Acceleration and plunge from a quasi-spherical orbit near a rotating black hole

Vladimír Karas

in collaboration with **Ondřej Kopáček**

Astronomical Institute, Czech Academy of Sciences

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The model: rotating black hole

Kerr metric describing the geometry of the spacetime around the rotating black hole is expressed in Boyer-Lindquist coordinates $x^\mu = (t,\ r,\ \theta,\ \varphi)$ as follows (Misner et al. 1973):

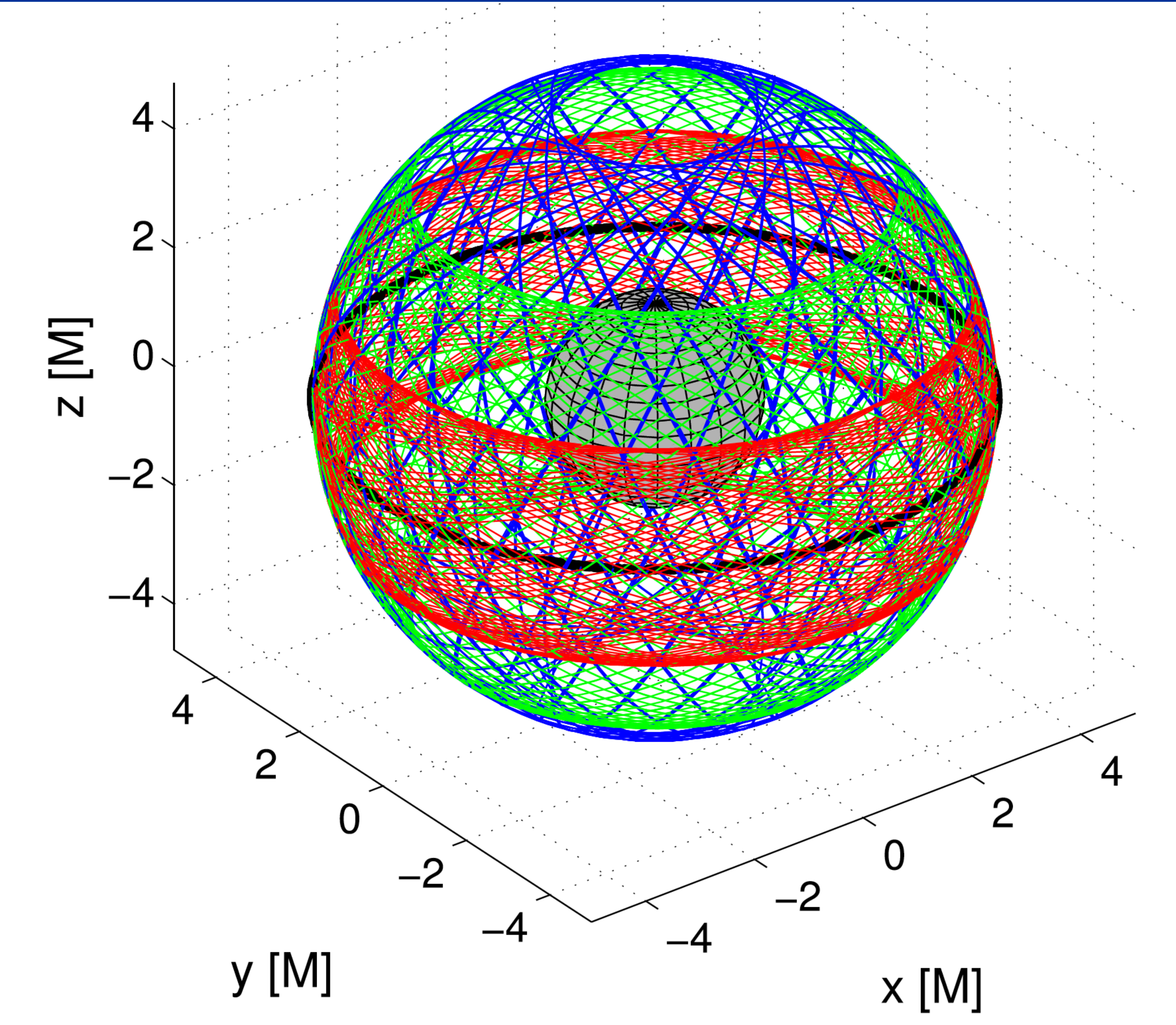
$$ds^2 = -\frac{\Delta}{\Sigma}\left[dt - a\sin\theta\, d\varphi\right]^2 + \frac{\sin^2\theta}{\Sigma}\left[(r^2+a^2)d\varphi - a\, dt\right]^2 + \frac{\Sigma}{\Delta}\, dr^2 + \Sigma d\theta^2, \qquad (1)$$

