Radiation-pressure instability is an artifact of constant-α closure

Implications for AGN disk tensions

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The classical problem: Radiation pressure dominated thin disks are unstable — or are they?

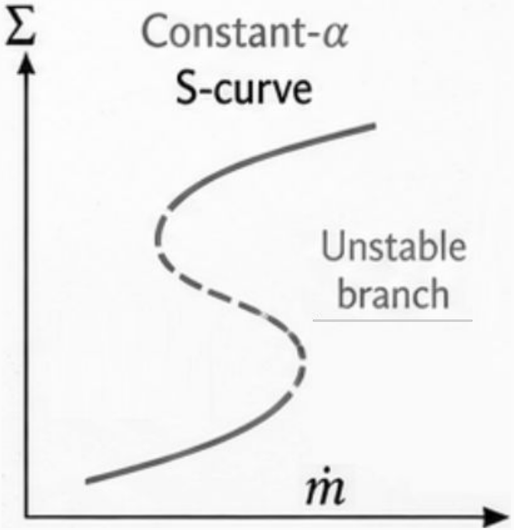
Standard thin disk: turbulent stress parameterized by α.

Classical assumption: α = constant

In the radiation-pressure-dominated regime:

- thermal instability
- viscous instability
- negative-slope branch in the $\dot{M}$–Σ equilibrium curve

This is the classical Lightman–Eardley / Shakura–Sunyaev instability.



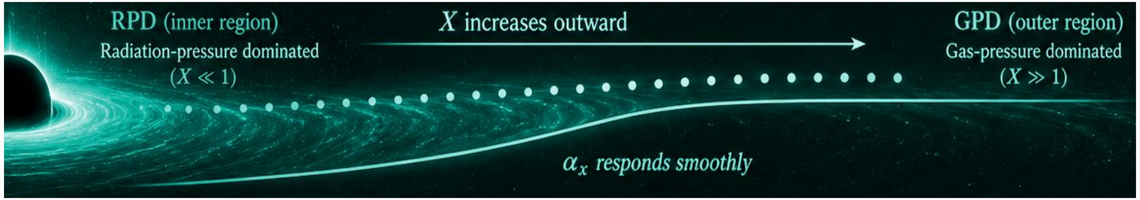
Is radiation pressure itself responsible, or the closure imposed on the stress?

Shakura & Sunyaev 1973; Lightman & Eardley 1974; Shakura & Sunyaev 1976; Piran 1978.

Why question constant α? It is an effective closure, not a fundamental constant!

- **In ionized disks, transport is expected to arise from MRI-driven MHD turbulence.**
- **α represents the vertically averaged nonlinear stress response.**
- **There is no fundamental reason for it to remain unchanged between:**
  - **gas-pressure domination**
  - **radiation-pressure domination**
- **Radiation-MHD simulations already show behavior more complex than a rigid constant α.**

Therefore let the effective stress respond to the thermodynamic state.



$$\alpha_x \equiv \alpha(X), \qquad X \equiv \frac{P_{\rm gas}}{P_{\rm rad}}$$

Balbus & Hawley 1991; Hawley et al. 1995; Hirose et al. 2009; Jiang et al. 2013, 2019; Penna et al. 2013.

Minimal modification: Keep the thin disk. Change only the closure.

$$\alpha_x \equiv \alpha(X), \quad X \equiv \frac{P_{\rm gas}}{P_{\rm rad}}$$

$$\boxed{\eta_X \equiv \frac{d \ln \alpha_x}{d \ln X}}$$

- Same height-integrated thin-disk equations.
- Same hydrostatic balance, angular-momentum transport and radiative cooling.
- No new wind, magnetic-support branch or slim-disk solution is introduced.

What response η_X is required for stability?

Deriving the response function η_X

- **Thermal stability gives the condition:**

$$\eta_X \equiv \frac{d \ln \alpha_x}{d \ln X} > 4/7$$

This is not an assumed value of α. It is a condition on how rapidly the effective stress must respond to the pressure partition.

