Colliding Relativistic Shells: New Insights

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Rahaman, Granot & Beniamini 2024a, b (MNRAS, 258, L45; MNRAS, 258, 160)

Granot & Rabinovich 2024 (Physics of Fluids, 36, 016142)

Bera, Granot, Rabinovich & Beniamini 2024 (Physics of Fluids, 36, 016141)

Charlet, Granot & Beniamini 2025 (submitted)

Relativistic Fluids around Compact Objects, Warsaw, Poland, 5 May 2025

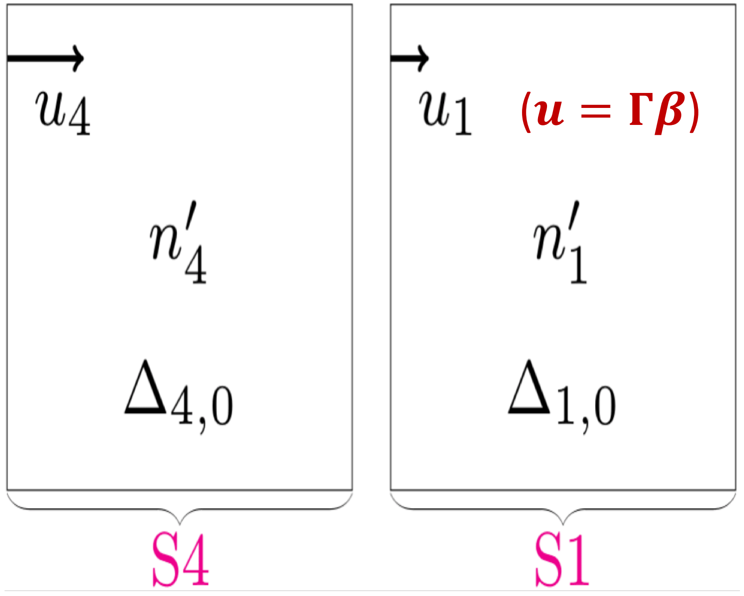
Relativistic Bulk Collisions in Astrophysics:
a fast shell overtakes and collides with a slower shell

- Blazars: internal shocks (Rees 1978) $u = \Gamma\beta \sim 10 - 30$
- Gamma-Ray Bursts (GRBs): internal shocks (Rees & Meszaros 1994) $u \sim 10^2 - 10^3$
- Magnetar Giant Flare (GF): outflow & external shell (JG et al. 2006) $u \sim 1 - 10^2$
- Superluminous Supernovae (SLSNe): ejecta & external massive shell (Smith & McCray 2007) $u \sim 10^{-2}$
- Fast Radio Bursts (FRBs): GF outflow + MWN or another GF outflow (Lyubarsky 2014; Metger+ 2017; Beloborodov 2017) $u \sim 10^2 - 10^4$

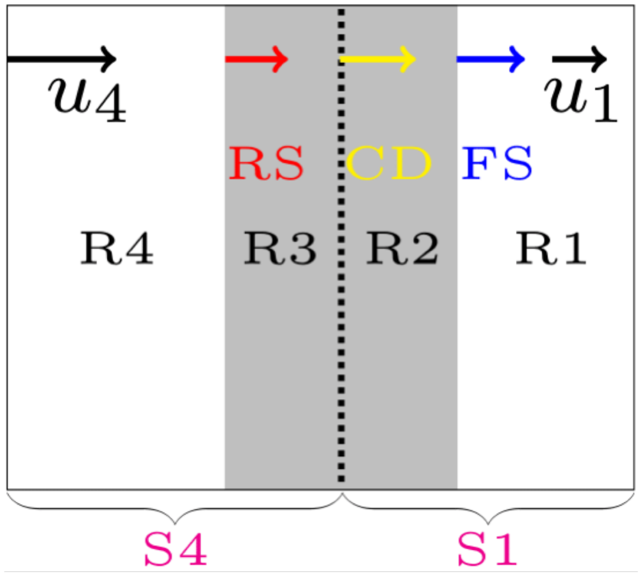
A Two Shell Collisions: (Rahaman, JG & Beniamini 2024b)

