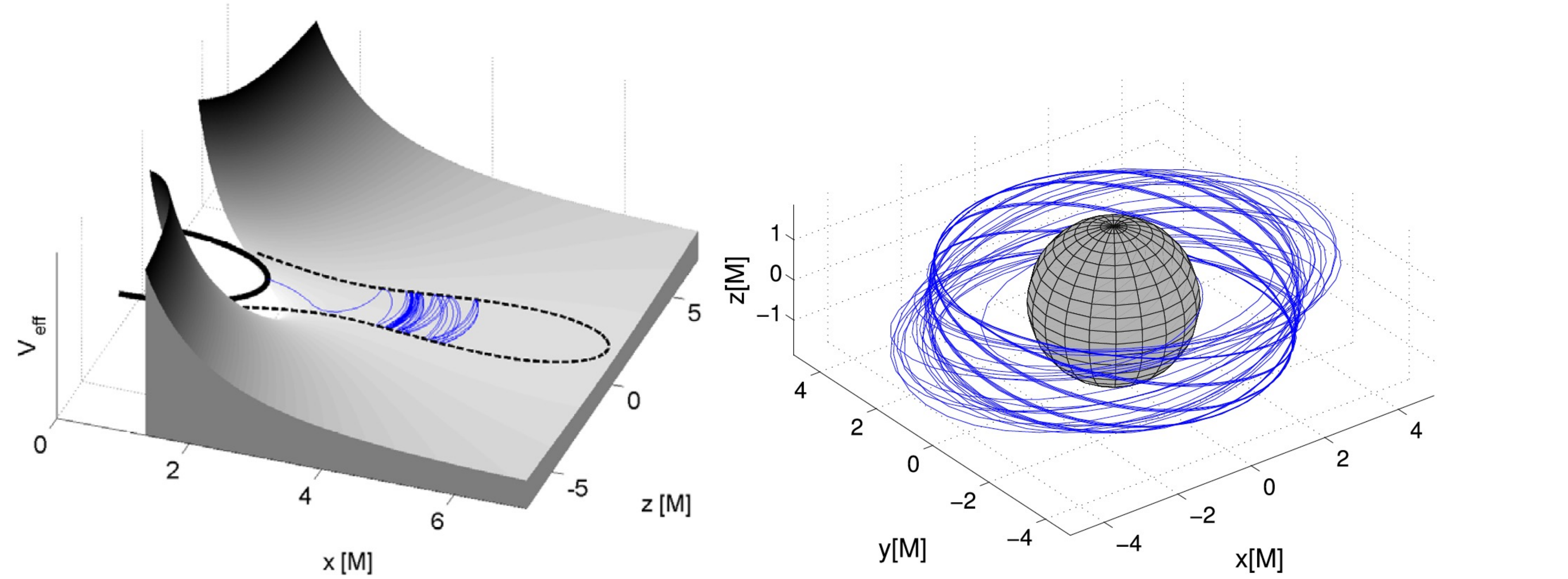
(Karas & Dovčiak 2000)

Escape in oblique configuration

Once the charge is introduced, the value of effective potential (12) changes accordingly. In order to study trajectories of escaping particles, we examine the behavior of the potential for $r \gg M$. In particular, for the initially neutral particle with energy $E_{\rm Kep}$ ionized in the equatorial plane at r_0 we obtain the following relation valid in the asymptotic region:

$$E - V_{\rm eff}|_{r \gg M} = E_{\rm Kep} - 1 - \frac{qB_z a}{r_0} + \mathcal{O}\left(r^{-1}\right). \tag{18}$$

Motion is possible only for $E \geq V_{\rm eff}$. Since $E_{\rm Kep} < 1$ with finite r_0 and $a \geq 0$ is considered, we observe that (i) particles may only escape for $qB_z < 0$, (ii) the escape is possible only for $a \neq 0$, (iii) asymptotic velocity of escaping particles is an increasing function of parameters $|qB_z|$ and a, and a decreasing function of r_0, and (iv) the escape is not allowed for the perpendicular inclination, $\alpha \equiv \arctan\left(B_x/B_z\right) = \pi/2$.



Example of an unstable plunging spherical orbit launched below the ISSO. The following parameters were employed: $r_0 = 4$, $Q^{1/2} = 0.75$, and $a = 0.5$.

Below the Innermost Stable Circular Orbit (ISSO)

Plunge from an unstable spherical orbit

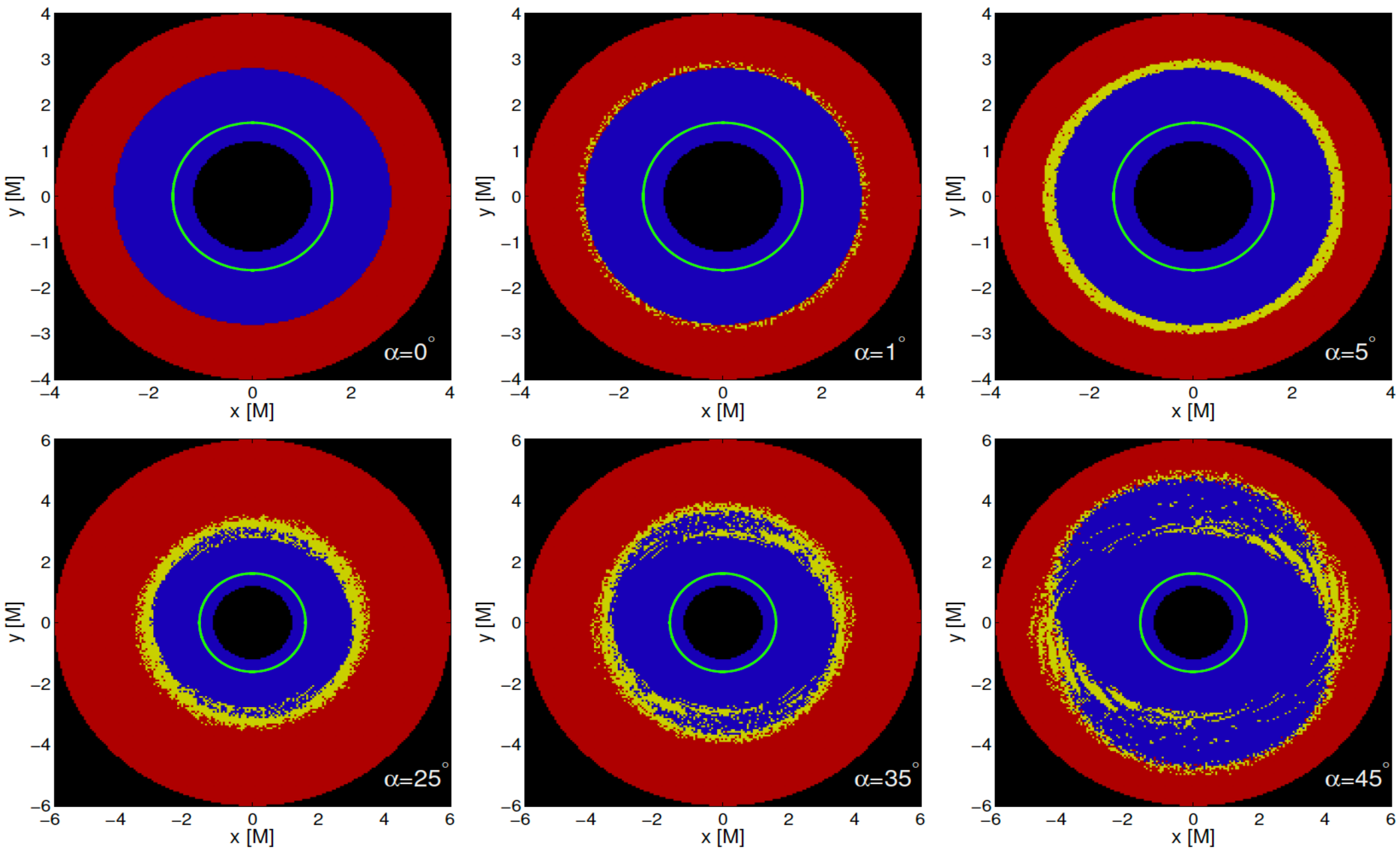


Figure 1. Different types of the trajectories launched from the equatorial plane (x, y) are plotted with respect to the magnetic inclination angle α. Color-coding: blue for plunging orbits, red for stable ones (bound to the black hole) and yellow for escaping trajectories. Green circle denotes the ISCO. Inner black region marks the horizon of the black hole. Parameters of the system are $a = 0.98$ and $qB = -5$. Magnetic field is inclined in the positive x-direction.

