Charged accretion Tori around Schwarzschild black hole:

Self-generated electromagnetic fields and their effect on disk structure.

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Roadmap

- Introduction
- Self-generated magnetic field & external magnetic field
- Results
- Discussion
- Conclusions & future work

Motivations

- Thick, radiatively inefficient accretion flows are classically modelled as constant angular-momentum equilibrium tori.
- These disks are magnetized, and two physically distinct sources compete:
  - a **self-generated** field, sourced by the disk's own poloidal current;
  - a **large-scale external** field of environmental origin.

Open question: when both are present, which one actually shapes the torus and is comparing field strengths even the right question?

Approach: start from the classical *unmagnetized* torus and add both components self-consistently.

Initial model: the neutral relativistic torus

Schwarzschild spacetime ($G = c = M = 1$):

$$ds^2 = -\left(1 - \tfrac{2}{r}\right) dt^2 + \left(1 - \tfrac{2}{r}\right)^{-1} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta \, d\phi^2.$$

Stationary, axisymmetric perfect fluid, rigidly rotating, $u^\mu = (u^t, 0, 0, u^\phi)$.
For $\ell = \mathrm{const}$, $\ell = -u_\phi/u_t$, $\nabla_\mu T^{\mu\nu} = 0$ becomes:

$$\nabla_i \frac{p}{\rho + p} = -\nabla_i \ln |u_t| + \underbrace{\frac{\Omega \, \nabla_i \ell}{1 - \Omega \ell}}_{=0 \text{ for } \ell = \mathrm{const}} \implies \nabla_i \frac{p}{\rho + p} = -\nabla_i W_0,$$

$$W_0(r, \theta; \ell) = \ln |u_t| = \tfrac{1}{2} \ln \frac{g_{t\phi}^2 - g_{tt} g_{\phi\phi}}{\ell^2 g_{tt} + 2\ell g_{t\phi} + g_{\phi\phi}}.$$

For ℓ_0 slightly *above* the marginally stable (ISCO) value ℓ_{ms}, $W_0(r, \frac{\pi}{2})$ has a local **minimum** (torus centre) and a **saddle** (cusp, mass overflow) point.

Trova 2016; Boyer 1965

The Charged Accretion Disc (CAD) model

Equilibrium modified by the Lorentz force ($\sigma = 0$):

$$\frac{\nabla_i p}{\rho + p} = -\nabla_i \ln |u_t| + \frac{\Omega \nabla_i \ell}{1 - \Omega \ell} + \frac{q}{\rho + p} u^\mu F_{i\mu}.$$

Integrability of the resulting pressure equation requires the charge-to-enthalpy ratio to depend only on the flux function

$$\mathcal{R}_i = -\nabla_i \ln |u_t| + \frac{\Omega \nabla_i \ell}{1 - \Omega \ell} + \frac{q}{\rho + p} F_{i\mu} u^\mu \implies \boxed{\partial_\theta \mathcal{R}_r = \partial_r \mathcal{R}_\theta}$$

Hence ($\ell = \mathrm{const}$):

$$W_{\mathrm{ch}}(r, \theta; \ell) = W_0(r, \theta; \ell) - k(r, \theta)\left[A_\phi^H(r, \theta) + A_\phi^{\mathrm{ext}}(r, \theta)\right], \quad k(r, \theta) = \frac{q}{\rho + p}$$

Solving the *charged* case: we impose $q = q(A_\phi) \rightarrow$ minimum of $W_{\mathrm{ch}} \rightarrow$ centre, saddle $\rightarrow$ cusp $\rightarrow$ torus morphology.

Kovář et al. 2011, 2014; Trova et al. 2016.

