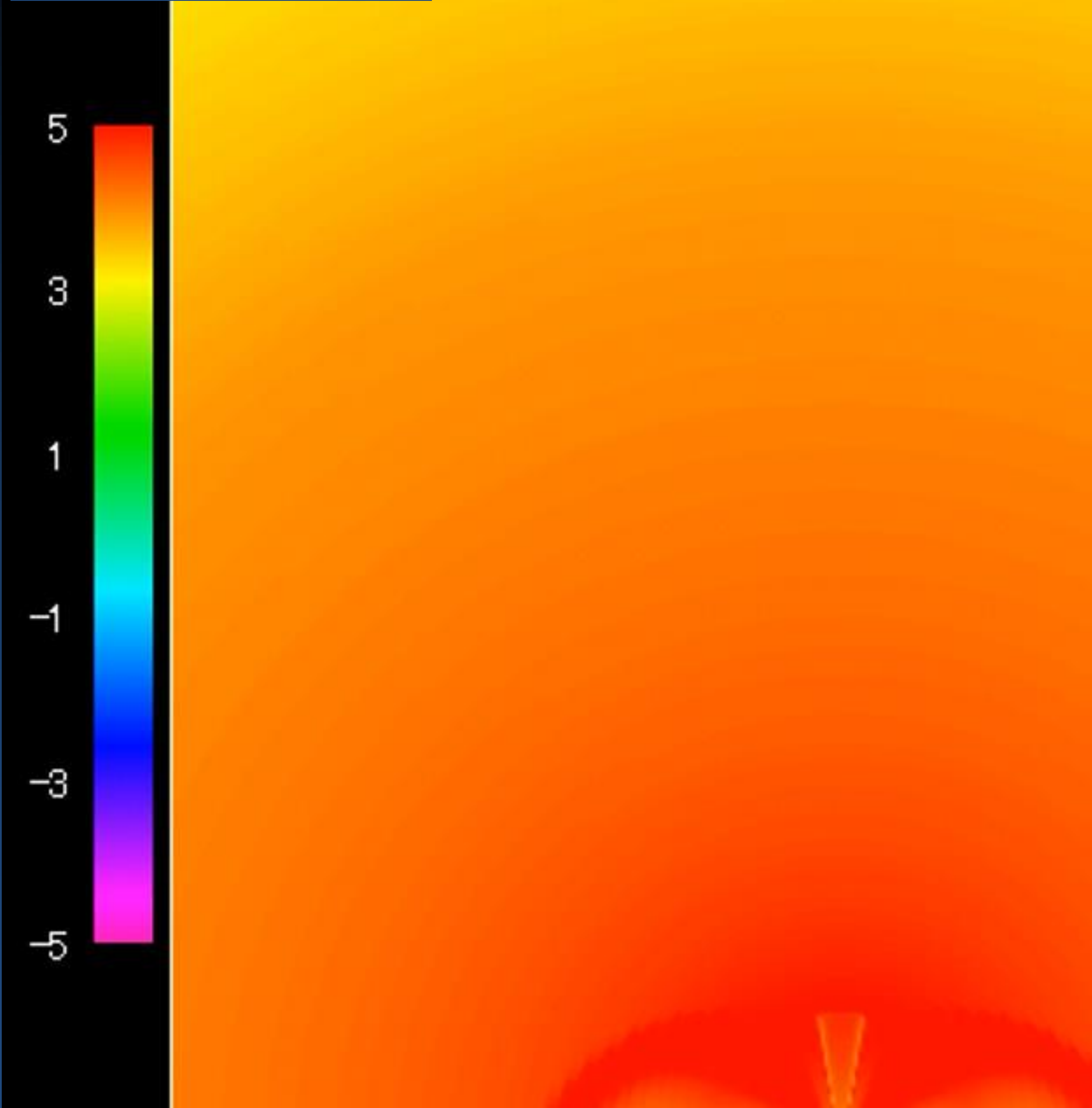
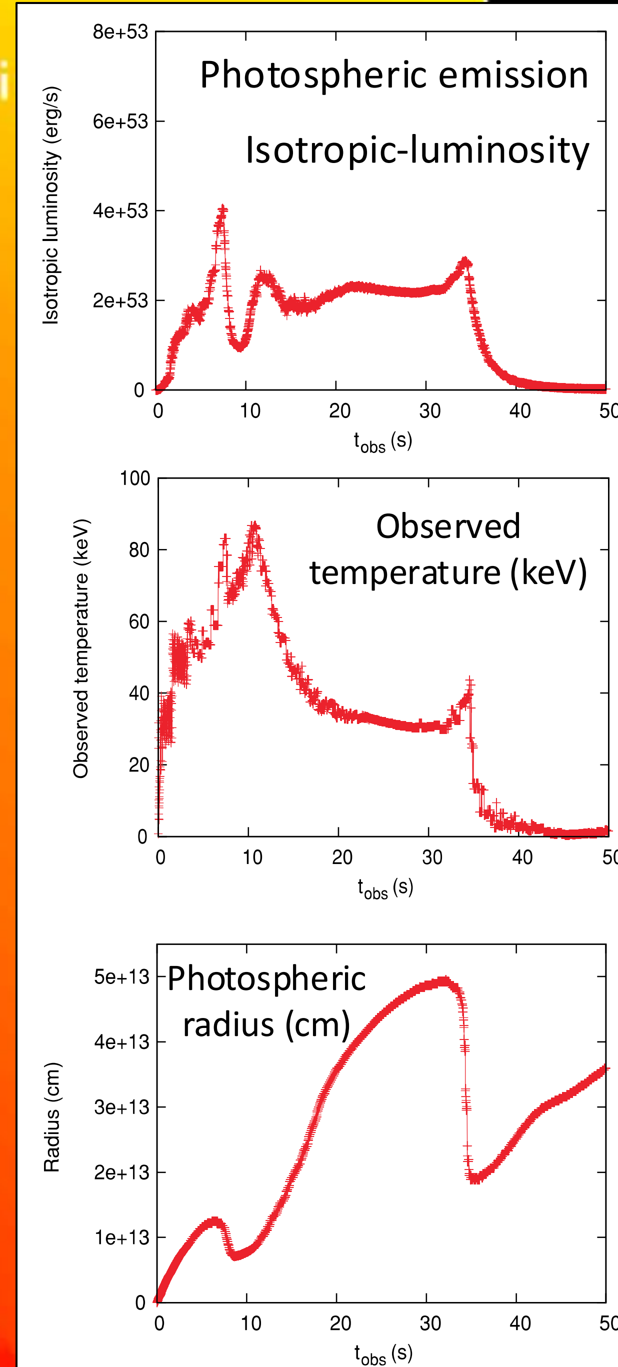
Quantum kinetics of neutrinos in dense astrophysical environments

Hiroki Nagakura
(National Astronomical Observatory of Japan)

GRB-jet (collapsar)