Terminal velocity of accelerated particles

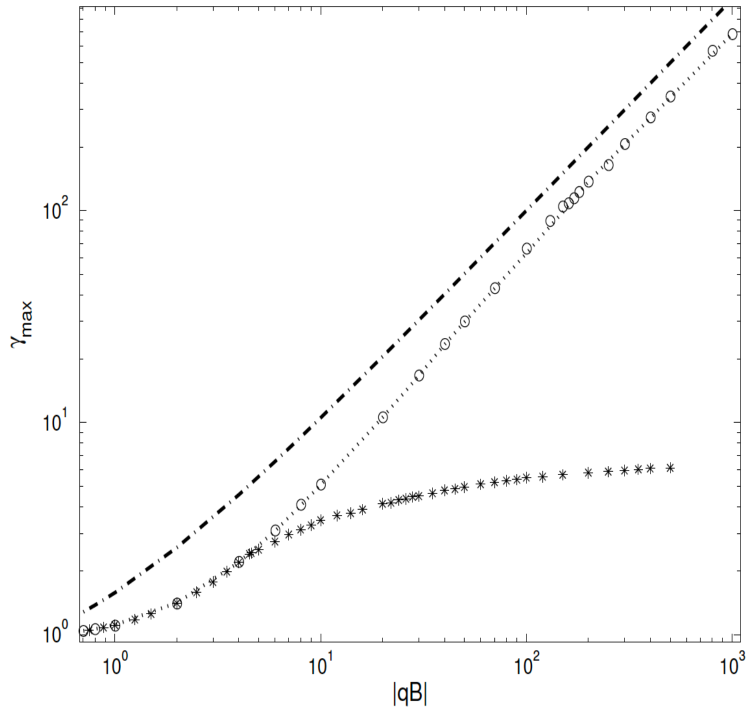


Figure 5. Maximal value of final Lorentz factor of escaping particles as a function of magnetization parameter $|qB|$. Circles denote values obtained numerically for trajectories in inclined magnetosphere ($B_x/B_z = 0.1$, i.e., $\alpha \approx 6^{\circ}$ with $\varphi_0 = \pi/2$), while stars show the aligned case ($B_x = 0$) analyzed in Paper I. Dashed line shows the expected value predicted by the Eq. (20) for numerically obtained value of $r_0^{\rm min}$ and dash-dotted is the theoretical maximum for $r_0 = r_+ = 1$.

Kopáček & Karas, ApJ (2020, 2024)

Combining magnetic fields and matter

Conservation of the particle number reads $(\rho_0 u^\alpha)_{;\alpha} = 0$, with $\rho_0 = mn$;, where m is the particle rest mass, n numerical density, u^α four-velocity. Here we do not consider a possibility of creation of pairs, which would break this conservation law. Normalization condition for four-velocity is written $u^\alpha u_\alpha = -1$. As originally shown by Znajek [719], by employing the explicit form of the energy-momentum conservation $T^{\alpha\beta}{}_{;\beta} = 0$, and the definition of energy-momentum tensor in terms of material density ρ and pressure P, one finds [537]

$$\boxed{T^{\alpha\beta} = T^{\alpha\beta}_{\text{matter}} + T^{\alpha\beta}_{\text{EMG}},} \tag{152}$$

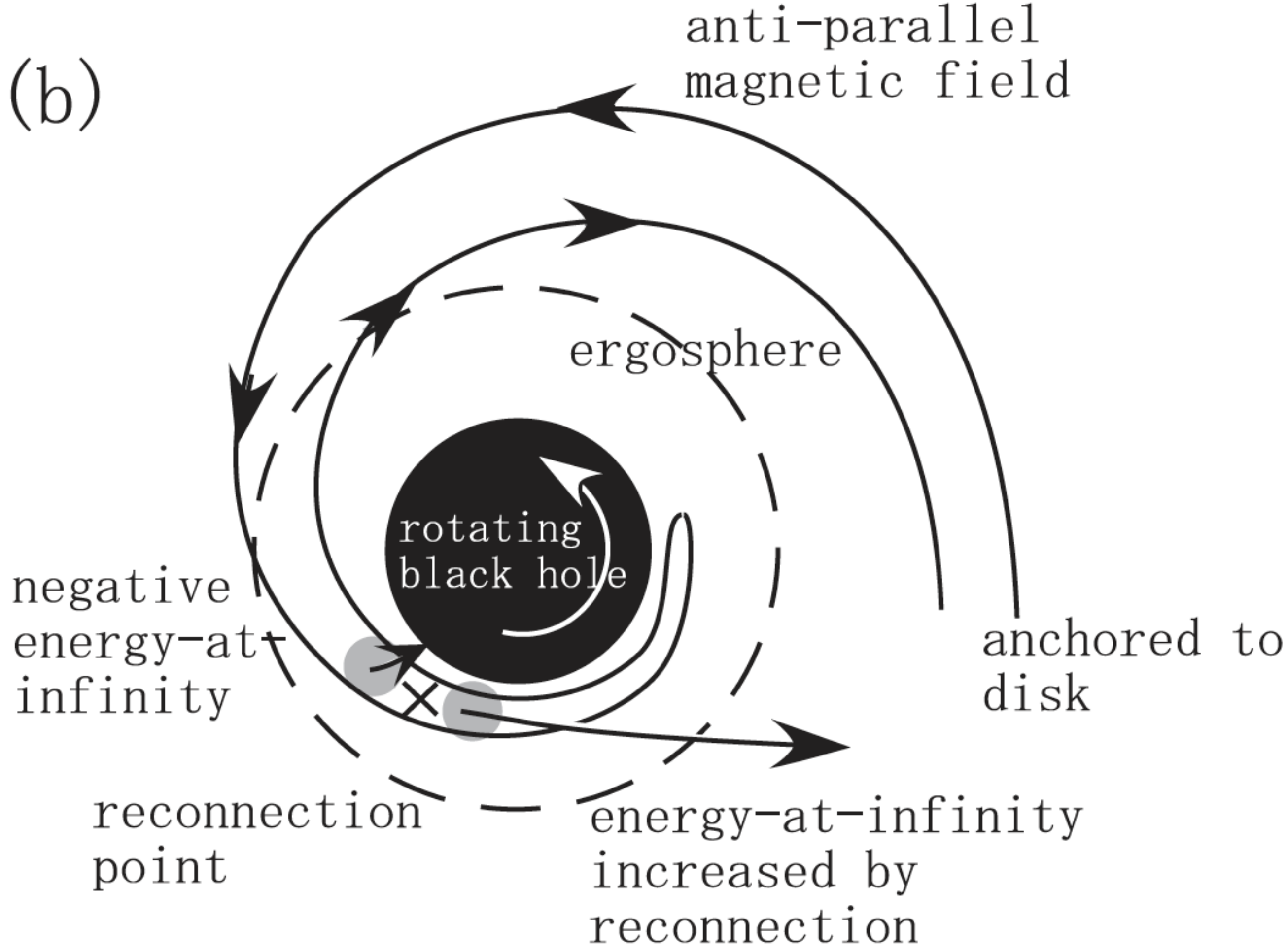
where the right-hand side contains the source term divided in two components,

$$T^{\alpha\beta}_{\text{matter}} = (\rho + p)u^\alpha u^\beta + p g^{\alpha\beta}, \tag{153}$$

