Detection of cosmic strings by gravitational wave lensing

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Plan of presentation

1. Introduction
2. Wave effects
3. Model
4. Detection
5. Mock observations

In this work the following convention is used : metric sign $(-, +, +, +)$ and $c = 1$

Cosmic Strings I

The cosmic string's metric is described by following metric

$$ds = -dt^2 + dz^2 + dr^2 + (1 - 4G\mu)^2 r^2 d\varphi^2 \tag{1}$$

where :$G\mu$ - tension of cosmic string

Cosmic Strings II

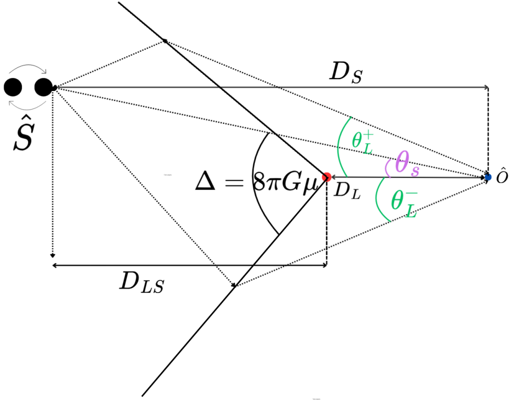


Figure 1: Lensing By cosmic string scheme. Where :D_S - angular distance to source; D_L - angular distance to lens; $D_{LS} = D_S - \frac{z_L+1}{z_S+1} D_S$; z_i - redshift; θ_L - inclination of image; θ_S - inclination of source; Δ-deficit angle;$G\mu$ - tension of cosmic string.

Cosmic Strings III

From (Vilenkin, 1984; Peter, 1994; Sazhin et al., 2007; Harari and Sikivie, 1988), we know that in our case the separation between images is constant,

$$\theta_+ + \theta_- = \Delta\theta = 8\pi G\mu \tag{2}$$

Based on this information, the characteristic angle (Einstein angle) θ_* can be derived (Xiao et al., 2022).

$$\alpha(\vec{\theta}) = \theta_* = 4\pi G\mu \frac{D_{LS}}{D_S} \tag{3}$$

From (Sazhin et al., 2007), one can easily see that lensing is possible for $\theta_S < 2\theta_*$. :

Wave Optics and GW Lensing I

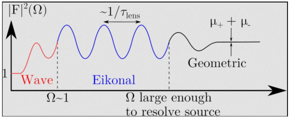


Figure 2: A schematic depicting different regimes in gravitational lensing as a function of angular frequency $\Omega = \frac{4\pi R_s}{\lambda}$ where R_s is Schwarzschild radius and λ is the wavelength of the gravitational wave. Source:(Leung et al., 2025)

One of the fundamentals of wave optics formalism is amplification factor $F(w, y)$ (Nakamura and Deguchi, 1999; Villarrubia-Rojo et al., 2025)

$$F(w, y) \equiv \frac{\tilde{h}(f)}{\tilde{h}_0(f)} \ . \tag{4}$$

Where, $\tilde{h}_0(f)$ and $\tilde{h}(f)$ are characteristic strain in frequency domain of unlensed and lensed gravitational wave.

Wave Optics and GW Lensing II

Further, we define dimensionless quantities based on (Nakamura and Deguchi, 1999; Yoo et al., 2013; Villarrubia-Rojo et al., 2025)

$$d_{eff} \equiv \frac{D_L D_{LS}}{(1+z_L) D_S} \tag{5}$$

$$\varphi = \theta_S - \theta_* \tag{6}$$

$$y = \frac{\varphi}{\theta_*} \tag{7}$$

$$w = 2\pi \frac{(D_L \theta_*)^2}{d_{eff}} f \tag{8}$$

$$\beta = \frac{D_{LS}}{D_S} \tag{9}$$

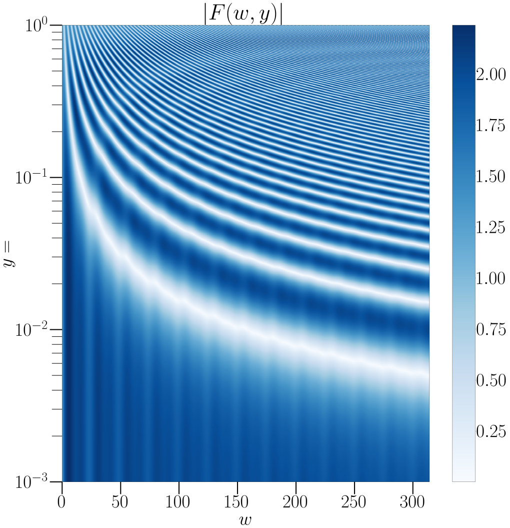
Wave Optics and GW Lensing III

Next with defined equations 5,7 and 8m, we can finally present amplification factor approximation

