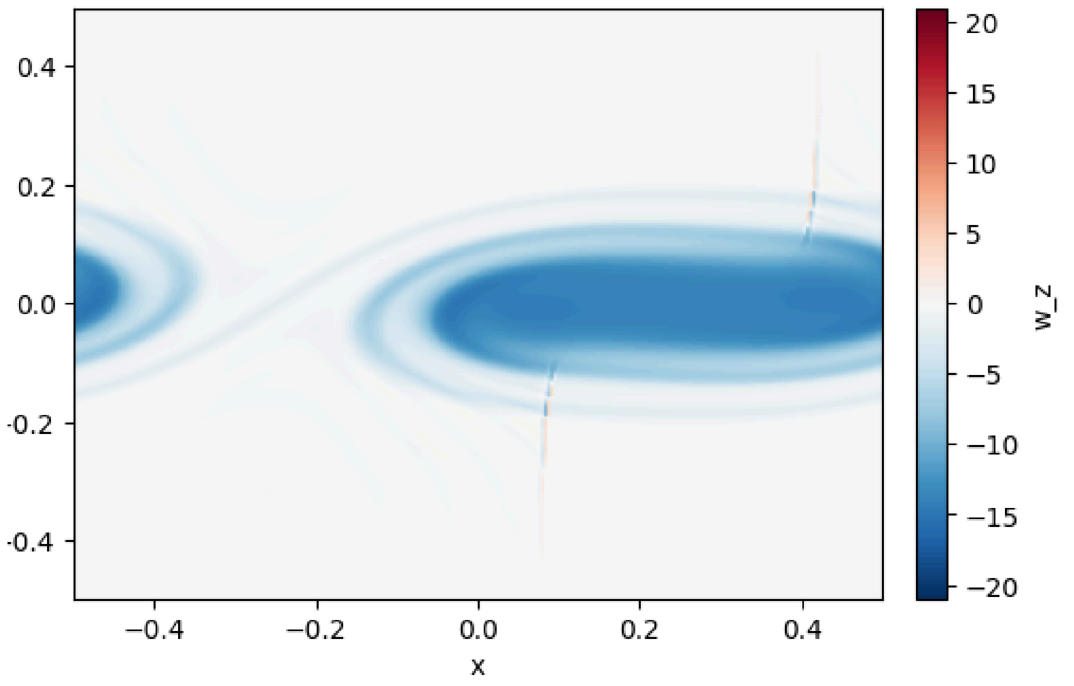
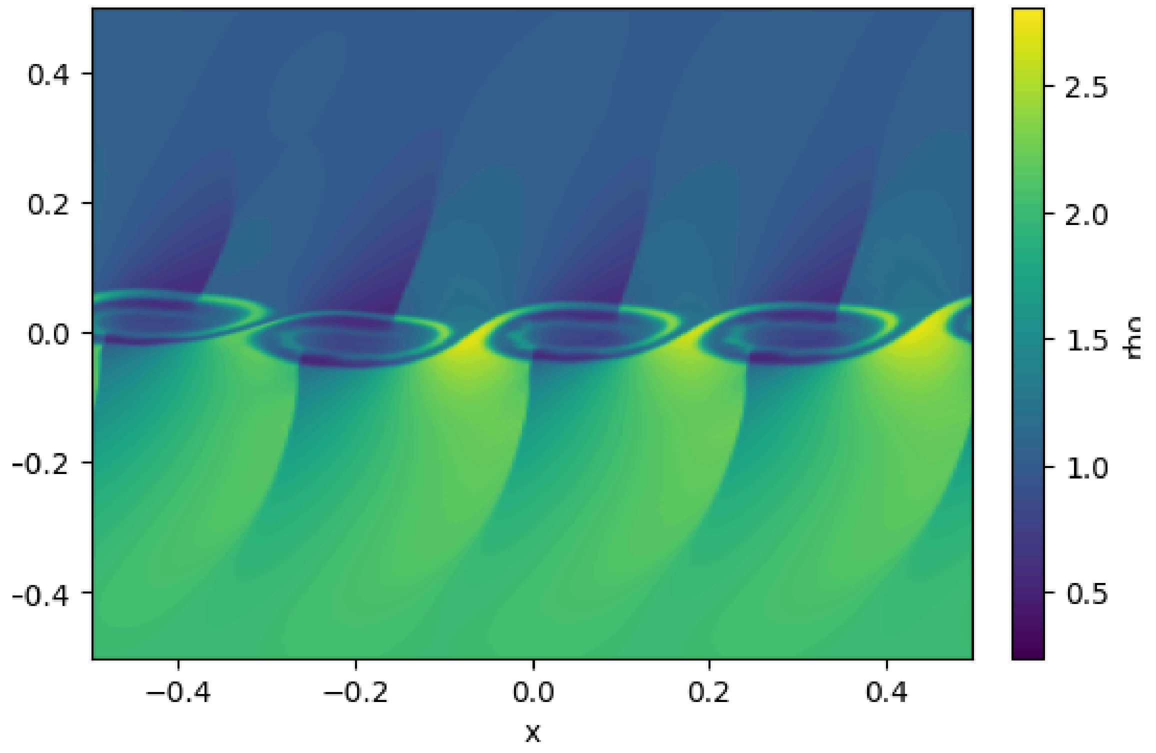
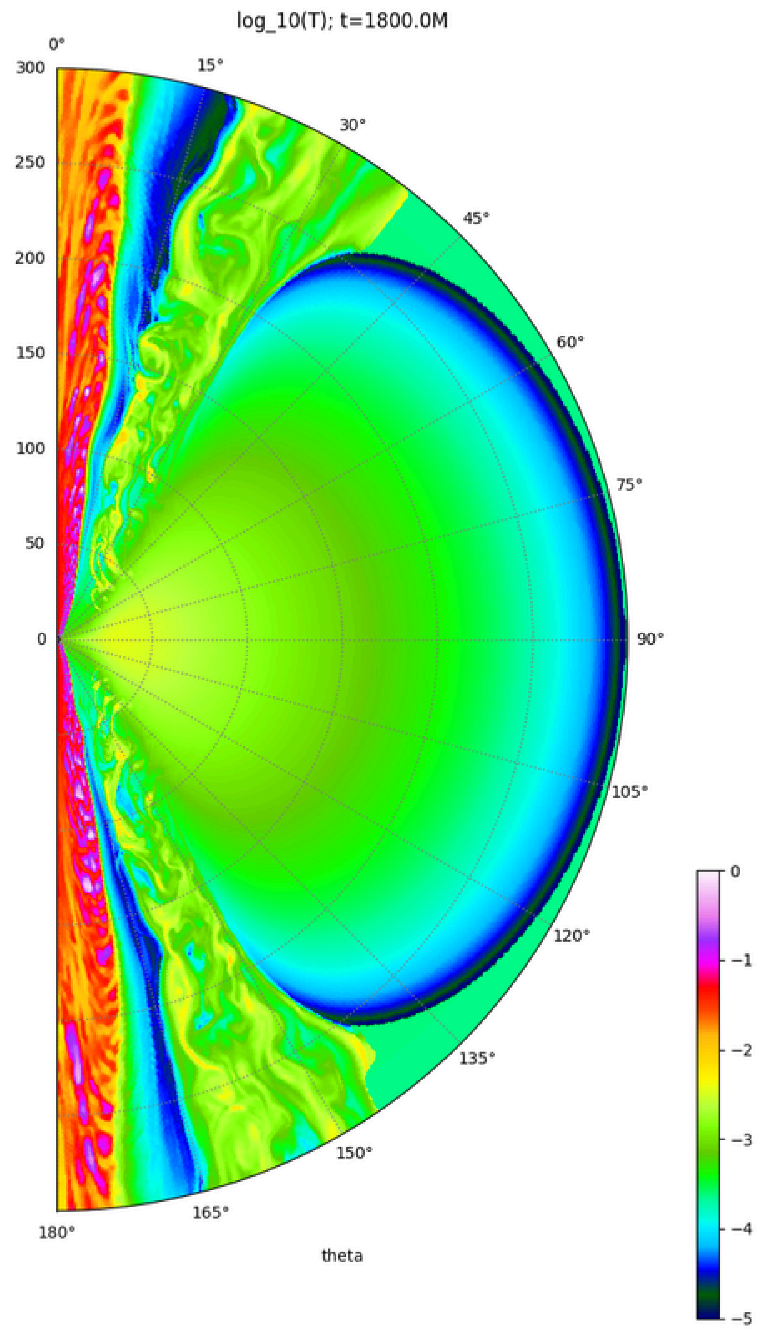
Numerical analysis of the Kelvin-Helmholtz instability (classic & GRMHD)