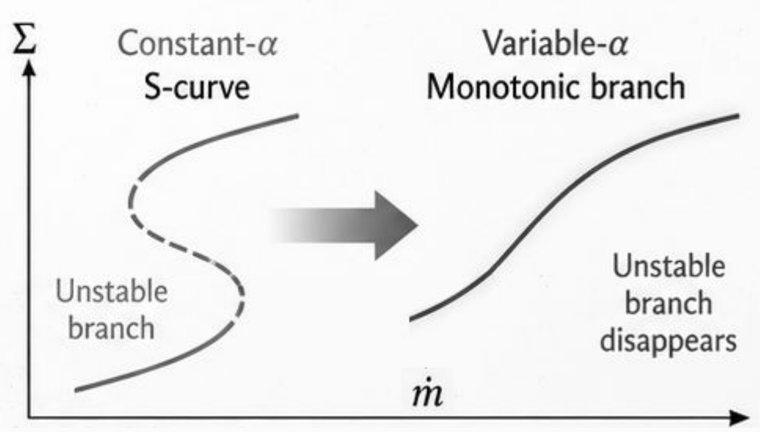
- **The $\dot{M}$–Σ relation with positive slope gives the same condition.**

Thermal stability and disappearance of the viscous negative-slope branch give the same threshold.

A simple realization: One sufficient closure

$$\boxed{\eta_X \equiv \frac{d \ln \alpha_x}{d \ln X} > 4/7}$$

$$\boxed{\alpha_x = \alpha_0 X^p, \text{ with } p > 4/7}$$

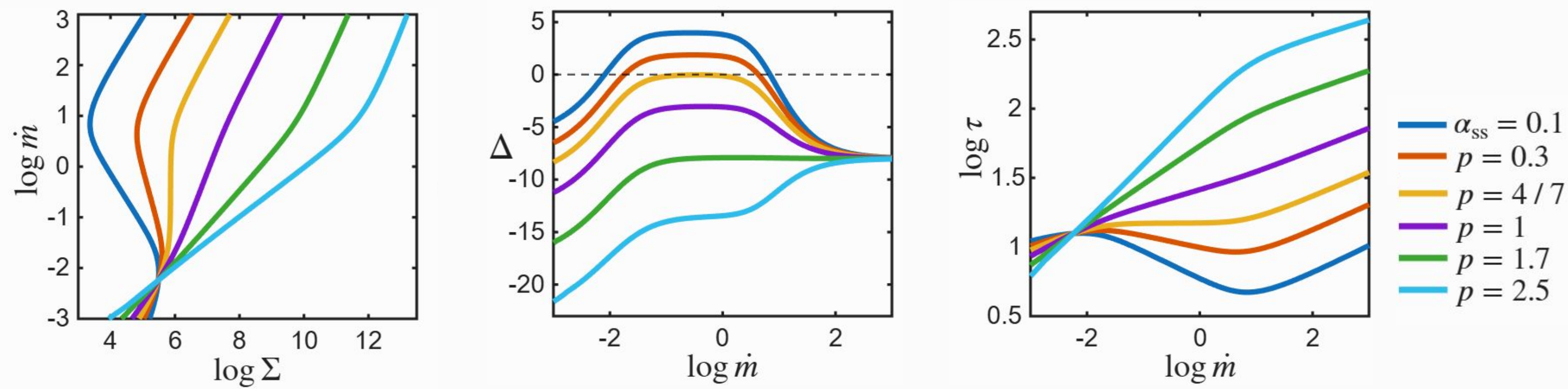


- X ≪ 1 in RPD regions.
- Therefore α decreases naturally toward strongly radiation-dominated regions.
- p = 1 approaches gas-pressure-like stress scaling in the RPD limit.
- The main result is not the power law itself.
- The fundamental result is the general condition $\eta_X > 4/7$.

The unstable branch disappears: η_X > 4/7 removes both thermal and viscous instability

- p = 0 → classical unstable branch
- p = 4/7 → marginal
- p > 4/7 → stable, single-valued solution

increasing p shrinks the unstable branch and that it disappears around p ≳ 4/7, while the thermal stability diagnostic parameter Δ becomes negative everywhere.