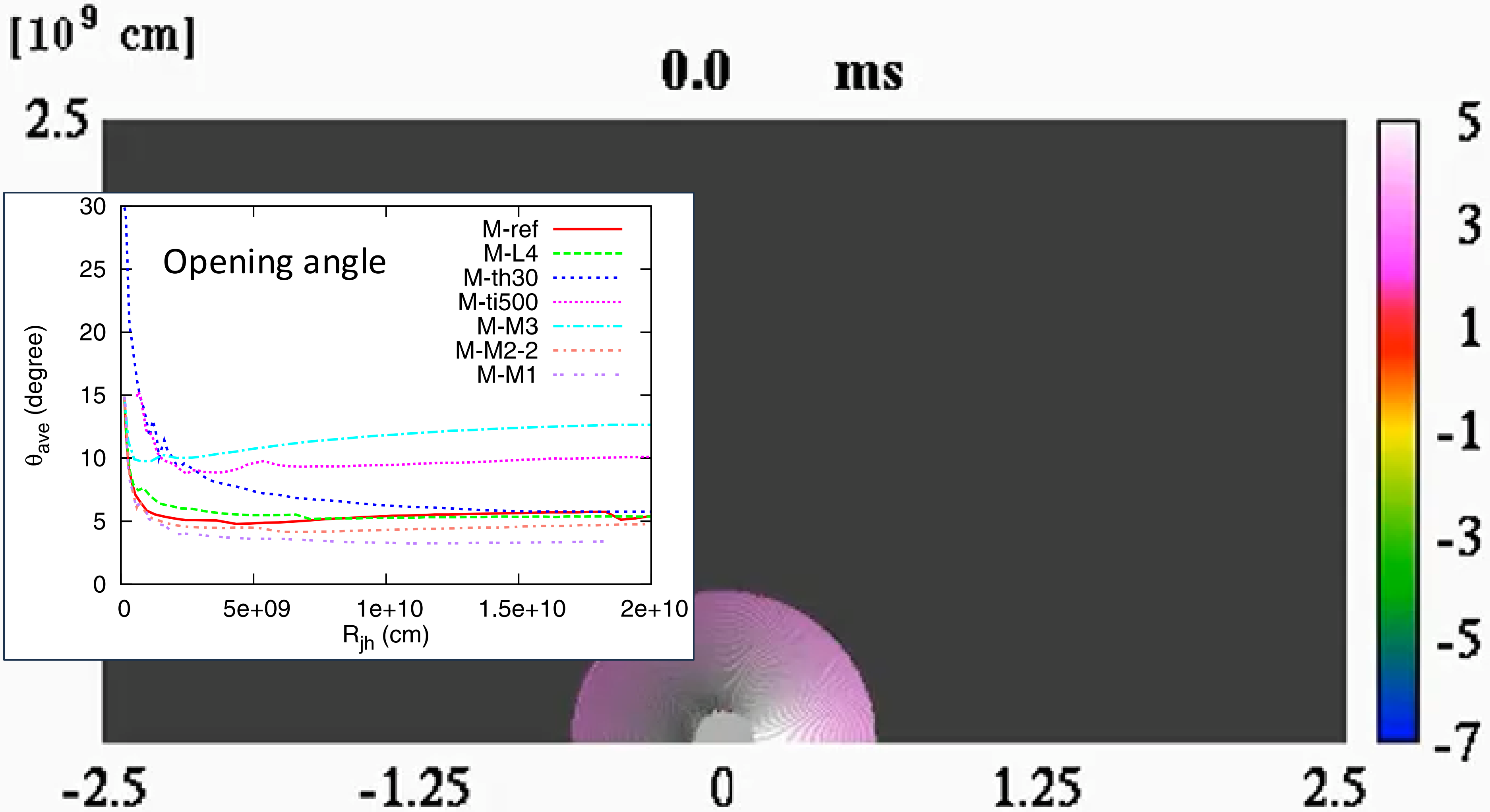
Nagakura et al. 2011





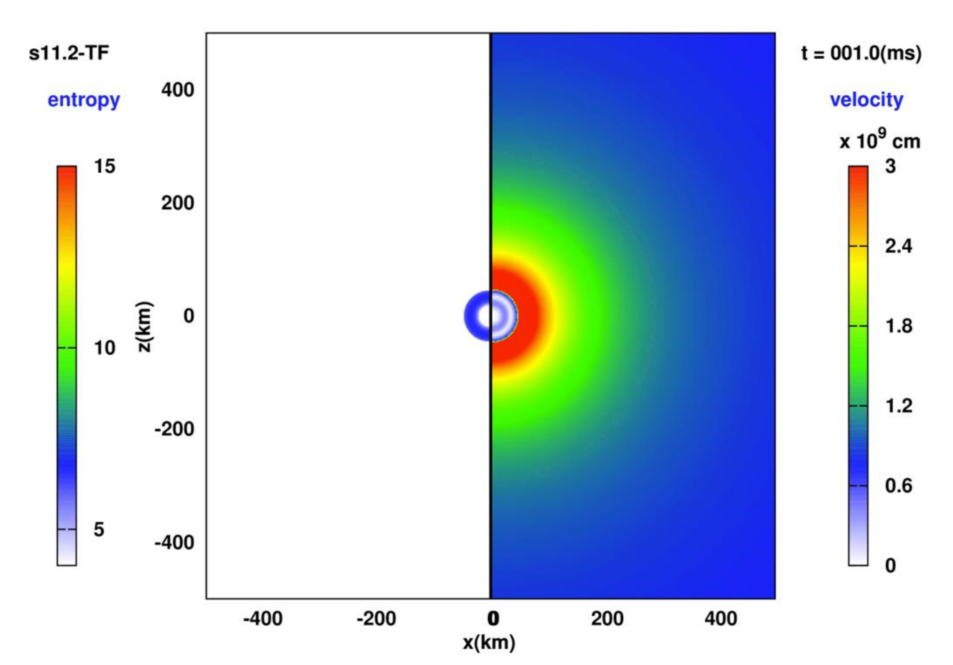
GRB-jet (penetrating binary neutron star merger ejecta)

Nagakura et al. 2014



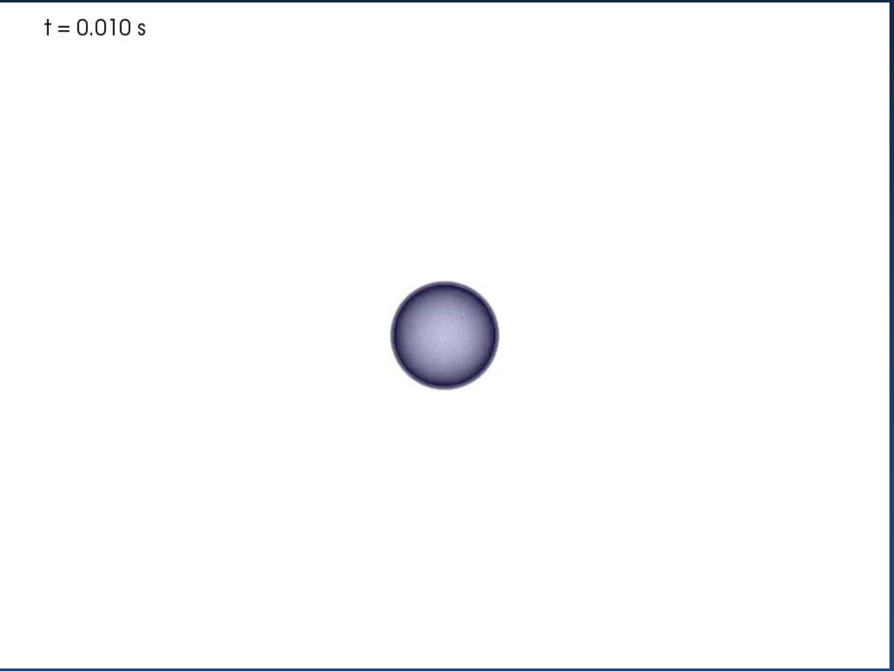
Multi-dimensional core-collapse supernova simulations (Neutrino-radiation-hydrodynamic simulations) 2014～

Full Boltzmann neutrino transport



Nagakura et al. ApJL 2019

Two-moment neutrino transport



Nagakura et al. MNRAS 2019

Today's topic:

"Quantum kinetics of neutrinos in dense astrophysical environments"

Neutrino transport with
neutrino oscillation (flavor conversion)

Core collapse supernova (CCSN)
Binary neutron star merger (BNSM)

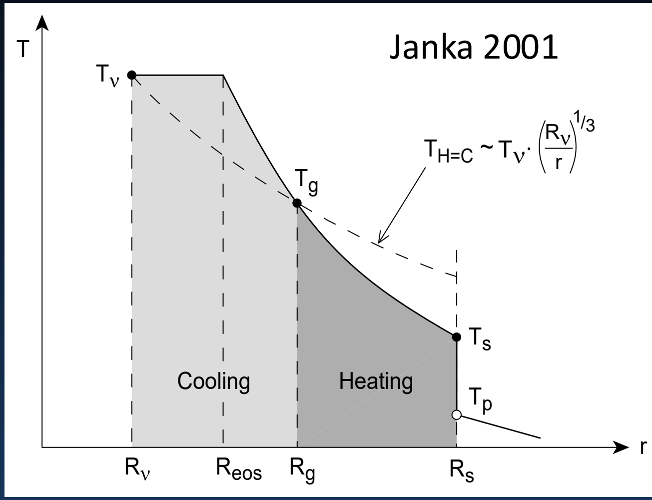
Neutrino-heating mechanism for CCSN explosions