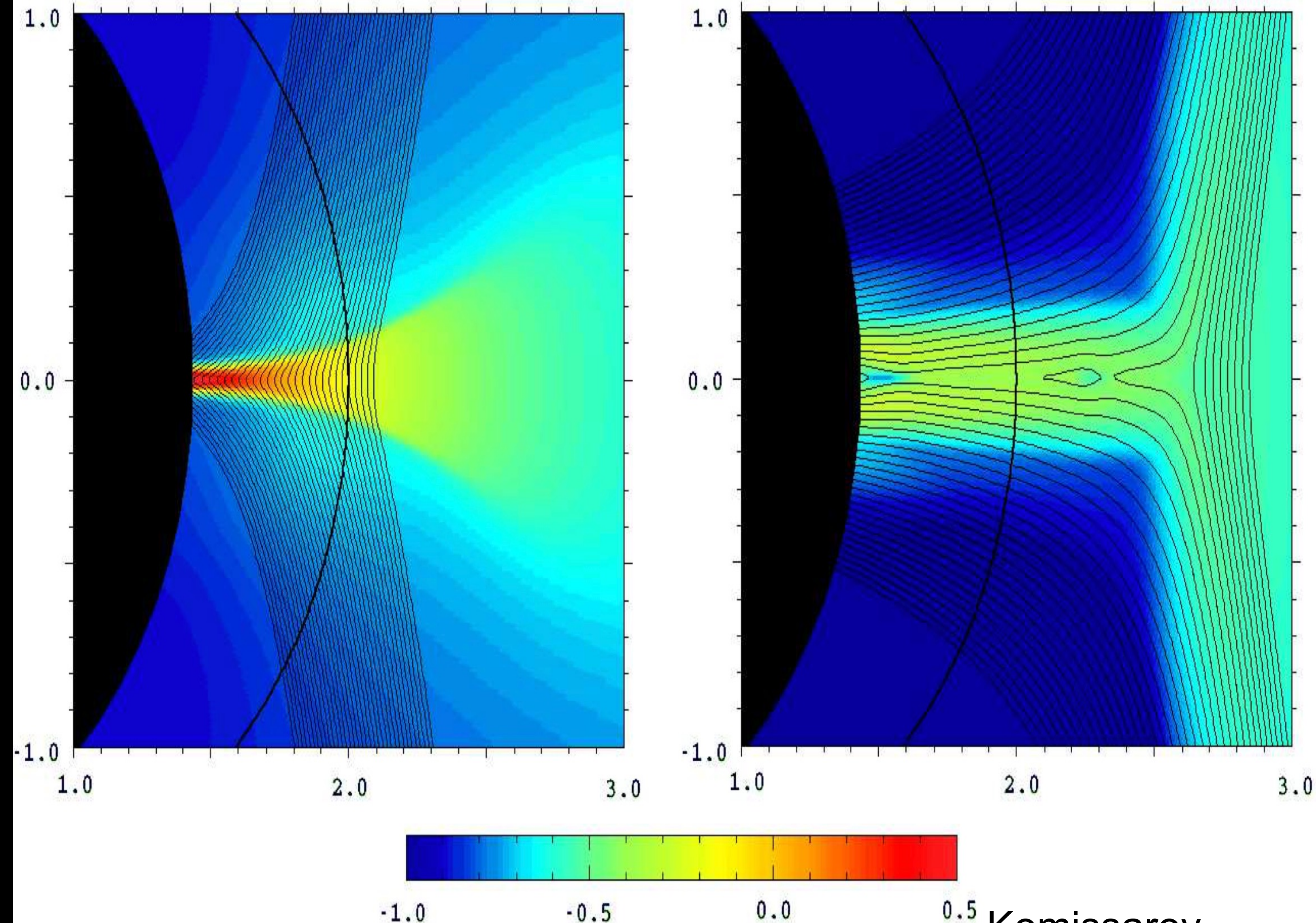
$$T^{\alpha\beta}_{\text{EMG}} = \frac{1}{4\pi}\left(F^{\alpha\mu}F^\beta_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}g^{\alpha\beta}\right), \tag{154}$$

with $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$.

Motivation: magnetic reconnection in ergosphere?

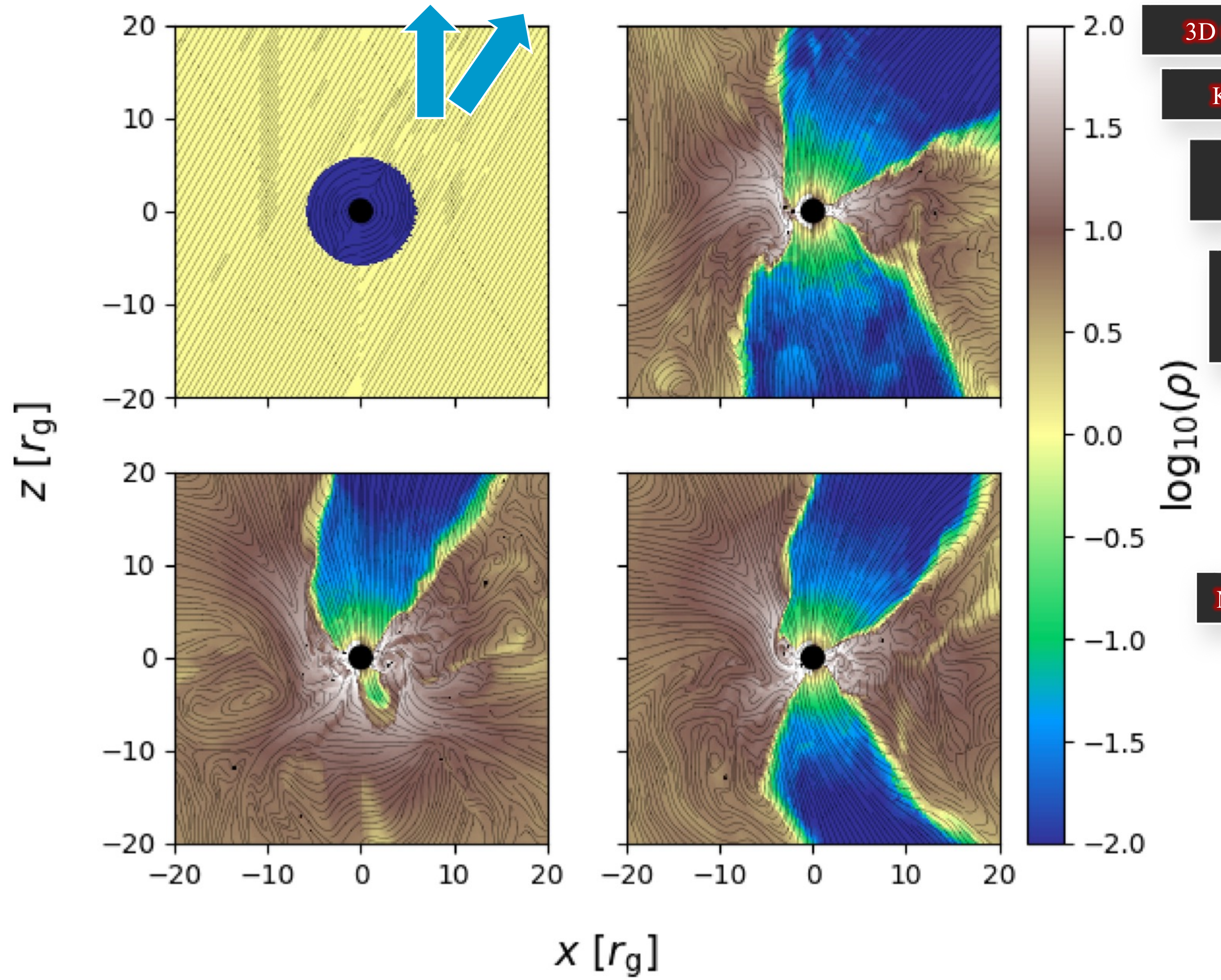


Koide & Arai (ApJ, 2008); Lyutikov (PRD, 2011); Morozova et al. (arXiv:1310.3575)



Komissarov

Accretion vs. jet launching



3D GRMHD

Kerr metric

Bondi-type (low angular mom.)