$$F(w,y) = \exp\left(\frac{w}{2i}(1+2y)\right)[1 - \frac{1}{2}\operatorname{erfc}(\sqrt{\frac{w}{2i}}(1+y))] + \exp\left(\frac{w}{2i}(1-2y)\right)[1 - \frac{1}{2}\operatorname{erfc}(\sqrt{\frac{w}{2i}}(1-y))] \quad (10)$$

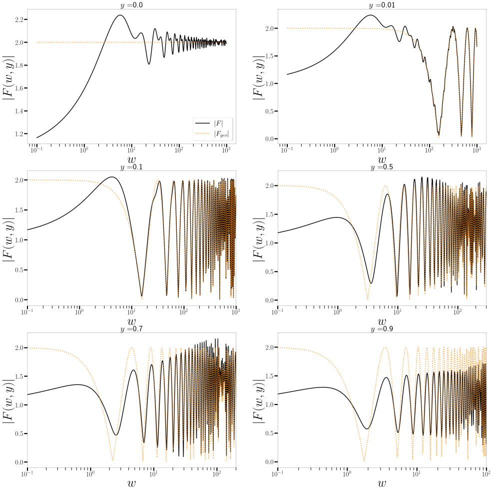
where: $\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}}\int_x^\infty e^{-t^2}dt$

Amplification value dependence on w and y (Fig. 9)



Amplification factor plots with $F_{geo} = \sqrt{2 + 2\cos(2wy)}$

$|F(w, y)|$

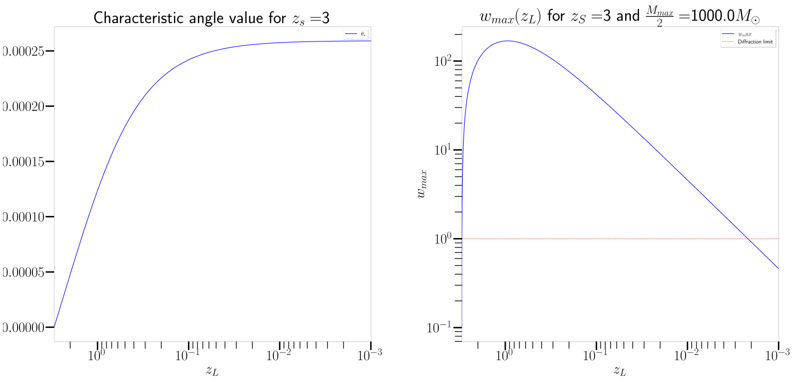


Initial Data

For the first part, assume:

- equal mass binary systems on circular orbits, placed at redshift $z_S = 3$,(Dominik et al., 2013)
- mass span of single Black hole from $\frac{M}{2} \in [5, 1000]$ solar mass (range of ET (Singh et al., 2022))
- $G\mu \approx 1 \times 10^{-10}$ based on the (Planck Collaboration, 2014; Lentati et al., 2015; Abbott et al., 2021)