Nothing catastrophic happens to the disk.

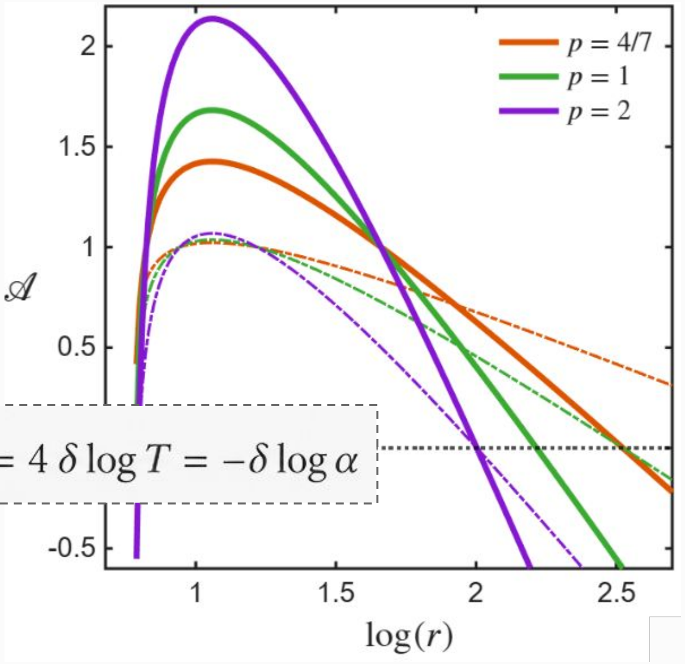
The multivalued branch simply collapses into a continuous stable solution.

What changes physically? Stability comes with higher Σ and τ — not a thicker disk

In radiation pressure dominated limit:

- α_X ↓
- Σ ↑
- τ ↑
- T_c ↑ moderately
- $H/R \approx$ unchanged to leading order

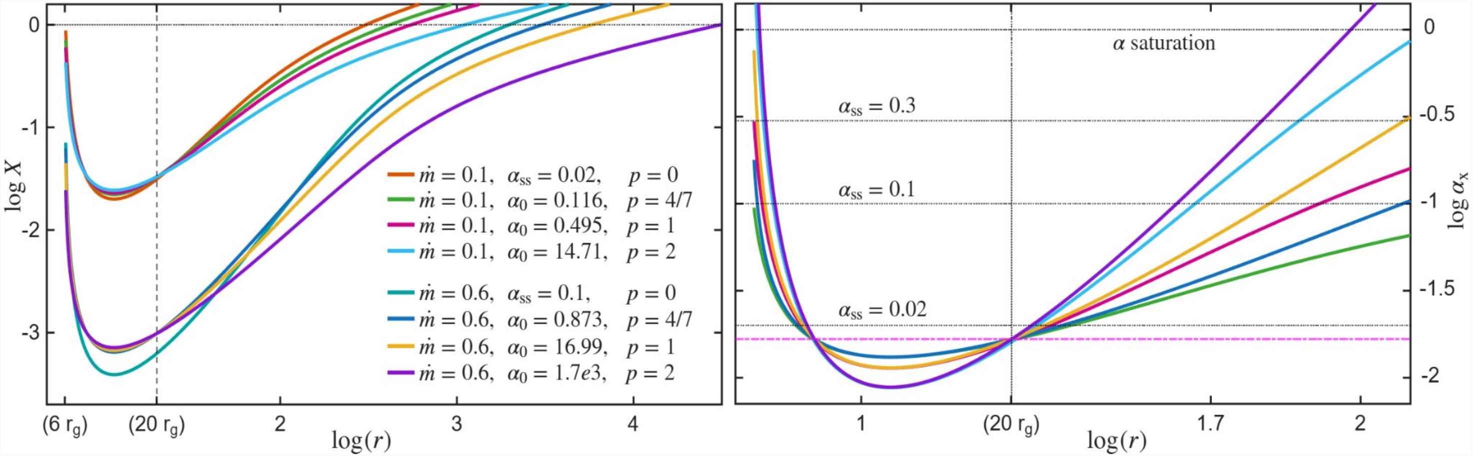
($H_{rad} \propto \kappa \dot{M} J$ with no explicit α_0 or p dependence)



$$\delta \log \Sigma = 4\, \delta \log T = -\delta \log \alpha$$

logarithmic departure from the standard SS solution shows stronger deviations in the inner RPD disk and recovery toward the standard branch outward.