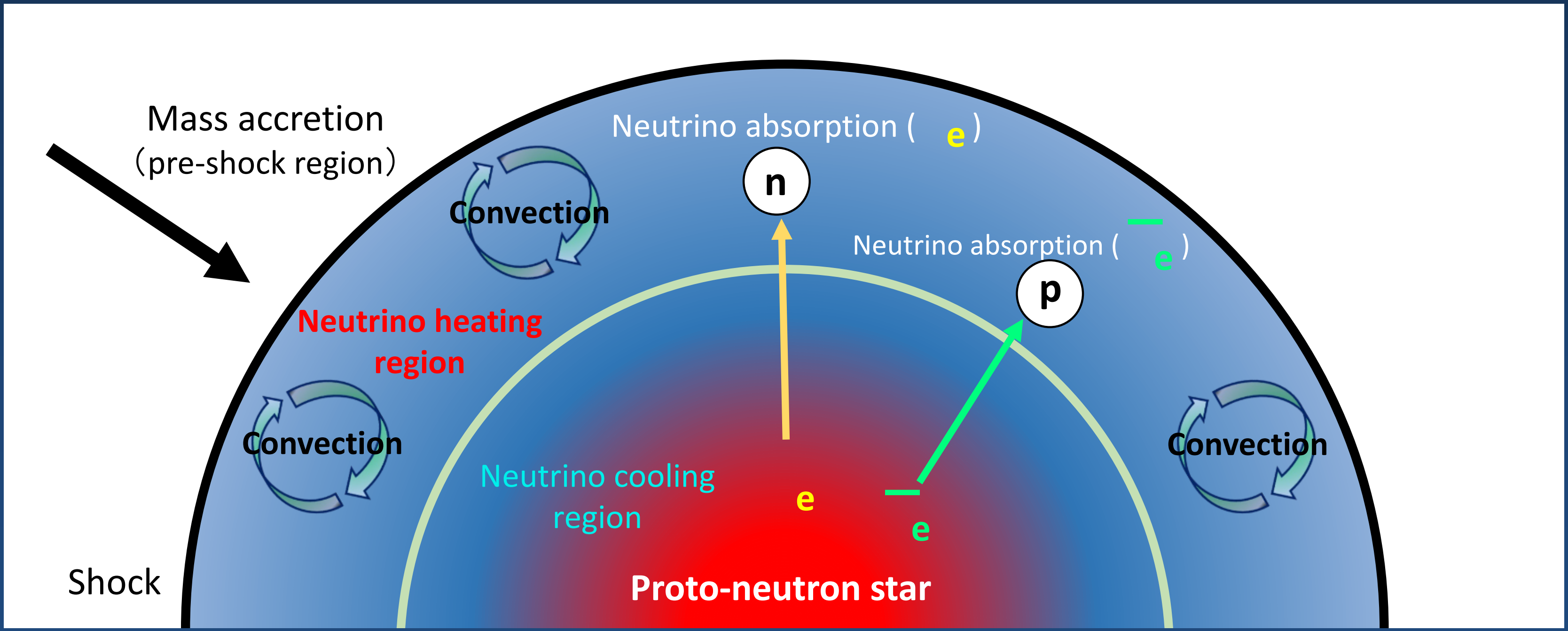
✓Neutrino Heating Rate

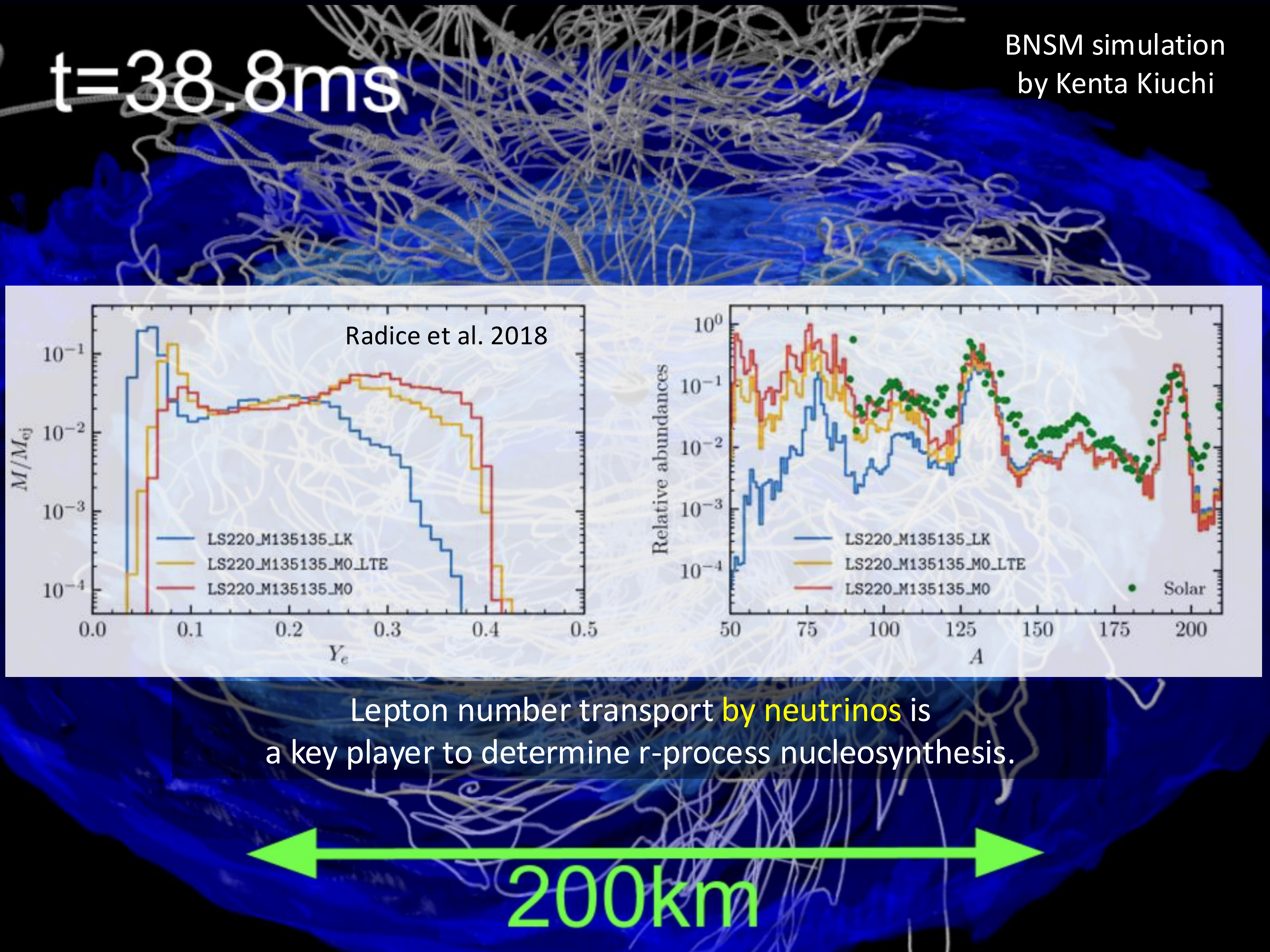
$$Q_\nu^+ \approx 160\mathrm{Mev/s}\frac{\rho}{m_a}\frac{L_{\nu_e,52}}{r_7^2\langle\mu_\nu\rangle}\left(\frac{T_{\nu_e}}{4\mathrm{MeV}}\right)^2$$

✓Neutrino Cooling Rate

$$Q_\nu^- \approx 145\mathrm{Mev/s}\frac{\rho}{m_a}\left(\frac{T}{2\mathrm{MeV}}\right)^6$$

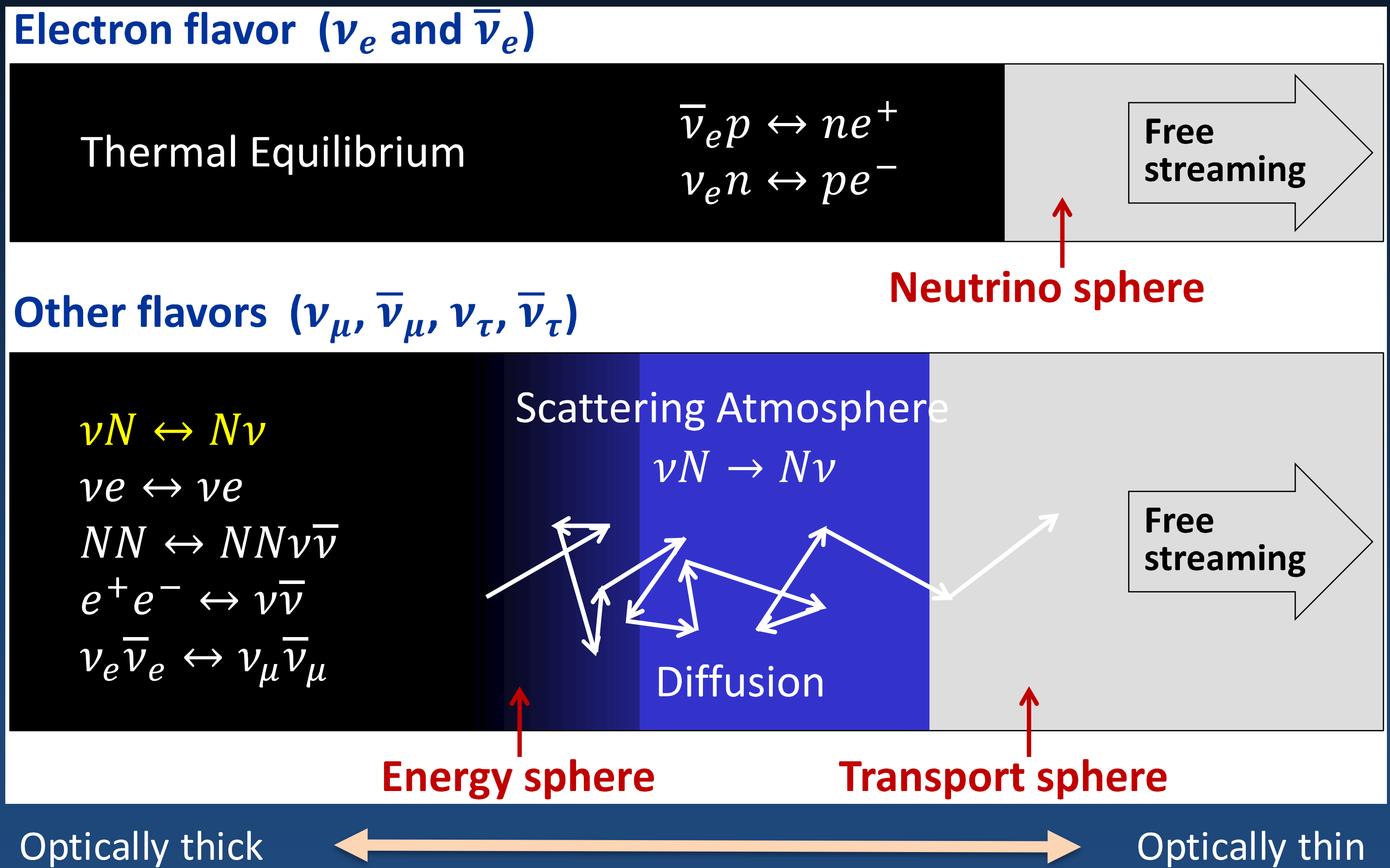





t=38.8ms
BNSM simulation
by Kenta Kiuchi
Radice et al. 2018
M/M_{ej}
Y_e
LS220_M135135_LK
LS220_M135135_M0_LTE
LS220_M135135_M0
Relative abundances
A
Solar
Lepton number transport by neutrinos is
a key player to determine r-process nucleosynthesis.
200km

A kinetic framework is essential for modeling of neutrino radiation field

Figure by Janka 2017

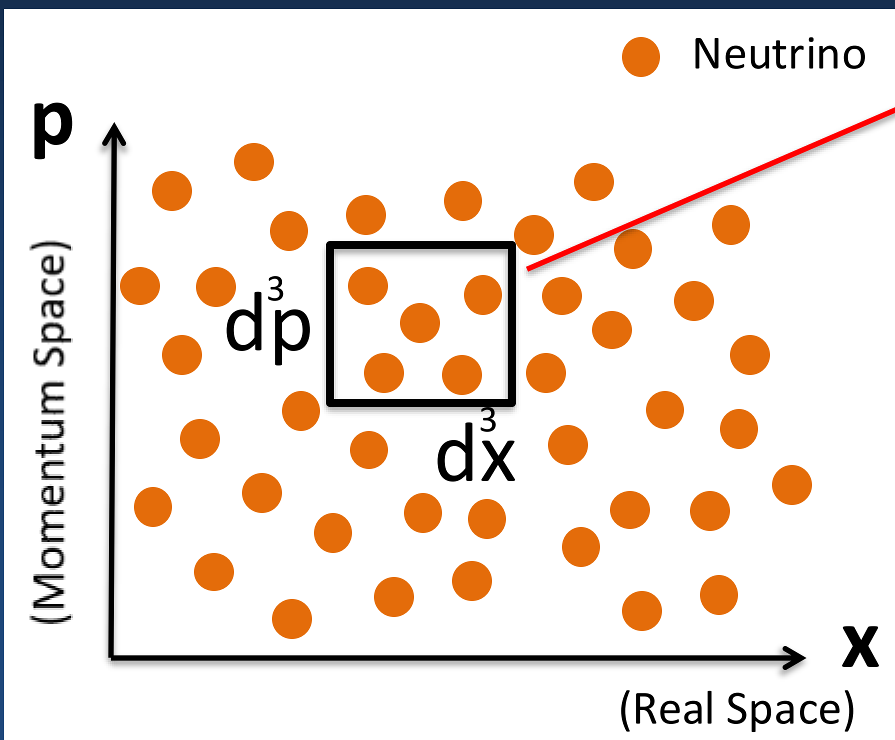


Boltzmann neutrino transport

$$p^{\mu}\frac{\partial f}{\partial x^{\mu}} + \frac{dp^{i}}{d\tau}\frac{\partial f}{\partial p^{i}} = \left(\frac{\delta f}{\delta \tau}\right)_{\mathrm{col}},$$

(Time evolution + Advection Term) (Collision Term)

6-dimensional phase space



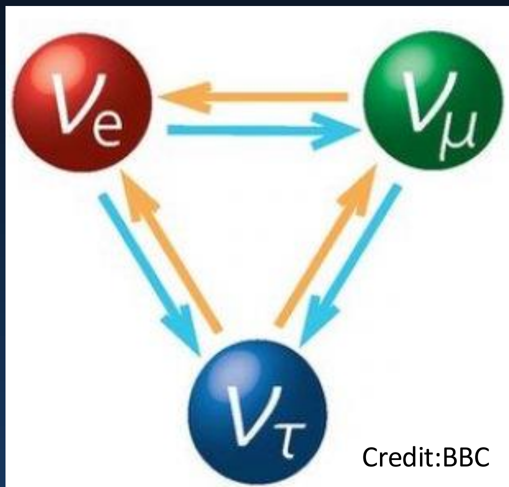
$$dN = f(t, \boldsymbol{p}, \boldsymbol{x})d^3p\,d^3x$$

Conservative form of GR Boltzmann eq.

