Aspects of X-ray data analysis for accreting compact objects: theory and results

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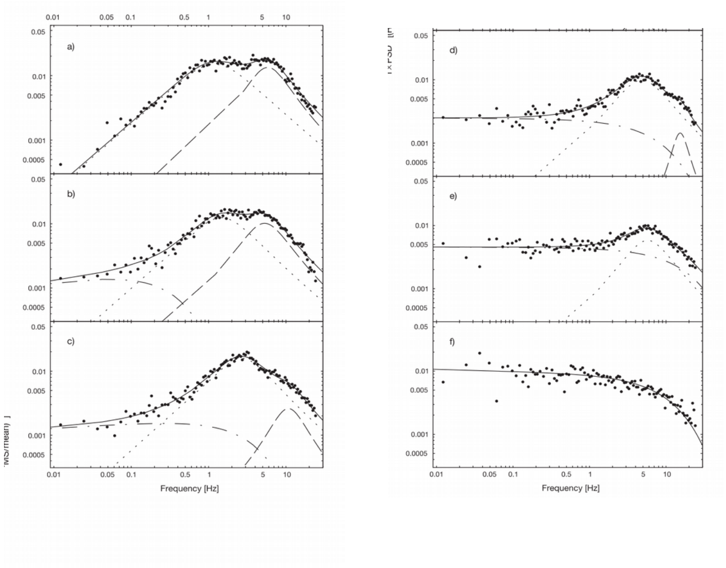
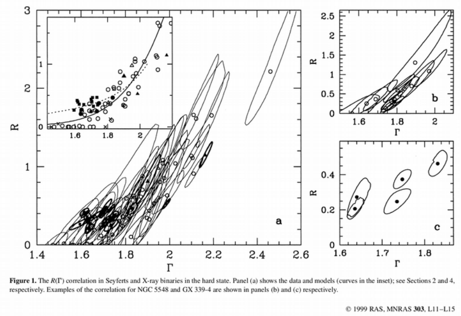
Previous lecture

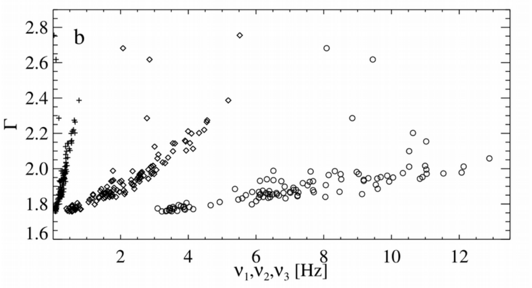
X-ray variability

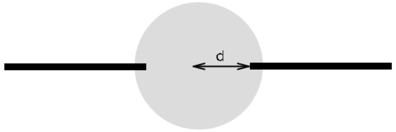


Evolution of PDS: from a multiple-Lorentzian to a cutoff power law

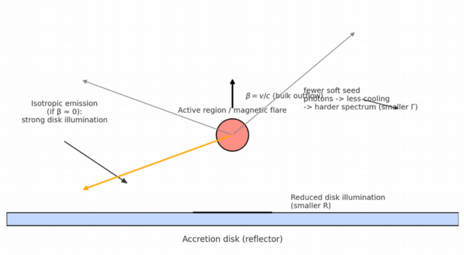
Correlations between parameters



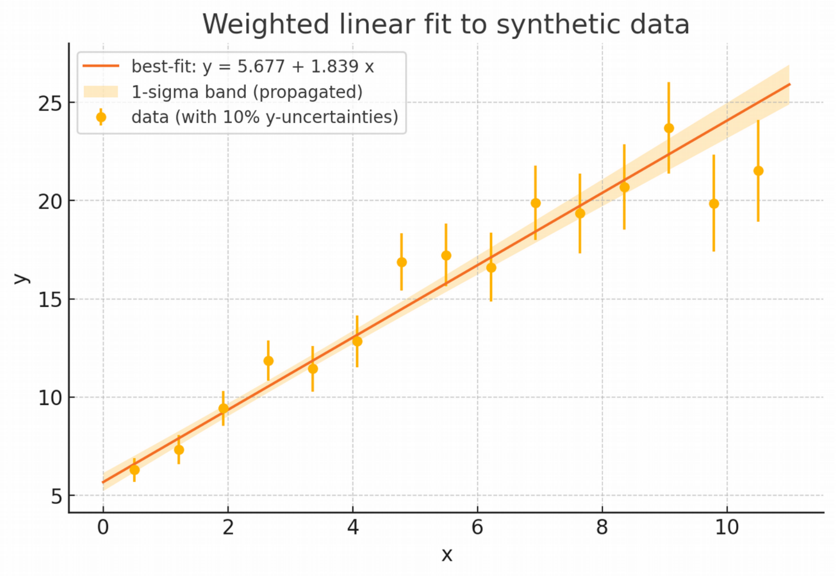




Schematic: Outflowing active region (Beloborodov 1999) — reduced reflection R and harder Γ