where

$$\Delta \equiv r^2 - 2Mr + a^2, \quad \Sigma \equiv r^2 + a^2\cos^2\theta. \qquad (2)$$

Coordinate singularity at $\Delta = 0$ corresponds to outer/inner horizon of the black hole $r_\pm = M \pm \sqrt{M^2 - a^2}$. Rotation of the black hole is measured by the spin parameter $a \in \langle -M, M\rangle$. Here we only consider $a \geq 0$ without the loss of generality.

Spherical orbits: their stability and binding



Geodesic motion in Kerr metric:

$a = 0.8$
$R = 5M$
$\Theta = 0, 1, 4, 9$

Kopáček & Karas, ApJ, 2024

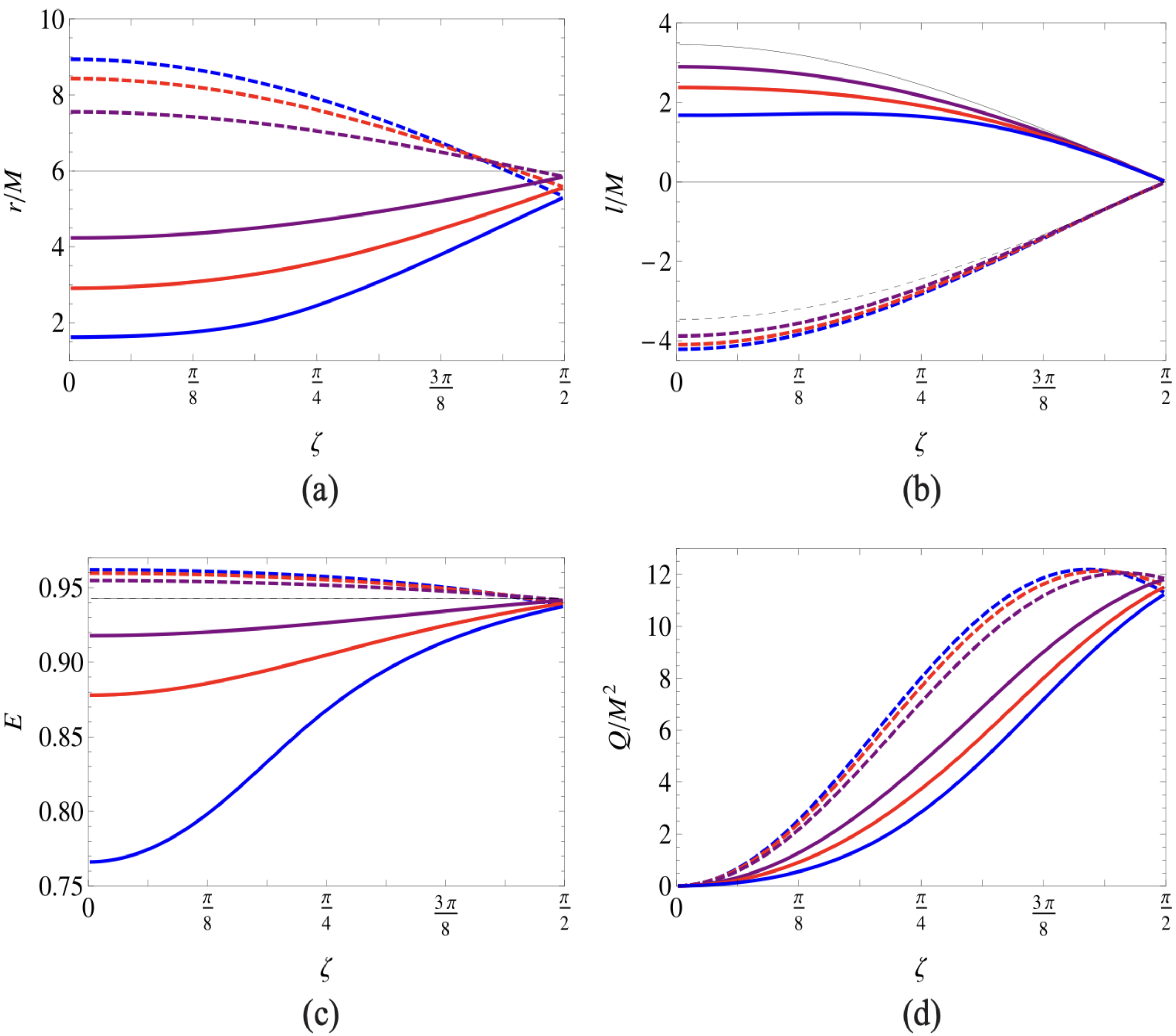


FIG. 4. Characteristic quantities of the ISSO as a function of the tilted angular ζ. (a) $r/M-\zeta$. (b) $l/M-\zeta$. (c) $E-\zeta$. (d) $Q/M^2-\zeta$. Black hole spin $a/M=-0.98, -0.8, -0.5, 0.5, 0.8, 0.98$ for these thick curves from top to bottom in (a), (c), (d), and from bottom to top in (b). The solid thin curves are for the case with $a/M=0$. Dashed and solid curves are for the retrograde and prograde ISSOs.

We employ the test-field solution of Maxwell's equations for a weakly magnetized Kerr black hole immersed in an asymptotically uniform magnetic field specified by the component B_z parallel to the spin axis and the perpendicular component B_x. The electromagnetic vector potential A_μ is given as follows (Bičák & Janiš 1985):

$$A_t = \frac{B_z a M r}{\Sigma}\left(1+\cos^2\theta\right) - B_z a + \frac{B_x a M \sin\theta\cos\theta}{\Sigma}\left(r\cos\psi - a\sin\psi\right), \tag{3}$$