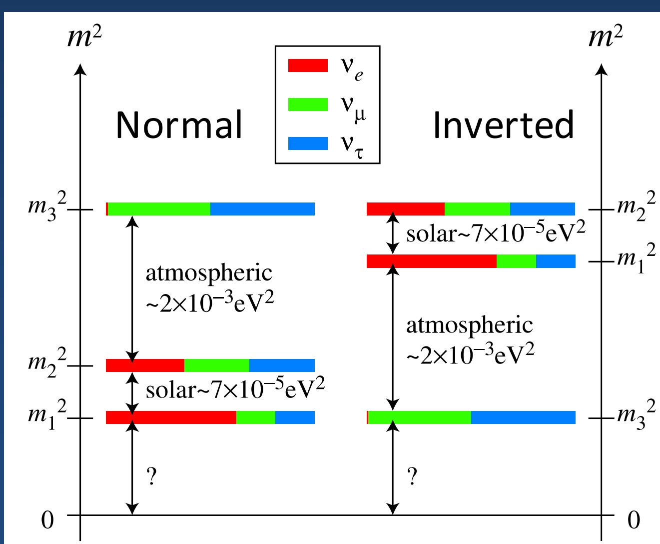
$$\frac{1}{\sqrt{-g}}\frac{\partial\left(\sqrt{-g}\,\nu^{-1}p^{\alpha}f\right)}{\partial x^{\alpha}}\Bigg|_{q_{(i)}} + \frac{1}{\nu^{2}}\frac{\partial}{\partial\nu}\left(-\nu f p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(0)}\right)$$
$$+\frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}\left(\nu^{-2}\sin\bar{\theta} f\sum_{j=1}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\theta}}\right)$$
$$+\frac{1}{\sin^{2}\bar{\theta}}\frac{\partial}{\partial\bar{\varphi}}\left(\nu^{-2} f\sum_{j=2}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\varphi}}\right) = S_{\mathrm{rad}},$$

Shibata, Nagakura, Sekiguchi, and Yamada (2014)

Neutrino oscillations



- There are many experimental evidences that neutrinos can go through flavor conversion.
- Neutrinos have at least three different masses.
- Flavor eigenstates are different from mass eigenstates.



Feruglio et al. 2003

$$|\nu_i\rangle = \sum_\alpha U^*_{\alpha i}\,|\nu_\alpha\rangle\,,$$
Mass state

$$|\nu_\alpha\rangle = \sum_i U_{\alpha i}\,|\nu_i\rangle\,,$$
Flavor state

U: Pontecorvo–Maki–Nakagawa–Sakata matrix (PMNS matrix)

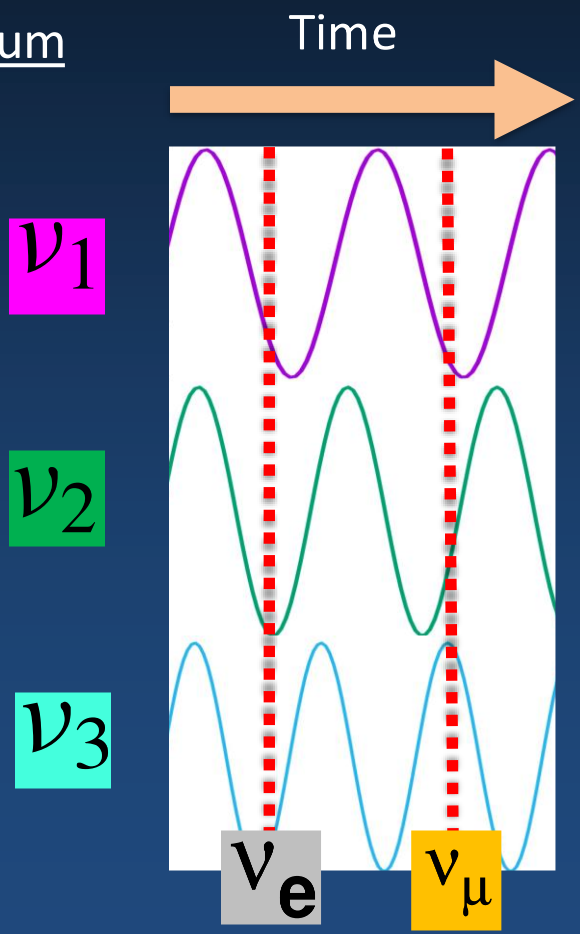
$$U = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{bmatrix}$$
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{bmatrix} \begin{bmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{bmatrix} \begin{bmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$= \begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix} \begin{bmatrix} e^{i\alpha_1/2} & 0 & 0 \\ 0 & e^{i\alpha_2/2} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

Neutrino oscillation with a plane-wave picture

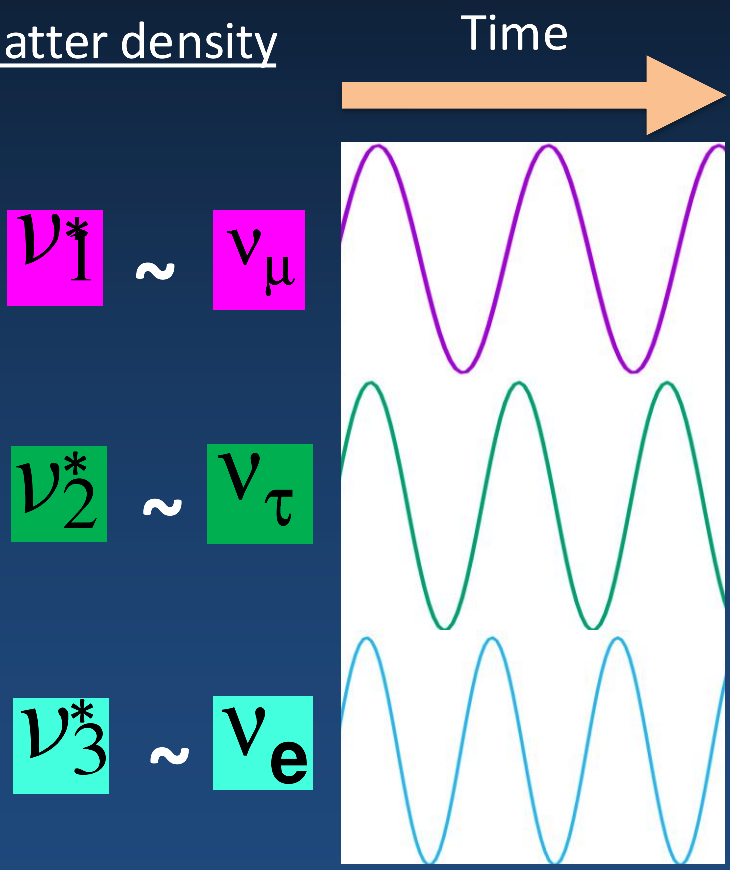
$$\psi_\ell(0,0) \equiv |\nu_\ell\rangle = \sum_{i=1}^{3} U_{\ell i}^{*} \, |\nu_i\rangle$$

$$\psi_\ell(t,x) = \sum_{i=1}^{3} U_{\ell i}^{*} \, |\nu_i\rangle \, e^{-i\phi_i}, \qquad \text{phase } \phi_i = E_i \, t - p_i \, x$$

Vacuum



High matter density

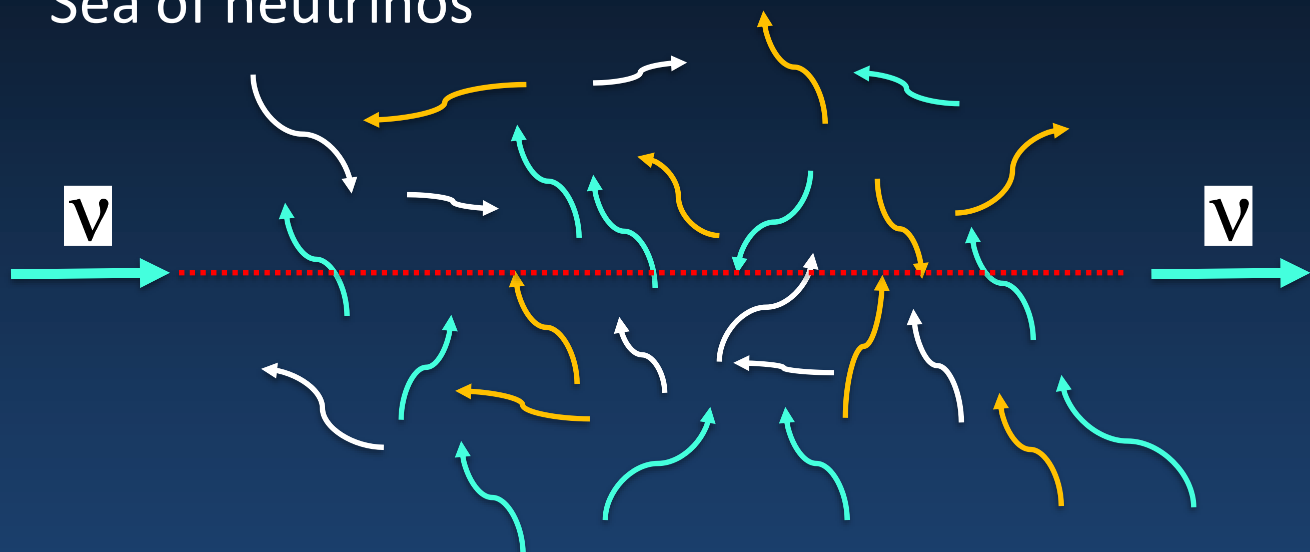


Boltzmann transport becomes a reasonable approximation.