The variability consequence: The model provides a likely possible connection



quasar optical variability amplitude depends on Edd ratio and BH mass, and Arévalo+ find that the dependence itself changes with timescale; in particular variability amplitude decreases with increasing accretion rate.

$t_{thermal} \propto 1/\alpha_X$ and $t_{inflow} \sim\propto 1/\alpha_X$ (because H/R remains unchanged)

$\alpha_X \downarrow$ in RPD $\rightarrow$ longer characteristic timescales, thus slower inner disk

For a **$M_{BH} = 10^9\ M_\odot$** at **$r = 20\ r_g$** $\rightarrow$ **$t_{dyn} \approx 5$ days** and **300-day** characteristic var. time $\rightarrow$ **$\alpha_X \approx 0.017$**

Kelly et al. 2009; Stone et al. 2022; Arévalo et al. 2023.

AGN implications: A stable RPD disk can remain thin, optically thick, and slowly evolving

Variability

- avoids mandatory large-amplitude radiation-pressure limit cycles
- longer thermal/inflow times, therefore a potentially useful AGN-variability prediction.

Geometry / BLR

- no requirement for strong radiation-pressure puffing
- altered illumination / shielding geometry
- possible consequences for BLR and winds

Continuum sizes

- T_{eff} profile remains essentially standard
  therefore this does not directly solve the disk-size problem.
  but higher Σ and τ may change thermalization, color correction and lag normalization.

Wang et al. 2014; Du et al. 2018; Morgan et al. 2010; Jha et al. 2022; Hutsemékers & Sluse 2025.

What this result is — and is not

What we claim

✓ Within the standard radiatively cooled thin-disk framework, constant α is what generates the classical RPD unstable branch.

✓ A sufficiently state-dependent effective stress, $\eta_X > 4/7$, restores a thermally stable, single-valued solution.

✓ Stability against the classical radiation-pressure runaway does not imply a steady disk. Our result changes the characteristic response timescale; stochastic MRI-driven variability can remain.

What we do not claim

✗ $\alpha(X)$ is not yet a first-principles MRI saturation law.

✗ Magnetic vertical support is not modeled explicitly.

✗ Turbulent / magnetic vertical heat transport is not modeled explicitly.

✗ The criterion applies to the radiatively cooled RPD thin-disk regime.

The next test is numerical: measure

$$d \ln \left[W_{R\varphi} \,/\, (P_{gas} + P_{rad}) \right] \,/\, d \ln X$$

directly in radiation-MHD simulations.

Take-home message

1. **Radiation pressure alone does not require an unstable thin disk.**
2. **Within the same equations that produce the classical instability,**

   $\eta_X \equiv d \ln \alpha \,/\, d \ln (P_{gas}/P_{rad}) > 4/7$

   removes both the thermal and viscous unstable branch.

3. **The resulting disk remains approximately thin but becomes:**

   denser • optically thicker • slower

4. **This provides a new closure-level framework for testing AGN disk variability and structure.**

Thank you

A.2. Thermal stability analysis with α_x

For an infinitesimal temperature perturbation while Σ is taken to be constant, thermal stability requires

$$(\partial [Q_{vis} - Q_{rad} - Q_{adv}] / \partial T)_{\Sigma} < 0, \tag{A.14}$$

or, since $T > 0$, simply

$$\delta Q_{vis} - \delta Q_{rad} - \delta Q_{adv} < 0. \tag{A.15}$$

Defining

$$f_a \equiv Q_{adv}/Q_{vis}, \quad f_r \equiv Q_{rad}/Q_{vis}, \quad f_r + f_a = 1, \tag{A.16}$$

Then

$$Q_{vis} [\delta \ln Q_{vis} - f_r \delta \ln Q_{rad} - f_a \delta \ln Q_{adv}] < 0 \tag{A.17}$$

Since $Q_{vis} > 0$, the sign is set only by the bracket.