$$A_r = -B_x(r-M)\cos\theta\sin\theta\sin\psi, \tag{4}$$

$$A_\theta = -B_x a(r\sin^2\theta + M\cos^2\theta)\cos\psi - B_x(r^2\cos^2\theta - Mr\cos 2\theta + a^2\cos 2\theta)\sin\psi, \tag{5}$$

$$A_\varphi = B_z\sin^2\theta\left[\frac{1}{2}(r^2+a^2) - \frac{a^2 M r}{\Sigma}(1+\cos^2\theta)\right] - B_x\sin\theta\cos\theta\left[\Delta\cos\psi + \frac{(r^2+a^2)M}{\Sigma}(r\cos\psi - a\sin\psi)\right], \tag{6}$$

where ψ denotes the azimuthal coordinate of Kerr ingoing coordinates, which is expressed in Boyer–Lindquist coordinates as follows:

$$\psi = \varphi + \frac{a}{r_+ - r_-}\ln\frac{r-r_+}{r-r_-}. \tag{7}$$

Conditions for the escape of charged particles

Effective potential expressing the minimal allowed energy of charged test particles in a non-axisymmetric magnetosphere of a rotating black hole may be derived in the rest frame of a static observer (Kopáček & Karas 2018c). Tetrad vectors of this frame are given as (Semerák 1993):

$$e^{\mu}_{(t)} = \left[\frac{\Sigma^{1/2}}{\chi}, 0, 0, 0\right], \quad e^{\mu}_{(r)} = \left[0, \frac{\Delta^{1/2}}{\Sigma^{1/2}}, 0, 0\right], \tag{10}$$

$$e^{\mu}_{(\theta)} = \left[0, 0, \frac{1}{\Sigma^{1/2}}, 0\right], \quad e^{\mu}_{(\varphi)} = \frac{\chi}{\sin\theta\,\Delta^{1/2}\Sigma^{1/2}}\left[\frac{-2aMr\sin^2\theta}{\chi^2}, 0, 0, 1\right], \tag{11}$$

where $\chi^2 \equiv \Delta - a^2\sin^2\theta$.