Data modelling



$$\chi^2 = 11.5 \text{ for } 13 \text{ d.o.f} (15 \text{ data points} - 2 \text{ parameters})$$

Chat GPT

$$\chi^2(\theta) = \sum_i \frac{[y_i - f(x_i, \theta)]^2}{\sigma_i^2}$$

θ – parameters of the model

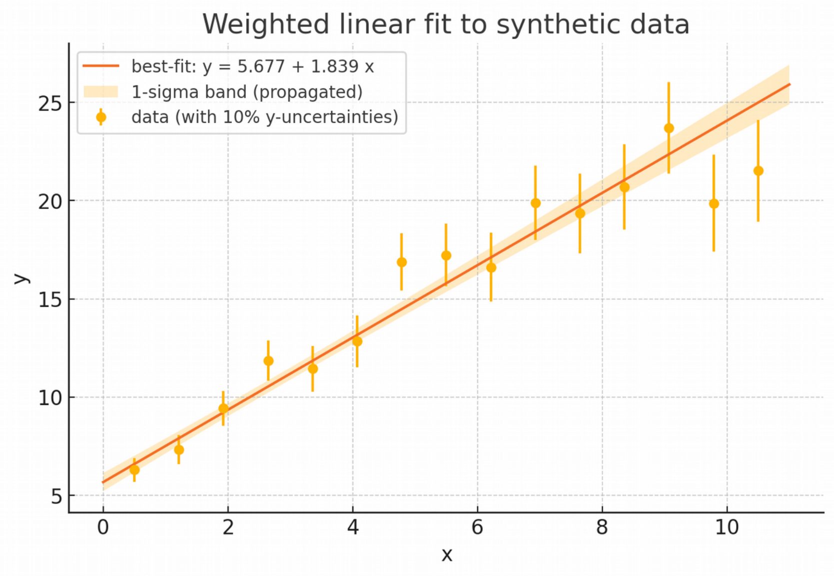
We want to find the parameters of the model function that minimize χ^2.

Simple case: $f(x) = ax + b$

In this simple case there exist analytical formulae for *a* and *b*.

In more complex cases (non-linear model functions) we need to use numerical algorithms to find the minimum of χ^2.

Data modelling



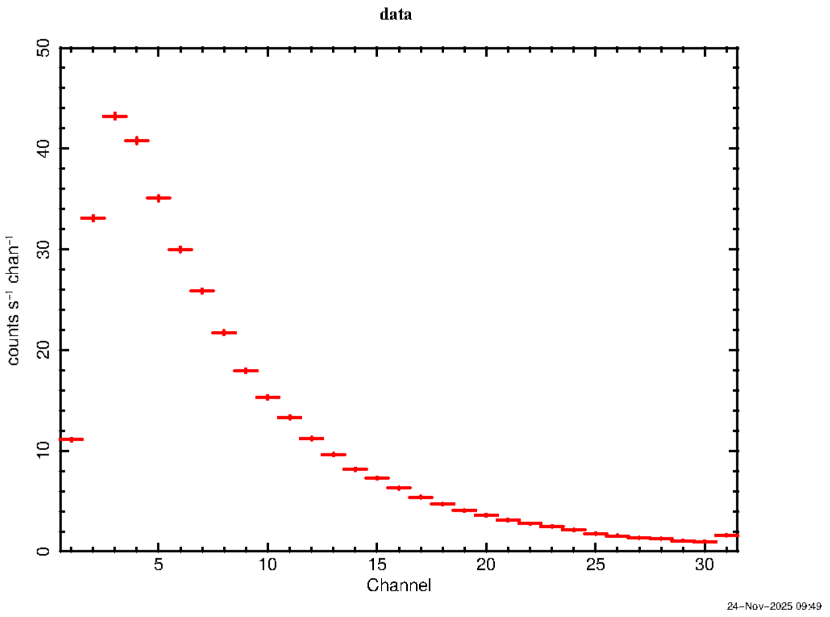
The error bars of the data points represent uncertainty of the location of the real y-value for a given x.