Neutrino oscillation induced by self-interactions

Pantalone 1992

Sea of neutrinos



1. Refractions by self-interactions induce neutrino flavor conversions, which is analogy to matter effects (e.g., MSW resonance).

2. The oscillation timescale is much shorter than the global scale of CCSN/BNSM.

3. Collective neutrino oscillation induced by neutrino-self interactions commonly occurs in CCSNe and BNSM environments.

Rich flavor-conversion phenomena driven by neutrino-neutrino self-interactions

Vacuum: $\omega = \frac{\Delta m^2}{2E_\nu}$,
Matter: $\lambda = \sqrt{2} G_F n_e$,
Self-int: $\mu = \sqrt{2} G_F n_\nu$,

- Slow-mode (Duan et al. 2010)

- Energy-dependent flavor conversion occurs.
- The frequency of the flavor conversion is proportional to $\sqrt{\omega\mu}$

- Fast-mode (FFC) (Sawyer 2005)

- Collective neutrino oscillation in the limit of ω → 0.
- The frequency of the flavor conversion is proportional to μ
- Anisotropy of neutrino angular distributions drives the fast flavor-conversion.

- Collisional flavor instability (CFI) (Johns 2021)

- Asymmetries of matter interactions between neutrinos and anti-neutrinos drive flavor conversion.

$$\mathrm{Im}\,\Omega \simeq \pm \frac{\Gamma - \bar{\Gamma}}{2} \frac{\mu S}{\sqrt{(\mu D)^2 + 4 \mu S}} - \frac{\Gamma + \bar{\Gamma}}{2}$$

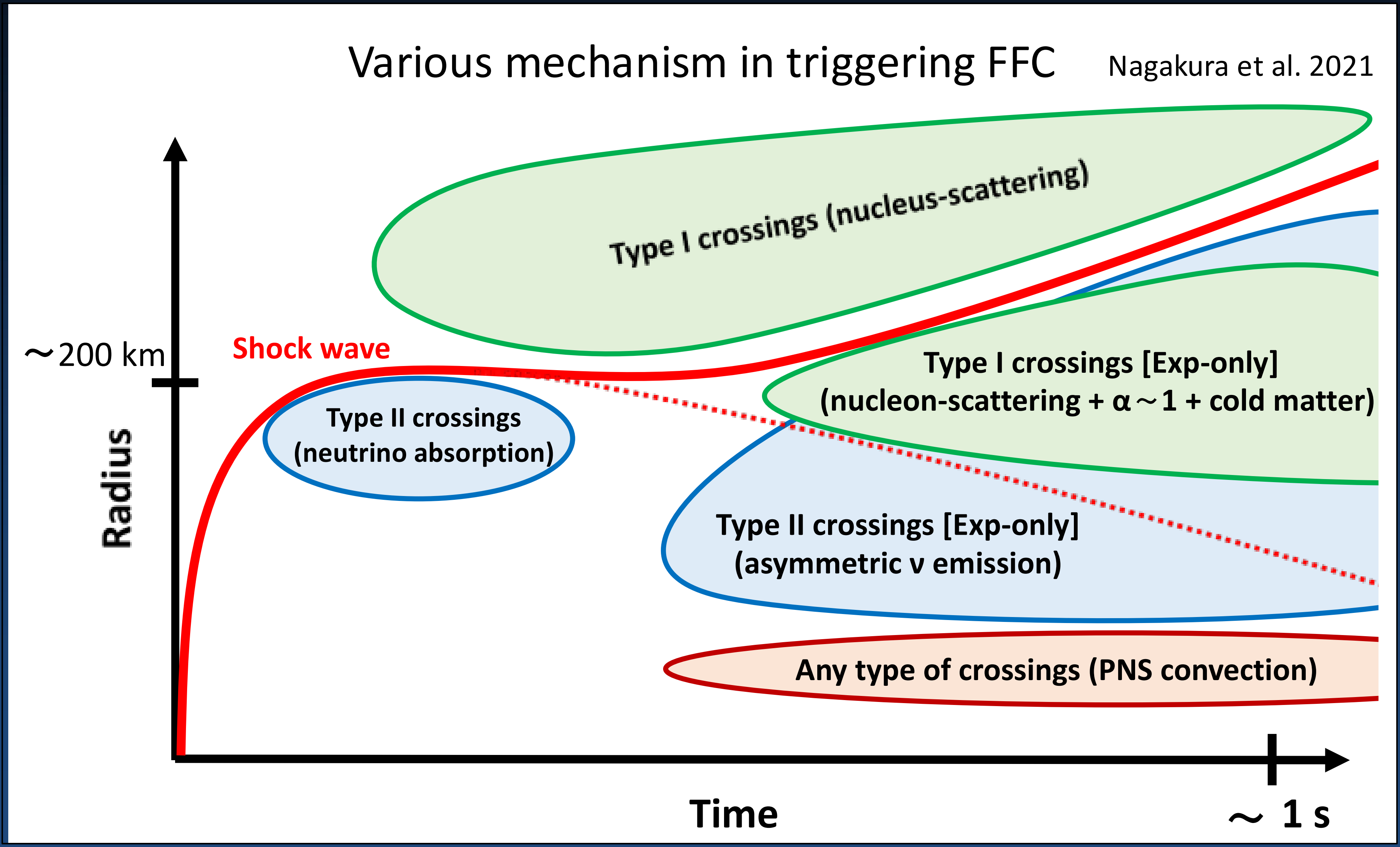
Γ: Matter-interaction rate

- Matter-neutrino resonance (Malkus et al. 2012)

- The resonance potentially occur in BNSM/Collapsar environment (but not in CCSN).
- Essentially the same mechanism as MSW resonance.