(cold & uniform shells, in 1D planar symmetry)

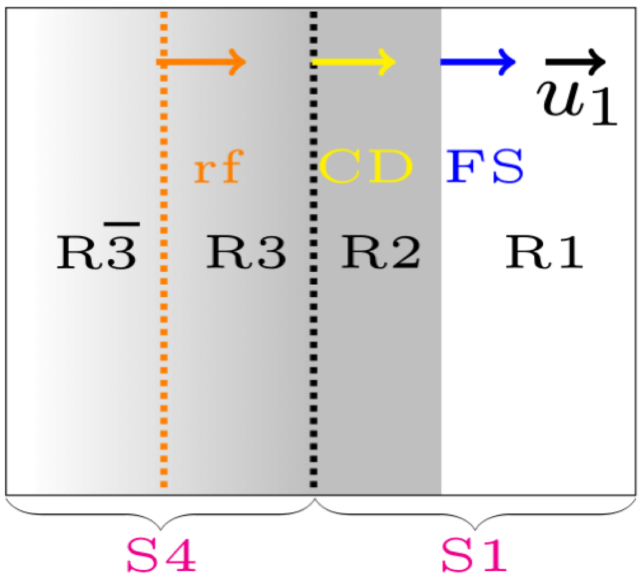
Pre-collision



Post collision



Post reverse shock crossing (for $t_{RS} < t_{FS}$)



Proper density ratio: $f = n'_4/n'_1$

Proper speed ratio: $a_u = u_4/u_1 > 1$

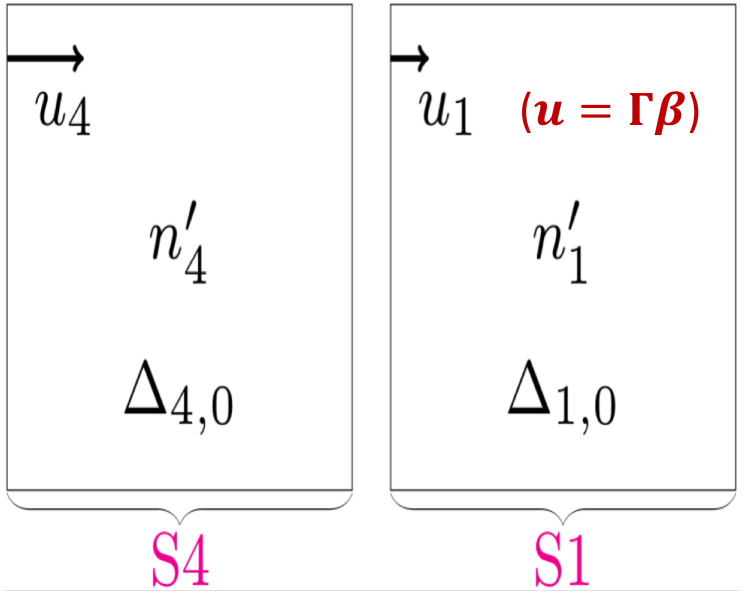
Radial width ratio: $\chi = \frac{\Delta_{1,0}}{\Delta_{4,0}} \sim 1$

Collision radius: $R_0 = \frac{\beta_1 \beta_4 c t_{off}}{\beta_4 - \beta_1}$

A Two Shell Collisions: (Rahaman, JG & Beniamini 2024b)

(cold & uniform shells, in 1D planar symmetry)

Pre-collision



- 10 unknowns $(p_2, n_2, e_2, \beta_2, p_3, n_3, e_3, \beta_3, \beta_{FS}, \beta_{RS})$
- 10 constraints (2 EoS - TM, 2 CD, 3 FS, 3 RS)
- First Solve in the rest frame of Shell 1 :

$$u_{21} = u_{31} = u_{41}\sqrt{\frac{2f^{3/2}\Gamma_{41} - f(1+f)}{2f(u_{41}^2 + \Gamma_{41}^2) - 1 - f^2}}$$

