

# Magnetic field dynamics in isolated neutron stars: insights from GRMHD simulations

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# Motivation

- **Neutron stars (NS)** host extreme **magnetic fields** up to  $10^{16}\text{G}$ : impact on NS structure, dynamics and evolution.
- Effects also on **observables**: EM radiation, outbursts, FRBs and gravitational waves.
- **Long-term equilibrium** configuration unclear.
- Pulsar observations point towards a large-scale **dipolar** field [Chung & Melatos 2011].
- **Purely poloidal** fields shown to be **unstable** [Taylor 1957, 1973, Wright 1973, Markey & Tayler 1973, 1974] and verified via **numerical simulations** [Kiuchi+ 2008, Cioffi+ 2011,2013, Lasky+ 2011, Sur+ 2021, Cheong+ 2024].
- We study the **stability** of **equilibrium configurations** by performing long numerical simulations.

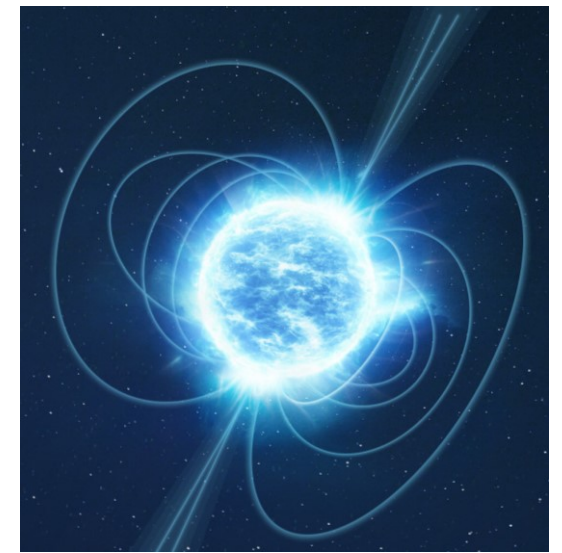
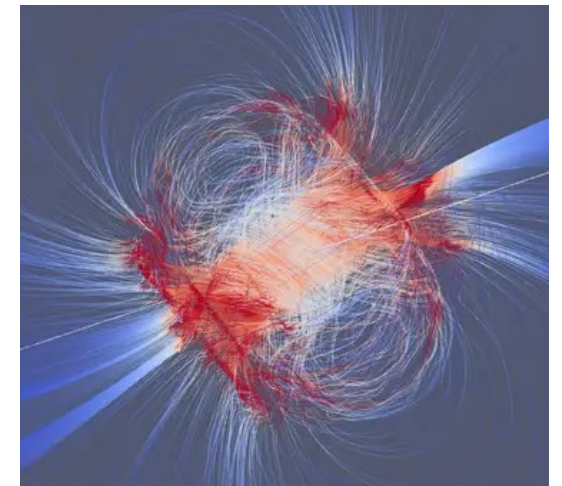


Figure: ESA



# Numerical Setup: Athenak

- Open-source C++ code designed for solving **magneto-hydro-dynamics** (MHD) equations with **dynamical spacetime solver** [Stone+ 2024].
- Solves MHD equations with **constrained transport algorithm** [White+ 2016].
- Extension of Athena++ [Stone+ 2020] , rewritten in KOKKOS library [Trott+ 2022]  
→ can run on both CPUs and GPUs.
- Excellent **scaling** properties, **portability** and **speedup** efficiency [Fields+ 2024].
- MeshBlock-based **mesh refinement** [Zhu+ 2024].

Publically available at: <https://github.com/IAS-Astrophysics/athenak>



# Simulations

- **Static TOV star** with evolution in **Cowling** approximation.
- Fluid initial data set with gamma law EOS
- 3D domain of  $\pm 80$  km and **resolution 156m** [For comparison see Laski+ 2011, Tsokaros+ 2021]
- **Dipole external field** with maximum strength  $B = 10^{15}$  G at the surface [Sur+ 2020].
- We investigate internal magnetic field set with **various initial toroidal strengths**.