Therefore, the parameters of our model, *a* and *b*, are also uncertain. The fitting procedure should also give us these uncertainties.

In the simple case the uncertainties are also given by analytical formulae. **For more complex cases, the procedures to estimate the uncertainties are more complex.**

Meaning of the 1-σ band: imagine making N measurements, so we have N sets of points. The model is fitted to the data and 1-σ band is computed. For a given x, the 1-σ band contains the true value 68% of the times.

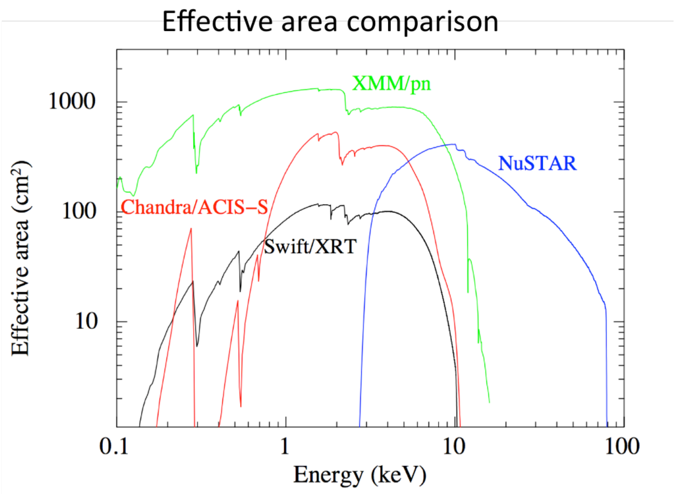