CAMK PAN

The Young Astronomers Meeting
18.03.2026, CAMK, Warsaw

Yauheni Stsefanenka
Astronomical Observatory of the Jagiellonian University

Plan of the presentation

- Theoretical aspects of Kelvin-Helmholtz instability (KHI) + a bit of history
- Numerical simulations and analysis (Athena++, psecas)
- GRMHD global simulation of jet production - KHI identification and importance

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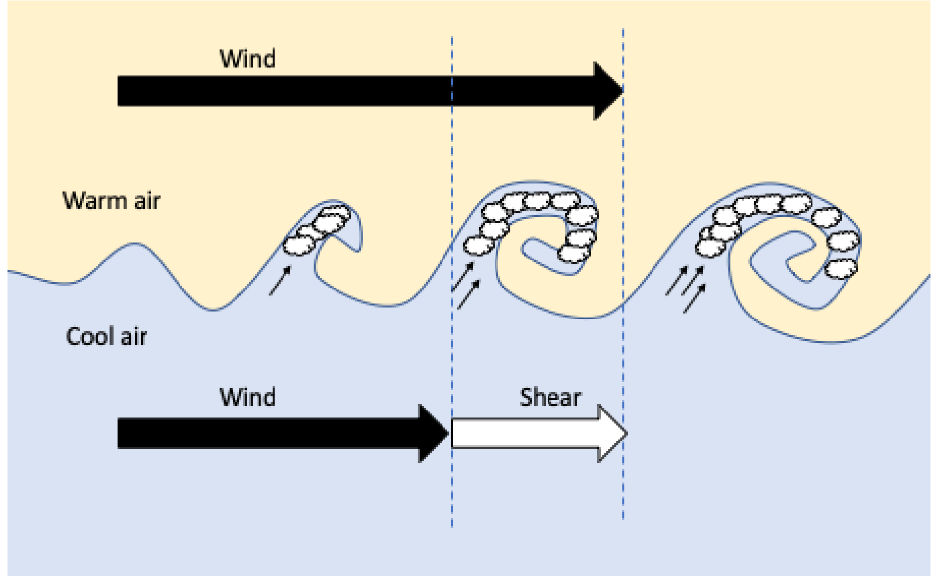
Kelvin-Helmholtz instability - history

“vortex” structures
in the clouds

Jupiter’s KHI: interaction of two bands in the atmosphere

Kelvin and Helmholtz (independently):
formation of the sea waves

$$|U_1 - U_2| \geq 650\,\mathrm{cm/s}$$



hydrodynamical instability,
velocity shear in the different layers

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Kelvin-Helmholtz instability - theory

S. Chandrasekhar (1961), “Hydrodynamic and hydromagnetic stability”

- comprehensive KHI study: linear regime, different velocity profiles, effects of surface tension, magnetic field, rotation etc.

Hereafter, we decompose perturbations into Fourier modes:

$$\delta f(x,z,t) = f_k(z)\exp(\mathrm{i}kx - \mathrm{i}\omega t)$$

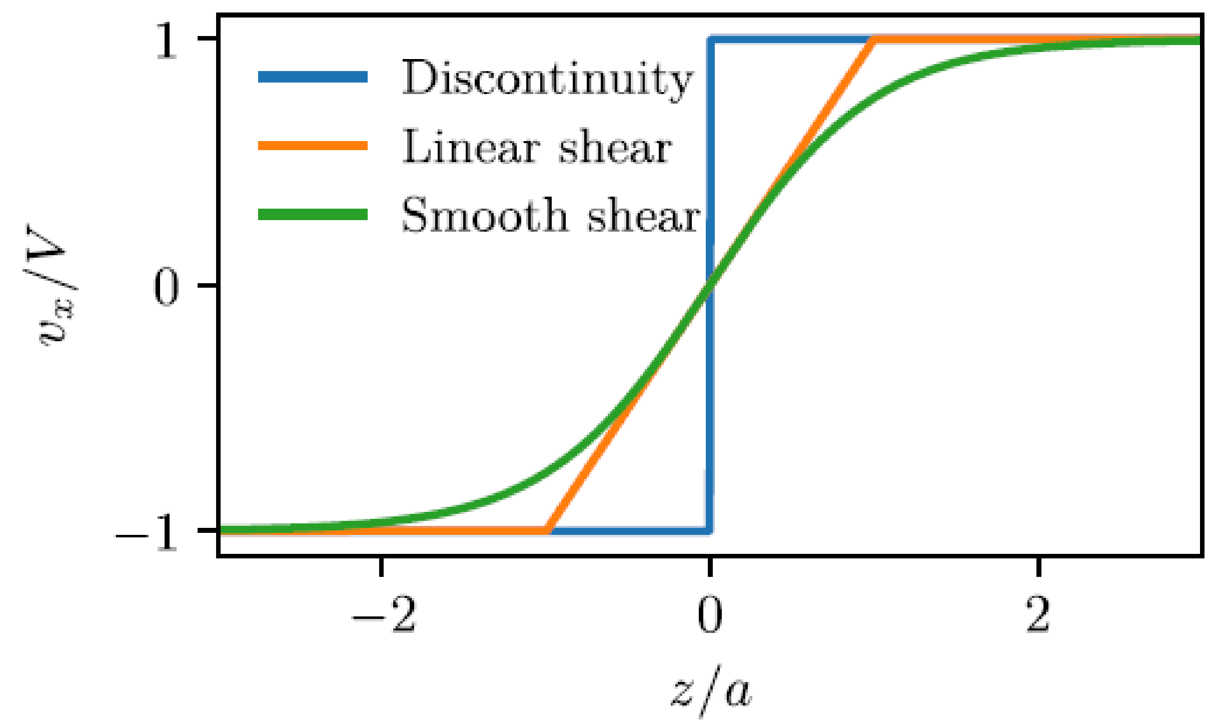
real (k) — $\omega = \omega_r + \mathrm{i}\omega_i$

temporal stability analysis

One can obtain the dispersion relation $\omega(k)$

and the growth rate of the instability $\sigma = \mathrm{Im}(\omega)$

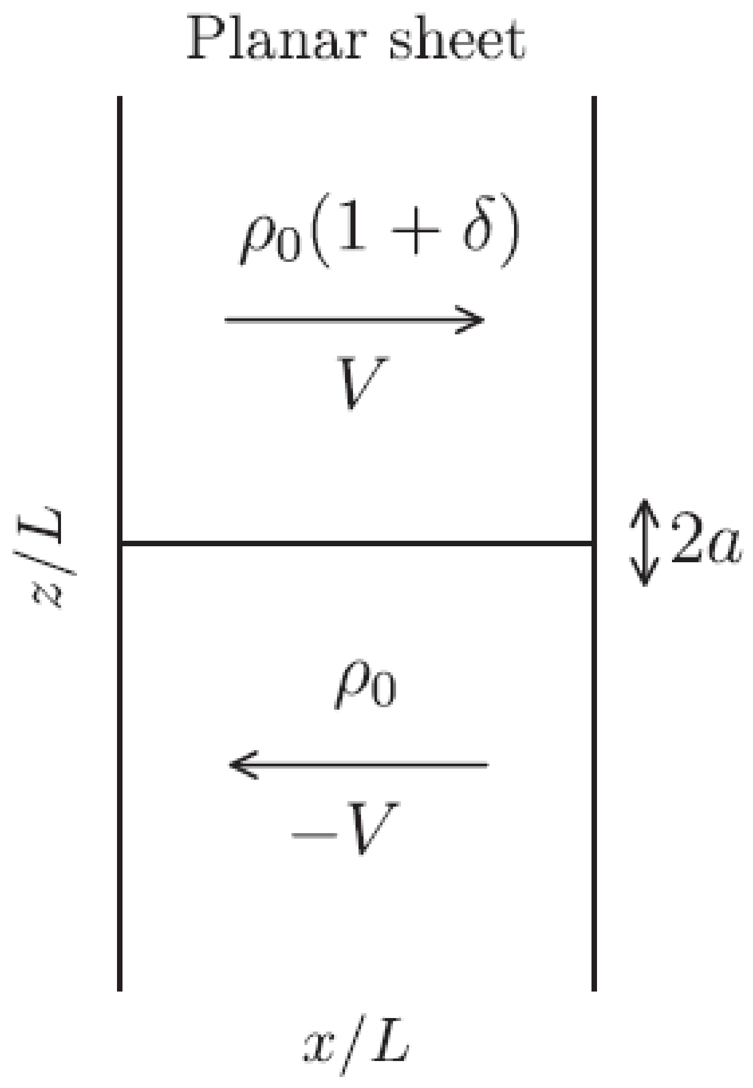
Kelvin-Helmholtz instability - theory

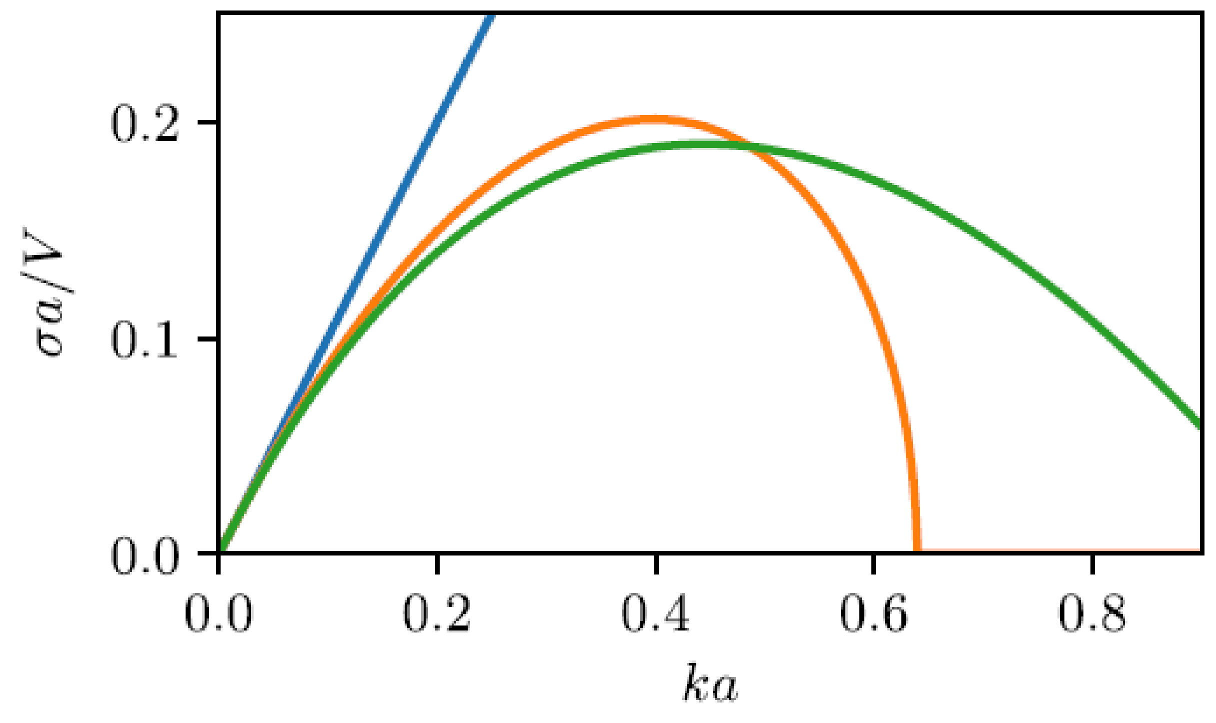


Chandrasekhar's result for the discontinuity profile is analytical, but unresolved in numerical simulations