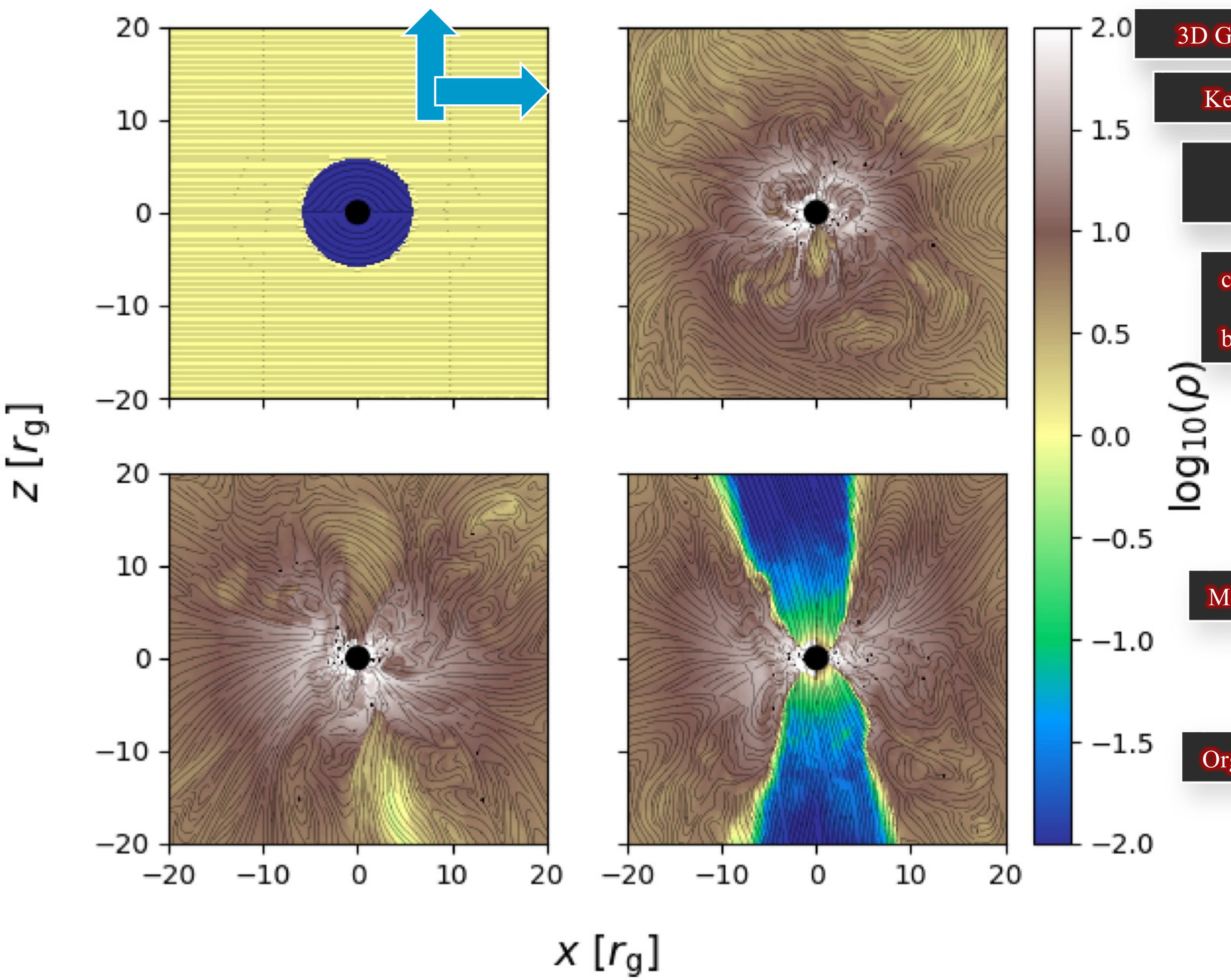
color: gas rest mass density

black lines: magnetic field

Magnetically dominated, β<1, below ~10R_g

Ressler et al, MNRAS (2021)

Accretion vs. jet launching



3D GRMHD

Kerr metric

Bondi-type (low angular mom.)

color: gas rest mass density

black lines: magnetic field

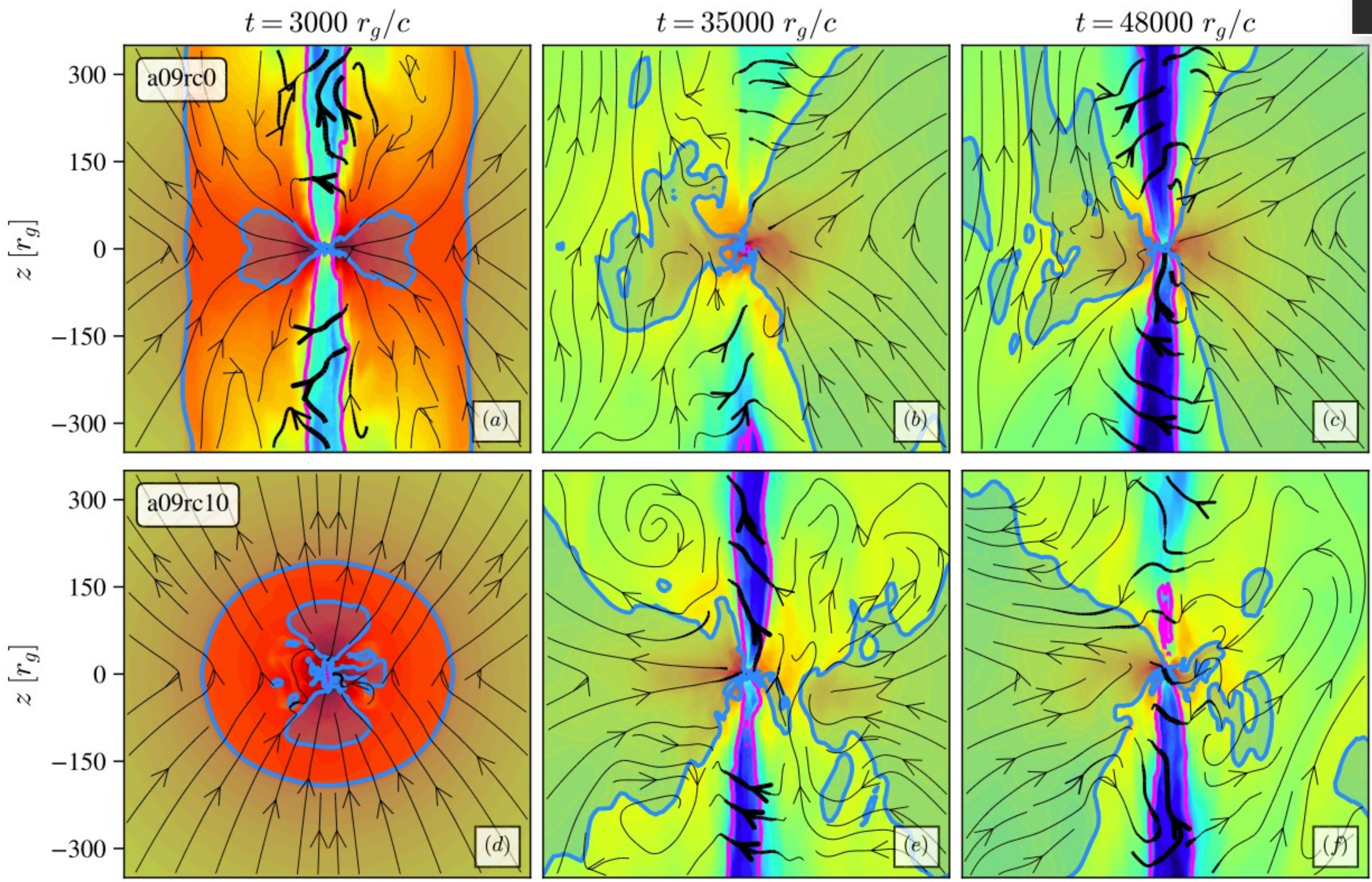
Magnetically dominated, $\beta<1$, below ~$10R_g$