The static frame is employed to express the effective potential:

$$V_{\rm eff}(r,\theta,\varphi) = \left(-\beta + \sqrt{\beta^2 - 4\alpha\gamma}\right)/2\alpha, \tag{12}$$

with the coefficients defined as:

$$\alpha = \left[e^{t}_{(t)}\right]^2, \quad \beta = 2qA_t e^{t}_{(t)}, \quad \gamma = q^2\left[e^{t}_{(t)}\right]^2 A_t^2 - 1. \tag{13}$$

Escape in oblique configuration

Once the charge is introduced, the value of effective potential (12) changes accordingly. In order to study trajectories of escaping particles, we examine the behavior of the potential for $r \gg M$. In particular, for the initially neutral particle with energy E_{Kep} ionized in the equatorial plane at r_0 we obtain the following relation valid in the asymptotic region:

$$E - V_{\mathrm{eff}}|_{r \gg M} = E_{\mathrm{Kep}} - 1 - \frac{qB_z a}{r_0} + \mathcal{O}\left(r^{-1}\right). \tag{18}$$

Motion is possible only for $E \geq V_{\mathrm{eff}}$. Since $E_{\mathrm{Kep}} < 1$ with finite r_0 and $a \geq 0$ is considered, we observe that (i) particles may only escape for $qB_z < 0$, (ii) the escape is possible only for $a \neq 0$, (iii) asymptotic velocity of escaping particles is an increasing function of parameters $|qB_z|$ and a, and a decreasing function of r_0, and (iv) the escape is not allowed for the perpendicular inclination, $\alpha \equiv \arctan\left(B_x/B_z\right) = \pi/2$.