$$\omega \propto ikV$$

(incompressible, inviscid, homogeneous fluid)





Introducing a smoothing scale (e.g. linear, tanh) prevents the KHI with λ<2πa from growing

↓

With sufficient resolution, one can resolve the full spectrum of KHI with smooth profile!

T. Berlok, C. Pfrommer, MNRAS, 485, 908, 2019

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Kelvin-Helmholtz instability - numerics

In order to numerically study KHI, one could either perform
eigenmode analysis (psecas) or study local HD/MHD simulations (Athena++)

pseudospectral analysis

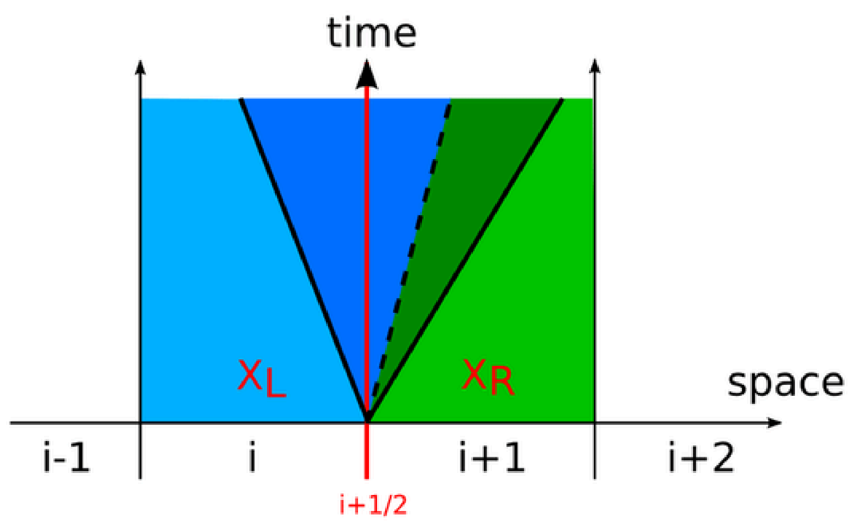
set of d linearized equations forced at N points (collocation points)

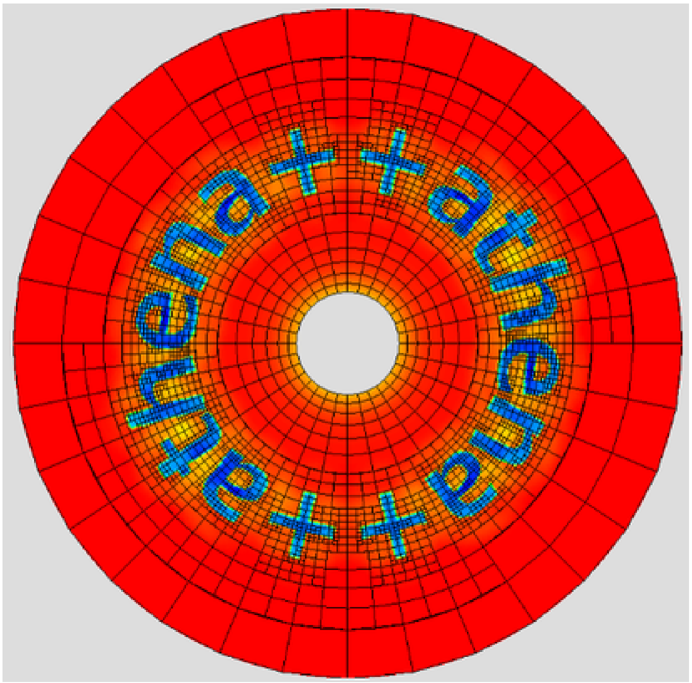
generalized eigenvalue problem with matrix dimension $Nd \times Nd$

eigenvalues (ω) and eigenvectors($f_k(z)$)

finite volume (approximate Riemann solver)

study the evolution of the system (both eigen- and non-eigenmodes)