X-ray data



Response function (matrix) – redistribution of incoming photons in energy

Effective area – energy dependent

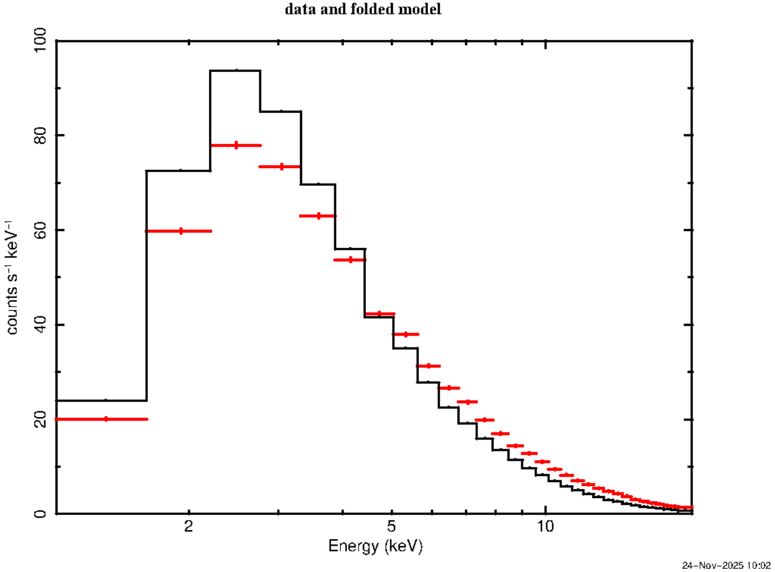
Energy channels

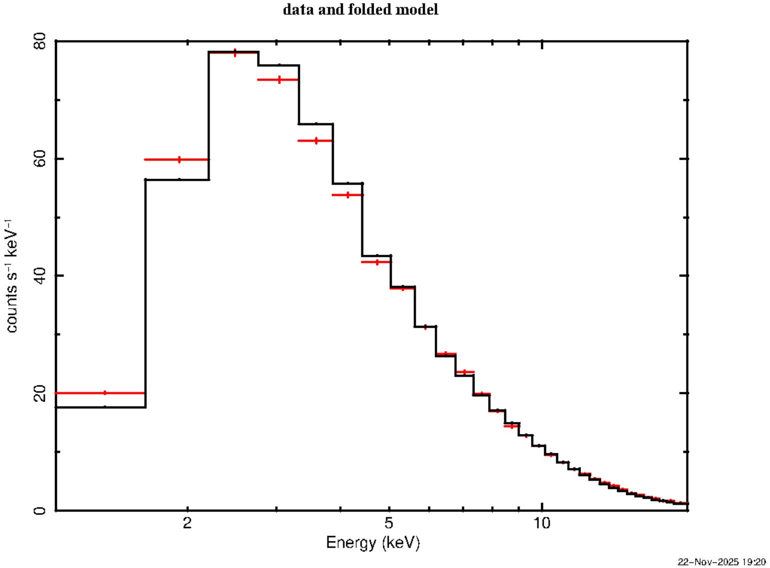


Ginga (1990-ies) – effective area of 4000 cm^2, poor energy resolution

Modelling X-ray data

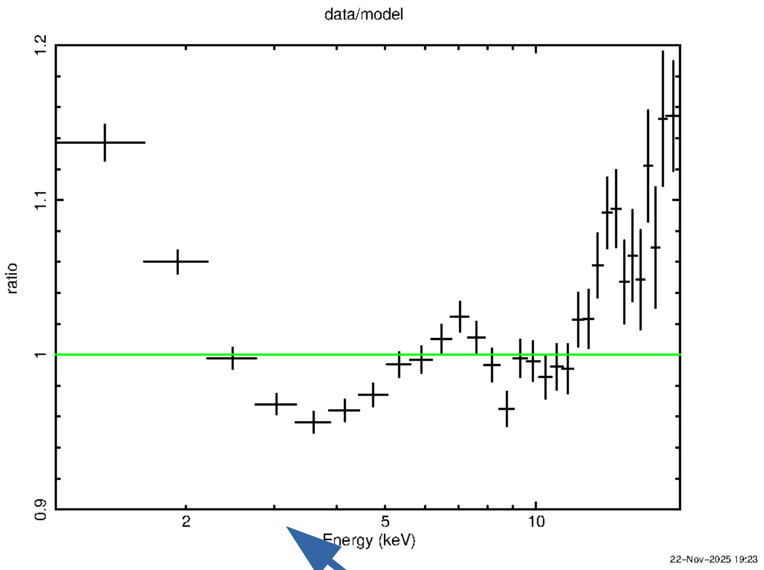
GS 1124-68 – Nova Muscae – a soft X-ray transient

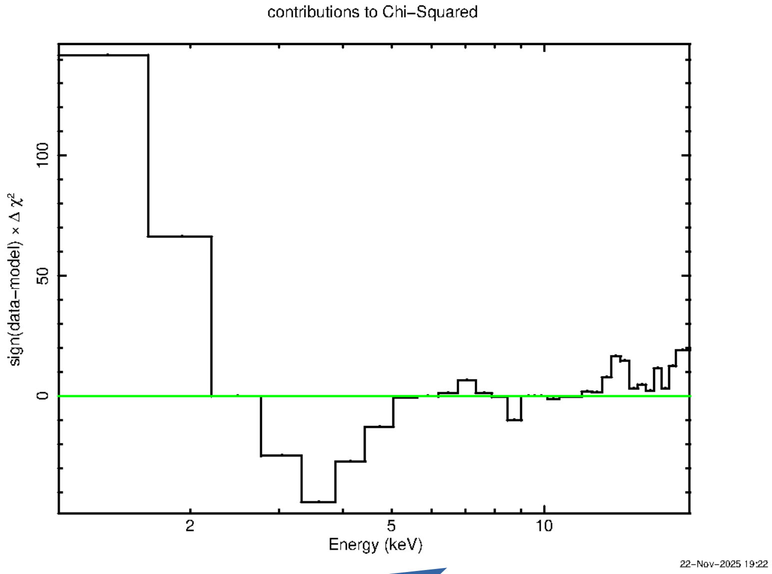




Initial guess of the model and parameters → not optimum description of data → fitting (minimizing χ^2) → best fit