Magnetic field expressions

Internal self-generated magnetic field: current loop heuristic potential of radius r_0 (azimuthal current)

$$A_\phi^H = \frac{B_H\, r_0^2\, r \sin\theta}{(r_0^2 + r^2)^{3/2}}.$$

External magnetic field: Wald potential for a asymptotically uniform magnetic field

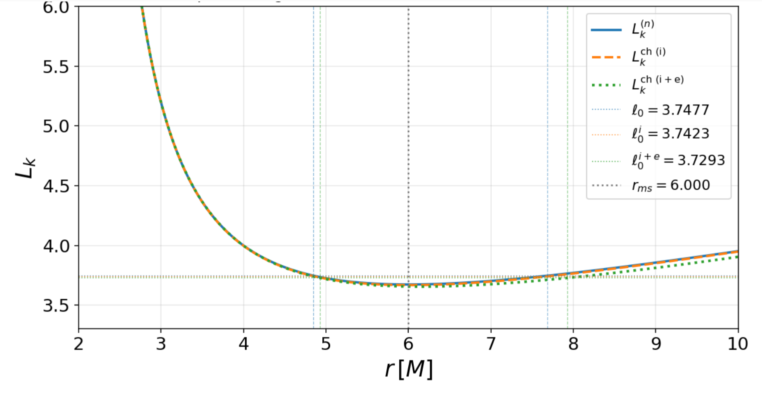
$$A_\phi^{\rm ext} = \tfrac{1}{2} B_{\rm ext}\, r^2 \sin^2\theta\,.$$

Procedure: $k(r)$ depends only on (B_H, r_0). The loop position is iterated, $r_0 \leftarrow \sqrt{2}\, x_c^{\rm (current)}$, until the loop sits at the actual torus centre.

Reference run shown throughout: $B_H = 5$, $B_{\rm ext} = 0.75$, $\ell_0 = \lambda\, \ell_{\rm ms}$ with $\lambda = 1.02$.

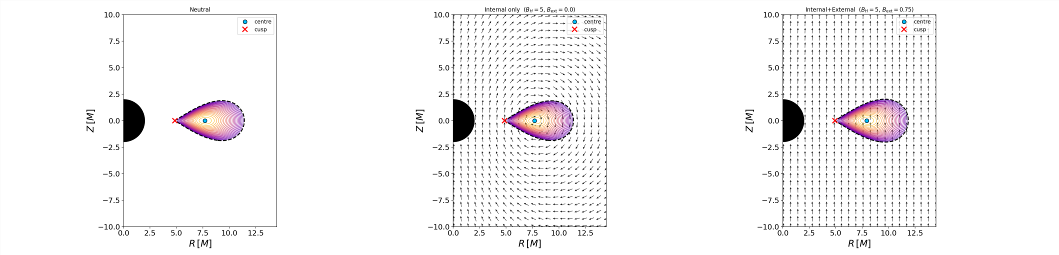
M. Kološ and D. Kofroň (2025).

Specific angular momentum profile



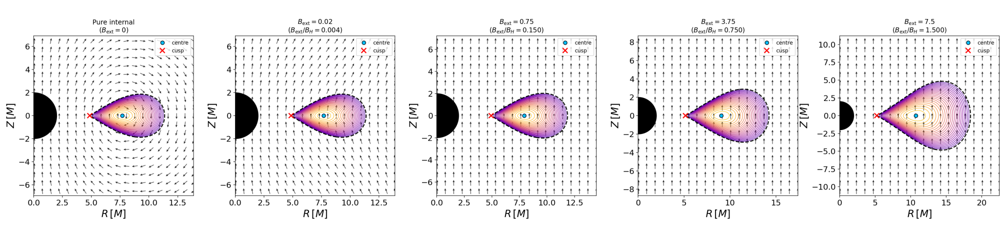
Specific angular momentum L_K in function of r for the neutral and the two charged cases.
$\ell_0 = \lambda\,\ell_{\rm ms},\ \lambda = 1.02.$

Equipotential surfaces



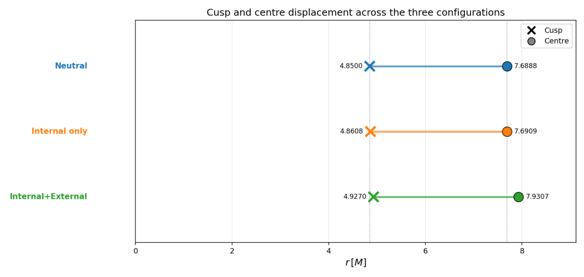
Equipotentials surfaces and $\vec{B}$ direction. Left to right: Neutral, Internal field, Internal+External field.