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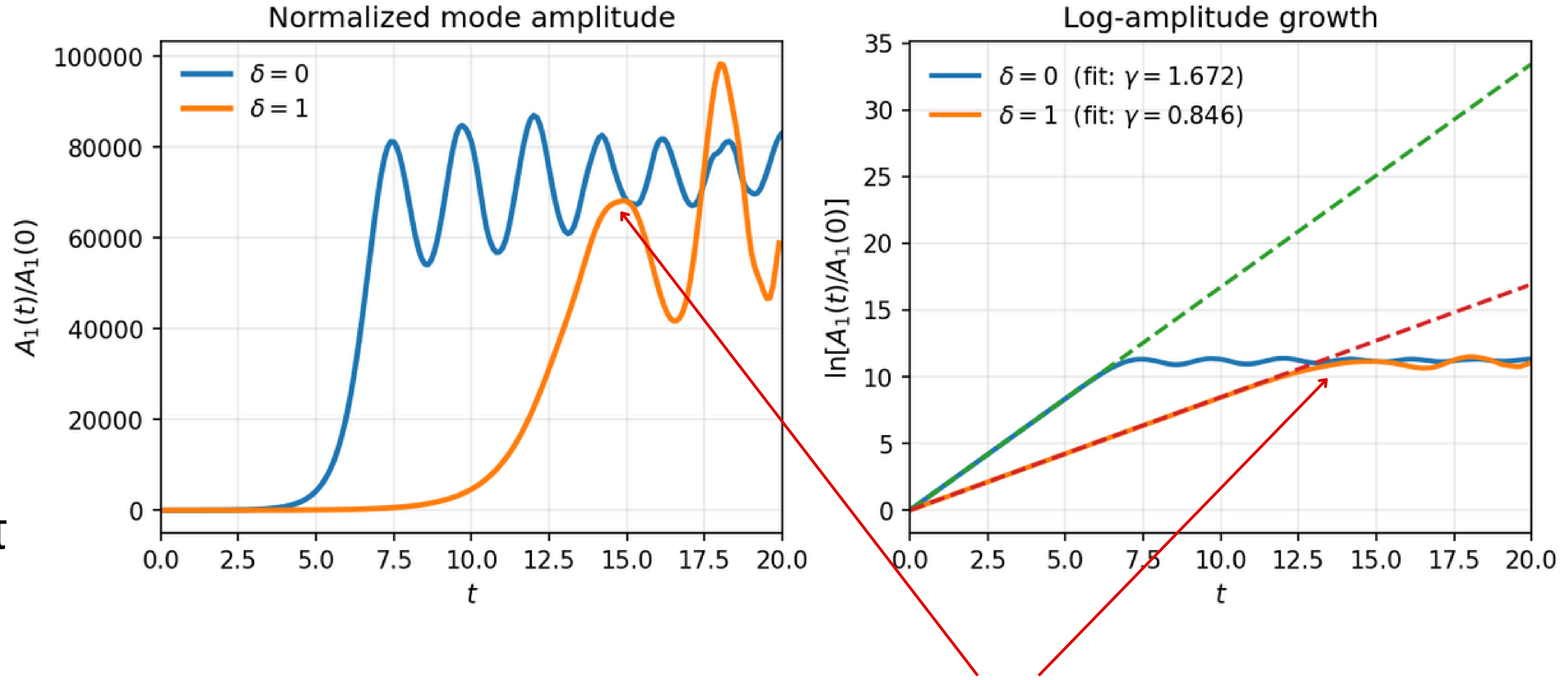
Kelvin-Helmholtz instability - numerics (example)

Let's say, domain is L in x direction in order to satisfy the periodicity of the solution, wavenumbers should be of the form

$$k = \frac{2\pi m}{L}, \quad m \in \mathbb{Z}$$

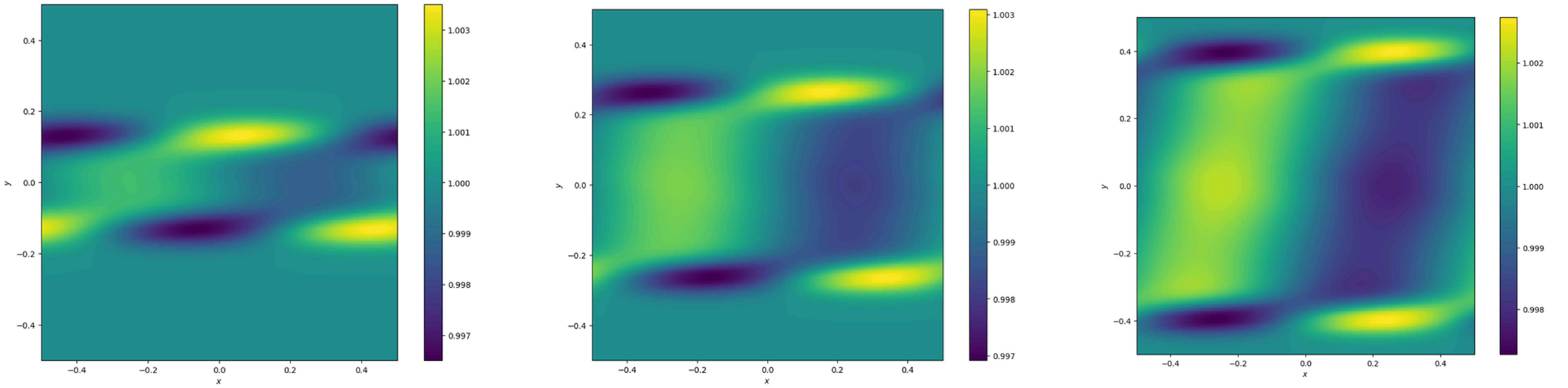
Athena++ setup:

- HLLC solver
- PLM reconstruction, vl2 time integrator (second order)
- Lx=1 periodic, nx=256 Lz=4 outflow, nz=512
- tanh velocity profile, a=0.05, ΔV=2, k=2π
- initial perturbation from psecas, m=1
- 2D hydrodynamics (δ=0,1)



non-linear phase

Kelvin-Helmholtz instability - numerics (example)



subsequent density maps (t=0.1,0.2,0.3), non-eigenmode perturbation (gaussian)

An arbitrary initial perturbation cannot be decomposed into a linear combination of eigenmodes → mix of modes:

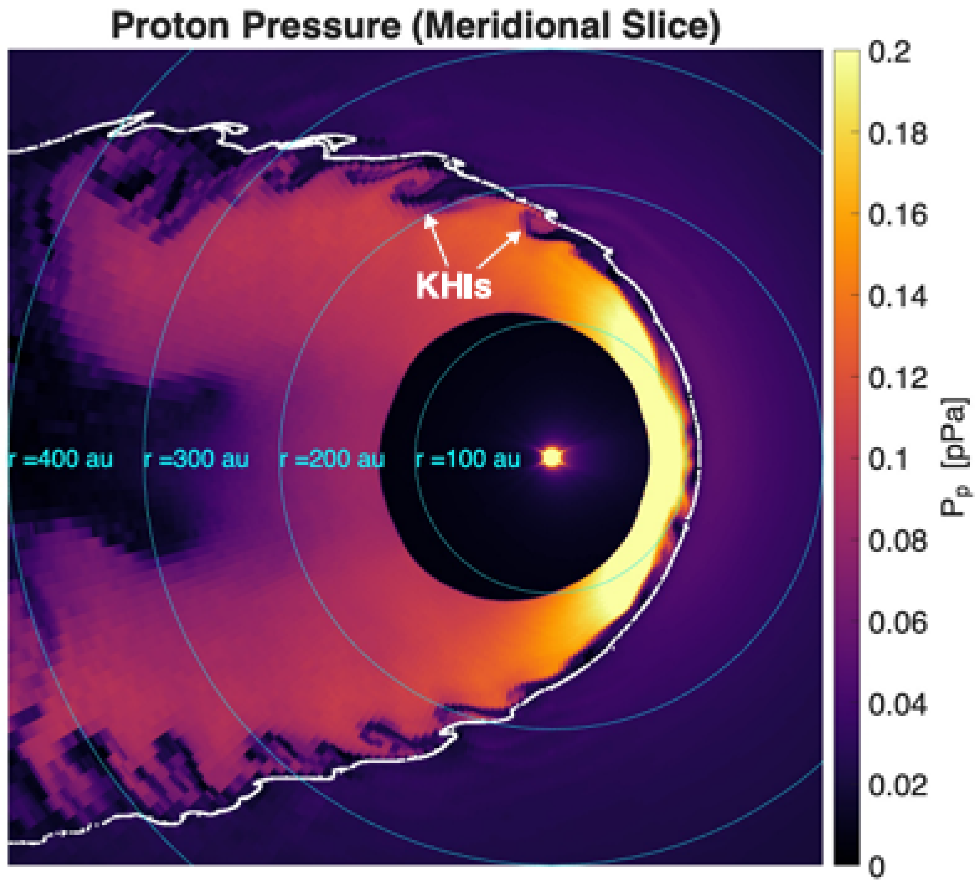
- unstable growing modes
- stable decaying modes
- residual - sound waves