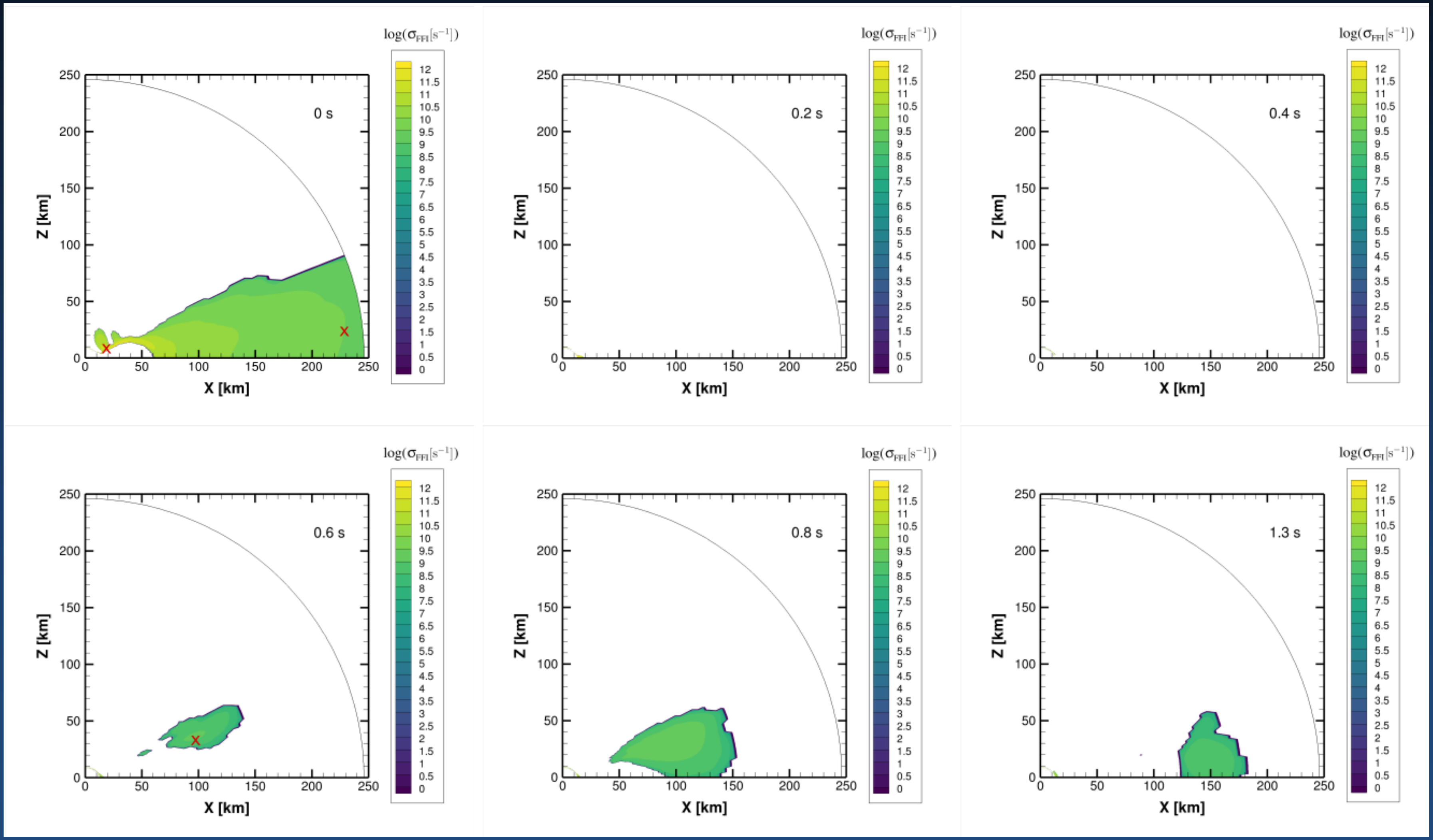
$$|\lambda + \mu| \sim |\omega|$$

FFC and CFI in CCSNe



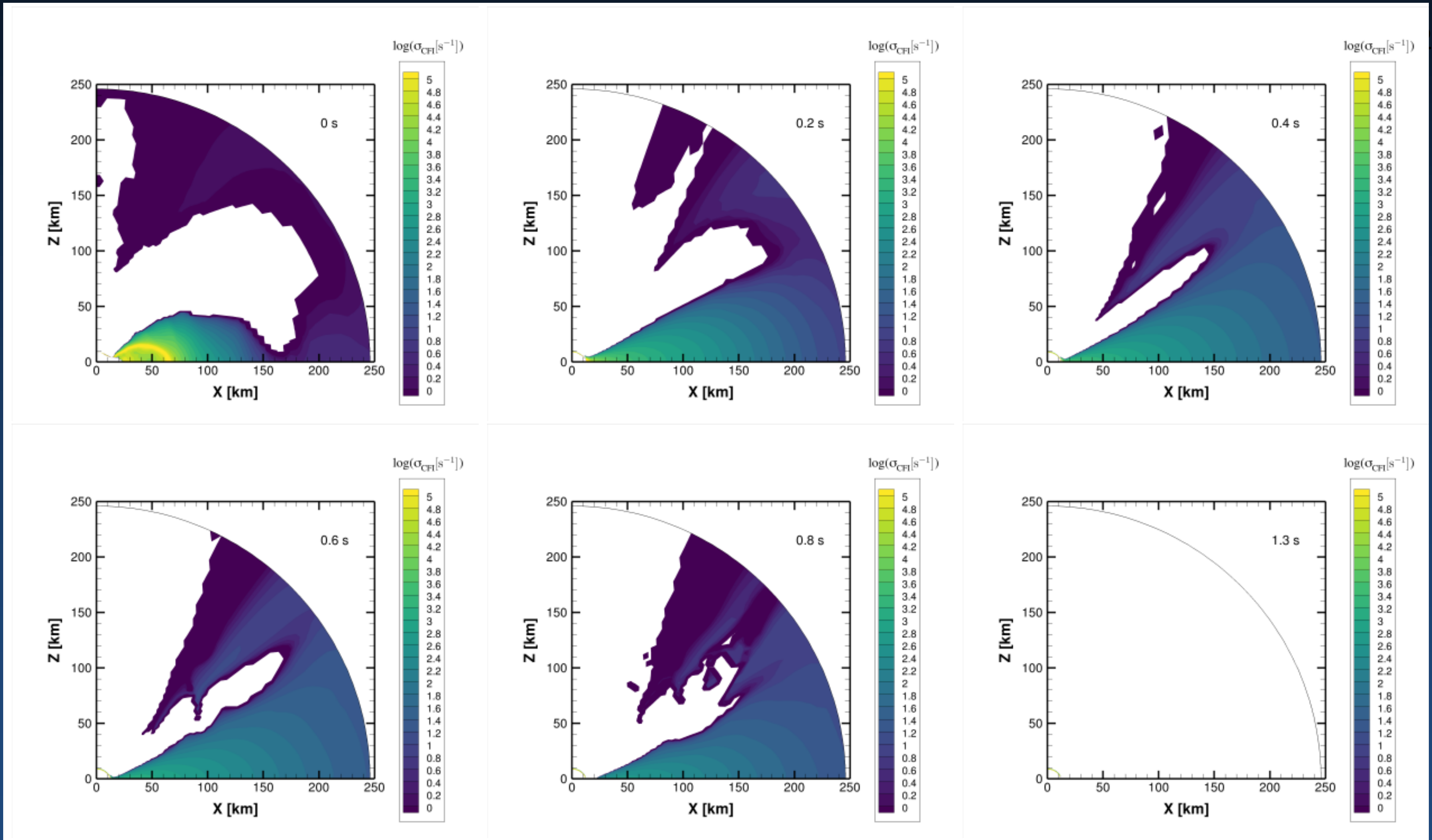
FFC in BNSMs

Nagakura et al. (arXiv:2504.20143)



CFI in BNSMs

Nagakura et al. (arXiv:2504.20143)



Quantum Kinetics neutrino transport:

Vlasenko et al. 2014, Volpe 2015,
Blaschke et al. 2016, Richers et al. 2019

$$\underline{p^\mu \frac{\partial \overset{(-)}{f}}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial \overset{(-)}{f}}{\partial p^i}} = \underline{-p^\mu u_\mu \overset{(-)}{S}_{\mathrm{col}}} + \underline{i p^\mu n_\mu [\overset{(-)}{H}, \overset{(-)}{f}]},$$

Advection terms
(Same as Boltz eq.)

Collision term

Oscillation term

f is not a
“distribution function”

Density matrix

$$\overset{(-)}{f} = \begin{bmatrix} \overset{(-)}{f}_{ee} & \overset{(-)}{f}_{e\mu} & \overset{(-)}{f}_{e\tau} \\ \overset{(-)}{f}_{\mu e} & \overset{(-)}{f}_{\mu\mu} & \overset{(-)}{f}_{\mu\tau} \\ \overset{(-)}{f}_{\tau e} & \overset{(-)}{f}_{\tau\mu} & \overset{(-)}{f}_{\tau\tau} \end{bmatrix}$$

Hamiltonian

$$\overset{(-)}{H} = \overset{(-)}{H}_{\mathrm{vac}} + \overset{(-)}{H}_{\mathrm{mat}} + \overset{(-)}{H}_{\nu\nu},$$

$$H_{\mathrm{vac}} = \frac{1}{2\nu} U \begin{bmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{bmatrix} U^\dagger,$$

$$H_{\mathrm{mat}} = D \begin{bmatrix} V_e & 0 & 0 \\ 0 & V_\mu & 0 \\ 0 & 0 & V_\tau + V_{\mu\tau} \end{bmatrix},$$

$$H_{\nu\nu} = \sqrt{2} G_F \int \frac{d^3 q'}{(2\pi)^3} (1 - \sum_{i=1}^{3} \ell'_{(i)} \ell_{(i)}) (f(q') - \bar{f}^*(q')),$$

- Global simulations:

General-relativistic quantum-kinetic neutrino transport (GRQKNT)

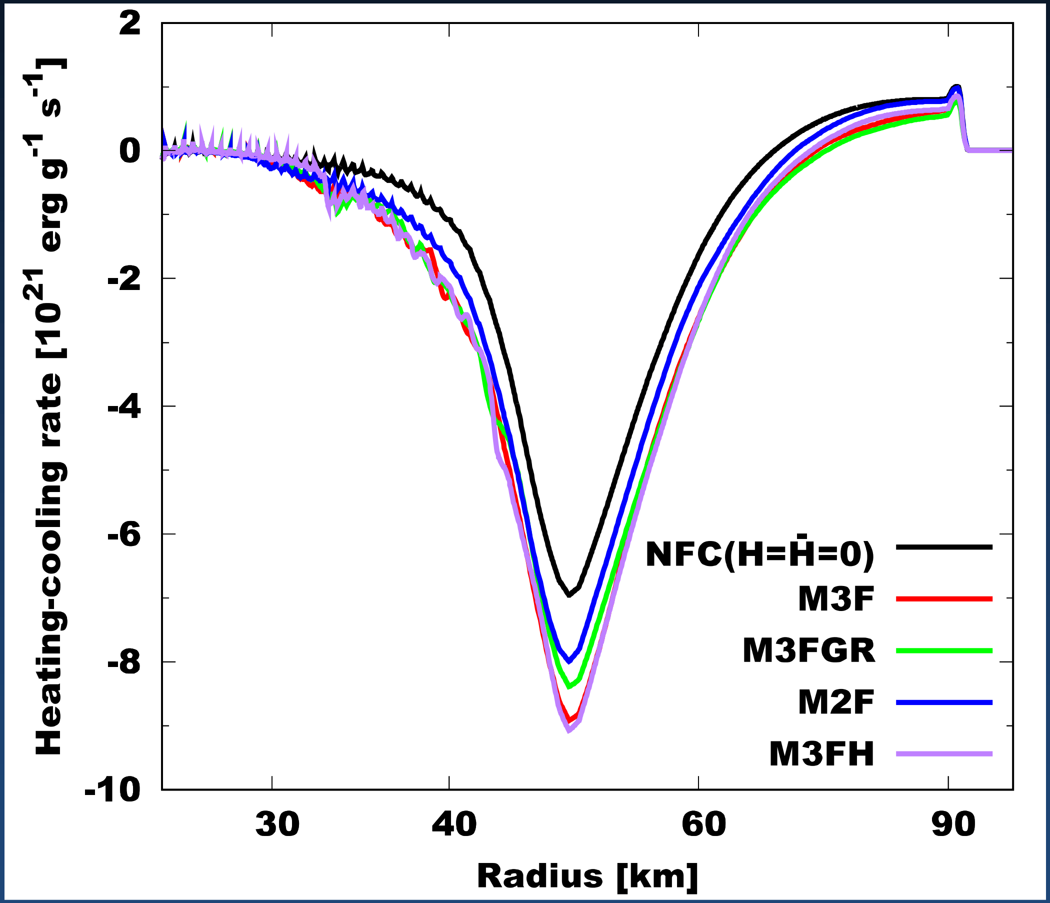
Nagakura 2022

$$p^{\mu} \frac{\partial \overset{(-)}{f}}{\partial x^{\mu}} + \frac{dp^{i}}{d\tau} \frac{\partial \overset{(-)}{f}}{\partial p^{i}} = -p^{\mu} u_{\mu} \overset{(-)}{S}_{\mathrm{col}} + i p^{\mu} n_{\mu} [\overset{(-)}{H}, \overset{(-)}{f}],$$

- Fully general relativistic (3+1 formalism) neutrino transport
- Multi-Dimension (6-dimensional phase space)
- Neutrino matter interactions (emission, absorption, and scatterings)
- Neutrino Hamiltonian potential of vacuum, matter, and self-interaction
- 3 flavors + their anti-neutrinos
- Solving the equation with Sn method (explicit evolution: WENO-5th order)
- Hybrid OpenMP/MPI parallelization

- FFC in CCSN

Nagakura PRL 2023



<u>Numerical setup:</u>

Collision terms are switched on.