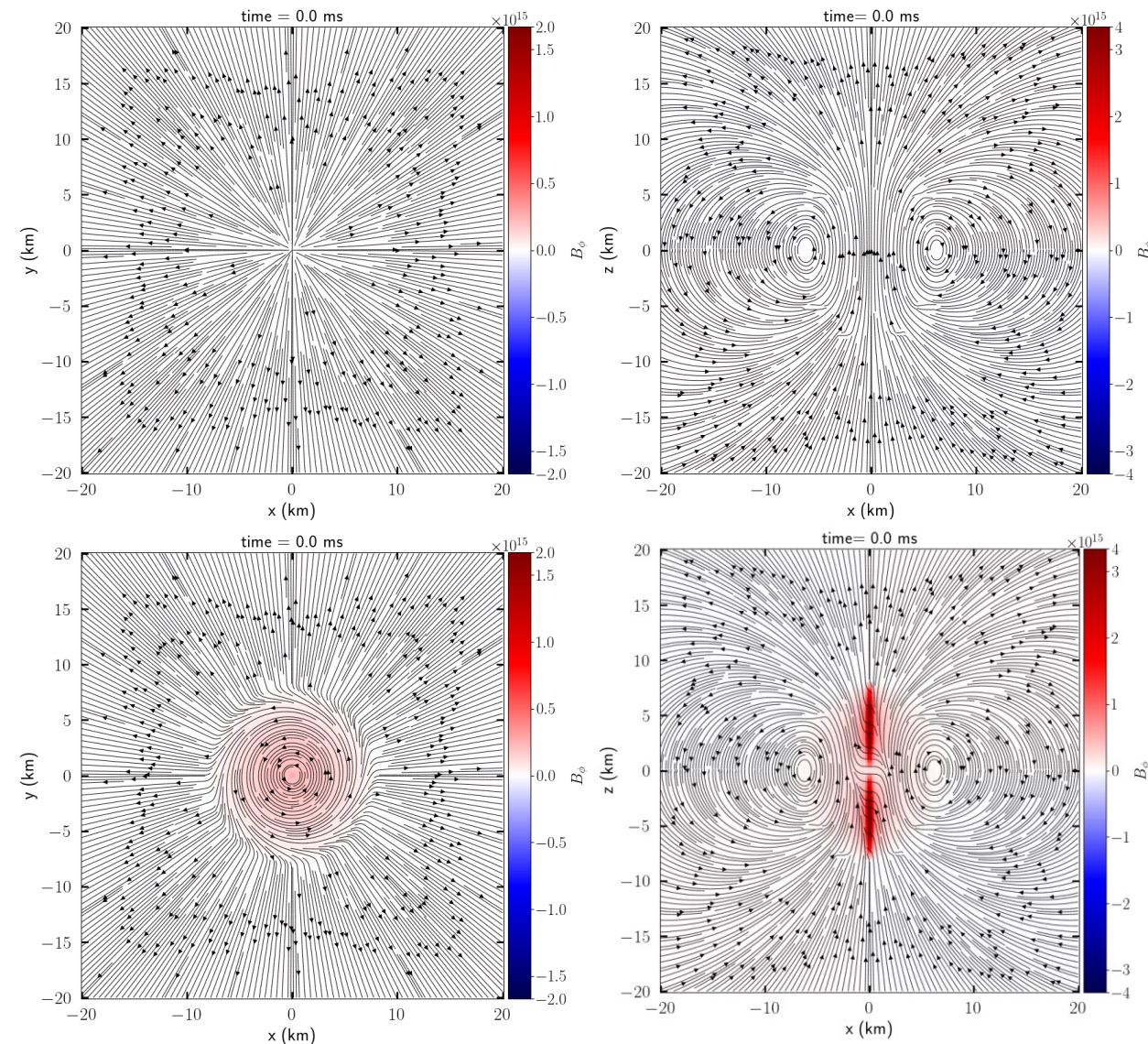
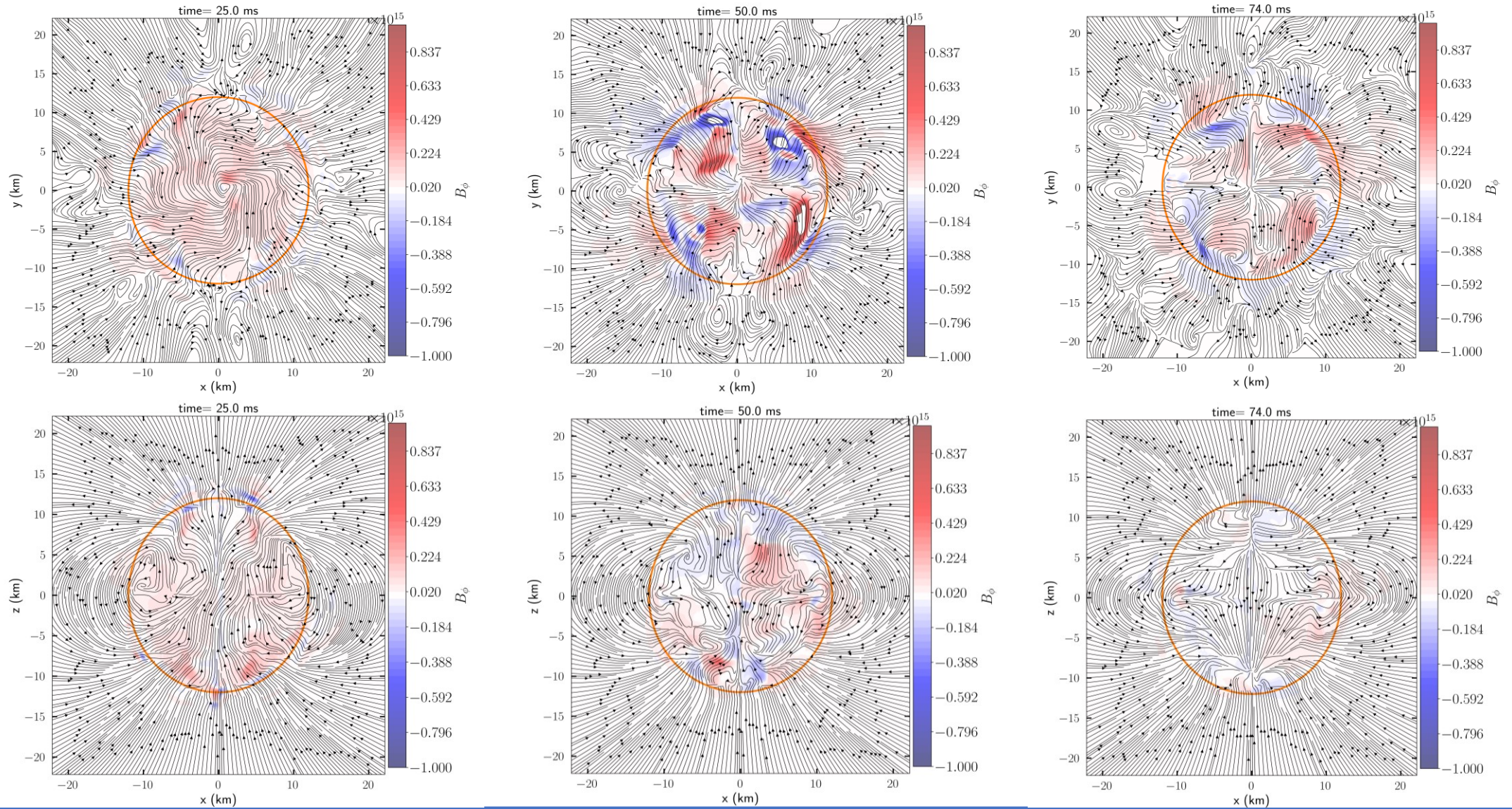


Figure:  $t = 0$  setup for  $BE_{tor} = 0\% BE_{tot}$  and  $BE_{tor} = 25\% BE_{tot}$