Mandelker et al., MNRAS, 463, 3921, 2016

Kelvin-Helmholtz instability - astrophysics

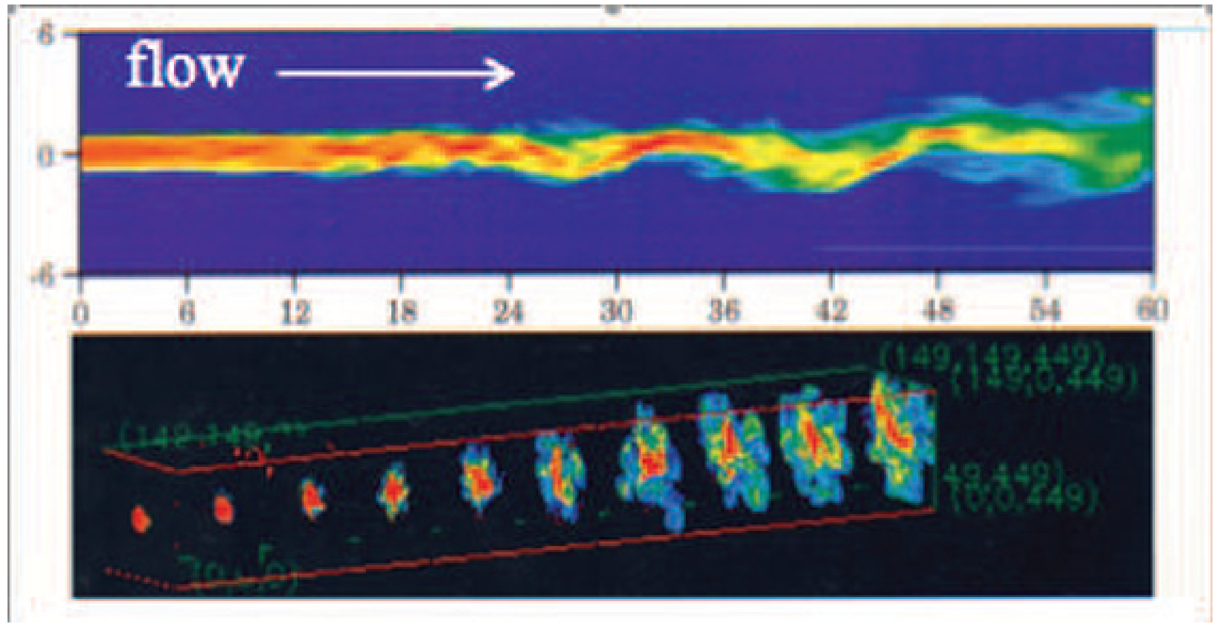
A few examples of KHI application in astrophysics:

- Magnetospheres of planets, heliosphere
- Cold fronts in galaxy clusters
- Jet instabilities
- Protoplanetary disks
- and many others