Effect of the external field strength



Equipotentials surfaces and $\vec{B}$ direction, internal field fixed ($B_H = 5$). Left to right: increasing $B_{\rm ext}$.

Effect of the fields on cusp & centre



Configuration	x_{cusp} [M]	x_c [M]	ℓ_0
Neutral	4.8500	7.6888	3.7477
Internal only	4.8608	7.6909	3.7423
Internal+External	4.9270	7.9307	3.7293

$M = 1,\ r_{\mathrm{EH}} = 2.0,\ r_{\mathrm{mb}} = 4.0,\ r_{\mathrm{ms}} = 6.0,\ \ell_0 = \lambda\ell_{\mathrm{ms}},$
$\lambda = 1.02;\ B_H = 5,\ B_{\mathrm{ext}} = 0.75.$

“External only” $\equiv$ Neutral, exactly

- In the external-only case, the field is non-zero, yet the torus is **numerically identical** to the neutral one ($\Delta x \sim 10^{-8}$).
- This is **not** a bug: $k(r)$ is sourced *only* by B_H. If $B_H = 0 \implies k(r) \equiv 0 \forall r$. Therefore

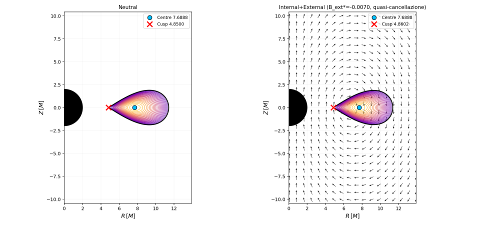
$$W_{\mathrm{ch}} = W_0 - \underbrace{k(r)}_{=0} \left[A_\phi^H + A_\phi^{\mathrm{ext}}\right] \equiv W_0 \qquad \forall\, B_{\mathrm{ext}}.$$

- A disk with no current of its own cannot feel the external Wald field in this model.

Robustness & a special sub-case

Sign test: $(B_H, B_{\rm ext}) \to (-B_H, -B_{\rm ext})$ leaves the torus invariant, as required by the field equations (parity check).

Mimicking the neutral configuration: for $B_H = 5,\ B^\star_{\rm ext} \approx -0.007$, restores the neutral *centre* exactly (but not the cusp simultaneously).



A full 2D search of $(B_H, B_{\rm ext})$ finds **no** strong-field configuration matching both: the potential profiles are too different.

Conclusions & Future Work

What we did

- Charged-torus model: internal self-generate and external magnetic fields around a Schwarzschild black hole.
- The internal self-generated field changes the topology and the morphology of the disc.
- **Relative potential profile** $(A_{\rm ext}/A_H)$, controls which component dominates.
- No strong-field configuration cancels *both* cusp and centre shifts at once.
- Validation against literature results.

Next steps

- Refine ℓ_0 as a fraction of $[\ell_{\rm ms}, \ell_{\rm mb}]$ for the *charged* case.
- Extend to a spinning (Kerr) hole.
- 2D (non-equatorial) study of the function $k(r)$.
- Testing other external fields.

Thank you!

Questions welcome.

Backup slides

Self-generated field: how $k(r)$ is fixed

Standard CAD approach (Kovář et al. 2011, 2014; Trova et al. 2016): integrability of the charged Euler equation only requires $k = k(A_\phi)$, a free function of the flux – chosen e.g. as a power law $\mu_n A_\phi^n$.