Organized vertical field forms a transient jet

Ressler et al, MNRAS (2021)

Magnetically Arrested Disks (MAD) vs. jet launching

The Astrophysical Journal Letters, 946:L42 (16pp), 2023 April 1



3D GRMHD (HARM)

Kerr metric

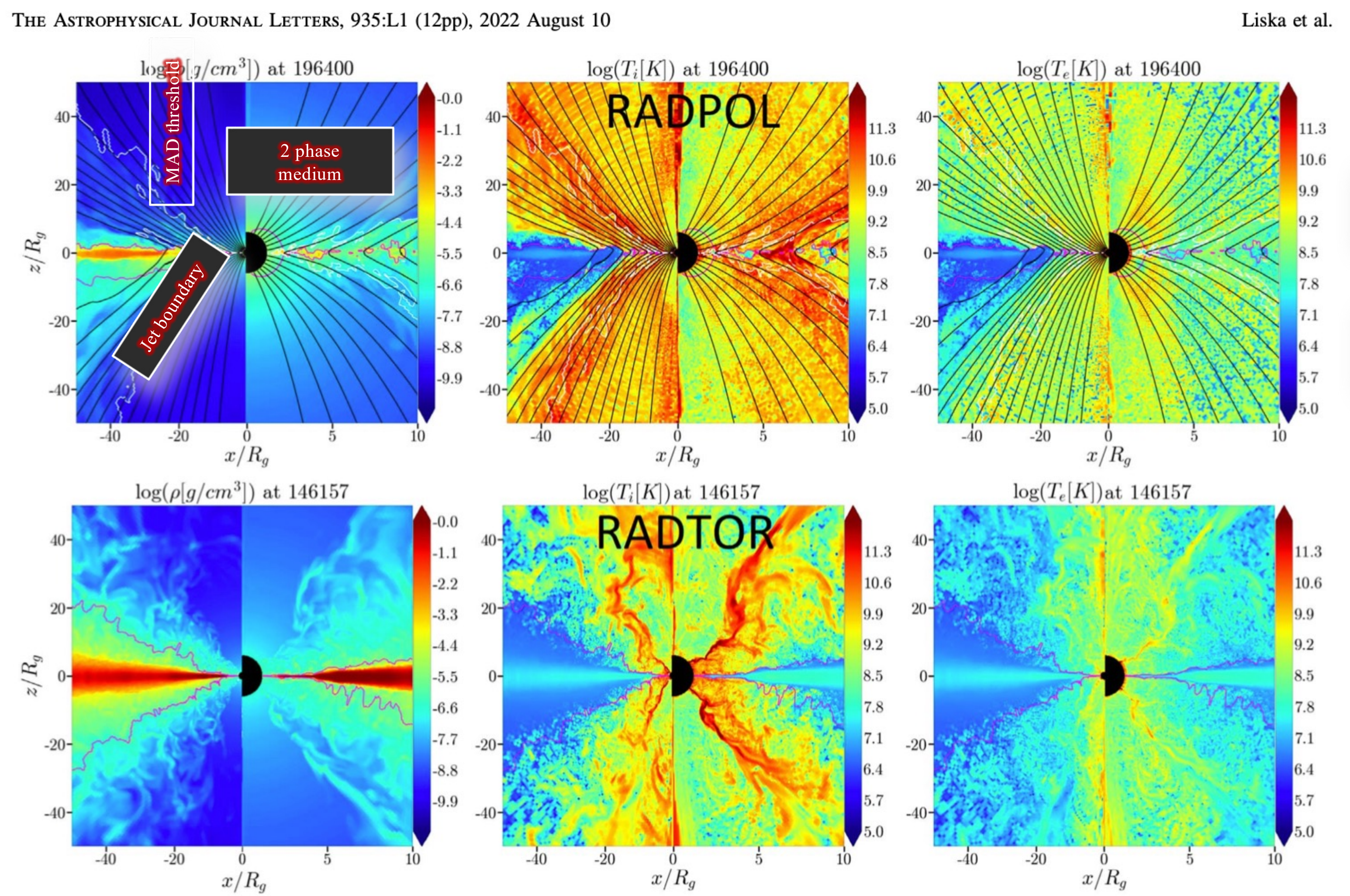
Bondi-type (low angular mom.)

color: gas rest mass density

black lines: magnetic field

Kwan et al, ApJ (2023)