Heliosphere,

E. J. Zirnstein et al., AJL, 997, 2026



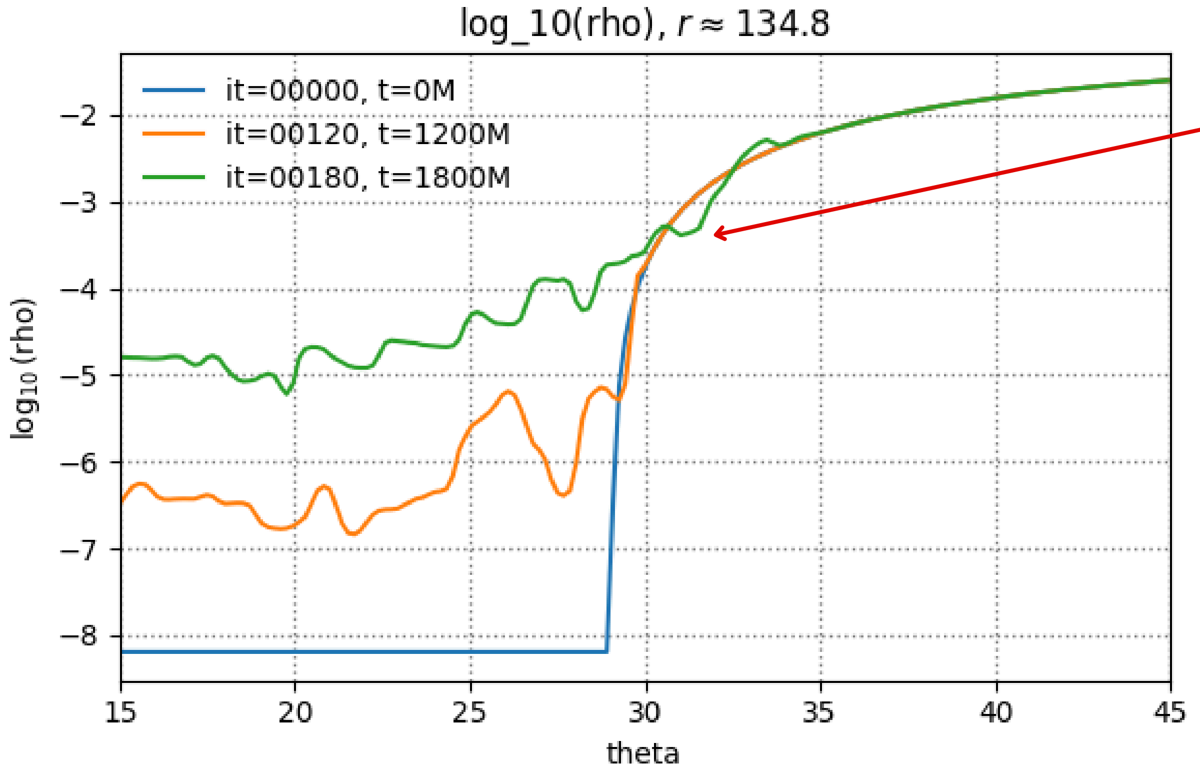
Jet with poloidal magnetic field,

P. E. Hardee, EPJ, 61, 2013

Kelvin-Helmholtz instability - GRMHD filaments

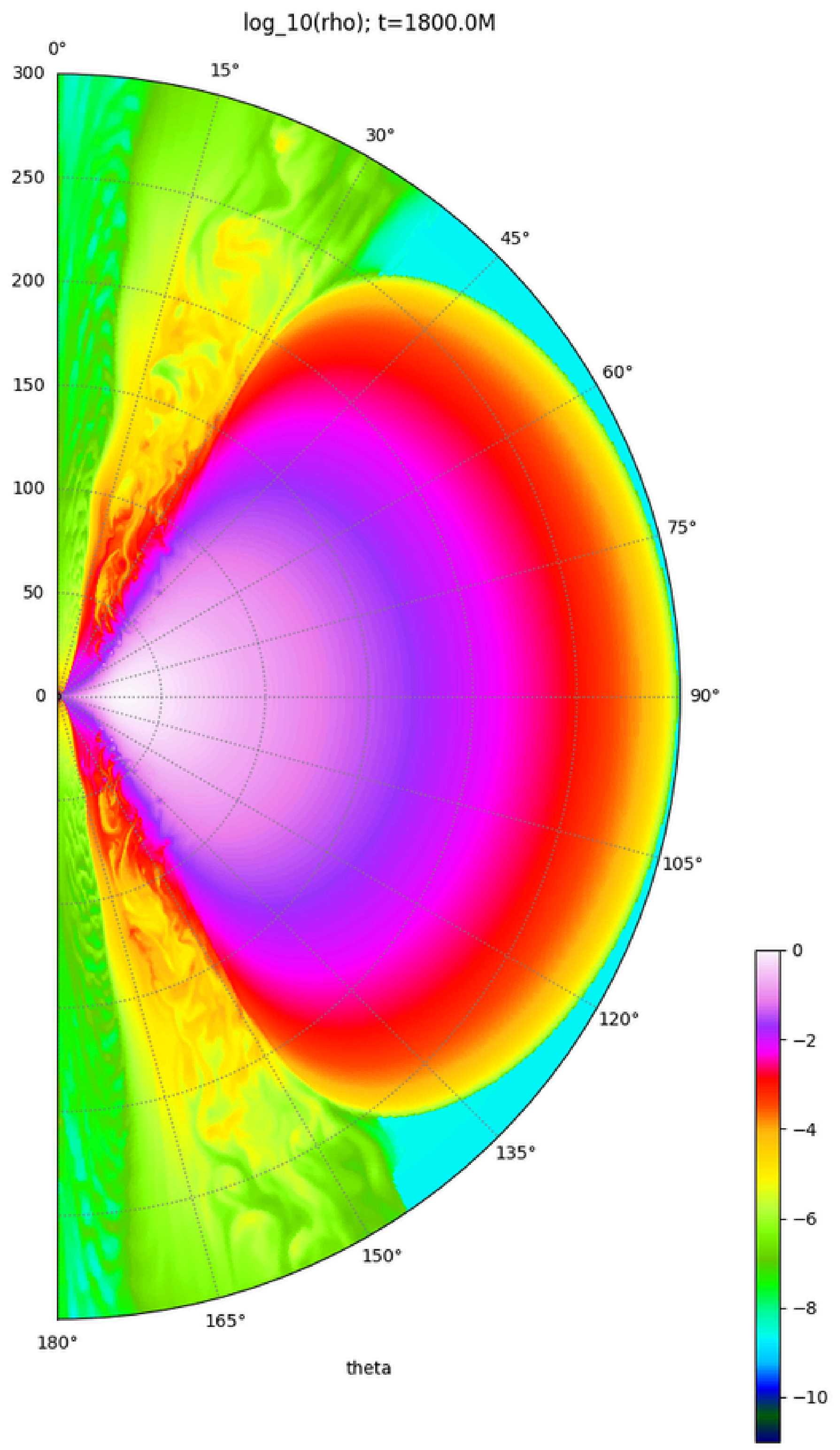
Global simulation of jet production around Kerr BH
Initial setup: Fishbone-Moncrief torus (HD equillibrium)

Expected growth of KH instability at the wind-torus interface?