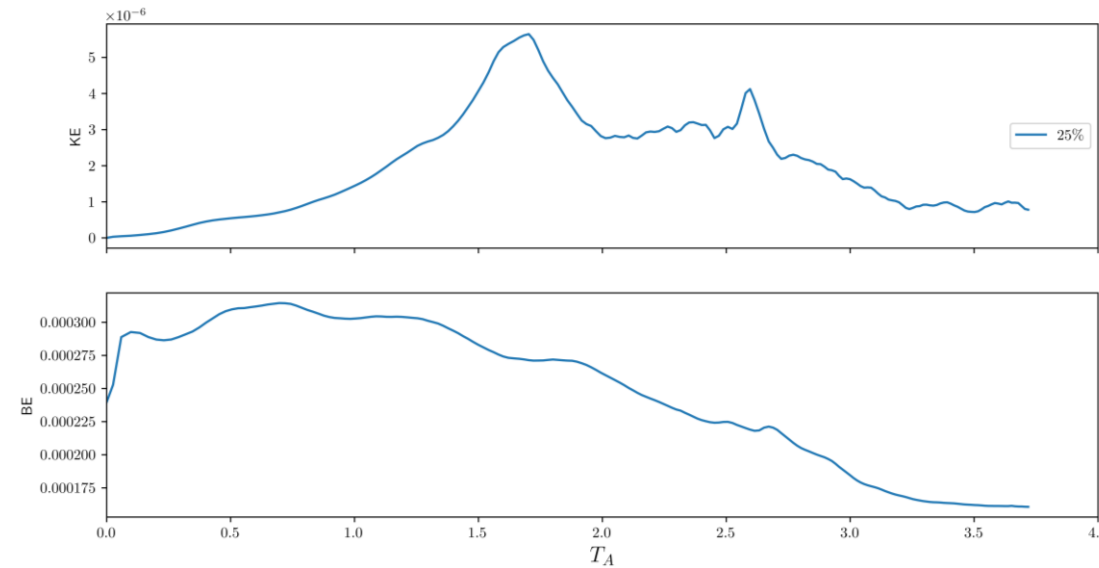
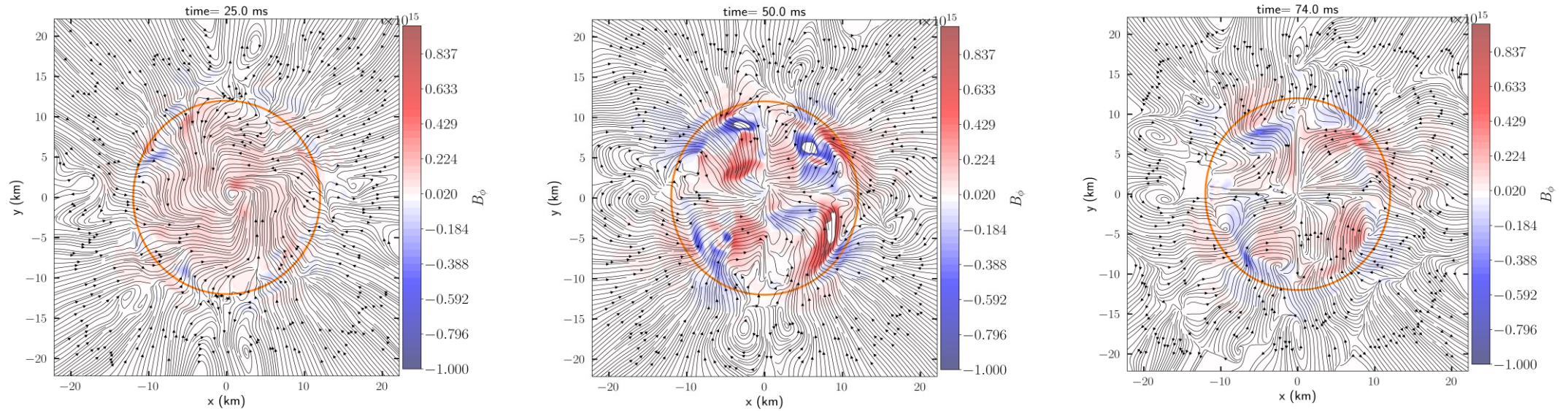


Figures: 2D evolution snapshots for  $BE_{tor} = 25\% BE_{tot}$





# Figures: 2D evolution snapshots for $BE_{tor} = 25\% BE_{tot}$

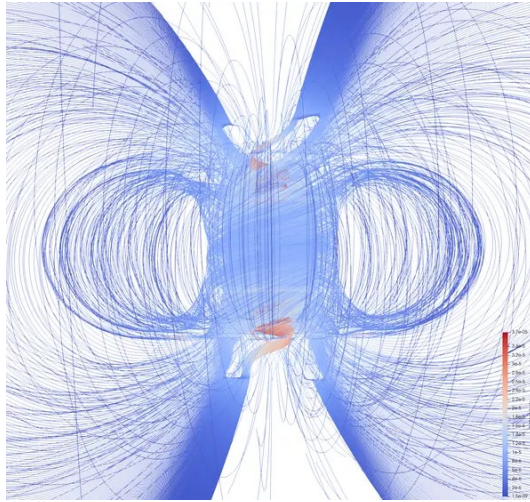


Energies obtained  
with Eq. B2, B4 from  
Cook+ 2025

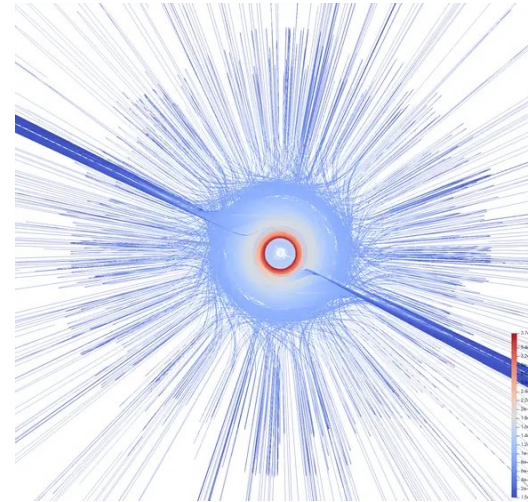
3D evolution  
snapshots for  
 $BE_{tor} = 80\% BE_{tot}$

$t = 0$  ms

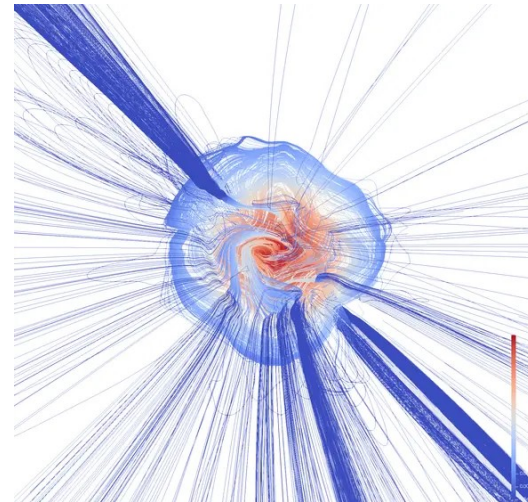
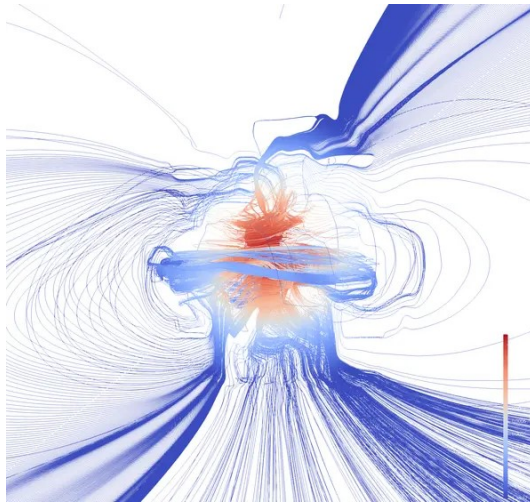
xy plane



xz plane



$t = 25$  ms





# Results:

- **Transient behaviour** after  $\sim 2$  Alfvén times.
- All configurations reach  $BE_{tor} \sim 6 - 18\% BE_{tot}$ .
- $\Delta m \sim 10^{-7}, \Delta IE \sim 10^{-5}$ .
- Consistent **quasi-equilibrium end state** and correlation with external dipole field.