Magnetically Arrested Disks (MAD) with radiation transport

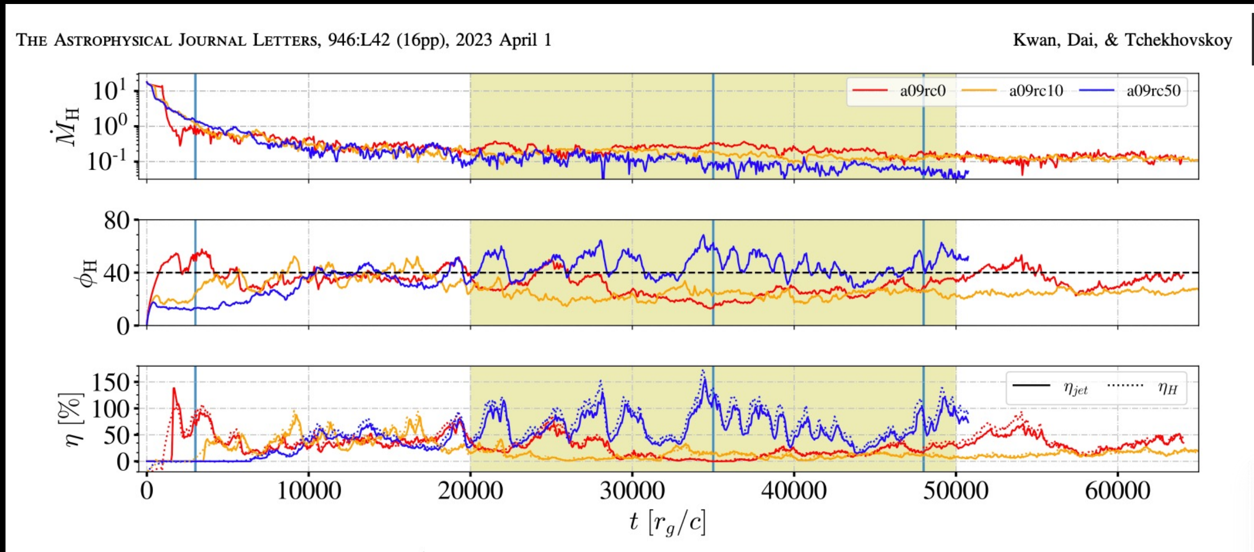


Large-scale poloidal magnetic field

Large-scale toroidal magnetic field

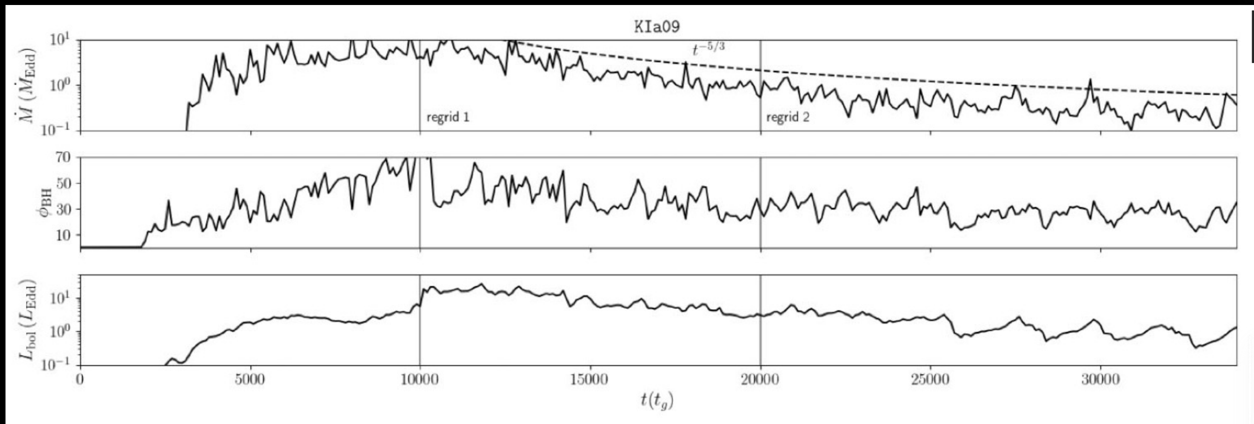
Liska et al, ApJ (2022)

Accretion rate, magnetic flux, efficiency



Kwan et al, ApJL (2023)