Fluid-profiles are taken from a CCSN simulation.

General relativistic effects are taken into account.

A wide spatial region is covered.

Three-flavor framework

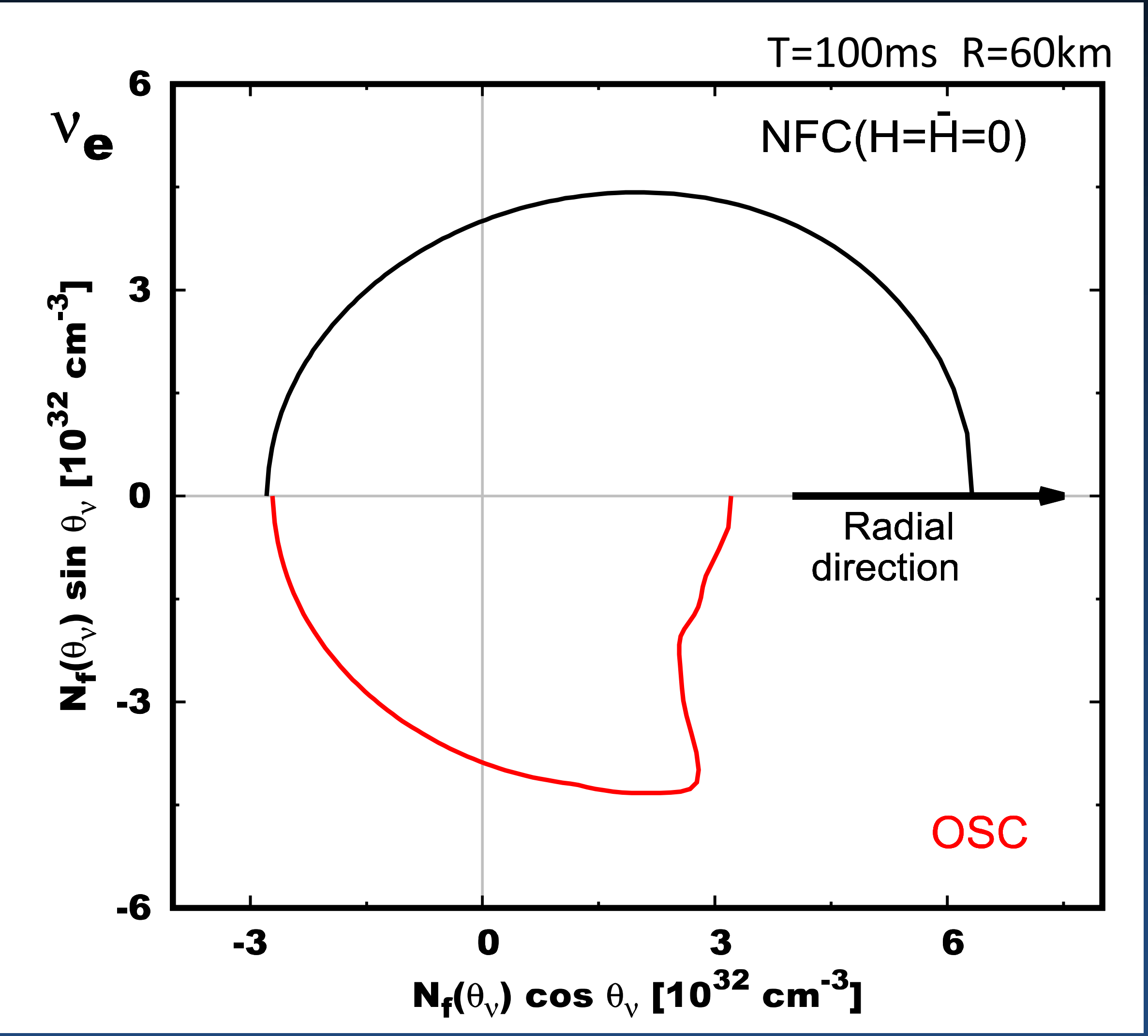
Neutrino-cooling is enhanced by FFCs
Neutrino-heating is suppressed by FFCs

Impacts on the explodability of CCSN

- FFC in CCSN

Nagakura PRL 2023

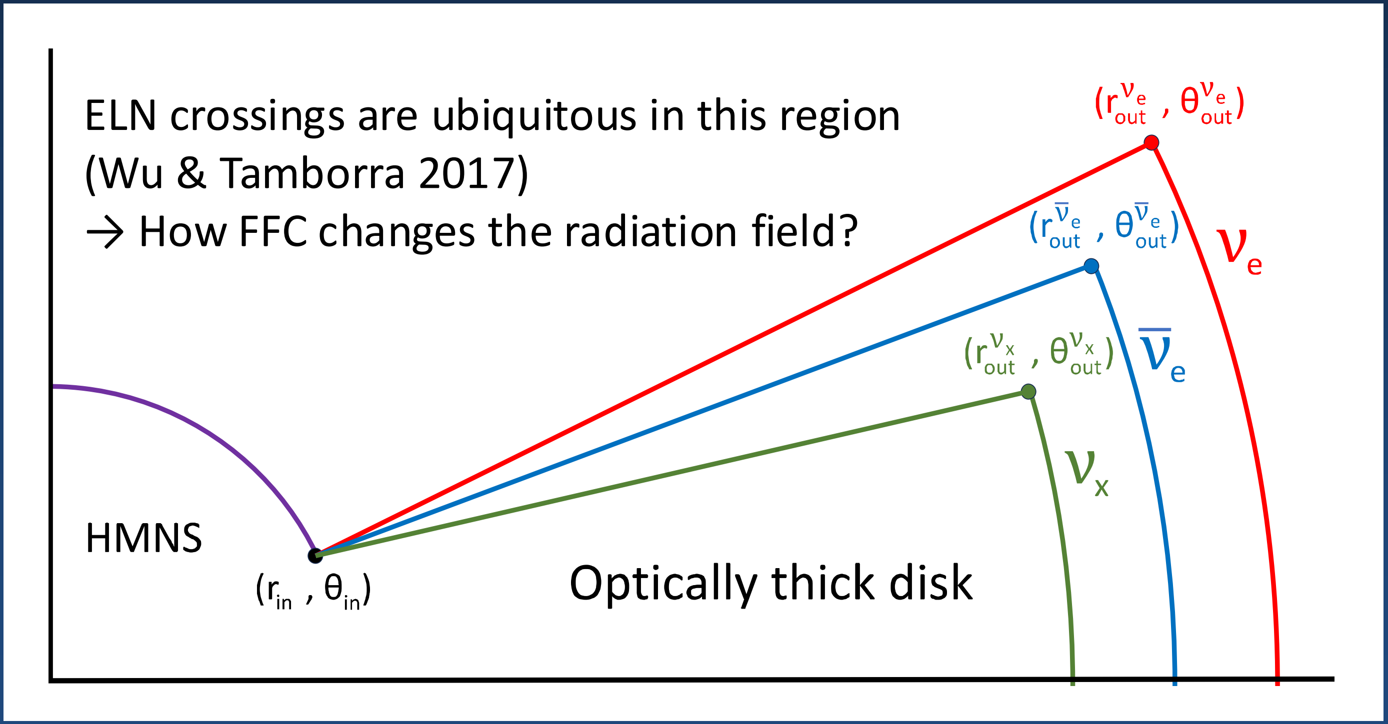
Neutrino angular distributions



- FFC in BNSM Nagakura 2023

- Setup:

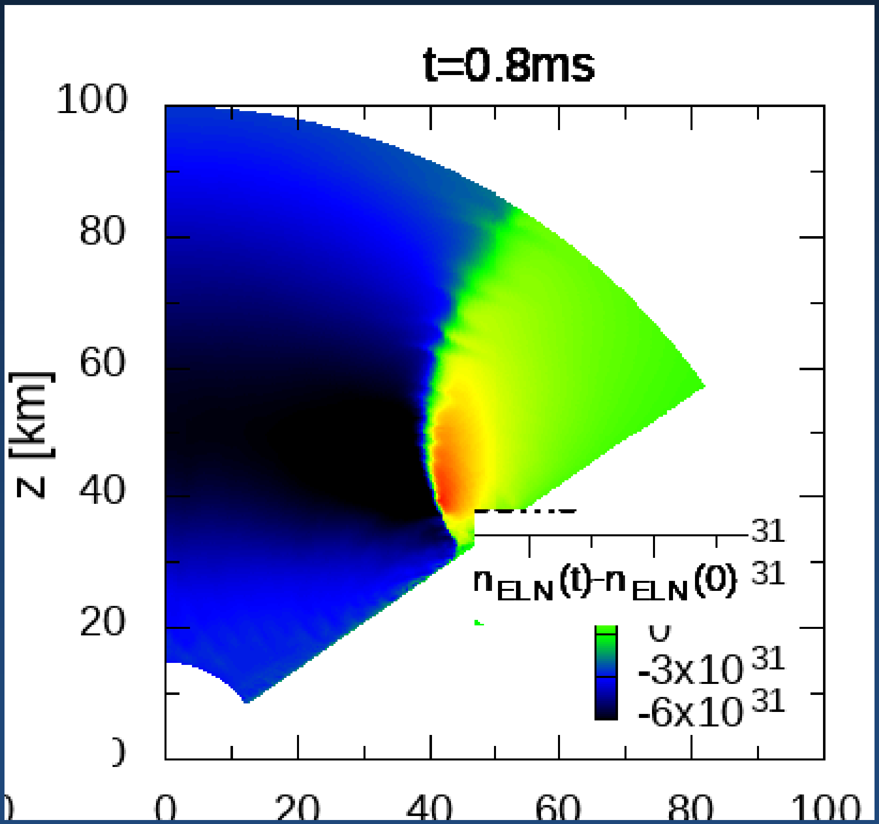
  - Hypermassive neutron star (HMNS) + disk geometry
  - Thermal emission on the neutrino sphere
  - QKE (FFC) simulations in axisymmetry
  - Resolutions: 1152 (r) × 384 (θ) × 98 (θ_ν) × 48 (ϕ_ν)