Estimation of z_L

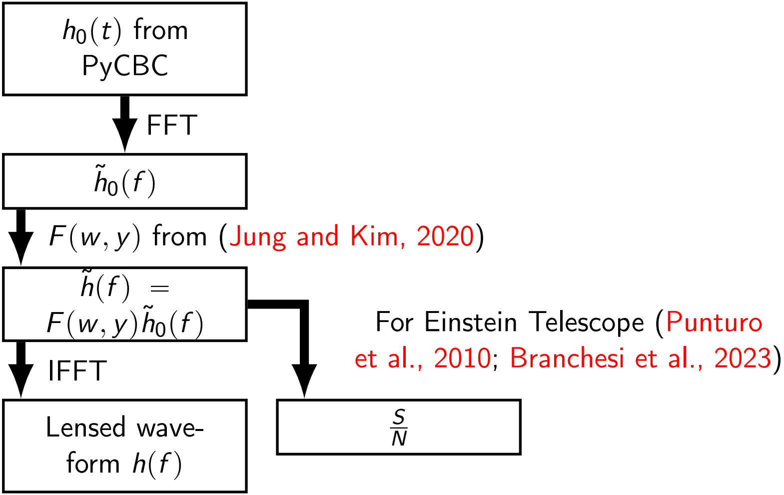


(a) Characteristic angle for source on $z_S = 3$

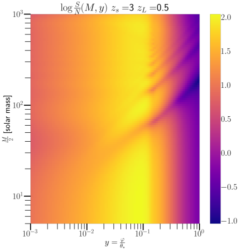
(b) w_{max} for source on $z_S = 3$ and $\frac{M}{2} = 1000\ \mathrm{M}_\odot$

Based on above figures we set redshift of cosmic string to $z_L \approx 0.5$.

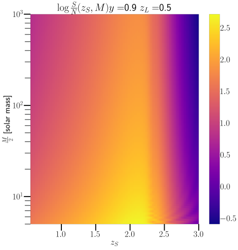
Pipeline of Method



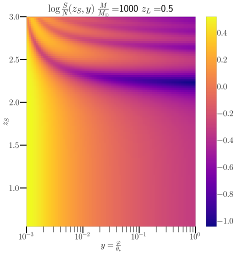
SNR value for ET



(a) $z_S = const$



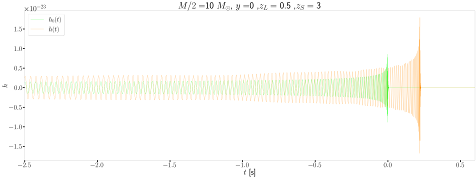
(b) $y = const$

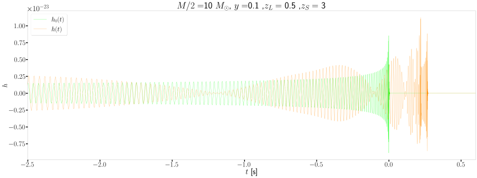


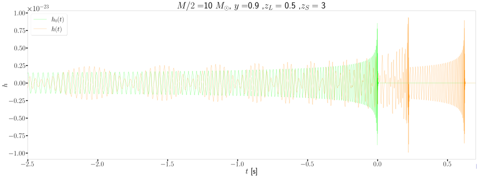
(c) $M = const$

Figure 4: SNR values

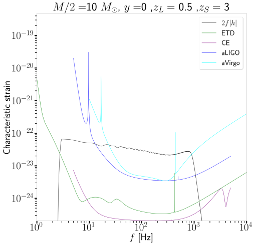
Example waveforms: Case of 10 $M_\odot$



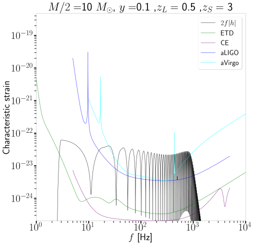




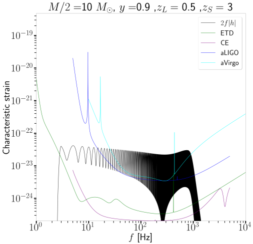
Example characteristic strains: case of 10 $M_\odot$



