cuHARM: general relativistic radiation magneto-hydrodynamics in the era of exascale computing

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With
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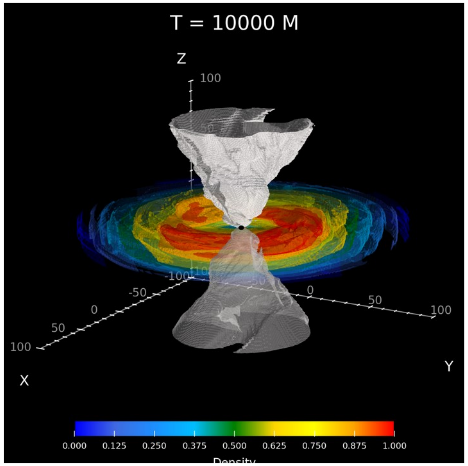

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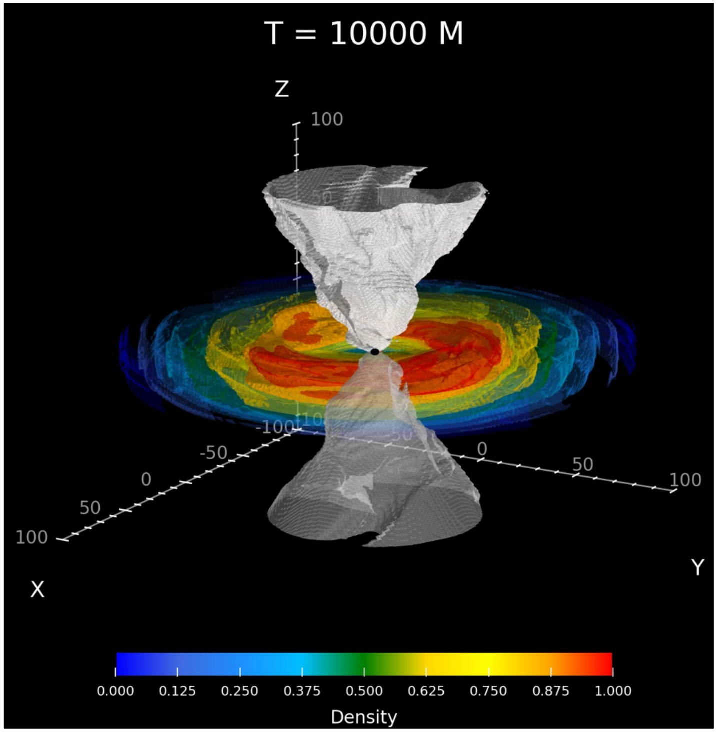
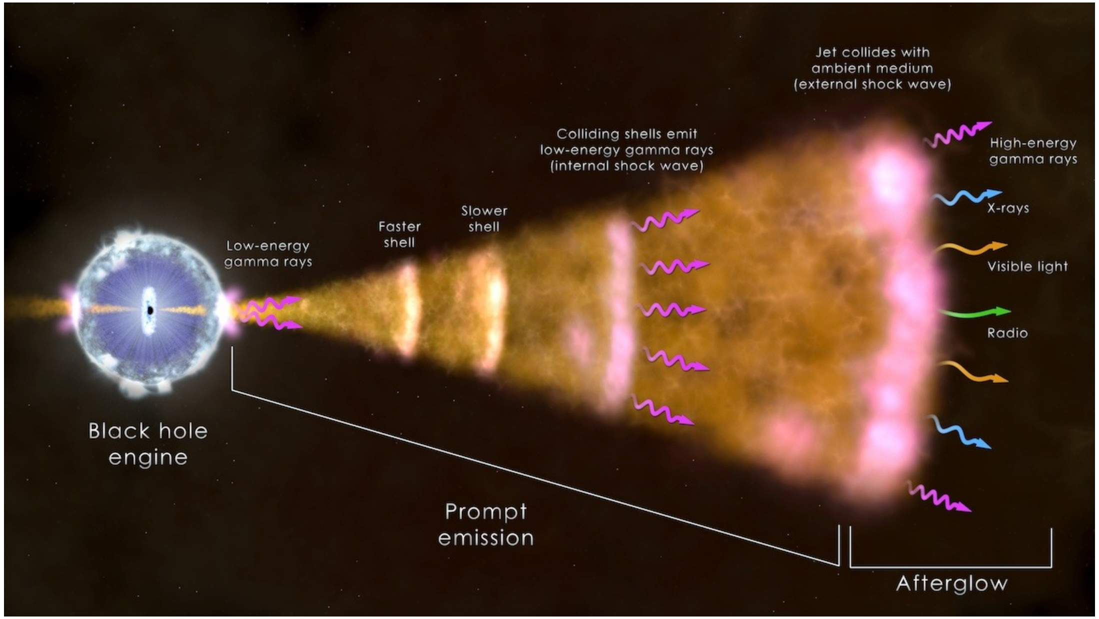
אוניברסיטת בר־אילן

ReFCO, Warsaw, 5th-7th of May 2025

Introduction

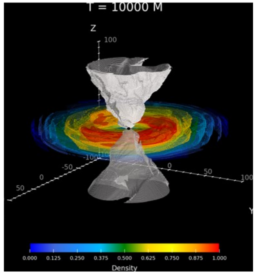
Dynamics **and** Radiation





Bégué, Pe'er et al. (2023) ; Zhang, **Bégué**, Pe'er et al. (2024) ; Singh, **Bégué** and Pe'er (ApJL 2025) ; Singh, **Bégué** and Pe'er (2025, in prep) ; Wallace, **Bégué** and Pe'er (2025, in prep)

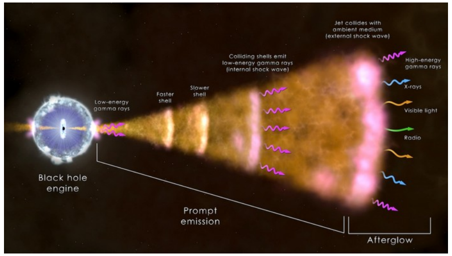
Introduction



Dynamics

- No emission
- Radiation usually neglected
- If not neglected: “simplified” treatment to only account for its dynamical contribution.

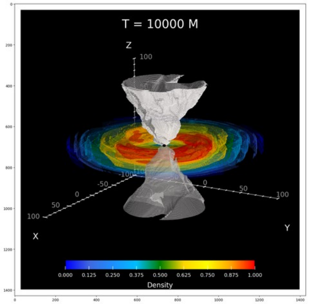
Emission



- Simplified dynamical model
  - “blob” model (blazar),
  - infinitely thin shell (GRBs)....
  - Self-similar motion (GRB afterglow)
  - Post-processing ray tracing.

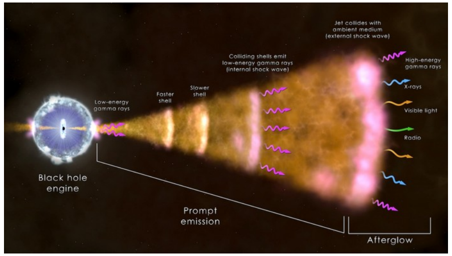
Yet .... **dynamics and radiation are tightly linked:**
- The dynamics is modified by radiative output.
- Observables (light curve, spectrum) strongly depends on the dynamics:
  - Geometry, cooling state of the gas, velocity field ...