The logarithmic responses of viscous heating, diffusive radiative cooling, and advective cooling are

$$\begin{aligned} \delta \ln Q_{vis} &= \delta \ln \alpha + \delta \ln P_{tot} + \delta \ln H, \\ \delta \ln Q_{rad} &= 4 \delta \ln T, \\ \delta \ln Q_{adv} &= \delta \ln \dot{M} + 2 \delta \ln H. \end{aligned} \tag{A.18}$$

In order to proceed, we define

$$\beta_r \equiv P_{rad}/P_{tot}, \quad \beta_g \equiv P_{gas}/P_{tot}, \quad \beta_r + \beta_g = 1. \tag{A.19}$$

Therefore, at a given radius for a fixed Σ,

$$\begin{aligned} \delta \ln P_{tot} &= \delta \ln H, \\ \delta \ln P_{tot} &= \beta_r \delta \ln P_{rad} + \beta_g \delta \ln P_{gas}, \\ \delta \ln P_{gas} &= \delta \ln T - \delta \ln H, \\ \delta \ln P_{rad} &= 4 \delta \ln T, \end{aligned} \tag{A.20}$$

$$\delta \ln H = A \, \delta \ln T / D, \tag{A.21}$$

where $A \equiv 4 - 3\beta_g$, and $D \equiv 1 + \beta_g$.

Using $X \equiv P_{gas}/P_{rad}$, we have

$$\delta \ln X = \delta \ln P_{gas} - \delta \ln P_{rad}, \tag{A.22}$$

which, combined with Eqs. A.2, A.20, and A.21, yields

$$\delta \ln \alpha = -7 \, \eta_x \, \delta \ln T / D \tag{A.23}$$

Using the angular-momentum equation

$$\begin{aligned} \delta \ln \dot{M} &= \delta \ln \alpha + \delta \ln P_{tot} + \delta \ln H, \\ \delta \ln \dot{M} &= (2A - 7\eta_x) \, \delta \ln T / D \end{aligned} \tag{A.24}$$

Now substituting into Eq. (A.18):

$$\begin{aligned} \delta \ln Q_{vis} &= (2A - 7\eta_x) \, \delta \ln T / D, \\ \delta \ln Q_{adv} &= (4A - 7\eta_x) \, \delta \ln T / D. \end{aligned} \tag{A.25}$$

Then

$$\left[\frac{2A - 7\eta_x}{D} - 4 f_r - f_a \frac{4A - 7\eta_x}{D} \right] \delta \ln T < 0. \tag{A.26}$$

For a positive temperature perturbation, $\delta \ln T > 0$, and as D is always positive, stability condition reads as

$$\Delta = 2A - 4D + 4 f_a (D - A) - 7(1 - f_a)\eta_x < 0, \tag{A.27}$$

Since the classical Lightman & Eardley (1974) instability is obtained in the radiatively cooled RPD branch, the critical bound is evaluated in the limit $\beta_g \to 0$ and $f_a \to 0$, thus