(a) $y = 0$



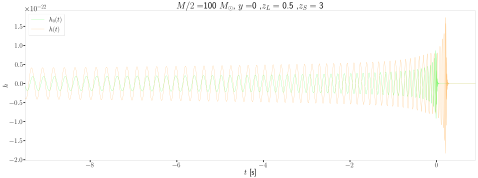
(b) $y = 0.1$

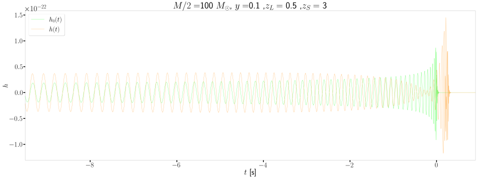


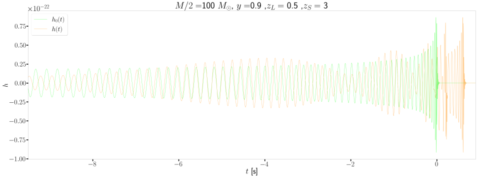
(c) $y = 0.9$

Figure 6: Characteristic strain plot for $\frac{M}{2} = 5$

Example waveforms: Case of 100 $M_\odot$







Example characteristic strains: case of 100 $M_\odot$

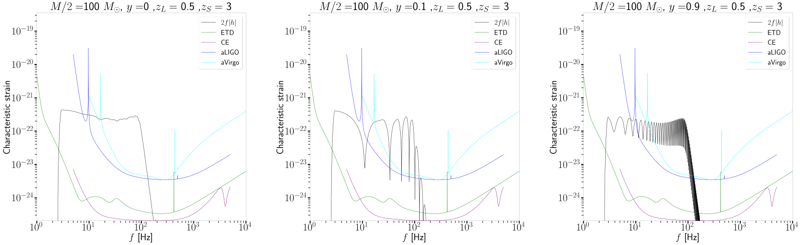
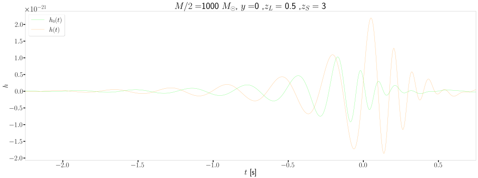
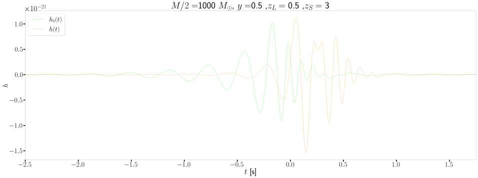
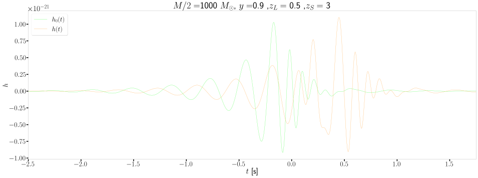


Figure 8: Characteristic strain plot for $\frac{M}{2} = 100$

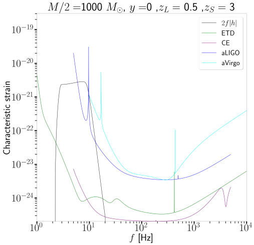
Example waveforms: Case of 1000 $M_\odot$



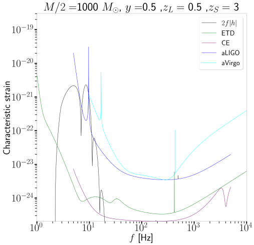




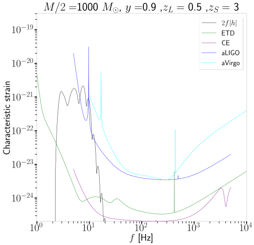
Example characteristic strains: case of 1000 $M_\odot$



(a) $y = 0$



(b) $y = 0.5$



(c) $y = 0.9$

Figure 10: Characteristic strain plot for $\frac{M}{2} = 1000$

Pipeline of code

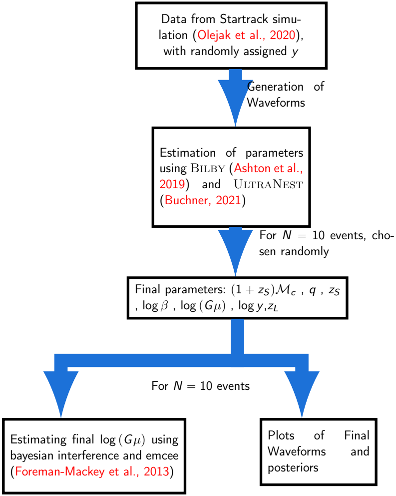


Figure 11: Pipeline of code used to estimate the simulated events with BILBY

Final estimation

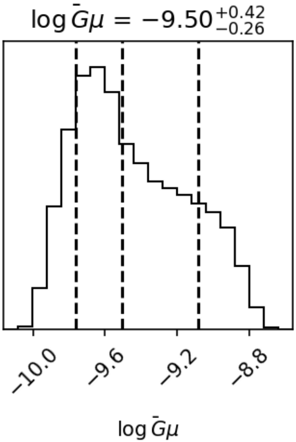


Figure 12: $\log(G\mu)$ estimation based on Bayesian interference

Conclusions

- The oscillating wave effects are clearly visible in amplification factor
- We can see interference patterns in SNR and Amplification factor
- Also they could be observed in waveforms, with characteristic beat patterns (Hou et al., 2021)
- Despite the tension values, which make the cosmic strings hard to observe via light, the gravitational waves lensing is a promisive way to search cosmic strings, especially in the era of the Einstein Telescope.

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