$$\tau_A = \frac{2R\sqrt{4\pi\langle\rho\rangle}}{\langle B \rangle}$$

$$T_A = \int_0^t \frac{dt}{\tau_A(t)}$$

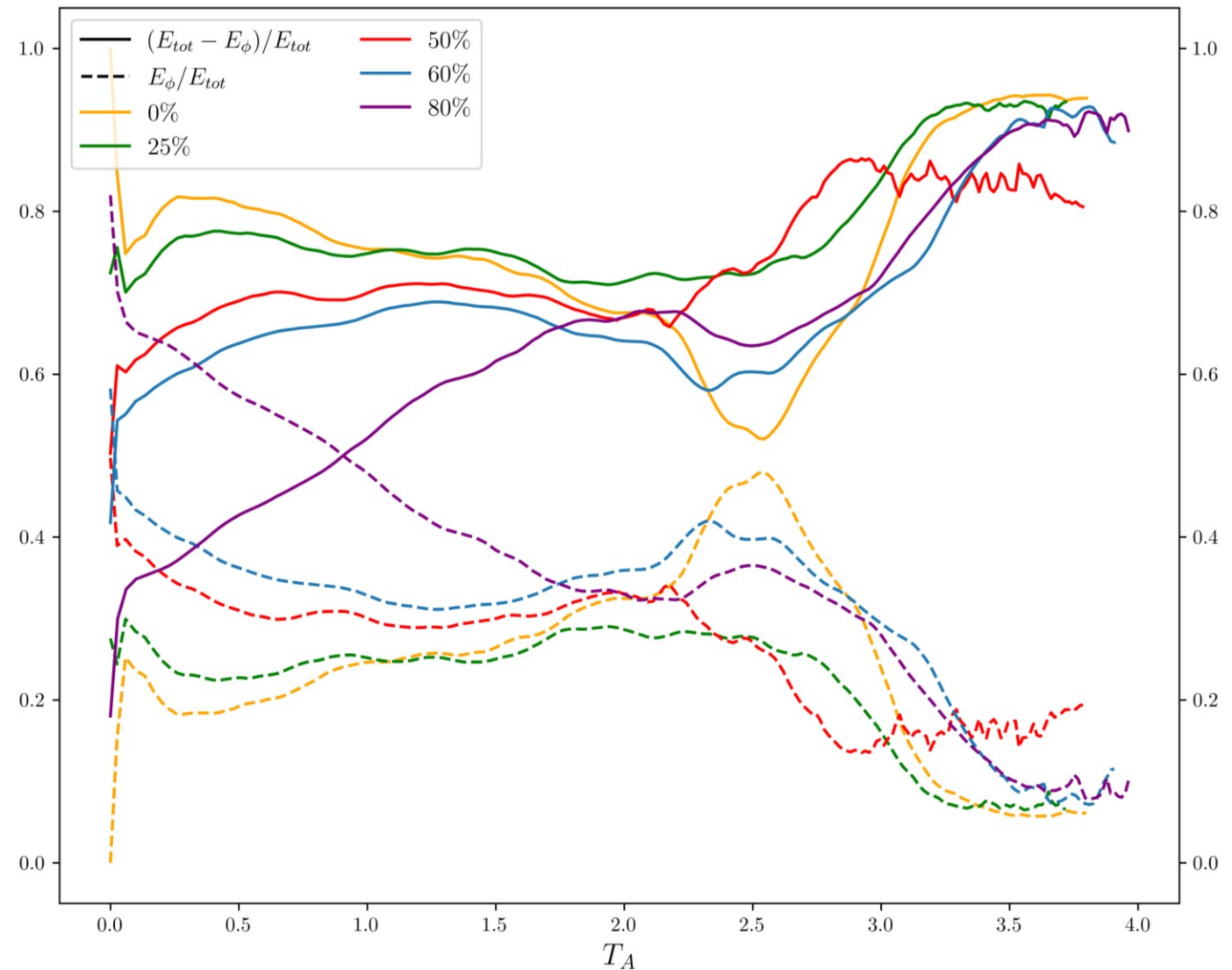


Figure: evolution of the magnetic field energy components (Capobianco+ in prep.)