$$\Delta = 4 - 7\eta_x < 0, \tag{A.28}$$

$$\eta_x > 4/7 \approx 0.571. \tag{A.29}$$

A.3. Constraining α_x from the $\dot{M}$–Σ relation

We now specialize to the RPD, radiatively cooled regime,

$$P_{\rm rad} \gg P_{\rm gas}, \qquad Q_{\rm adv} \ll Q_{\rm rad}, \tag{A.30}$$

From vertical hydrostatic equilibrium,

$$P_{\rm tot} \propto \Sigma H, \tag{A.31}$$

and in the radiatively cooled RPD limit,

$$P_{\rm tot} \approx P_{\rm rad} \propto T^4, \tag{A.32}$$

which gives

$$H \propto T^4 \Sigma^{-1}. \tag{A.33}$$

The gas pressure scales as

$$P_{\rm gas} \propto \rho T \propto \Sigma T H^{-1}, \tag{A.34}$$

and therefore

$$P_{\rm gas} \propto \Sigma^2 T^{-3}. \tag{A.35}$$

Combining these expressions, we obtain

$$X \equiv P_{\rm gas}/P_{\rm rad} \propto \Sigma^2 T^{-7}. \tag{A.36}$$

The viscous heating rate scales as

$$Q_{\rm vis} \propto \alpha P_{\rm tot} H \propto \alpha T^8 \Sigma^{-1}, \tag{A.37}$$

while radiative cooling gives

$$Q_{\rm rad} \propto T^4 \Sigma^{-1}. \tag{A.38}$$

Thermal equilibrium $Q_{\rm vis} = Q_{\rm rad}$ then implies $\alpha T^4 =$ const., hence, $T^4 \propto \alpha^{-1}$. Substituting into the expression for X yields

$$X \propto \Sigma^2 \alpha^{7/4}, \tag{A.39}$$

which determines $X(\Sigma)$ along the equilibrium branch

Taking the logarithm of the constraint equation,

$$\ln X - \frac{7}{4} \ln \alpha_x = 2 \ln \Sigma + \text{const}, \tag{A.40}$$

and differentiating with respect to $\ln \Sigma$, we obtain

$$d \ln X / d \ln \Sigma = 8/(4 - 7\eta_x). \tag{A.41}$$

From angular momentum transport,

$$\dot{M} \propto \alpha P_{\rm tot} H \propto \alpha T^8 \Sigma^{-1}. \tag{A.42}$$

Using the thermal equilibrium condition, we obtain

$$\dot{M} \propto \alpha_x^{-1} \Sigma^{-1}. \tag{A.43}$$

Thus, the full $\dot{M}(\Sigma)$ relation is determined by the implicit dependence $X(\Sigma)$.

Differentiating $\ln \dot{M}$ gives

$$d \ln \dot{M} / d \ln \Sigma = -1 - \eta_x \, d \ln X / d \ln \Sigma. \tag{A.44}$$

Substituting, we find

$$d \ln \dot{M} / d \ln \Sigma = (\eta_x + 4)/(7\eta_x - 4). \tag{A.45}$$

This equation gives the general slope of the $\dot{M}$–Σ equilibrium curve for an arbitrary α_x. The classical instability corresponds to the denominator changing sign, producing a negative-slope segment. Requiring $d \ln \dot{M} / d \ln \Sigma > 0$, yields a necessary condition for the absence of the unstable branch. For $\eta_x > 0$, this implies

$$\boxed{\eta_x > 4/7 \approx 0.571.} \tag{A.46}$$

Region A: RPD ($X \ll 1$), with $\kappa_{\rm es}$

$$
\begin{array}{lllll}
H & \propto & \dot{m}\mathcal{J} M_\bullet & & \\
H/R & \propto & \dot{m}\mathcal{J}\, r^{-1} & & \\
T_c & \propto & (M_\bullet^{-1} & (\dot{m}\mathcal{J})^{2p} & r^{-3p-3/2} \quad \alpha_0^{-1})^{w} \\
\Sigma & \propto & (M_\bullet^{p} & (\dot{m}\mathcal{J})^{7p-4} & r^{-9p+6} \quad \alpha_0^{-4})^{w} \\
\rho & \propto & (M_\bullet^{-4} & (\dot{m}\mathcal{J})^{6p-8} & r^{-9p+6} \quad \alpha_0^{-4})^{w} \\
P & \propto & (M_\bullet^{-1} & (\dot{m}\mathcal{J})^{2p} & r^{-3p-3/2} \quad \alpha_0^{-1})^{4w} \\
X & \propto & (M_\bullet^{-1} & (\dot{m}\mathcal{J})^{-8} & r^{21/2} \quad \alpha_0^{-1})^{w}
\end{array}
$$