Modelling X-ray data

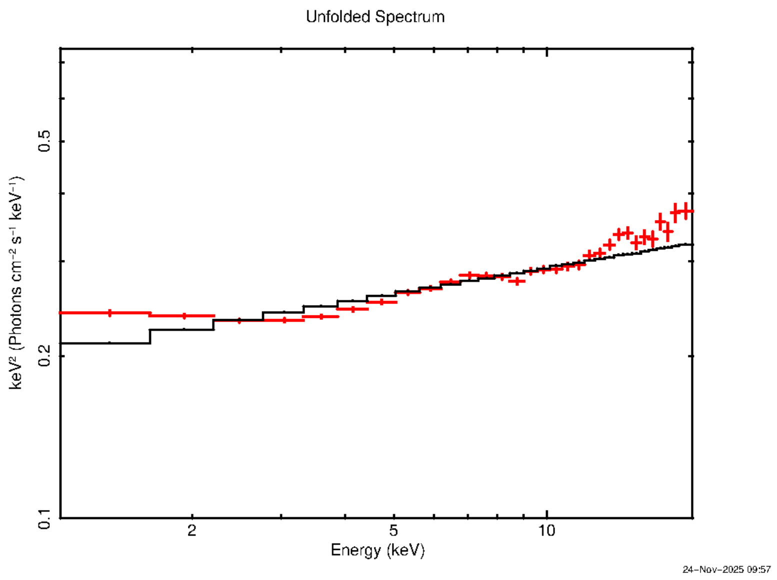




Diagnostics of the fit results:
data-to-model ratios,
contributions to χ^2

This fit is not so good, significant residuals are seen, χ^2 large.

Modelling X-ray data



Unfolded spectrum: how the real data would look like, assuming the best-fit model.

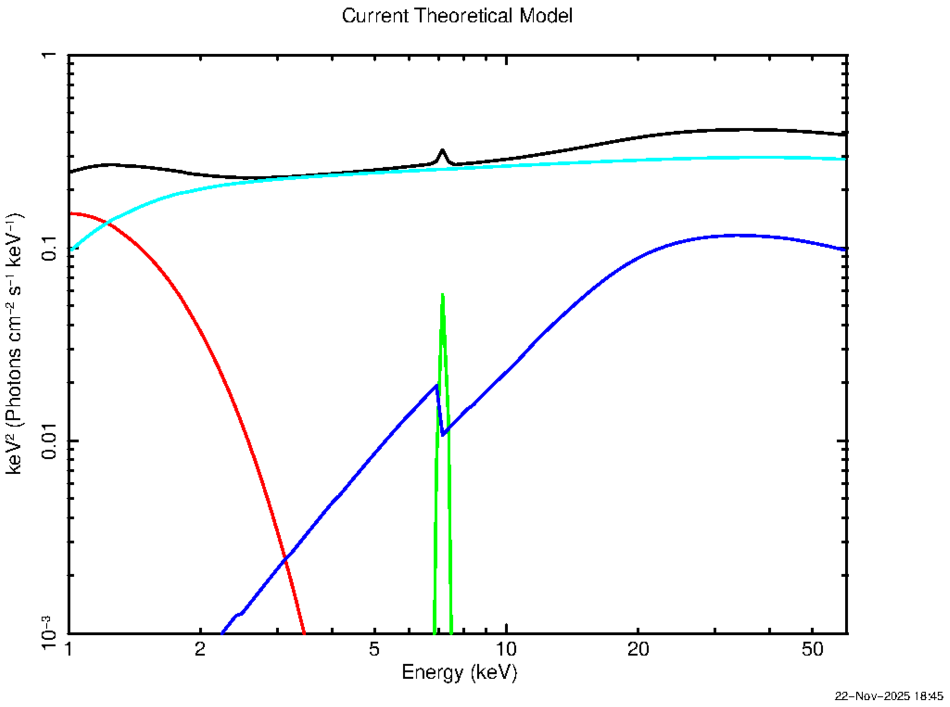
Key aspect: it is not possible to uniquely determine the real data (before they reach the detector), the unfolded spectrum depends on the model.

Finding a good fit

Need a better model. Using previous knowledge, physics, intuition, fit residuals, trial and error…

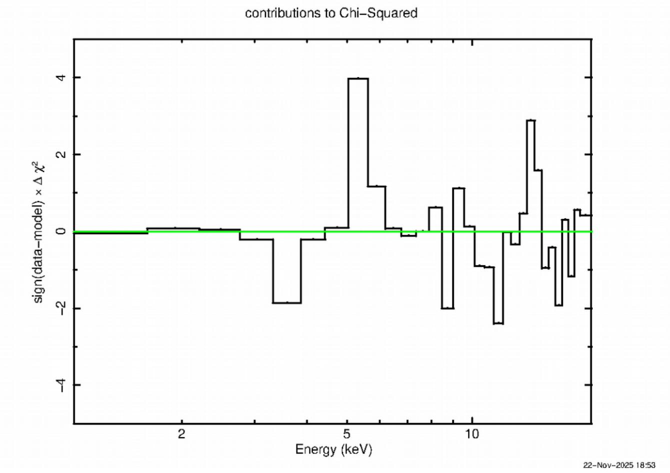
This model components are: disk blackbody emission, thermal Comptonization component, reflected/reprocessed component, with the Fe K_α line.