Introduction: how to bridge in a simulation radiation and dynamics ?



Dynamics

Emission



Goals:

1) Self-consistently calculate the radiative contribution to the dynamics:
   Cooling of the gas, radiation force, **radition anisotropy, radiation spectrum**.
2) Self-consistently integrate the dynamical effects on the produced radiation.

The dynamics

The dynamics: (GR) Magnetohydrodynamics (MHD)

Study the motion of magnetized plasma in the close vicinity of a black-hole

Study the motion of magnetized plasma in relativistic jets

We need a (GR)-MHD code

Conservation of gas density

$$\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\rho u^\mu\right)=0 \qquad (1)$$

Energy and momentum conservation

$$T^{\mu}_{\ \nu;\mu}=0 \qquad (2)$$

Maxwell Equations

$$\nabla_\mu({}^*F^{\mu\nu})=0 \qquad (3)$$

Komissarov (1999, 2001), Gammie et al. (2003), Font (2008), and Rezzolla & Zanotti (2013)

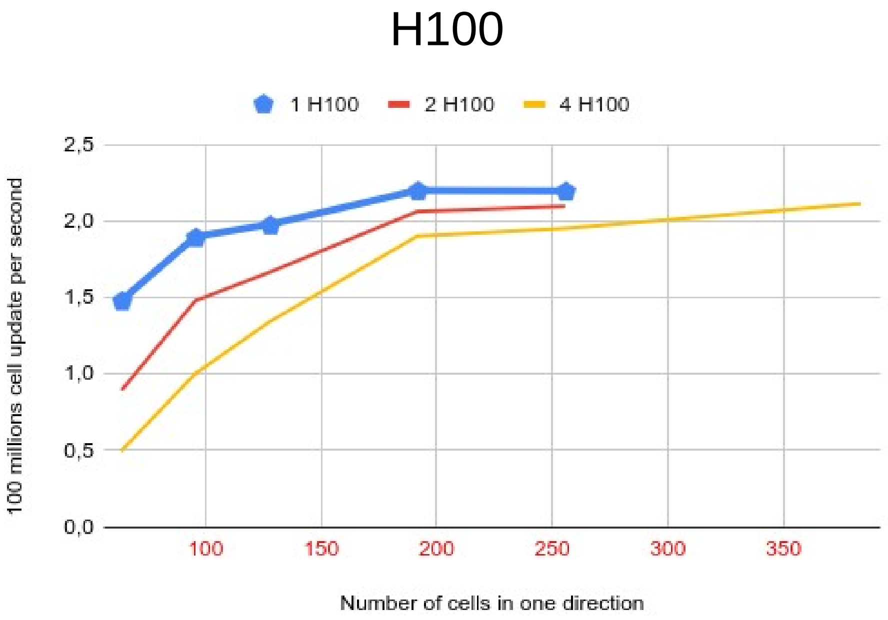
The dynamics: cuHARM in a nutshell

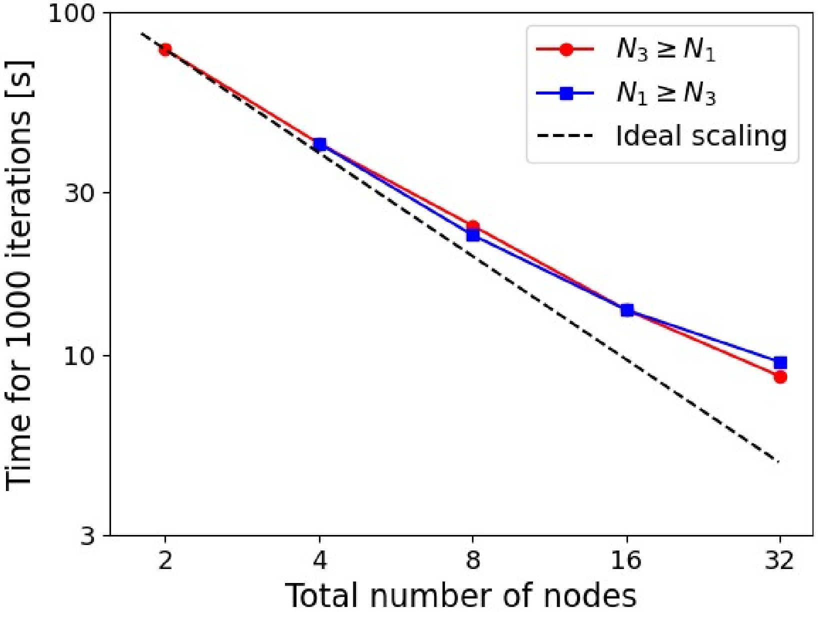
We wrote cuHARM.

- **3D GR-MHD code:**
  - Finite volume
  - Flux-CT to preserve div B = 0
- **Designed for multi-GPUs nodes:** CUDA-C / openMP / MPI
- **Thoroughly tested** (e.g. comparison with Porth et al. 2019)
- **Highly optimized:**
  - >~10^8 cell updates per second on a A100
  - Two versions: highly optimized vs easily modifiable

Bégué, Pe'er et al. (2023) ; Zhang, **Bégué**, Pe'er et al. (2024) ; Singh, **Bégué** and Pe'er (ApJL 2025) ; Singh, **Bégué** and Pe'er (2025, in prep)

Throughput of cuHARM





Strong scaling 512 x 256 x 512

~200 millions cell updates per second per H100 card.

Excellent calculation throughput and scaling.

The dynamics: recent results with cuHARM

1. Mass accretion rate

$$\dot{M} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \sqrt{-g}\,\rho u^r d\theta d\phi$$