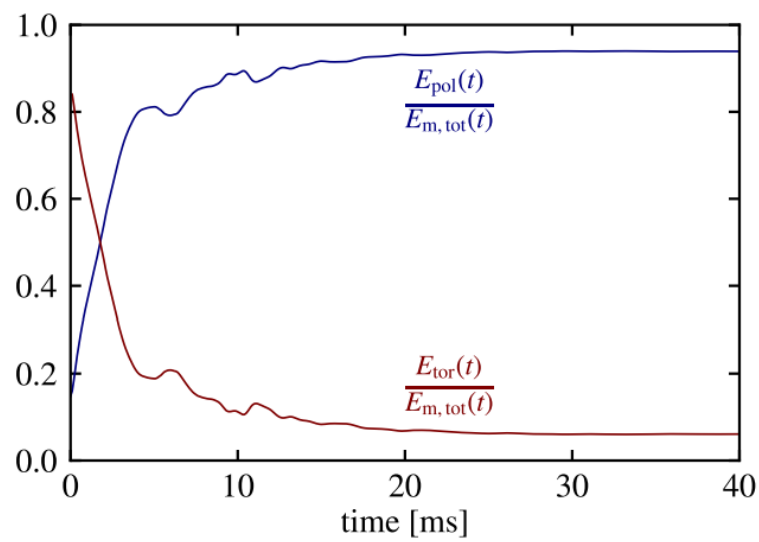


Figure 1 from Sur+ 2020

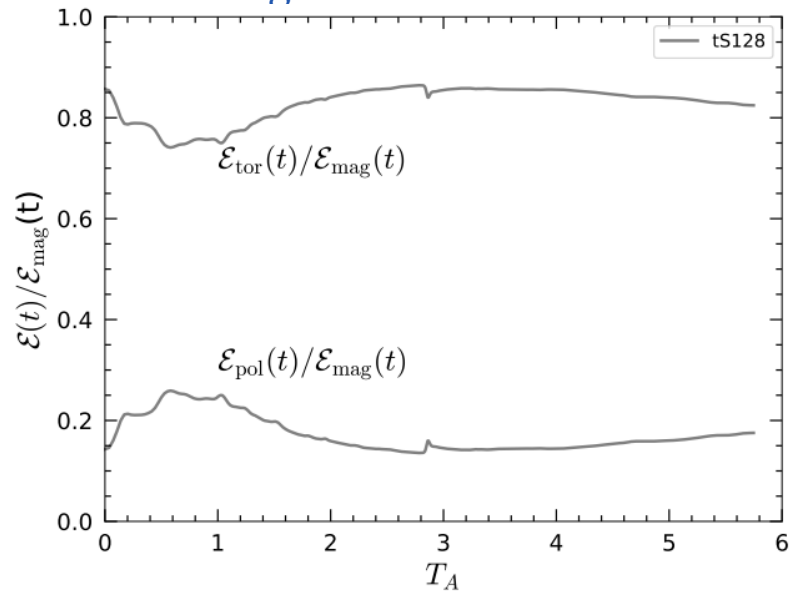


Figure 3 from Sur+ 2022

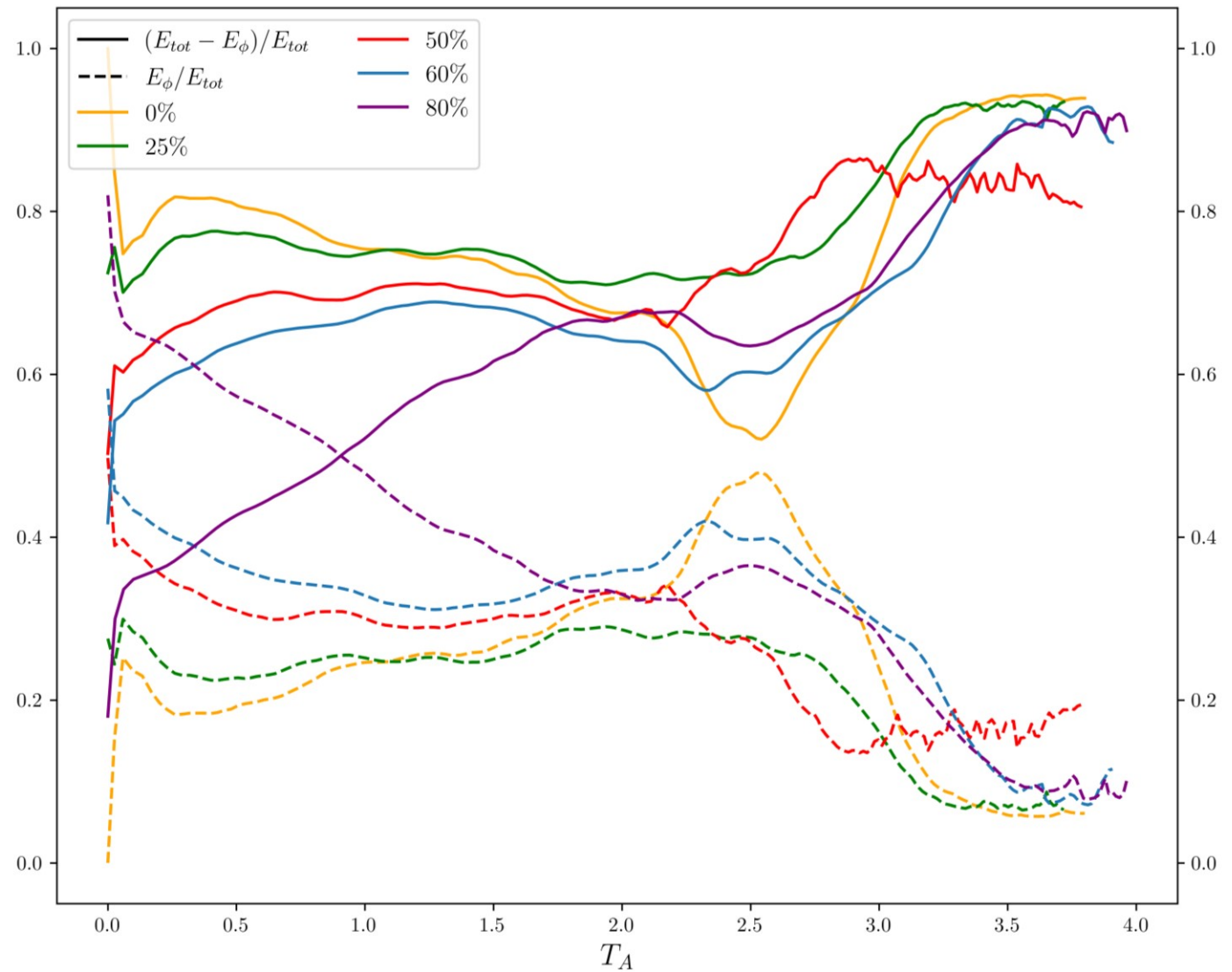


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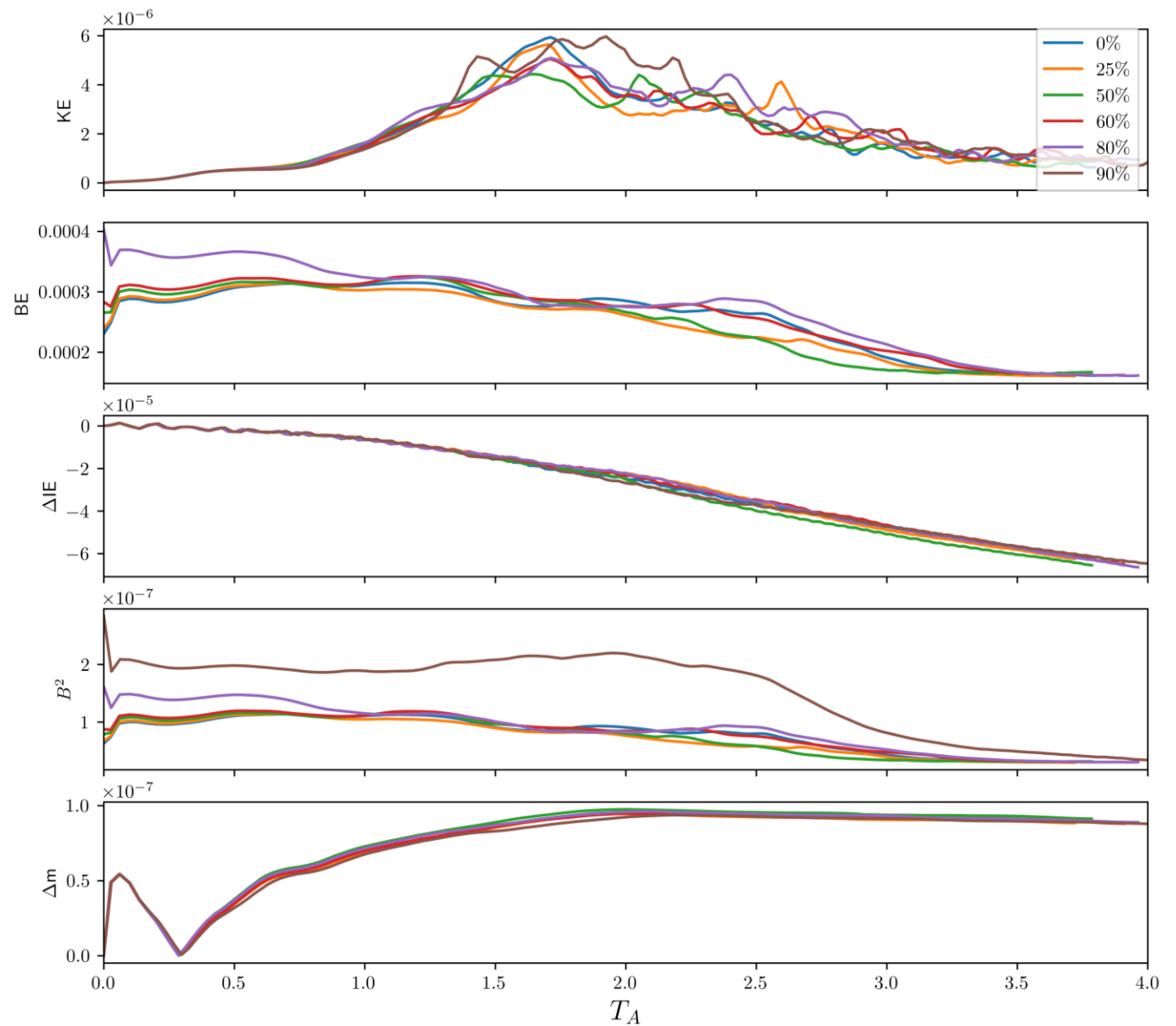
# Conclusions

- Toroidal magnetic energy decays to 6-18 % of total magnetic energy, independently of the initial  $BE_{tor} / BE_{pol}$  ratio.
- Ongoing further analysis to assess effects on helicity and modes.