High density and pressure contrast at the interface

Effects of magnetic field, rotation should be considered



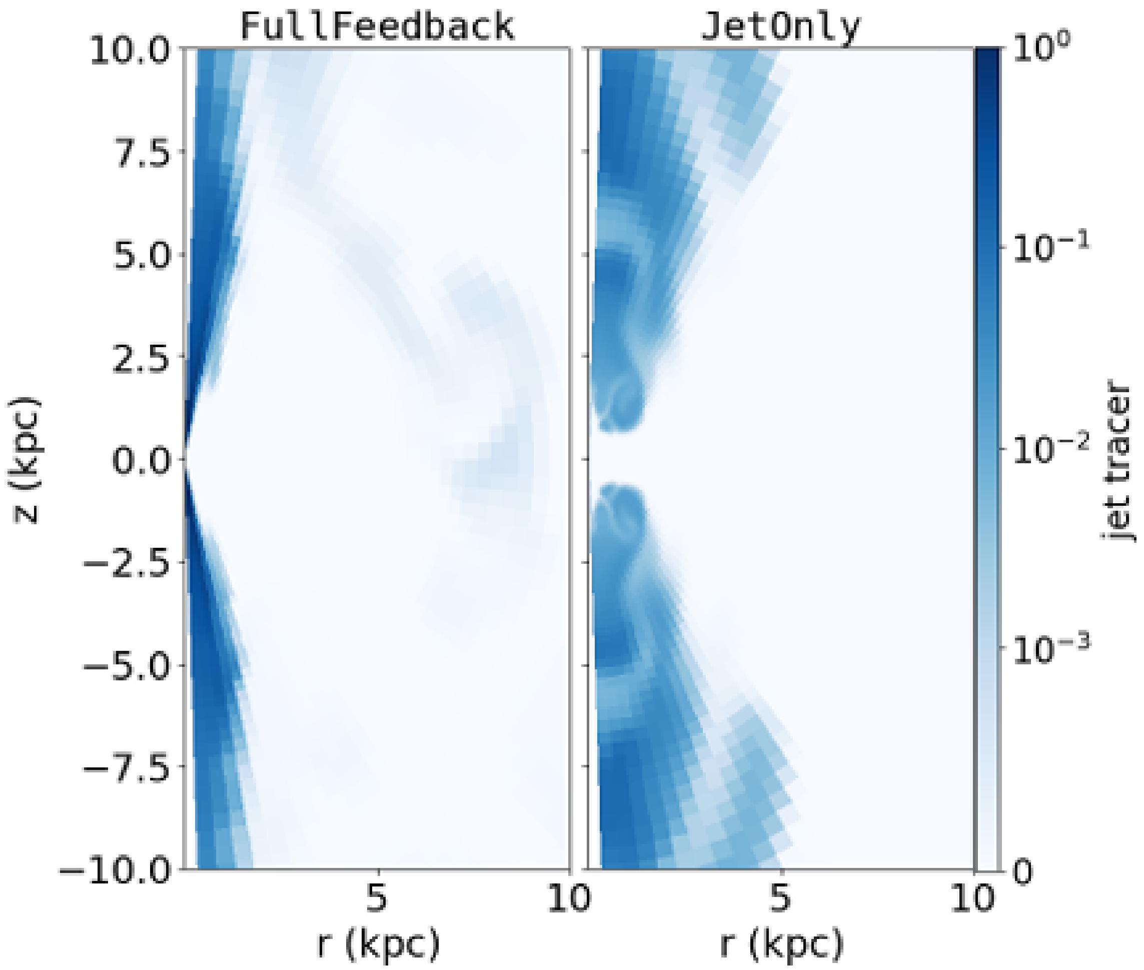
Kelvin-Helmholtz instability - GRMHD filaments

Importance of KHI:

The wind can contribute to jet collimation

(important observational consequences)

Jet + wind



Guo et al., AJ, 997, 2026

The wind may become turbulent, possibly due to shear-driven instabilities such as KHI

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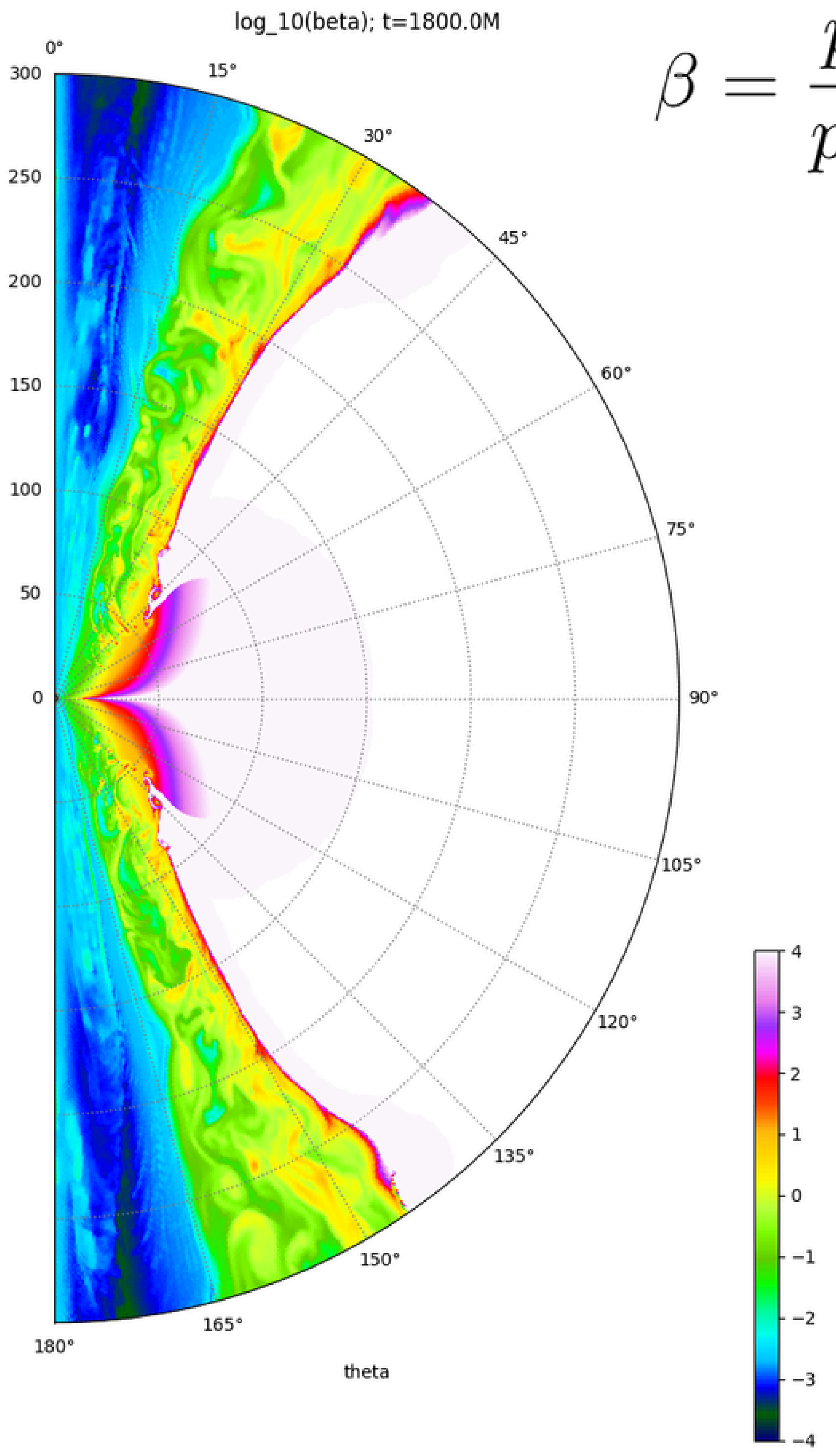
which may affect jet collimation



$$\beta = \frac{p_{gas}}{p_{mag}}$$

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Numerical analysis of the Kelvin-Helmholtz instability (classic & GRMHD)

Conclusions:

- Kelvin-Helmholtz instability is one of the main hydrodynamical instabilities, which provides the transition from laminar to turbulent flows

- Numerical analysis may resolve the full spectrum of instability

- In GRMHD may affect the wind and jet collimation

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Thank you for the attention!

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