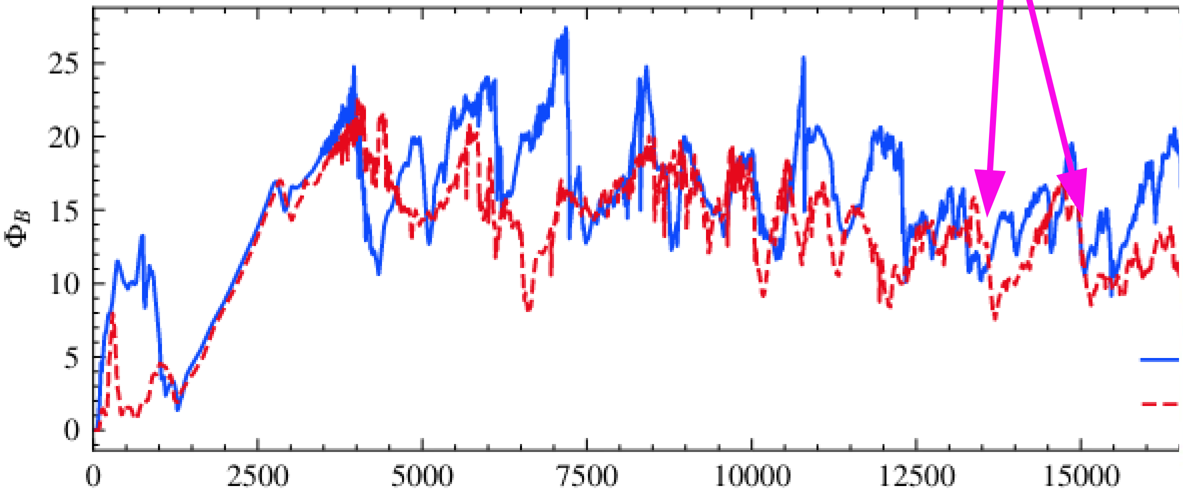
2. Magnetic flux threading the horizon

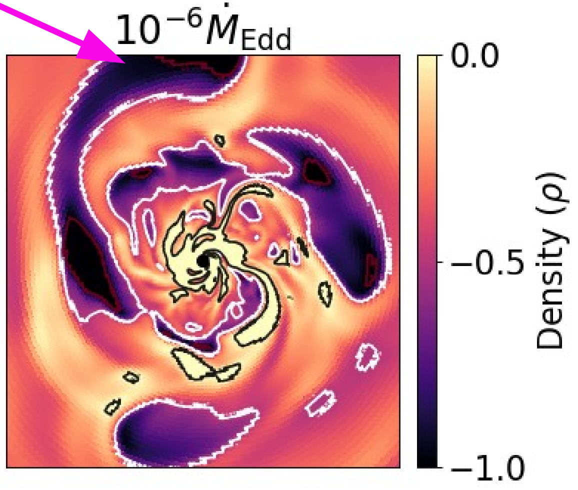
$$\varphi_B = \frac{1}{2}\int_{\theta=0}^{\pi}\int_{0}^{2\pi} \sqrt{-g}\,|{}^{*}F^{rt}|\; d\theta d\phi$$

The dynamics of MAD

Magnetic flux eruption

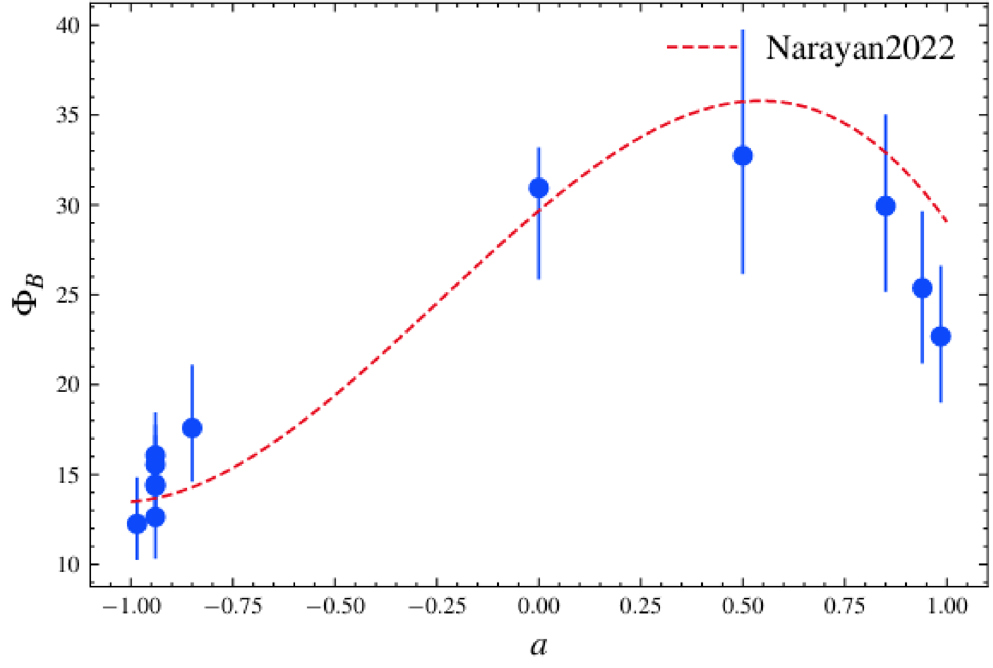
$$\phi_B = \frac{\varphi_B}{\sqrt{\dot{M}}}$$



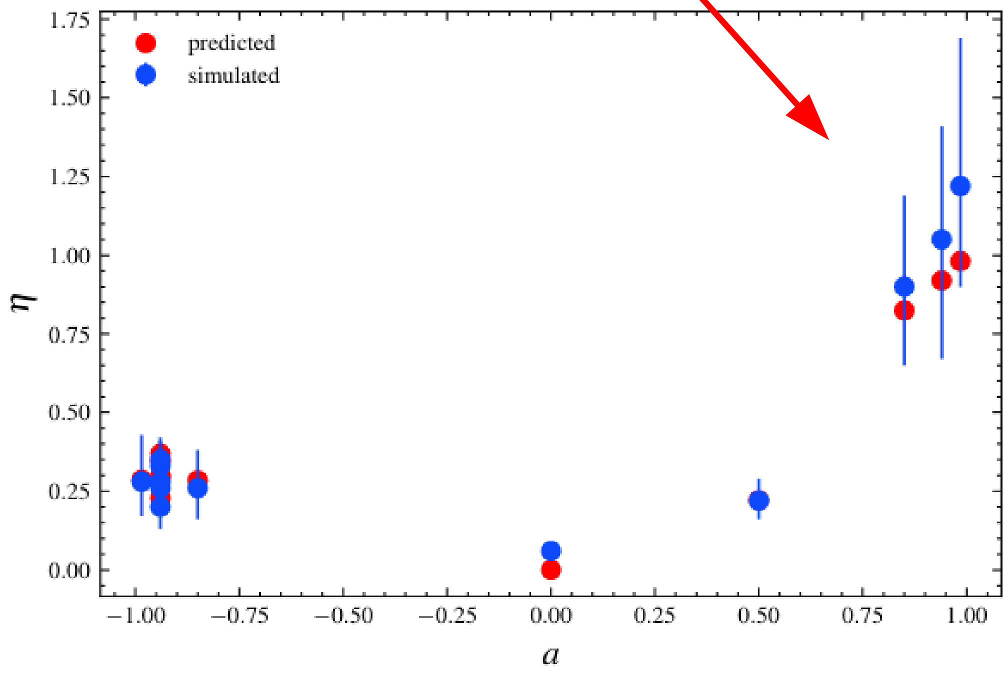


Zhang, **Bégué**, Pe'er et al. (2024) ; Singh, **Bégué** and Pe'er (ApJL 2025) ; Singh, **Bégué** and Pe'er (2025, in prep)