- Longer simulations (>100ms) needed to verify that this condition is stable.
- Higher resolution would improve authenticity of results.
- Future work will focus on rotating NS and evolution in full General Relativity.



# Further analysis

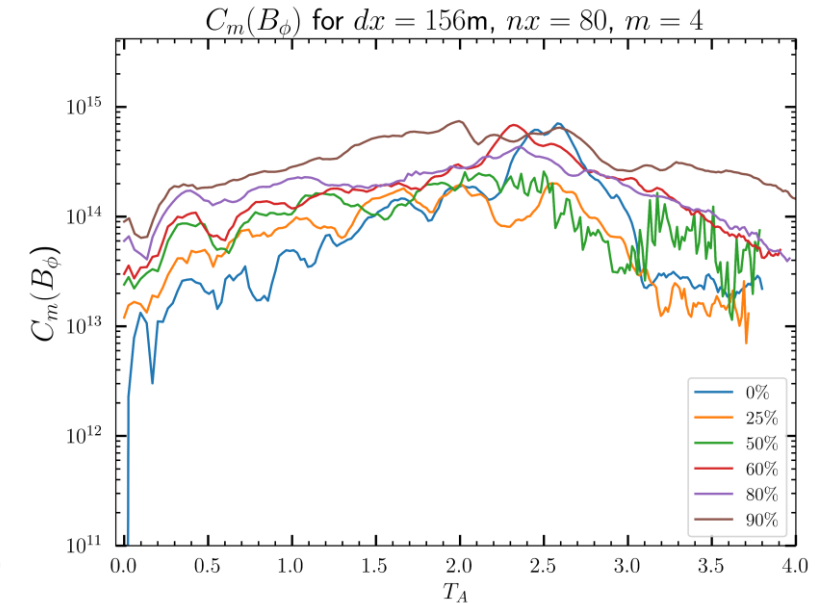
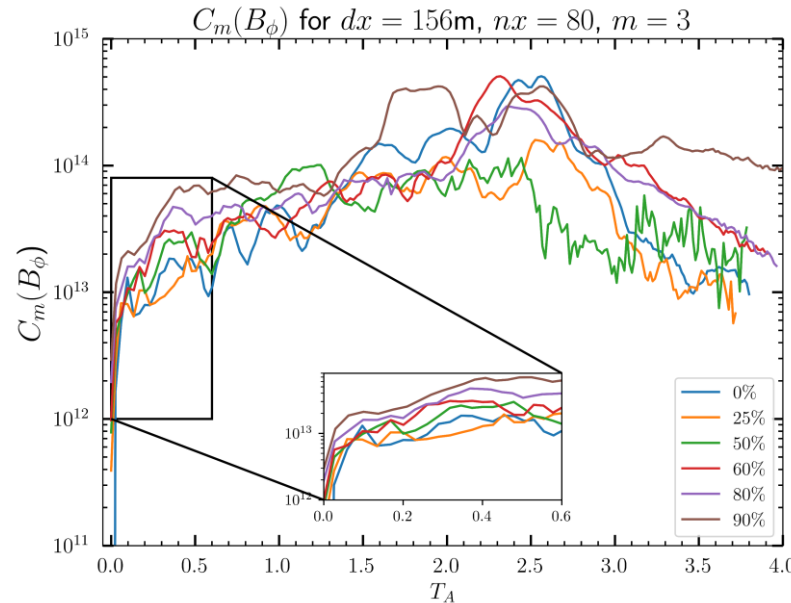
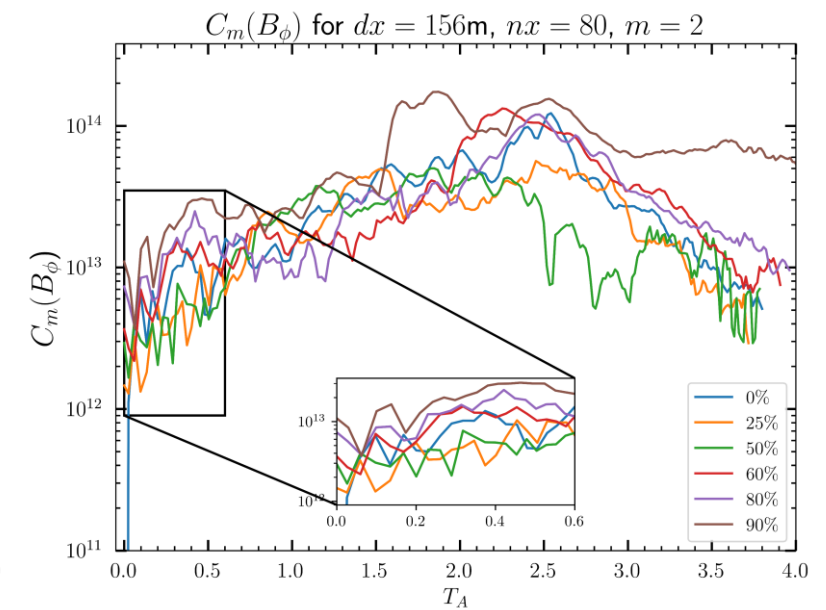
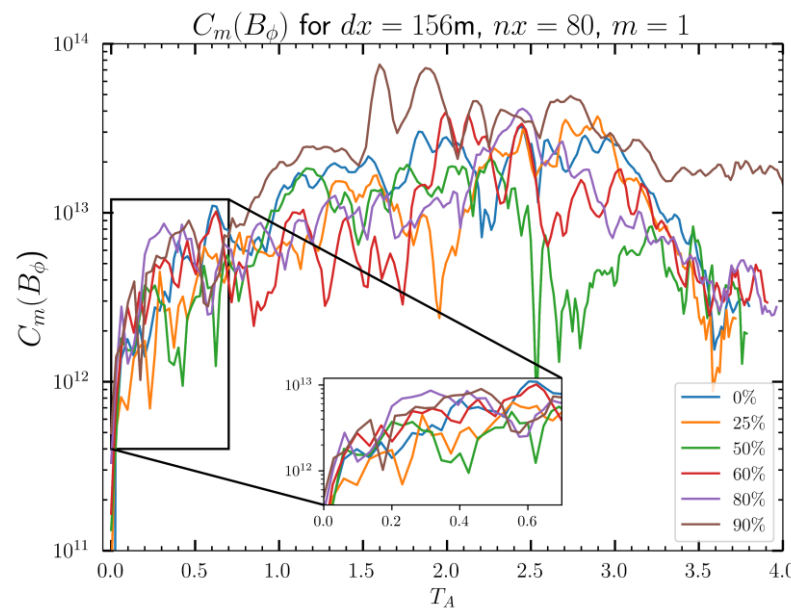
- Comparison of energy and mass conservation behaviour for all configurations