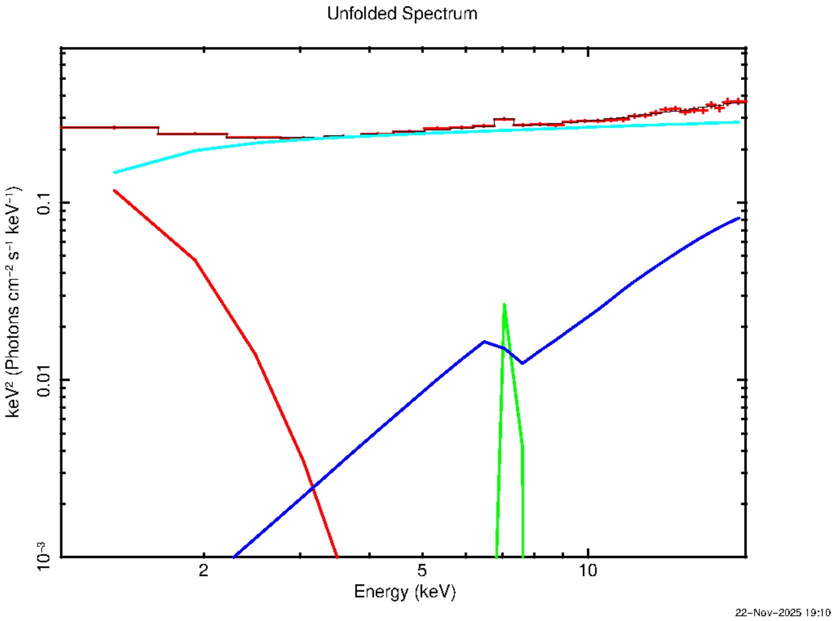
Best fit of this model is good: χ^2 = 27/24 d.o.f.



Finding a good fit

Best fit of this model is good: χ^2 = 27/24 d.o.f.