The dynamics: spin dependence for MAD



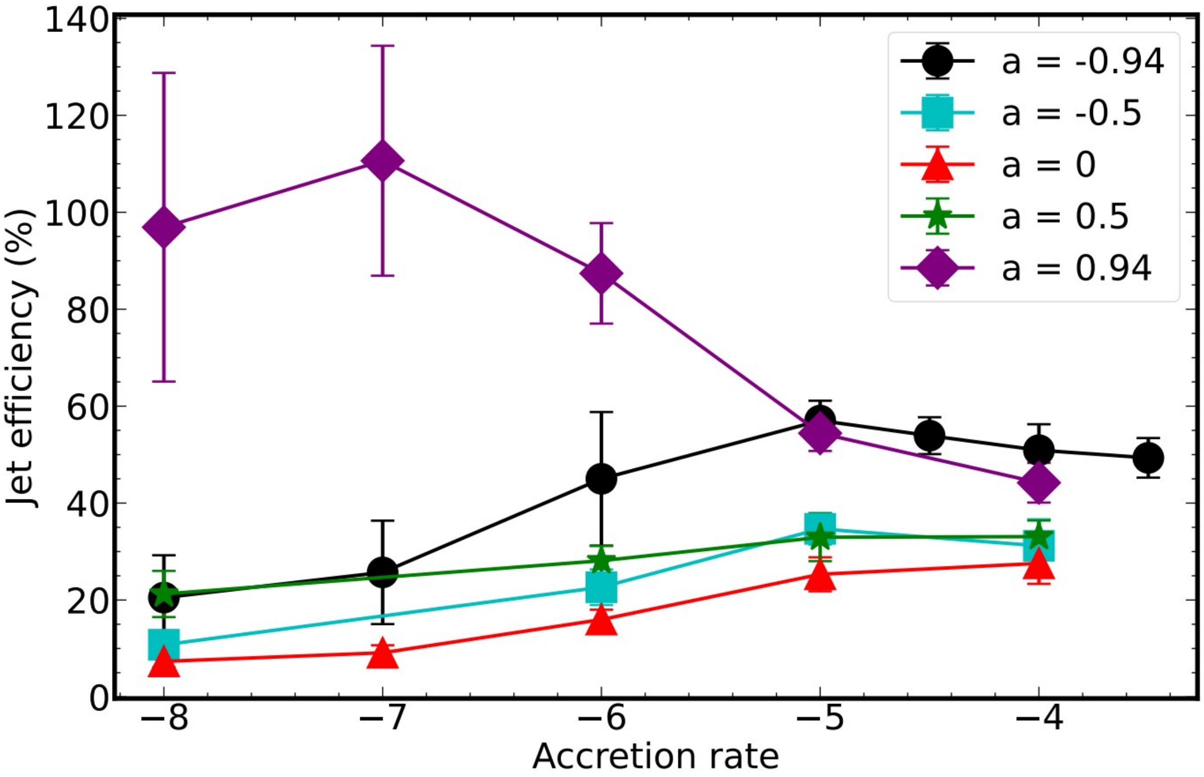
Evidence for the BZ mechanism



Bégué, Pe'er et al. (2023)
Zhang, **Bégué** et al. (2024)

Predicted = Tchekhovskoy et al. (2010)

The dynamics: MAD with radiative cooling



See talk by Akshay Singh this afternoon.

Singh, **Bégué** and Pe’er (ApJL 2025)
Singh, **Bégué** and Pe’er (2025, in prep)

The radiation

with J. Wallace and Asaf Pe’er

General relativistic radiation MHD

What happens when radiation affects the dynamics ?

- Supernovae explosion
- GRB jets (neutrino vs magnetic)

The radiative dynamical equations becomes:

$$(\rho u^\mu)_{;\mu} = 0,$$

$$(T^\mu_{\ \nu})_{;\mu} = G_\nu,$$

$$(R^\mu_{\ \nu})_{;\mu} = -G_\nu,$$

where

$$\widehat{R} = \begin{bmatrix} \widehat{E} & \widehat{F}^i \\ \widehat{F}^j & \widehat{P}^{ij} \end{bmatrix}$$

$$\widehat{E} = \int \widehat{I}_\nu \,\mathrm{d}\nu\,\mathrm{d}\Omega,$$

$$\widehat{F}^i = \int \widehat{I}_\nu \,\mathrm{d}\nu\,\mathrm{d}\Omega\, N^i,$$

$$\widehat{P}^{ij} = \int \widehat{I}_\nu \,\mathrm{d}\nu\,\mathrm{d}\Omega\, N^i N^j$$

Sadowski et al. (2013) ; McKinney et al. (2014), Jiang (2021), Asahina et al. (2020)

General relativistic radiation MHD

$$(\rho u^{\mu})_{;\mu} = 0,$$

In addition: $$G^{\nu} = \int (\chi_{\nu} I_{\nu} - \eta_{\nu})\,\mathrm{d}\nu\,\mathrm{d}\Omega\,N^{i},$$

$$(T^{\mu}_{\ \nu})_{;\mu} = G_{\nu},$$

In the comoving frame:

$$(R^{\mu}_{\ \nu})_{;\mu} = -G_{\nu},$$

$$\widehat{G} = \begin{bmatrix} \kappa(\widehat{E} - 4\pi\widehat{B}) \\ \chi\widehat{F}^{i} \end{bmatrix}.$$

- Assuming a closure relation gives P^{ij} (E, Fi). (loss of angular resolution).
- In that case, the specific intensity is not solved for.

Drawbacks
- Not suitable for characterizing observables (spectrum, light-curve)
- Not suitable for regions in which angular dependence of the radiation field becomes important.

What does it take to solve for I_ν ?

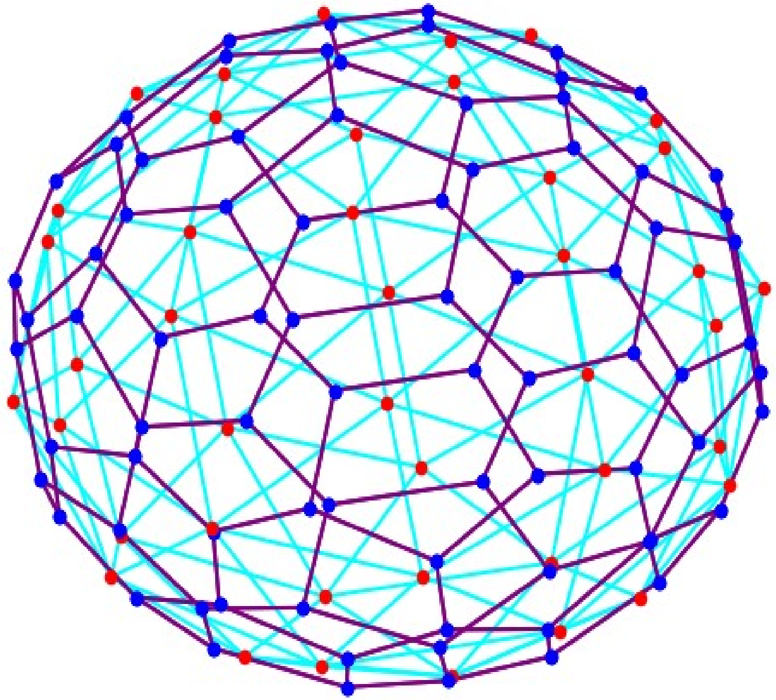
We have to resolve everywhere **in space,** the **angular** and **frequency** dependent quantity I_ν.

$$N_{tot} = N_{x_1} N_{x_2} N_{x_3} N_{angle} N_{frequency}$$

Tractable with GPUs

Angular grid: We use a geodesic grid

Pro: Isotropic discretization of the sphere
(avoid pole problem).
Con: Slightly complicated bookkeeping



Grid level 2: 162 hexagons and pentagons

Randall et al. (2000, 2002)

What it takes to solve for I_ν ?