# Further analysis

- Fourier modes calculated in star's interior [following Zink+ 2007, Lasky+ 2012]

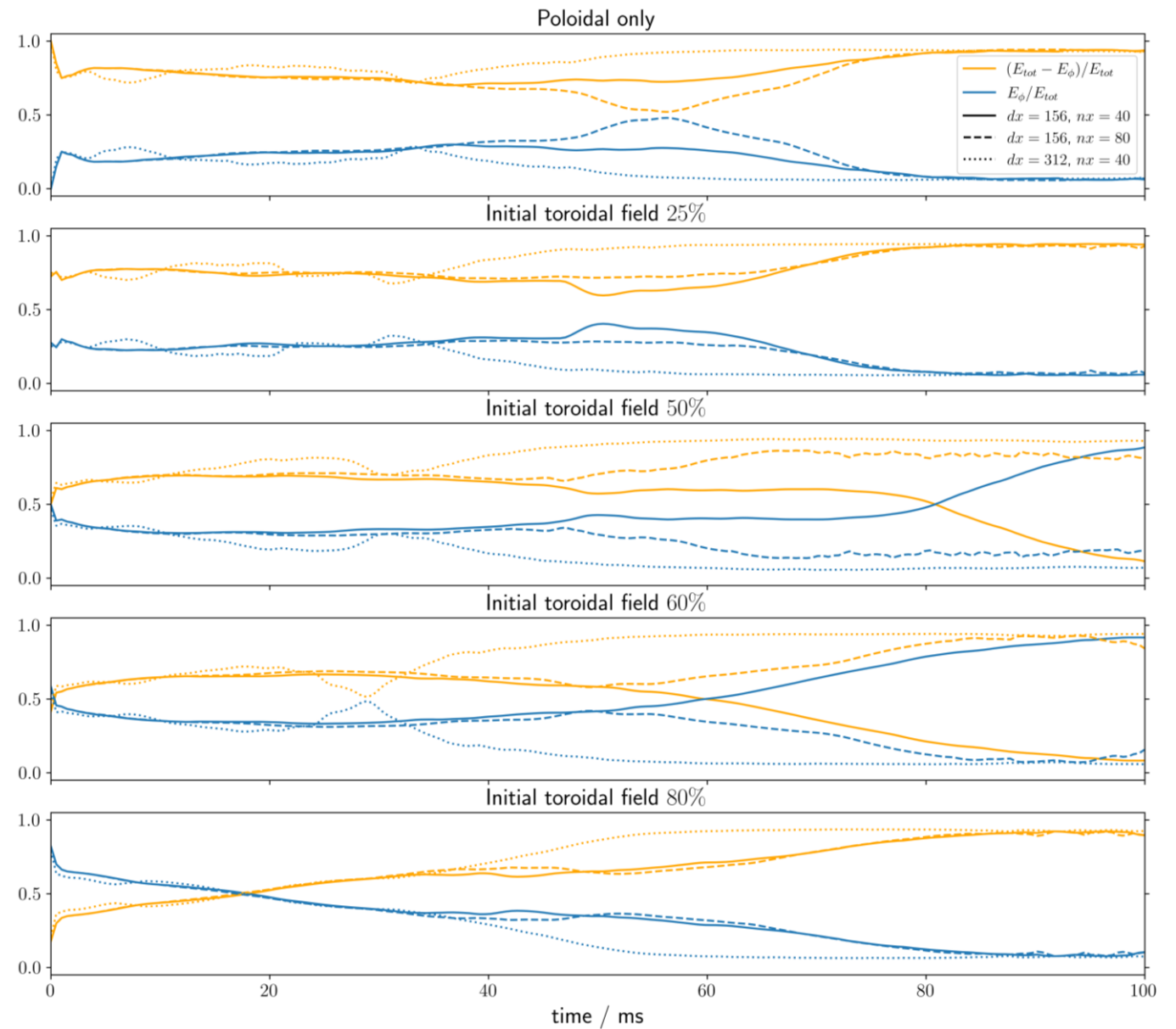
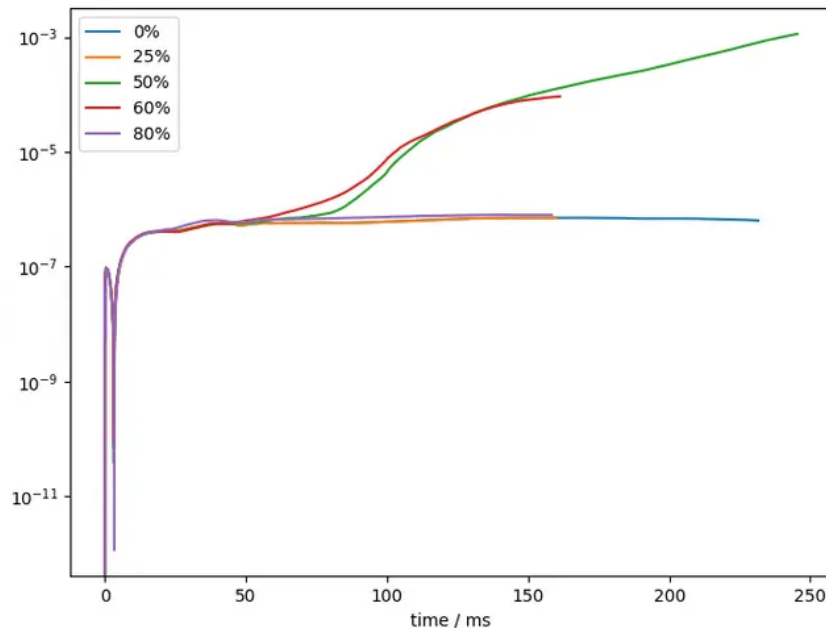
$$C_m = \int_0^{2\pi} B_\phi(\bar{\omega}, \phi, z=0) e^{im\phi} d\phi$$