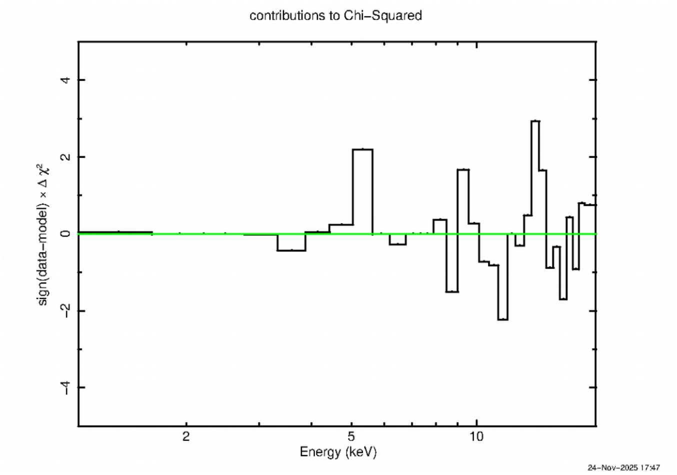


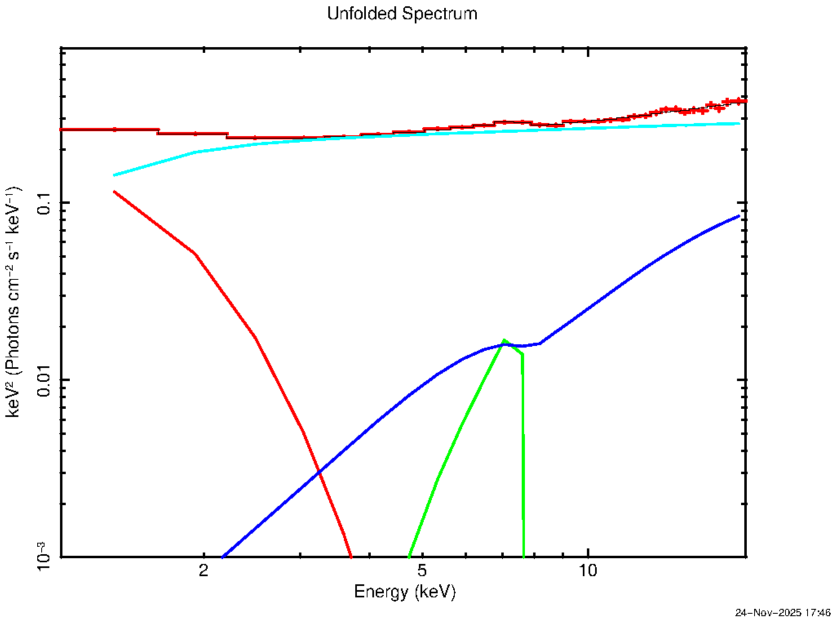


This is a good fit, although some residuals remain. They suggest a way to improve the fit even more, by applying relativistic effects.

Finding a better fit

Best fit of this model is good: χ^2 = 22/23 d.o.f.





A better fit, suggesting that the emission is coming from a rotating disk

Properties of the good fit

```
========================================================================
Model wabs<1>(diskbb<2> + gaussian<3> + pexrav<4> + nthComp<5>) Source No.: 1   Active/On
Model Model Component  Parameter  Unit     Value
 par  comp
   1    1   wabs       nH         10^22    0.160000     frozen
   2    2   diskbb     Tin        keV      0.279110     +/-  1.67868E-02
   3    2   diskbb     norm                6610.92      +/-  2501.30
   4    3   gaussian   LineE      keV      7.16468      +/-  0.131805
   5    3   gaussian   Sigma      keV      0.100000     frozen
   6    3   gaussian   norm                3.44840E-04  +/-  6.45261E-05
   7    4   pexrav     PhoIndex            1.89245      = p15
   8    4   pexrav     foldE      keV      300.000      frozen
   9    4   pexrav     rel_refl            -0.925049    +/-  0.150065
  10    4   pexrav     Redshift            0.0          frozen
  11    4   pexrav     abund               1.00000      frozen
  12    4   pexrav     Fe_abund            1.00000      frozen
  13    4   pexrav     cosIncl             0.500000     frozen
  14    4   pexrav     norm                0.139455     = p20
  15    5   nthComp    Gamma               1.89245      +/-  1.57632E-02
  16    5   nthComp    kT_e       keV      100.000      frozen
  17    5   nthComp    kT_bb      keV      0.279110     = p2
  18    5   nthComp    inp_type   0/1      0.0          frozen
  19    5   nthComp    Redshift            0.0          frozen
  20    5   nthComp    norm                0.139455     +/-  9.89552E-03
________________________________________________________________________

XSPEC12>sho fit

Fit statistic  : Chi-Squared                   26.99     using 31 bins.

Test statistic : Chi-Squared                   26.99     using 31 bins.
 Null hypothesis probability of 3.05e-01 with 24 degrees of freedom
```

Properties of the good fit

How to estimate the uncertainties of the fitted parameters?

Use χ^2 as a function of the parameters.

Confidence limits can be estimated from $\Delta\chi^2$ relative to the minimum. For one parameter $\Delta\chi^2=2.71$ corresponds to 90% confidence limit

```
XSPEC12>steppar 15 1.83 1.93 10

     Chi-Squared     Delta                 Gamma
                  Chi-Squared                 15

          43.846       16.854     0          1.83
           38.77       11.778     1          1.84
          34.613       7.6208     2          1.85
           31.38       4.3879     3          1.86
          29.056       2.0637     4          1.87
          27.618      0.62591     5          1.88
          27.018     0.025536     6          1.89
           27.21      0.21769     7           1.9
          28.175       1.1829     8          1.91
          29.886       2.8937     9          1.92
          32.326       5.3341    10          1.93
```

Properties of the good fit

The amplitude of reflected component

```
XSPEC12>steppar 9 -1.5 -0.5 10

     Chi-Squared    Delta           rel_refl
                Chi-Squared             9

          38.644       11.652     0          -1.5
          35.219       8.2273     1          -1.4
          32.288        5.296     2          -1.3
          29.933       2.9407     3          -1.2
          28.218       1.2258     4          -1.1
          27.222      0.23023     5            -1
          27.021     0.028399     6          -0.9
          27.685       0.6931     7          -0.8
          29.301        2.309     8          -0.7
          31.952       4.9598     9          -0.6
           35.71       8.7179    10          -0.5
```

Contours of χ^2 as function of spectral index and amplitude of reflection