We need an evolutionary equation (radiative transfer equation):

$$\nabla_\alpha(\hat{n}^\alpha \hat{n}_\beta \hat{I}_\nu) + \partial_{\hat{\nu}}(\hat{n}^{\hat{\nu}} \hat{n}_\beta \hat{I}_\nu) + \hat{s}^{-1}\partial_{\hat{\zeta}}(\hat{s}\hat{n}^{\hat{\zeta}} \hat{n}_\beta \hat{I}_\nu) + \partial_{\hat{\psi}}(\hat{n}^{\hat{\psi}} \hat{n}_\beta \hat{I}_\nu) = \hat{n}_\beta(\hat{j}_\nu - \hat{\alpha}_\nu \hat{I}_\nu)$$

Transport term

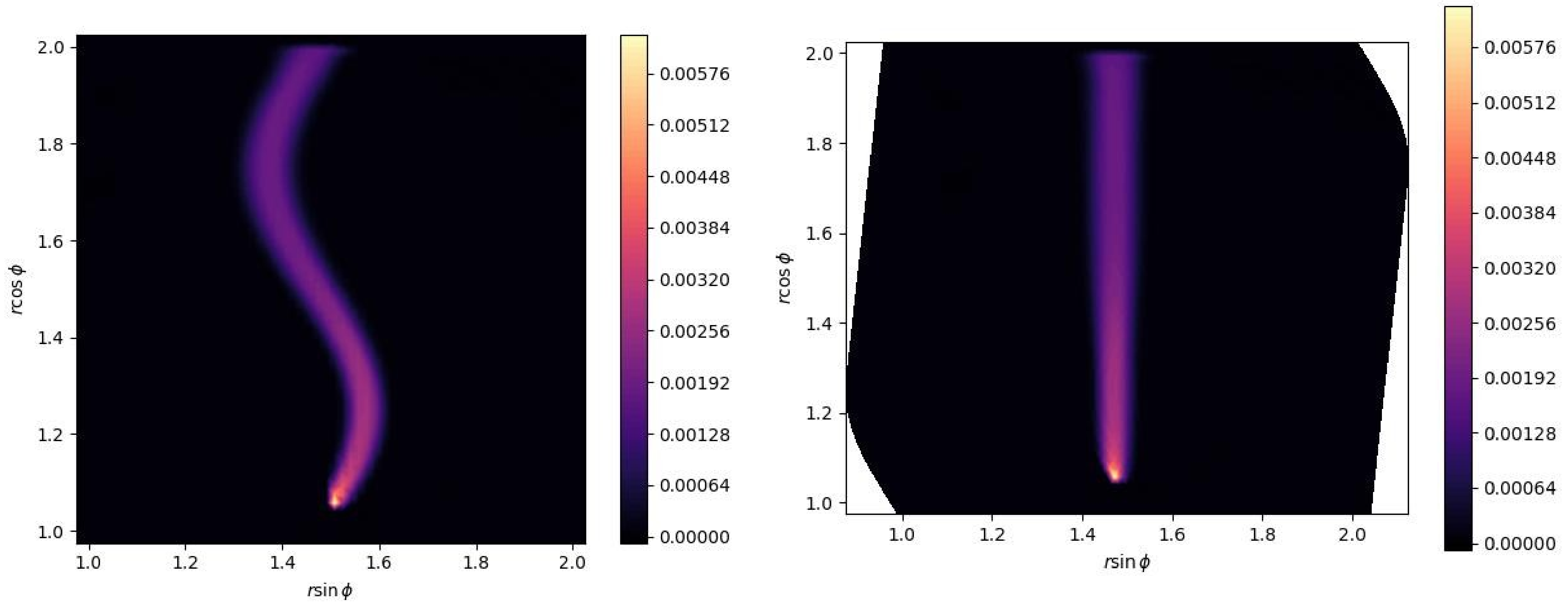
Gravitational Redshift

Change of direction with the coordinate system

Interaction term

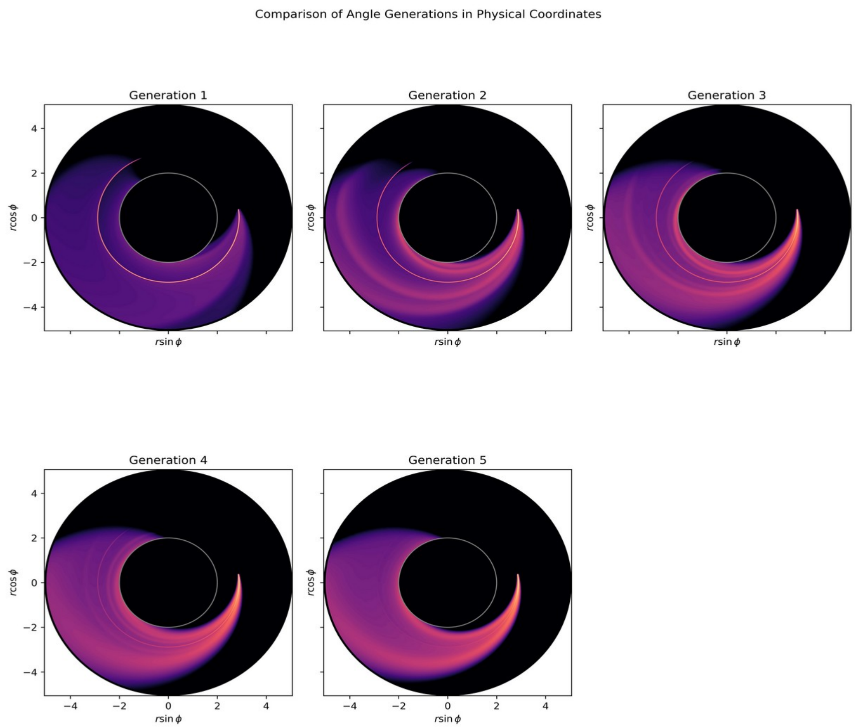
Davis and Gammie (2020), White et al. (2023), Asahina et al. (2020).

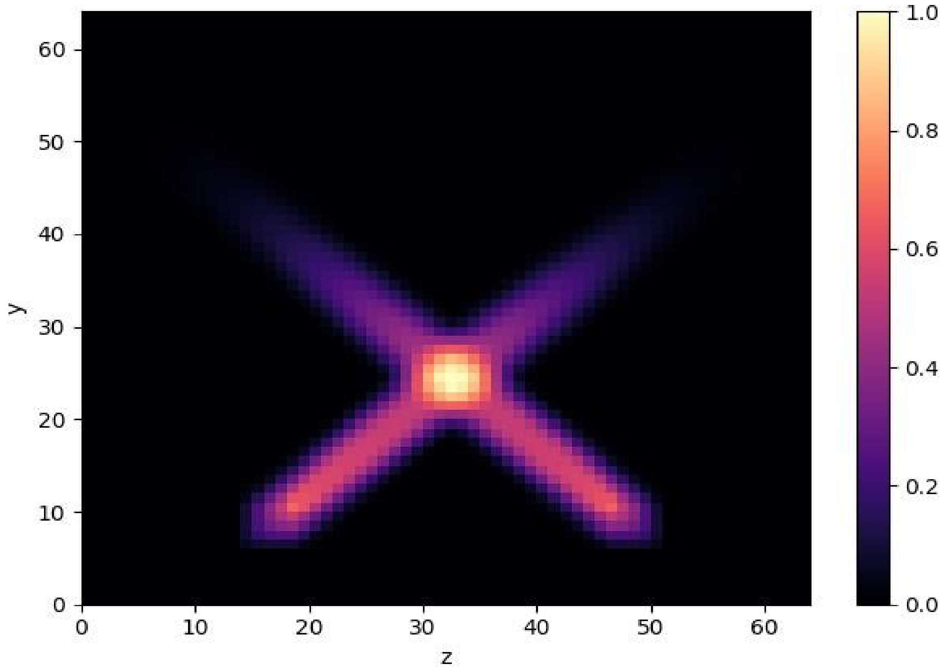
Transport tests: straight line



Curvilinear coordinates properly implemented and functionning for the radiation transport.