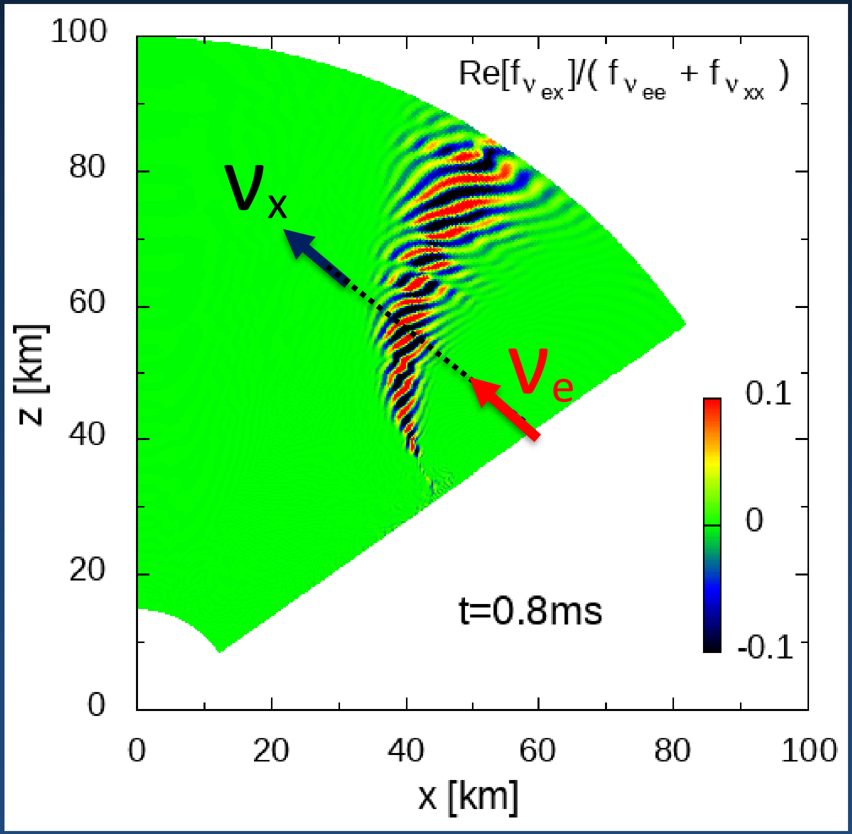
- FFC in BNSM
Nagakura 2023

- Appearance of <u>flavor swap and EXZS (ELN-XLN Zero Surface):</u>

ELN - XLN



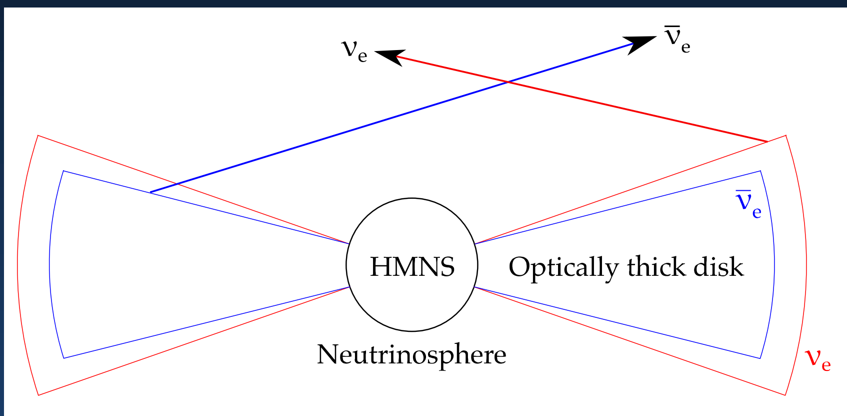
Flavor coherency

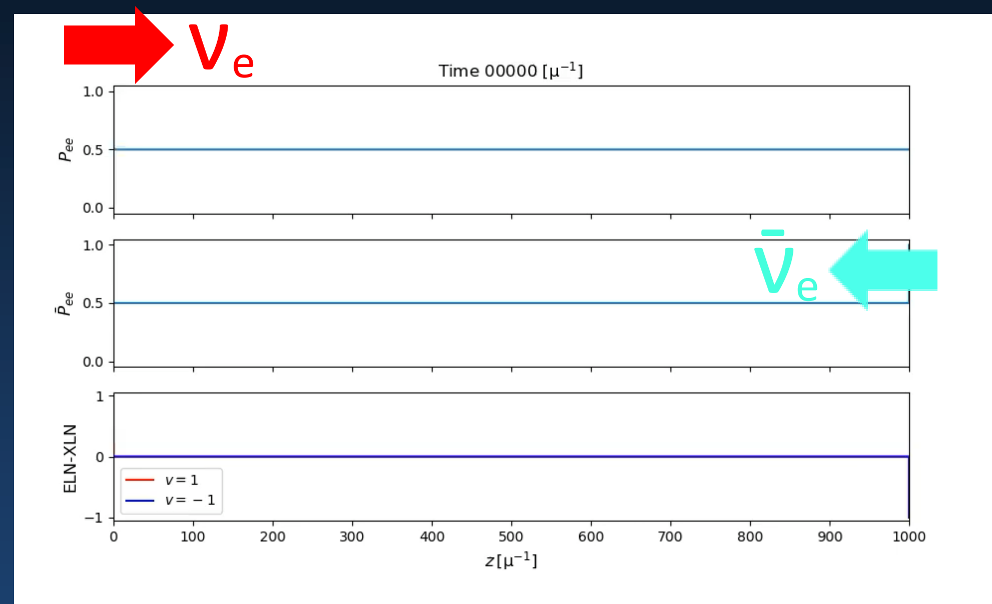


Fast flavor swap would be ubiquitous in BNSM

Zaizen and Nagakura 2024

- Colliding-beam model





$$\partial_{t'}^2 P_3 \sim -4\mu^2 \left(1 - (P_3)^2\right)$$

$P_3 = 1$: electron-type → Unstable

$P_3 = 0$: equipartition → Non-steady

$P_3 = -1$: heavy-lepton type → Stable

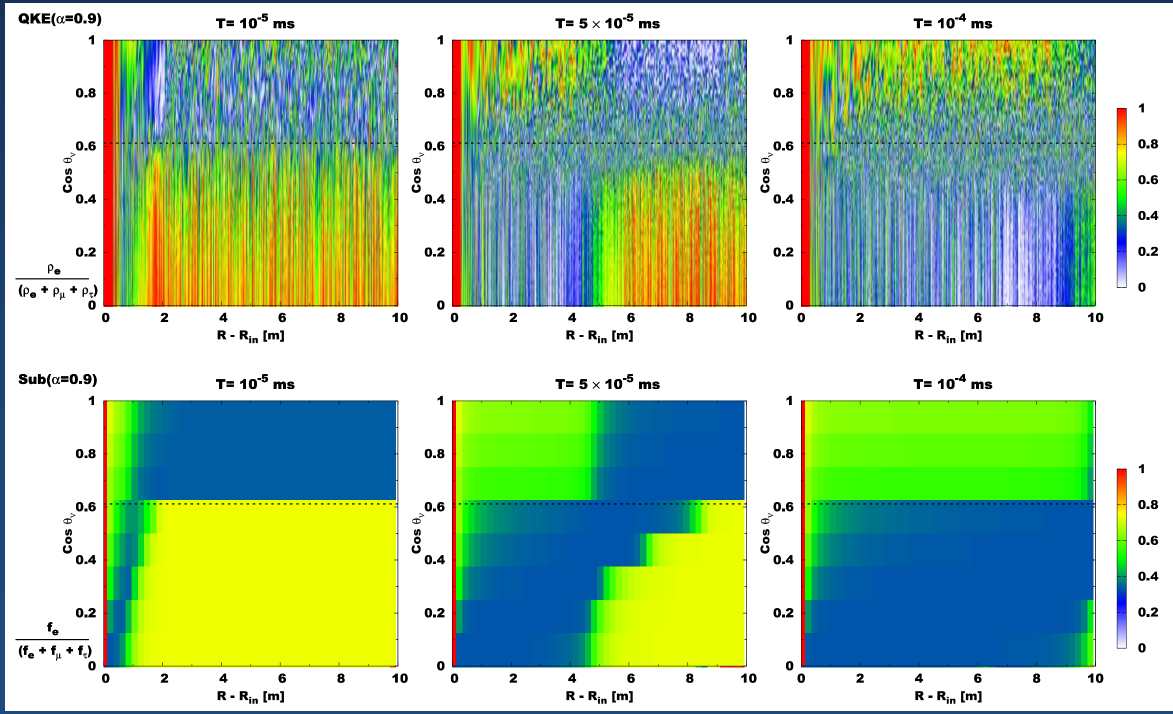
Neutrinos undergo flavor swaps in asymptotic states.

- BGK Subgrid model Nagakura et al. 2024

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + ip^\mu n_\mu [H, f]$$: Full QKE

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = -p^\mu u_\mu S + p^\mu n_\mu \frac{1}{\tau_a}(f - f^a)$$: Relaxation-time approximation

Radial-angular distributions for survival probability of electron-type neutrinos



Summary

- Radiation-hydrodynamic simulations under classical treatments of neutrino kinetics have been matured in CCSN and BNSM community.
- Neutrino flavor instabilities ubiquitously occur in CCSN and BNSM environments.
- We developed a new GRQKNT code for time-dependent global simulations of neutrino quantum kinetics (QKE).
- Many studies demonstrate that fast neutrino-flavor conversion (FFC) and collisional flavor instabilities give impact on fluid-dynamics and nucleosynthesis.
- Subgrid models of FFC/CFI have also been developed so as to incorporate effects of FFC/CFI into classical neutrino transport.
- This field has been developing at a rapid pace, promising to reveal new features of neutrino physics, as well as providing deeper insights into CCSN and BNSM.