Our approach is more constrained: instead of choosing $k(A_\phi)$ freely, we *derive* $k(r)$ by requiring the assumed current-loop potential A_ϕ^H to itself satisfy the vacuum Maxwell equation $\nabla \times \vec{B} = 4\pi \vec{J}$ (purely azimuthal current), evaluated on the equator:

$$k(r) \equiv k\left(r, \tfrac{\pi}{2}\right) = \frac{1}{4\pi r^2} \left\{ \frac{d}{dr} \left[\left(1 - \tfrac{2}{r}\right) \frac{\partial A_\phi^H}{\partial r} \bigg|_{\pi/2} \right] + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left[\frac{\partial_\theta A_\phi^H}{\sin\theta} \right]_{\pi/2} \right\}.$$

This removes a free function from the model: $k(r)$ is fixed uniquely by (B_H, r_0), and – crucially – *does not involve* $A_\phi^{\rm ext}$ at all (Finding 1).

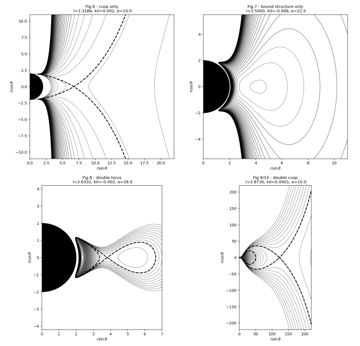
Numerical construction pipeline

1. Choose B_H, B_{ext}, and (for self-consistent runs) an initial r_0.
2. Build $A_\phi^H + A_\phi^{\mathrm{ext}}$ and derive $k(r)$ from the equatorial Maxwell equation.
3. Construct $W_{\mathrm{ch}}(r, \theta; \ell)$.
4. Determine ℓ_0 and locate x_{cusp}, x_c from $\left.\dfrac{dW_{\mathrm{ch}}}{dr}\right|_{\theta=\pi/2} = 0$.
5. Update $r_0 \leftarrow \sqrt{2}\, x_c$ (self-consistent cases) and iterate to convergence.
6. Reconstruct density from the polytropic Bernoulli integral; map equipotentials, $\vec{B}$-field and charge density.

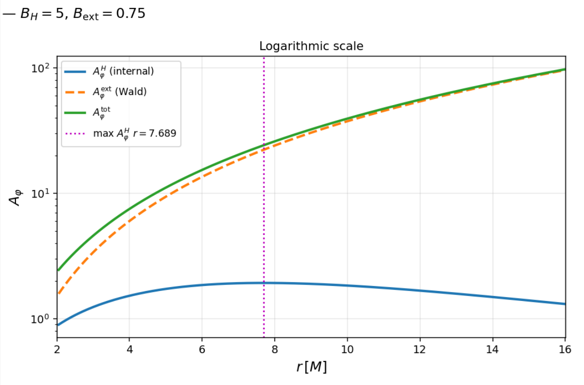
Validation against the literature

Benchmarked the metric/potential engine against Trova et al. 2020 (PRD 101, 083027, test charge + external uniform field), reproducing all 4 published example topologies (cusp-only, bound-only, double torus, double-cusp).

Plus two exact analytic checks in that paper: $\ell_{\mathrm{ISCO}} = \sqrt{13.5}$ and photon sphere $(r, \ell^2) = (3, 27)$ (reproduced to machine precision).



Why "Internal+External" shifts so much



- $B_H = 5 \gg B_{\rm ext} = 0.75$ – naively, internal should dominate.
- $A_\phi^{\rm ext} \sim r^2$, A_ϕ^H localized near r_0 and decaying.
- At the disk radius: $|A_\phi^{\rm ext}|/|A_\phi^H| \approx 12$.
- **The relevant ratio is $A_{\rm ext}/A_H$.**