Transport tests:

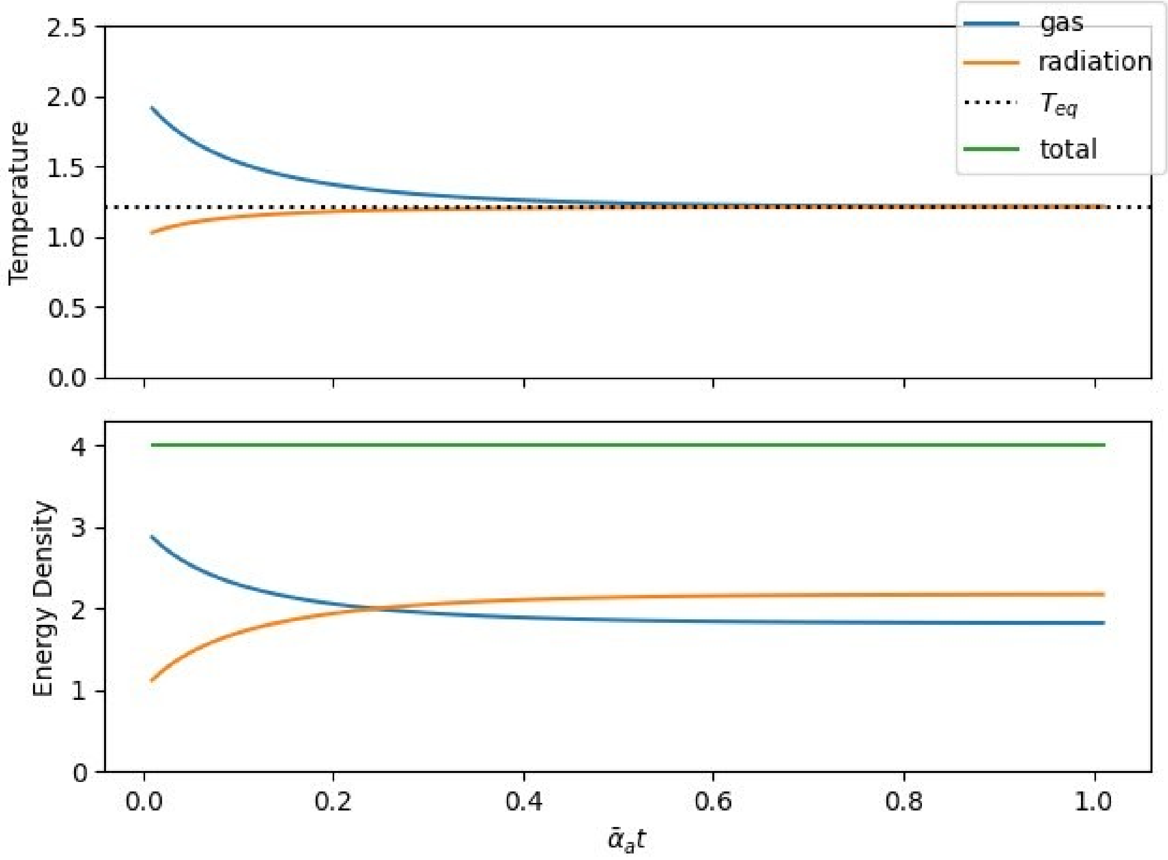




Circular orbit around a rotating BH

Crossing rays
(Would fail with M1 closure)

Interaction term: equilibration tests



1) The radiation and gas temperatures reach equlibrium on the proper timescale.

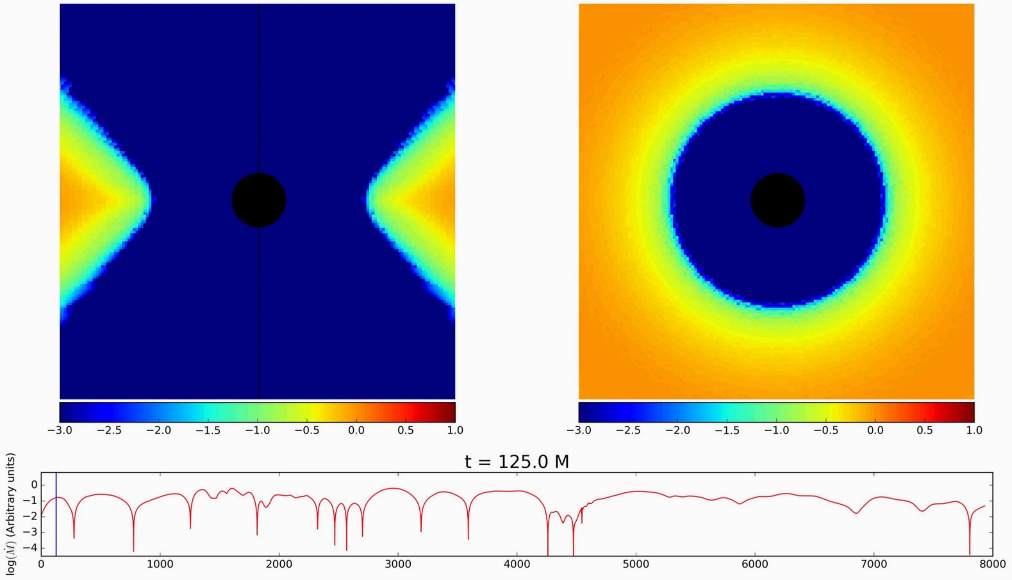
2) The total energy is conserved.

Rad-cuHARM: first full-scale GR-R-MHD simulation !

Spatial resolution: 128 x 64 x 64
Angular resolution: 162 angles (G2)

Initial setup:
- Fishbone and Moncrief disk
- $R_{in} = 6$; $r_{max} = 12$;
- Magnetic field: single loop for SANE
- Mass accretion rate: 0.1 M_{Edd}
- $M_{BH} = 10\ M_{sun}$
- Spin a = 0.94

Opacities:
- Scattering: $\kappa^s = 0.4\ cm^2\ g^{-1}$
- Absorption: $\kappa^a = 8 \times 10^{22}\ \rho\ T^{-3.5}\ cm^2\ g^{-1}$



We ran our first GR-R-MHD simulation !

Wallace, **Bégué**, Pe'er (in prep)

Conclusions

- We wrote cuHARM a 3D GR-MHD code.
  - Highly optimized
  - It uses GPUs for accelerating the computation.
- We just finalysed the addition of the radiation sector (Wallace et al., 2025 in prep)
- We just ran our first GR-R-MHD simulation.
- Applicability:
  - Systems with large accretion rate $10^{-4}\ L_{edd} < L < 10\ L_{edd}$
  - Transients with sub and super Eddington luminosity (e.g. GRBs, X-ray flares ...)
  - Transition from optically thick to optically thin regimes.