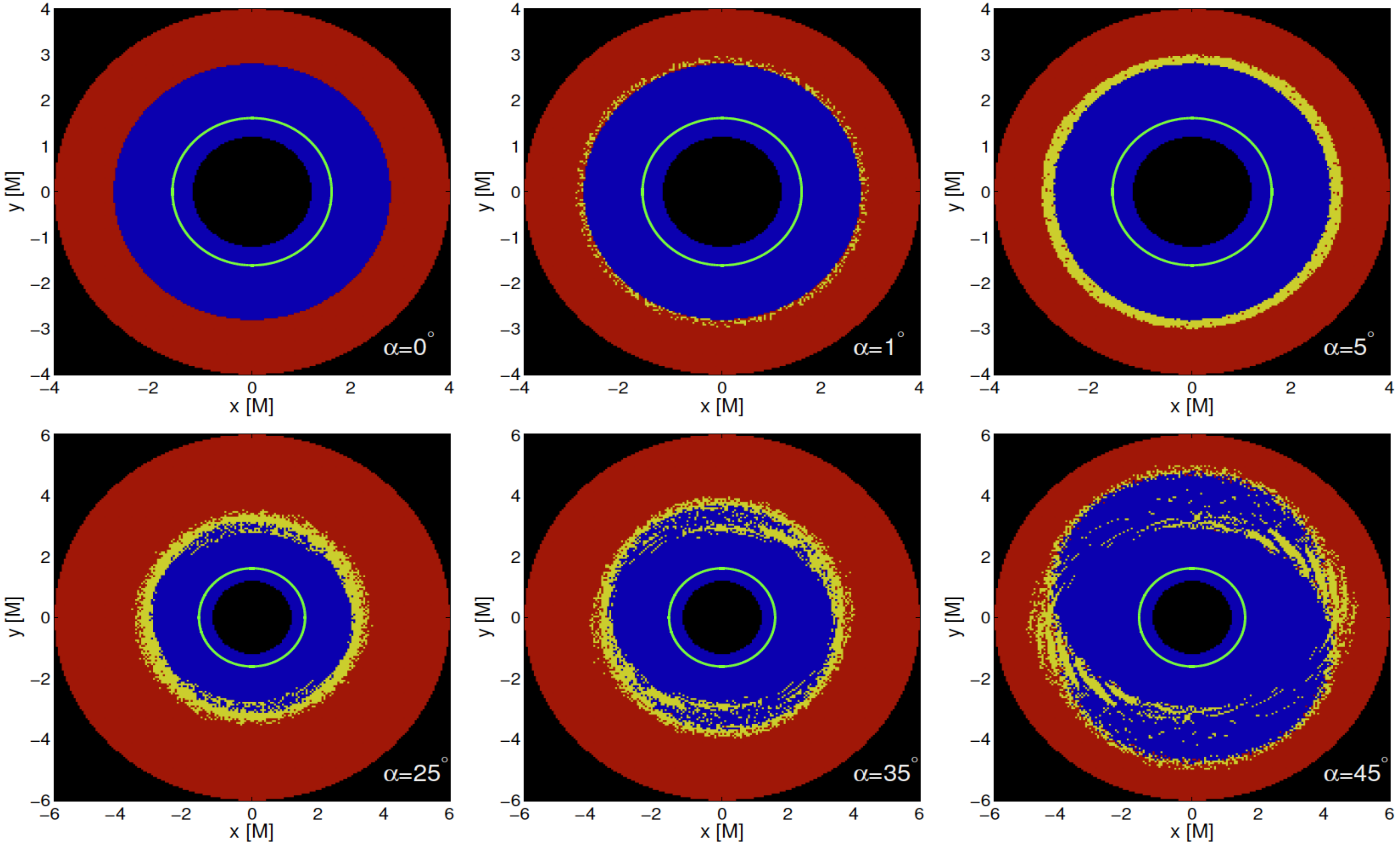


Figure 1. Different types of the trajectories launched from the equatorial plane (x, y) are plotted with respect to the magnetic inclination angle α. Color-coding: blue for plunging orbits, red for stable ones (bound to the black hole) and yellow for escaping trajectories. Green circle denotes the ISCO. Inner black region marks the horizon of the black hole. Parameters of the system are $a = 0.98$ and $qB = -5$. Magnetic field is inclined in the positive x-direction.