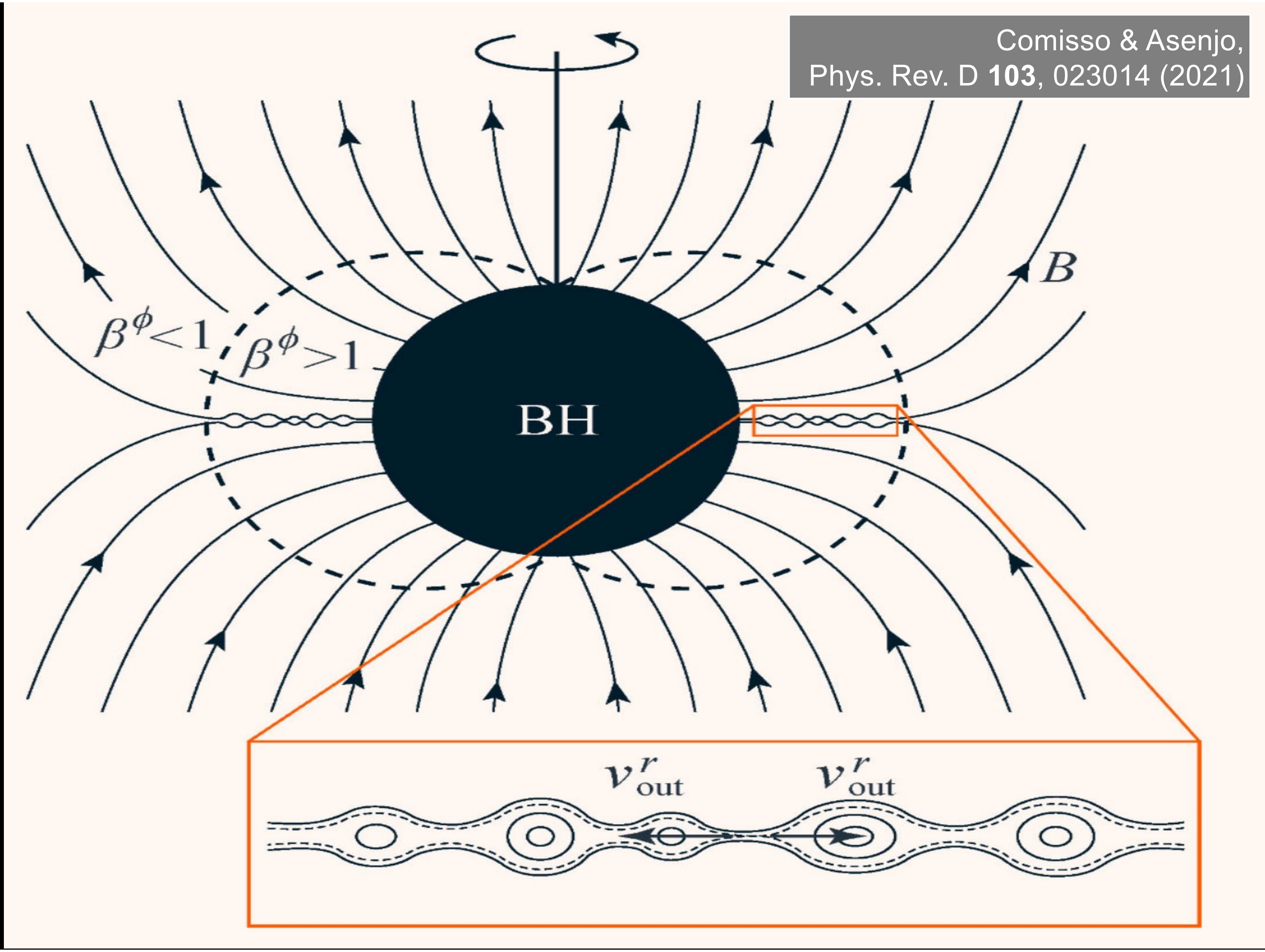
HARM GRMHD

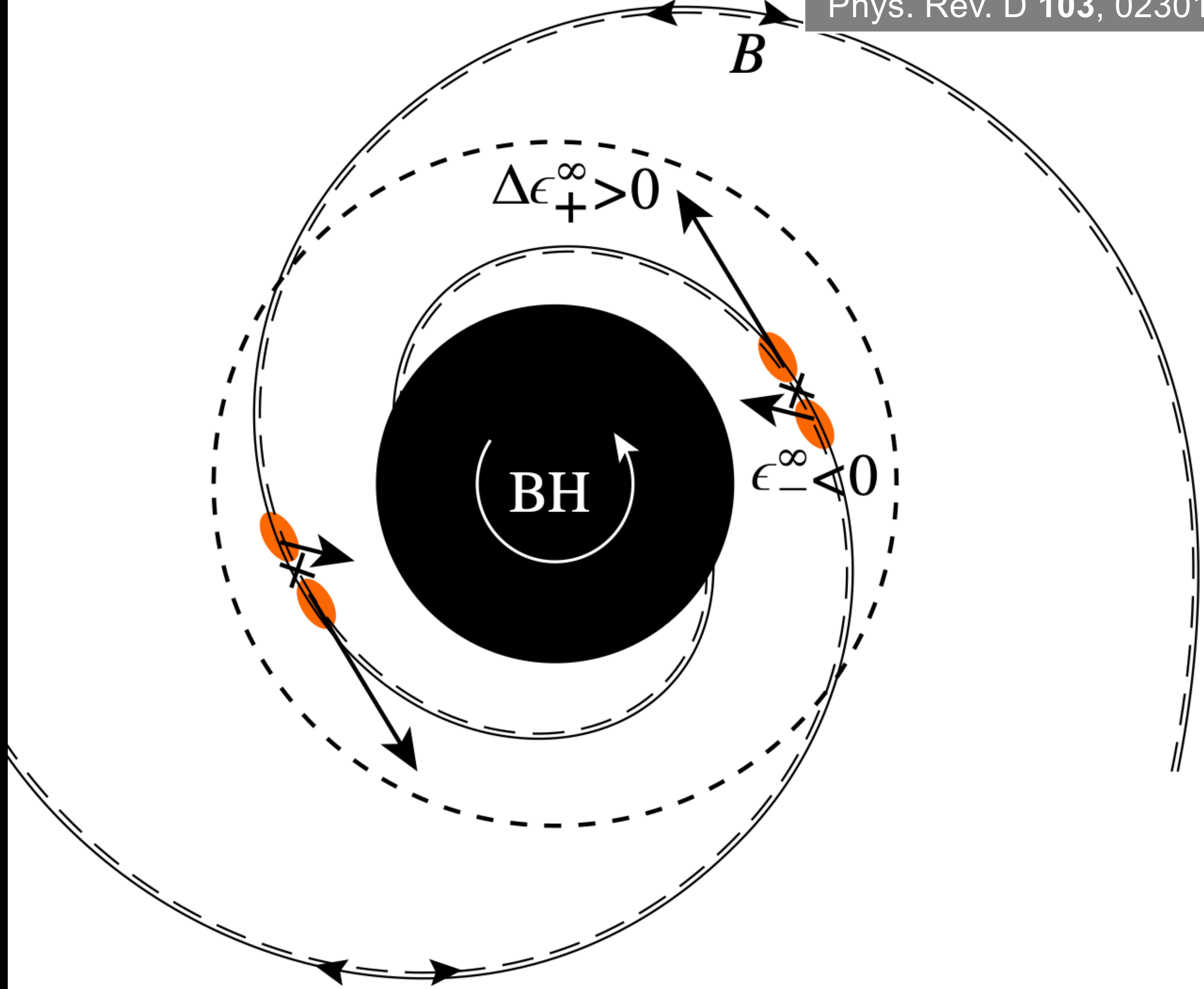


Curd & Narayan, MNRAS (2023)

==> Lančová, et al.

Koral GRRMHD


Comisso & Asenjo,
Phys. Rev. D **103**, 023014 (2021)
B
$\beta^{\phi}<1$
$\beta^{\phi}>1$
BH
$v^{r}_{\rm out}$
$v^{r}_{\rm out}$

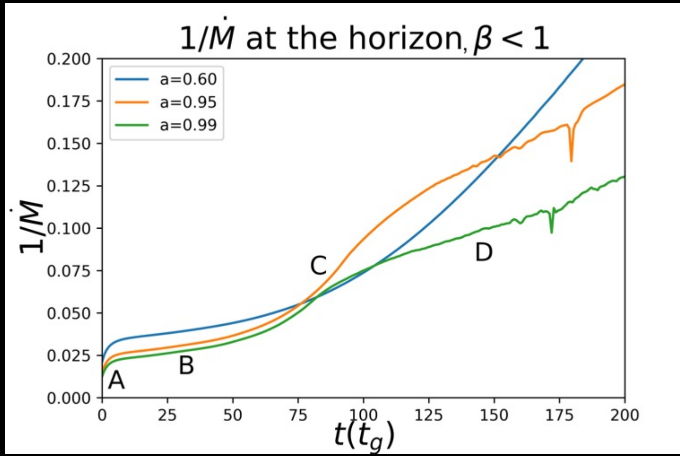


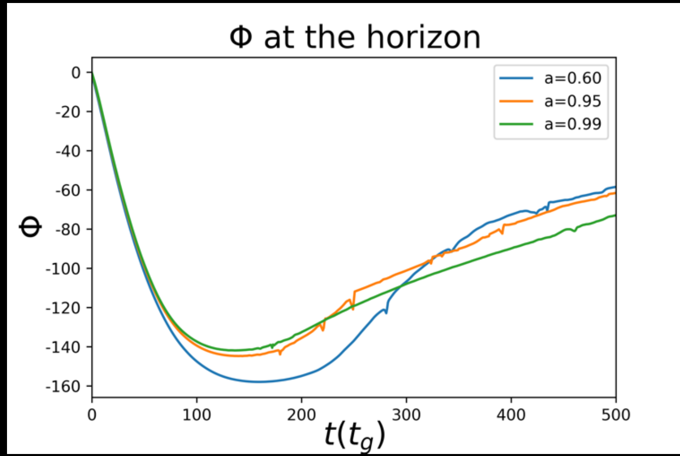
Comisso & Asenjo,
Phys. Rev. D **103**, 023014 (2021)

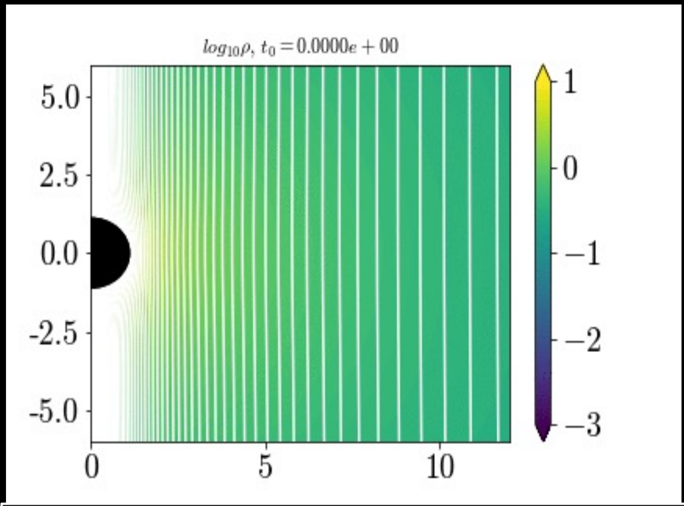
Equatorial outflow

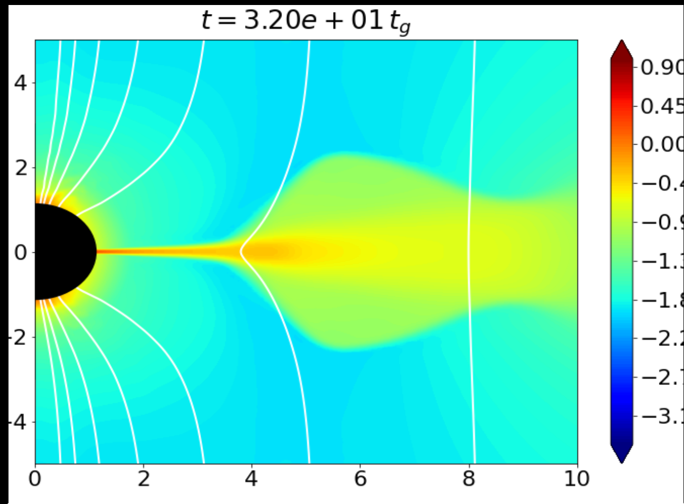
✓ 2D GRMHD (HARM)

✓ Initialized with Wald field









Sapountzis, Karas, Janiuk et al. (2021)

James, Janiuk, Karas (2024)

Thank you!

Discussion slides

$$B_{\rm SI} = \frac{q B_z c_{\rm SI}}{q_{\rm SI}\left(\frac{M}{M_\odot}\right) 1472\ {\rm m}}, \tag{B2}$$

where the quantities without subscript SI are dimensionless (expressed in geometrized units and scaled by mass of the black hole M) and the length 1472 m reflects the solar mass in geometrized units, $M_\odot = 1472$ m. Inserting the value $q B_z = 1$ and fixing the specific charge $q_{\rm SI}$ to electron, i.e., $q_{\rm SI} = 1.76 \times 10^{11}\ {\rm C\,kg^{-1}}$, we find that for the black hole of $M = 10\,M_\odot$, the corresponding magnetic field reads $B_{\rm SI} = 1.16 \times 10^{-7}$ T. Such a value is consistent with non-thermal filaments in the Galactic Center (Ferrière 2010; LaRosa et al. 2004).