# Resolution & boundaries

## Mass violation for domain $\pm 40$



External dipole field:

$$B_r = \frac{B_p R^3 \cos \theta}{r^3}$$

$$B_\theta = \frac{B_p R^3 \sin \theta}{2r^3}$$

Interior configuration:

$$B_r = \frac{B_p \cos \theta}{\pi(\pi^2 - 6)} [y^3 + 3(y^2 - 2) \sin y + 6y \cos y]$$

$$B_\theta = \frac{B_p \sin \theta}{2\pi(\pi^2 - 6)} [-2y^3 + 3(y^2 - 2)(\sin y - y \cos y)]$$

$$B_\phi = B_t \frac{\sin y \sin \phi}{\pi} \quad \text{with} \quad y = \frac{\pi r}{R_\star}$$

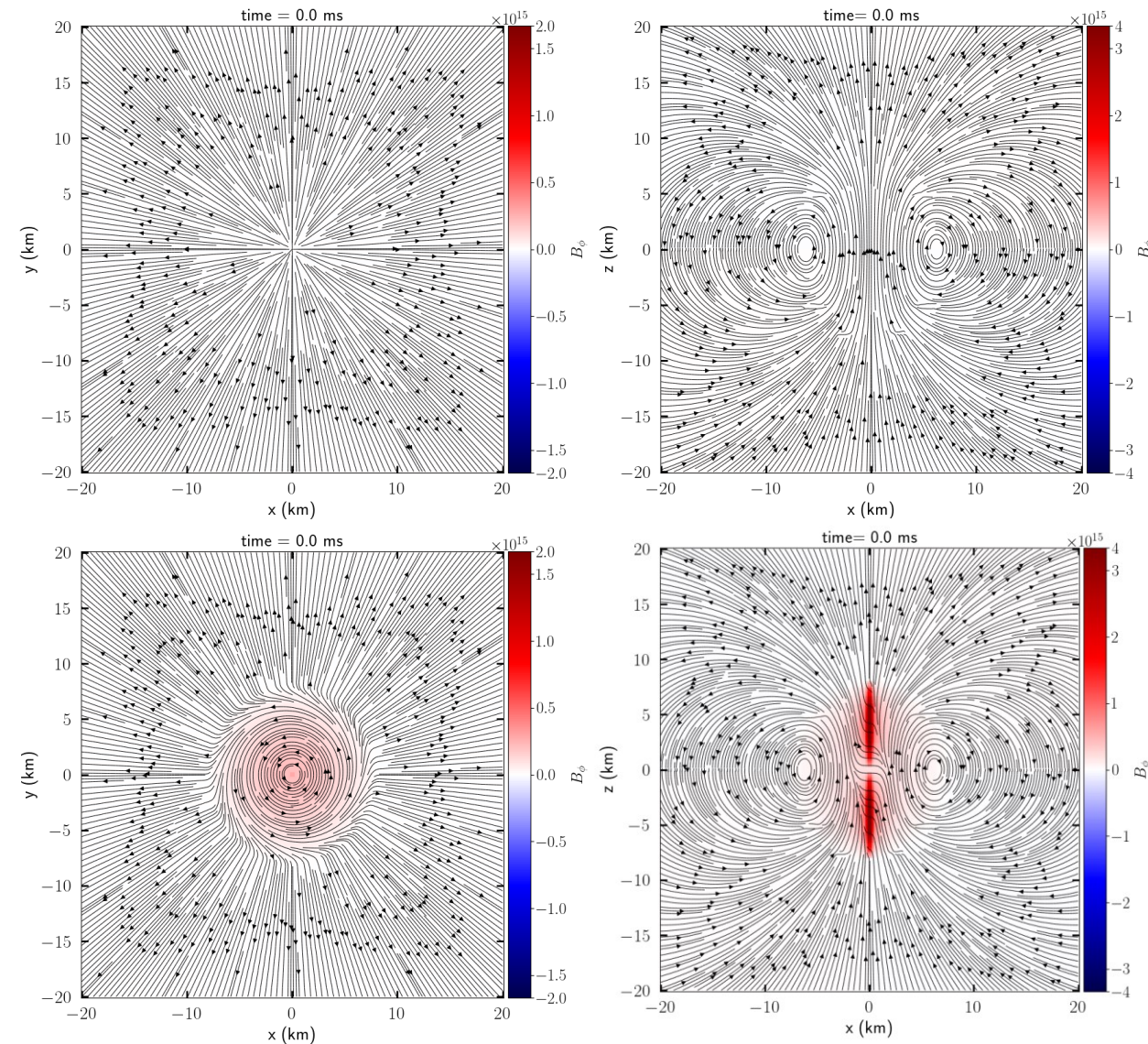


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