with $w = 1/(4+p)$

Region B: GPD ($X \gg 1$), with $\kappa_{\rm es}$

$$
\begin{array}{lllll}
H & \propto & (M_\bullet^{p+9} & (\dot{m}\mathcal{J})^{p+2} & r^{21/2} \quad \alpha_0^{-1})^{w} \\
H/R & \propto & (M_\bullet^{-1} & (\dot{m}\mathcal{J})^{p+2} & r^{1/2-p} \quad \alpha_0^{-1})^{w} \\
T_c & \propto & (M_\bullet^{-2} & (\dot{m}\mathcal{J})^{2p+4} & r^{-3p-9} \quad \alpha_0^{-2})^{w} \\
\Sigma & \propto & (M_\bullet^{p+2} & (\dot{m}\mathcal{J})^{7p+6} & r^{-9p-6} \quad \alpha_0^{-8})^{w} \\
\rho & \propto & (M_\bullet^{-7} & (\dot{m}\mathcal{J})^{6p+4} & r^{33/2-9p} \quad \alpha_0^{-7})^{w} \\
P & \propto & (M_\bullet^{-9} & (\dot{m}\mathcal{J})^{8p+8} & r^{15/2-12p} \quad \alpha_0^{-9})^{w} \\
X & \propto & (M_\bullet^{-1} & (\dot{m}\mathcal{J})^{-8} & r^{21/2} \quad \alpha_0^{-1})^{w}
\end{array}
$$

with $w = 1/(10+p)$

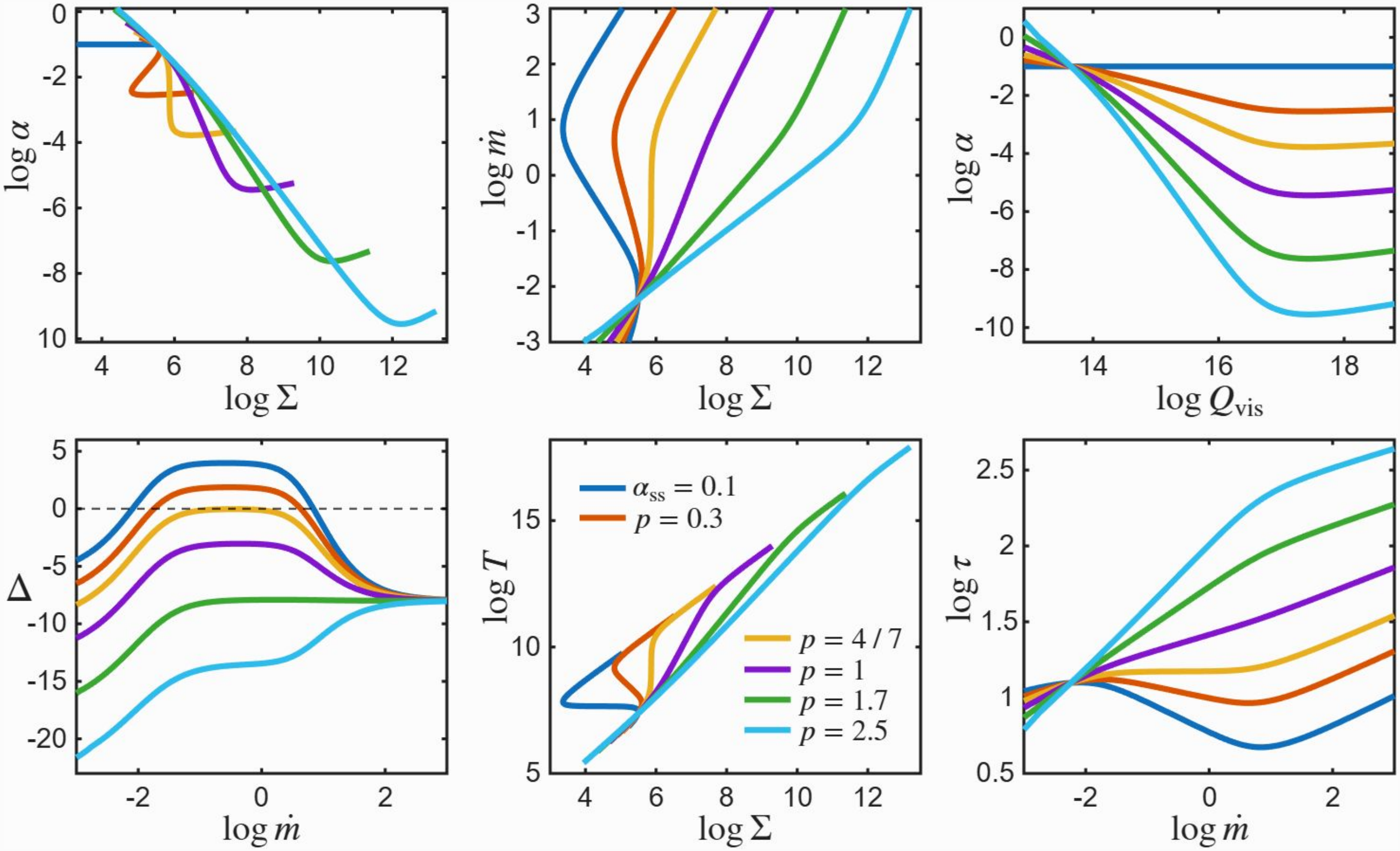
Region C: GPD ($X \gg 1$), with $\kappa_{\rm ff}$