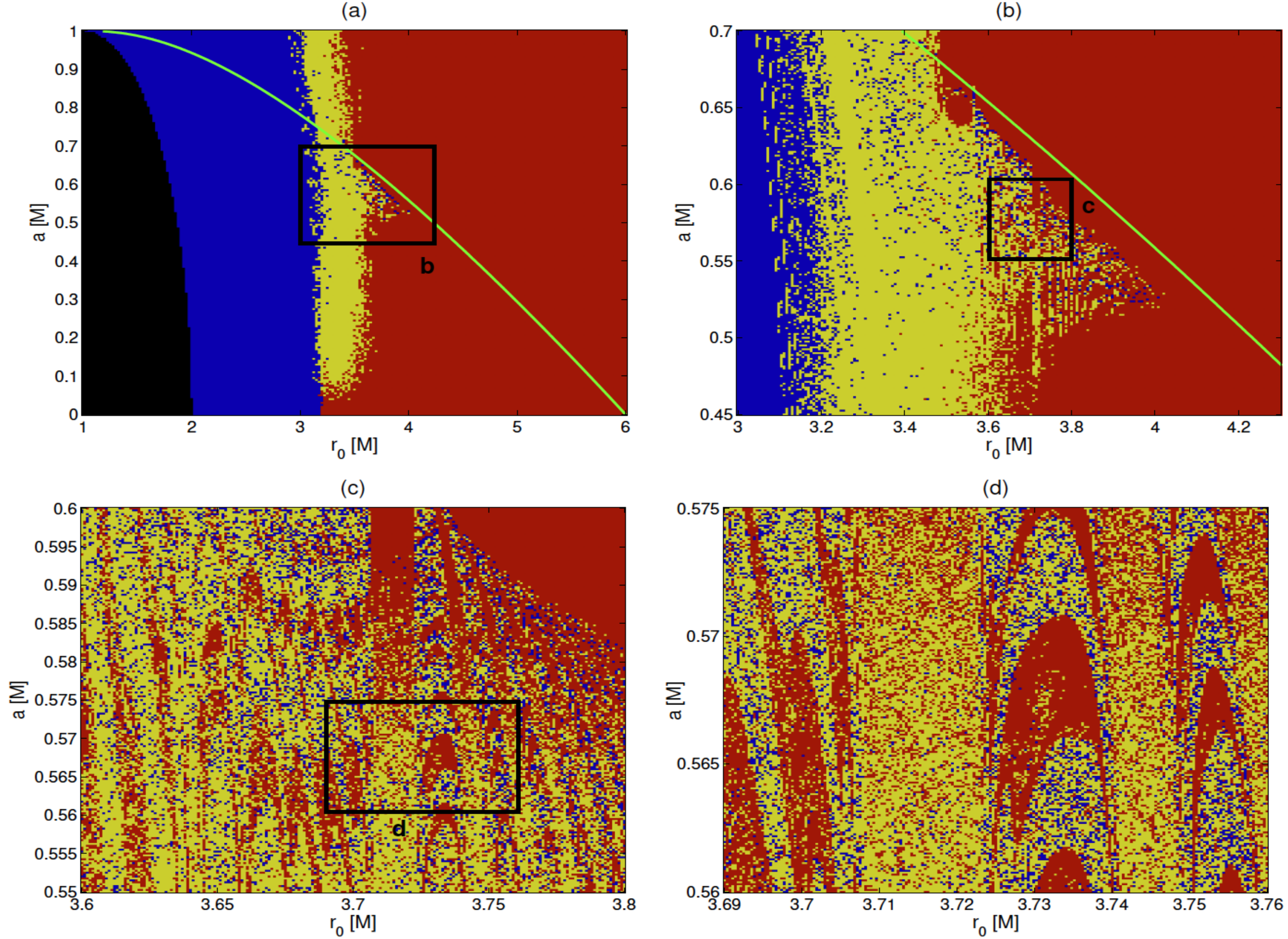


Figure 6. The structure of the escape zone ($qB = -4.1$, $\alpha = 14^{\circ}$ and $\varphi_0 = 0$) in the relevant range of the spin parameter a and initial radius r_0 is explored. Going from the top-left to the bottom-right panel, the portions of the plots (marked by black rectangles) are magnified progressively in the subsequent panels revealing the complex structure. Green line represents the ISCO, black color shows the horizon of the black hole. Color-coding of trajectories as in Figs. 1-3.

To explore the effects of the black hole charge, we begin with the aligned (Wald) field.