The Larmor radius of the particles' gyration $r_{\rm L}$ is then given as follows:

$$(r_{\rm L})_{\rm SI} = \frac{1472\ {\rm m}}{q B_z}\left(\frac{v_\perp}{c}\right)\left(\frac{M}{M_\odot}\right), \tag{B3}$$

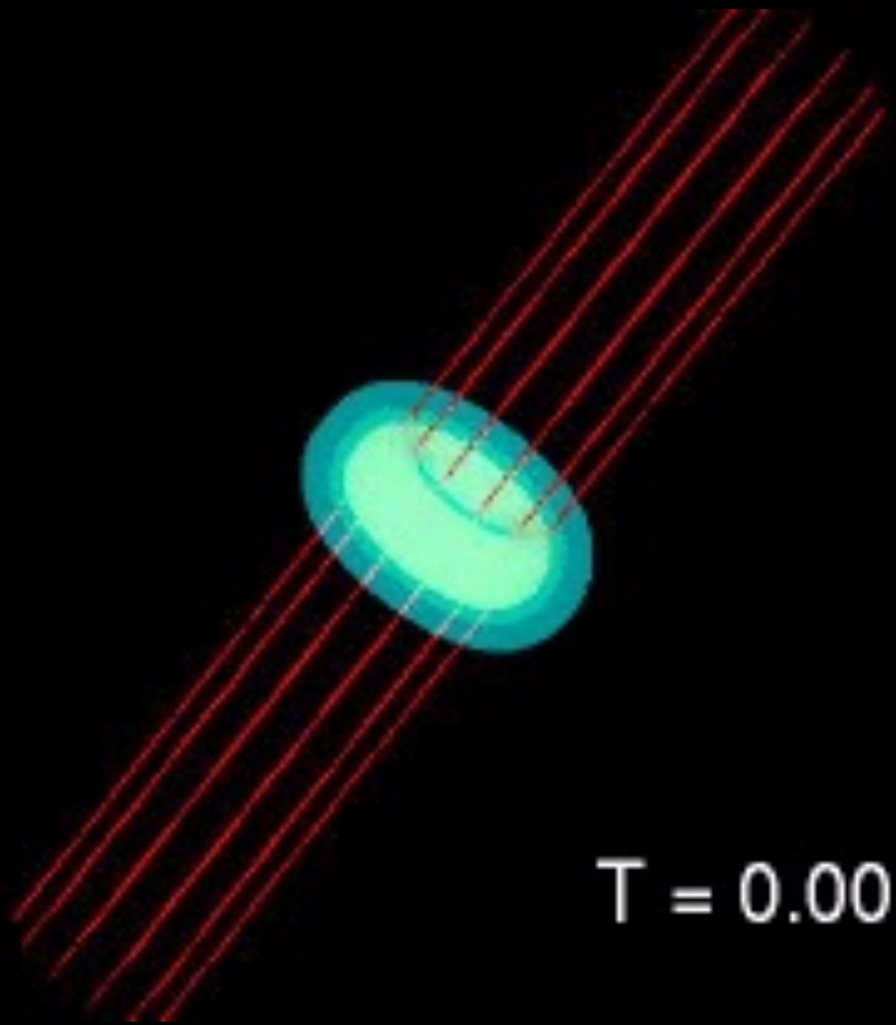
which reads $(r_{\rm L})_{\rm SI} = 1472$ m for electron with $v_\perp = 0.1c$

Electromagnetic formation of jets

For a quasar jet:

B ~ 10^4 G;
U ~10^{20} V
I ~ 10^{18} A
P ~ 10^{38} W

Blandford & Znajek



Matsumoto ==> Nathanail, et al.

Equatorial outflow

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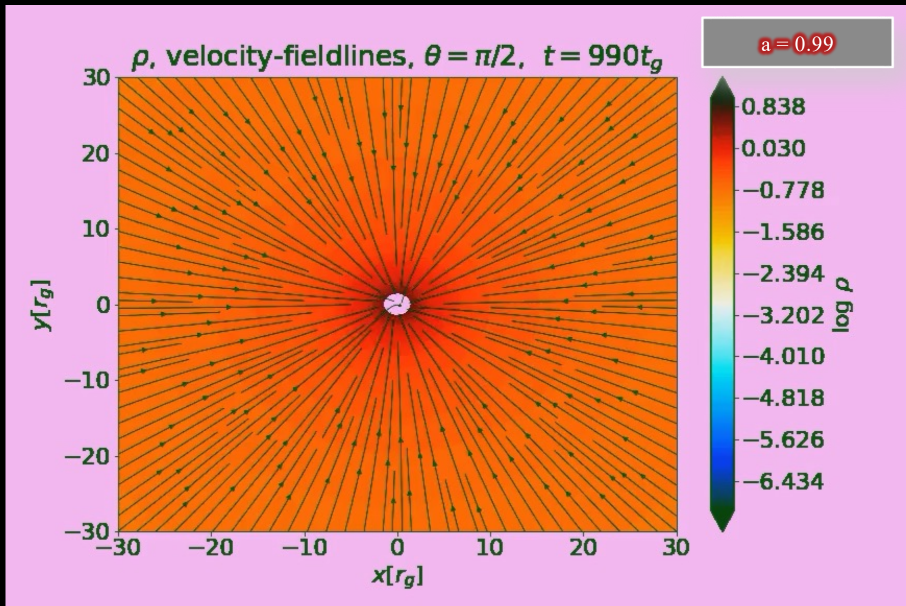

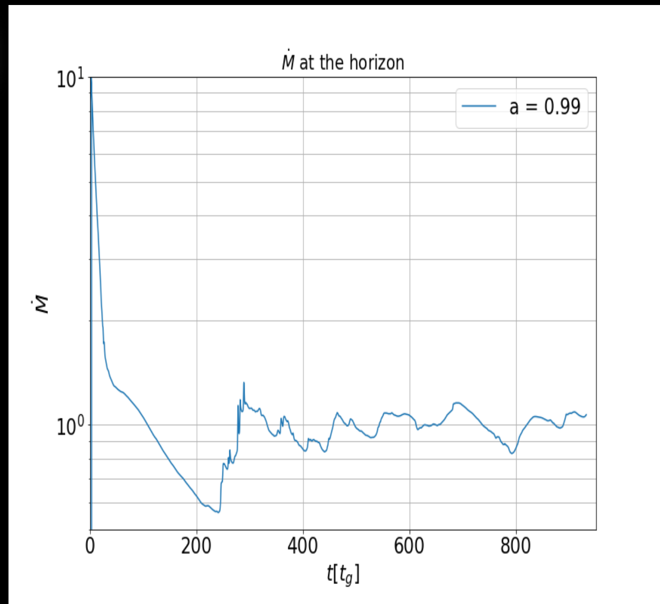

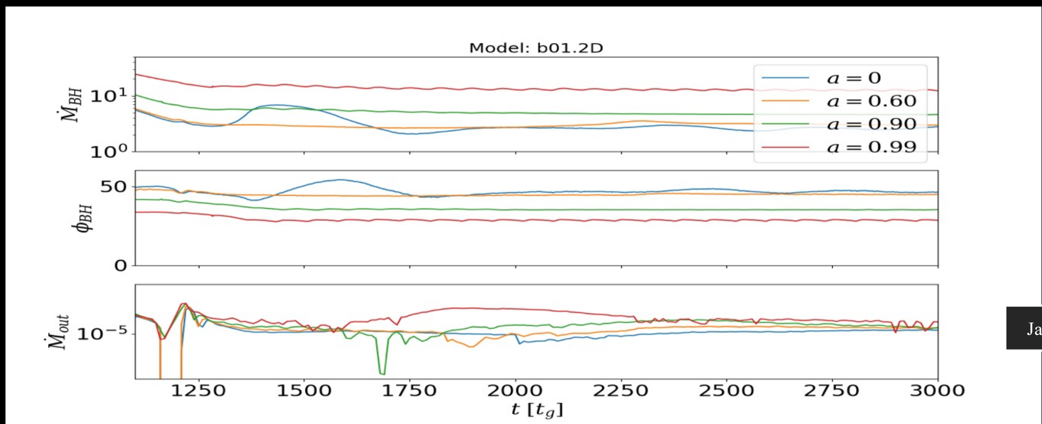

James, Janiuk, Karas (2024)