$$
\begin{array}{lllll}
H & \propto & (M_\bullet^{p+9} & (\dot{m}\mathcal{J})^{(p+3)/2} & r^{3(p+15)/4} \quad \alpha_0^{-1})^{w} \\
H/R & \propto & (M_\bullet^{-1} & (\dot{m}\mathcal{J})^{(p+3)/2} & r^{-(p-5)/4} \quad \alpha_0^{-1})^{w} \\
T_c & \propto & (M_\bullet^{-2} & (\dot{m}\mathcal{J})^{p+3} & r^{-3(p+5)/2} \quad \alpha_0^{-2})^{w} \\
\Sigma & \propto & (M_\bullet^{p+2} & (\dot{m}\mathcal{J})^{7(p+2)/2} & r^{-15(p+2)/4} \quad \alpha_0^{-8})^{w} \\
\rho & \propto & (M_\bullet^{-7} & (\dot{m}\mathcal{J})^{3p+11/2} & r^{(-18p-75)/4} \quad \alpha_0^{-7})^{w} \\
P & \propto & (M_\bullet^{-9} & (\dot{m}\mathcal{J})^{4p+17/2} & r^{-6p-105/4} \quad \alpha_0^{-9})^{w} \\
X & \propto & (M_\bullet^{-1} & (\dot{m}\mathcal{J})^{-7/2} & r^{15/4} \quad \alpha_0^{-1})^{w}
\end{array}
$$

with $w = 1/(10+p)$

The classic SS asymptotic behavior is recovered for $p = 0$, and the TD become globally stable for $p > 4/7$.

It also reduces to the prescription of Sakimoto & Coroniti (1981) for $p = 1$, because in the RPD regime, $P_{\rm rad} \gg P_{\rm gas}$, so $P_{\rm tot} \simeq P_{\rm rad}$, and therefore $T_{r\phi} \propto \alpha_0 P_{\rm gas}$. The key point, however, is that unlike Sakimoto & Coroniti (1981), who directly imposed gas-pressure scaling, the present closure, $\alpha_{\rm x} = \alpha_0 X^p$, follows from the closure-level stability condition and recovers a gas-pressure-like stress only as the $p = 1$ RPD-limit case.


log α
log Σ
log ṁ
log Σ
log α
log Q_vis
Δ
log ṁ
log T
log Σ
α_ss = 0.1
p = 0.3
p = 4 / 7
p = 1
p = 1.7
p = 2.5
log τ
log ṁ