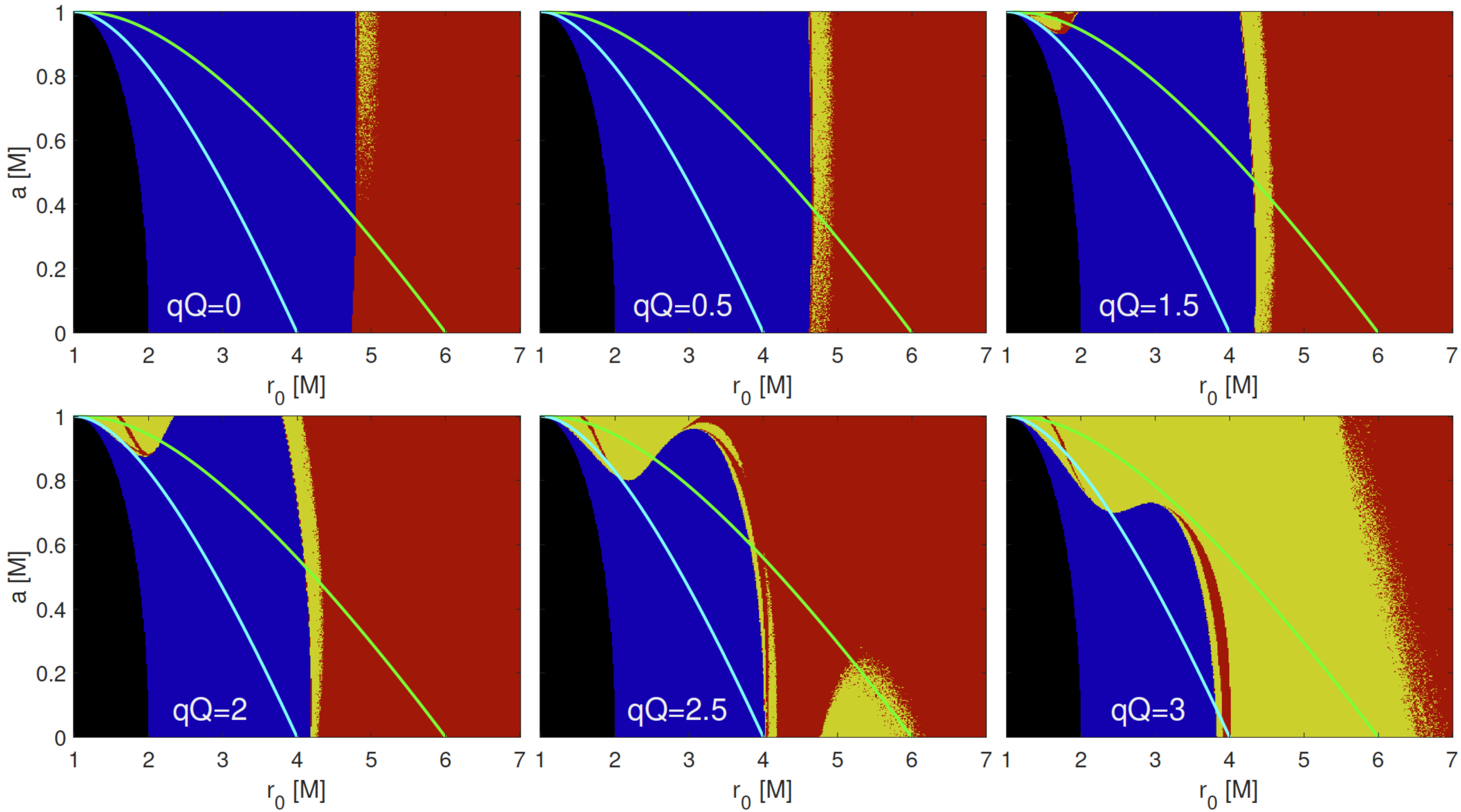


Figure 1. Evolution of escape zones of charged particles affected by the Coulomb repulsion of charged black hole. Color-coding of trajectories: blue for plunging orbits, red for stable ones (bound to the black hole) and yellow for escaping trajectories. The inner black region marks the horizon of the black hole. The green line denotes the ISCO of neutral particles, while the cyan line marks MBSO. Parameters of the system are $qB = -1$ and inclination $\alpha = 0$ while qQ varies as indicated.

Summary

We have computed the final Lorentz factor γ of escaping particles confirming that the highest γ is achieved in the innermost region of the primary escape zone (with the lowest allowed r_0). For a particular (realistically small; $\alpha \approx 6°$) value of inclination and fixed value of spin ($a = 0.98$), we searched for the maximal γ. Increasing the value of magnetization up to $|qB| = 10^3$ we confirmed that (unlike axisymmetric configuration) ultrarelativistic velocities with $\gamma \gg 1$ may be achieved. While the acceleration of the particle is actually powered by the parallel component B_z, the perpendicular component B_x acts as an extra perturbation which considerably increases the probability of sending the particles on escaping trajectories and allows the outflow also in cases which are excluded in the aligned setup.

References:

- Kopáček & Karas, The Astrophysical Journal, 966, 226 (2024)
- ---------"---------, The Astrophysical Journal, 900, 119 (2020)