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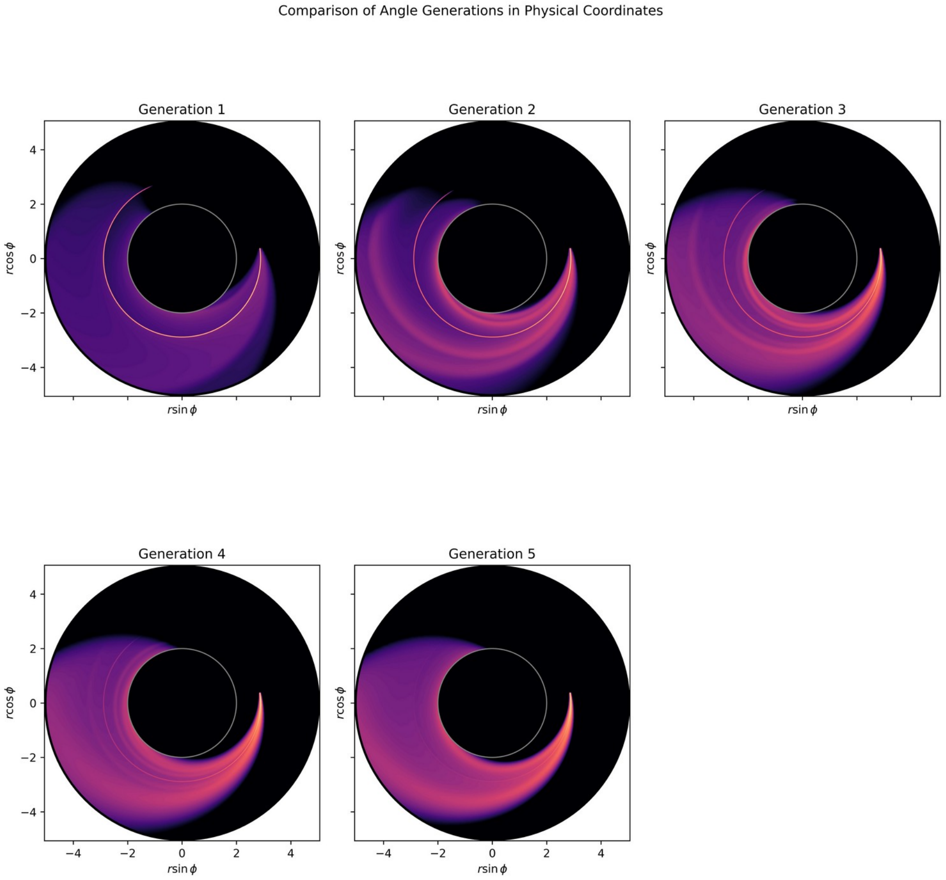
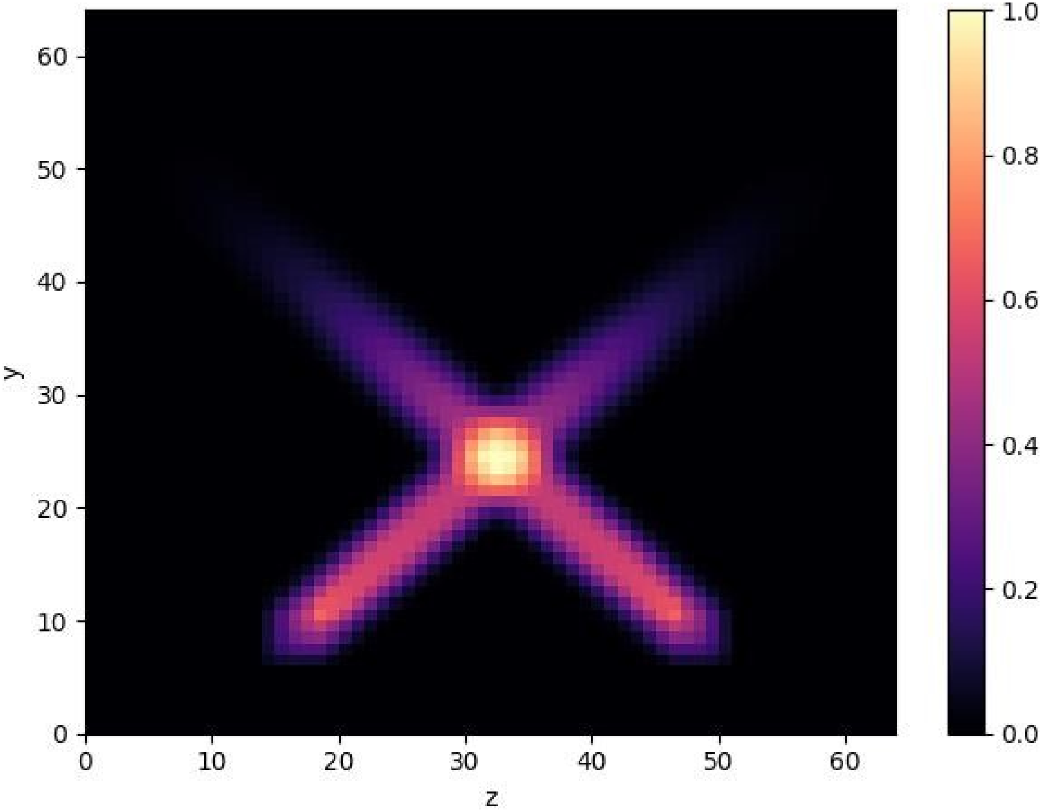
Backup slides

Transport:



Comparison of Angle Generations in Physical Coordinates
Generation 1
Generation 2
Generation 3
Generation 4
Generation 5
$r\cos\phi$
$r\sin\phi$



y
z

Next challenges:

Next challenges:

- Anisotropic and energy dependent interactions on the geodesic grid (CS, synchroton, ...)
- Electron temperature model: 2-T plasma evolution, proton/electron coupling.
- Numerical optimization of the radation modules.

General relativistic radiation MHD

All the physics is hidden in the definition of the radiation stress energy tensor.

$$(\rho u^{\mu})_{;\mu} = 0,$$

$$(T^{\mu}_{\ \nu})_{;\mu} = G_{\nu},$$

$$(R^{\mu}_{\ \nu})_{;\mu} = -G_{\nu},$$

$$\widehat{R} = \begin{bmatrix} \widehat{E} & \widehat{F}^i \\ \widehat{F}^j & \widehat{P}^{ij} \end{bmatrix}$$

How to write G_ν ?

Mihalas and Mihalas (84)

$$G^{\nu} = \int (\chi_{\nu} I_{\nu} - \eta_{\nu})\, \mathrm{d}\nu\, \mathrm{d}\Omega\, N^{i},$$

In the comoving frame:

$$\widehat{G} = \begin{bmatrix} \kappa(\widehat{E} - 4\pi\widehat{B}) \\ \chi\widehat{F}^i \end{bmatrix}.$$

General relativistic radiation MHD

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$$\widehat{G} = \begin{bmatrix} \kappa(\widehat{E} - 4\pi\widehat{B}) \\ \chi\widehat{F}^i \end{bmatrix}.$$

Questions:

What did we gain?

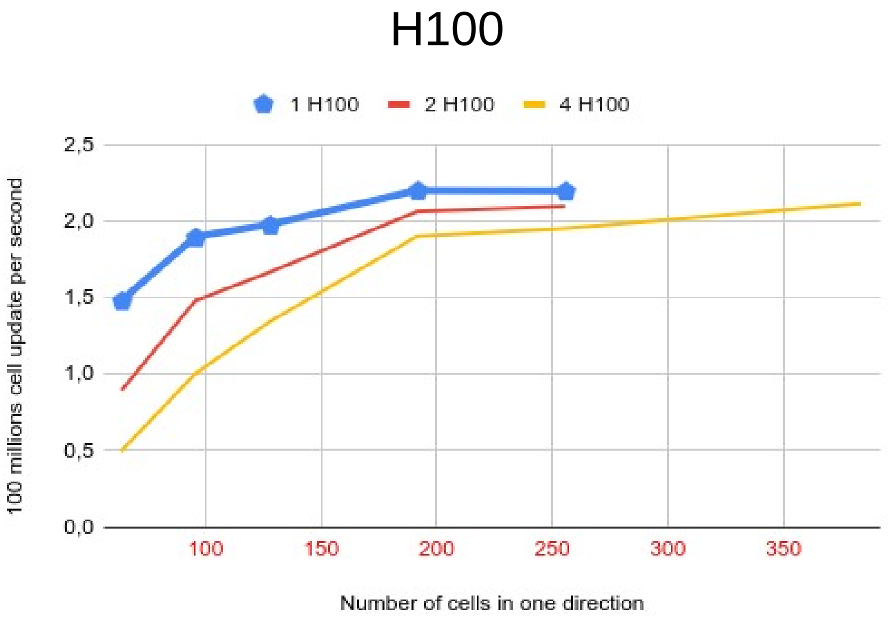
- relative simplicity

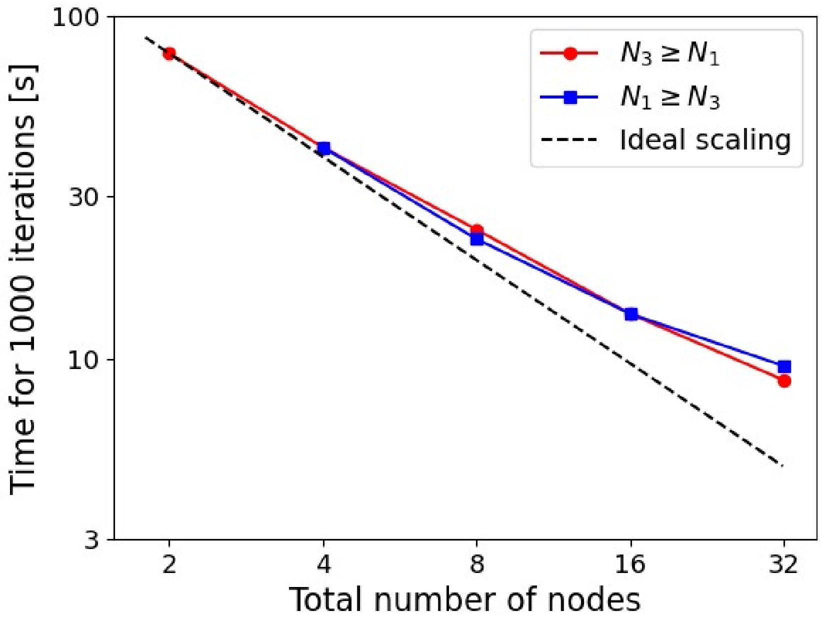
What did we lose:

- accurate description of the anistropy of the radiation field
- averaged emissivity and absorption factor
- the use of a closure relation.

Why not solve for the evolution of I_ν?

Throughput





~200 millions cell update per second per H100 card.

Strong scaling 512 x 256 x 512

Numerical study of SANE/MAD accretion disks

SANE:

Standard And Normal Evolution

Magnetic field **does not** regulate the accretion

Why SANE ?

- Numerically simpler than MAD
- Well-studied
- EHT code comparison paper: we can check our results.

MAD:

Magnetically Arrested Disk

Magnetic field **does** regulate the accretion

Why MAD ?

- More interesting phenomenology,
- Produces powerful jets,
- More relevant for EHT results,
- Role of magnetic fields not fully understood.

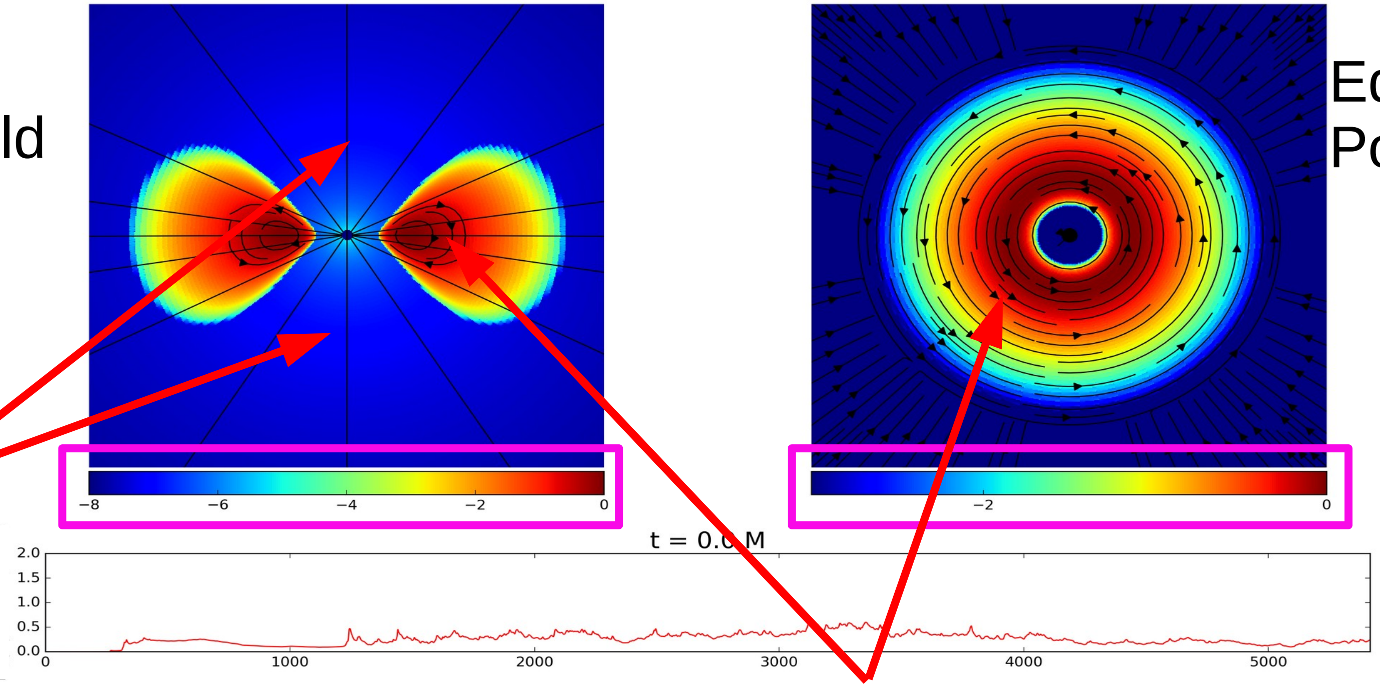
Initial setup:

Fishbone and Moncrief (1976)

Matter density

Polar cut
Toroidal field

Equatorial cut
Poloidal velocity



Jets

Turbulence