- Then transform to the lab frame

Proper density ratio: $f = n'_4/n'_1$

Proper speed ratio: $a_u = u_4/u_1 > 1$

Radial width ratio: $\chi = \frac{\Delta_{1,0}}{\Delta_{4,0}} \sim 1$

Collision radius: $R_0 = \frac{\beta_1\beta_4 c t_{off}}{\beta_4 - \beta_1}$

A Two Shell Collisions:

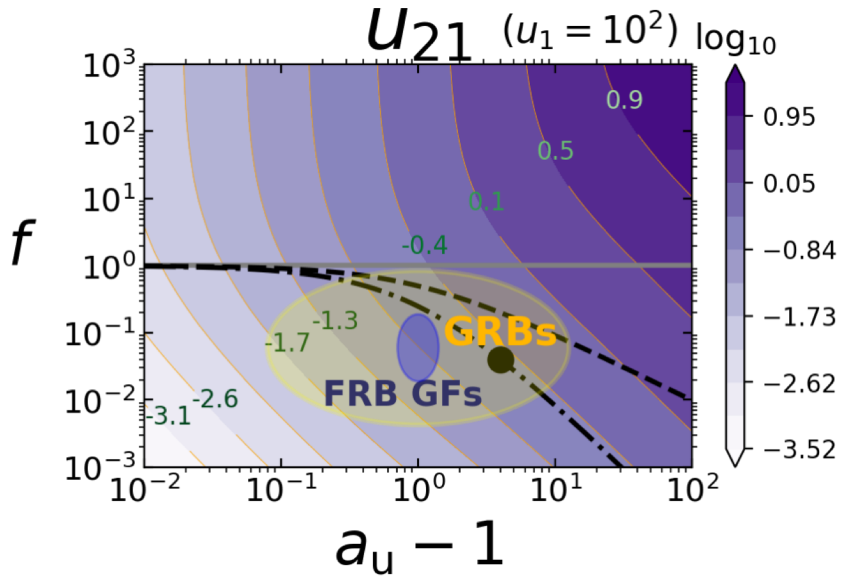
- Gamma-Ray Bursts (GRBs): internal shocks
- Fast Radio Bursts (FRBs): GF outflow + MWN or GF outflow

$$u_{41} = \left(a_u\Gamma_1 - \sqrt{1 + a_u^2 u_1^2}\right) u_1$$

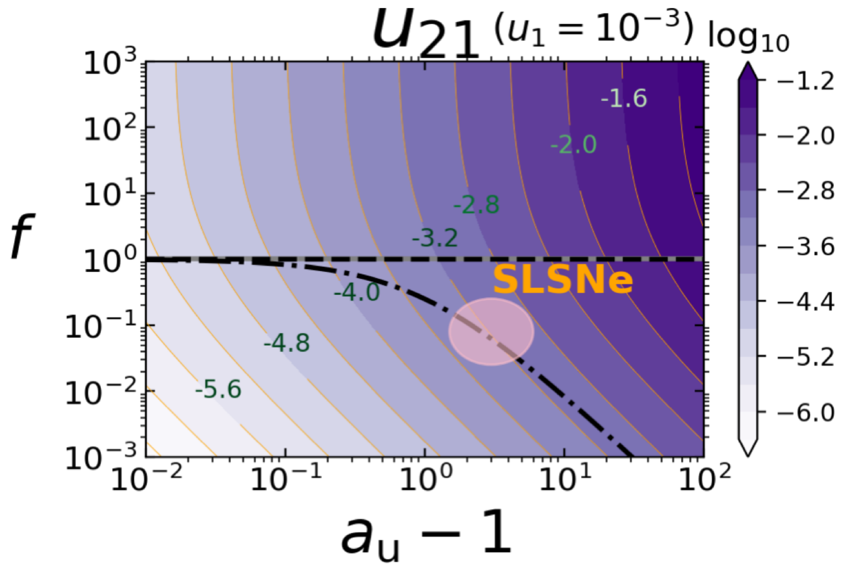
$$u_{21} = u_{41}\sqrt{\frac{2f^{3/2}\Gamma_{41} - f(1+f)}{2f\left(u_{41}^2 + \Gamma_{41}^2\right) - 1 - f^2}}$$

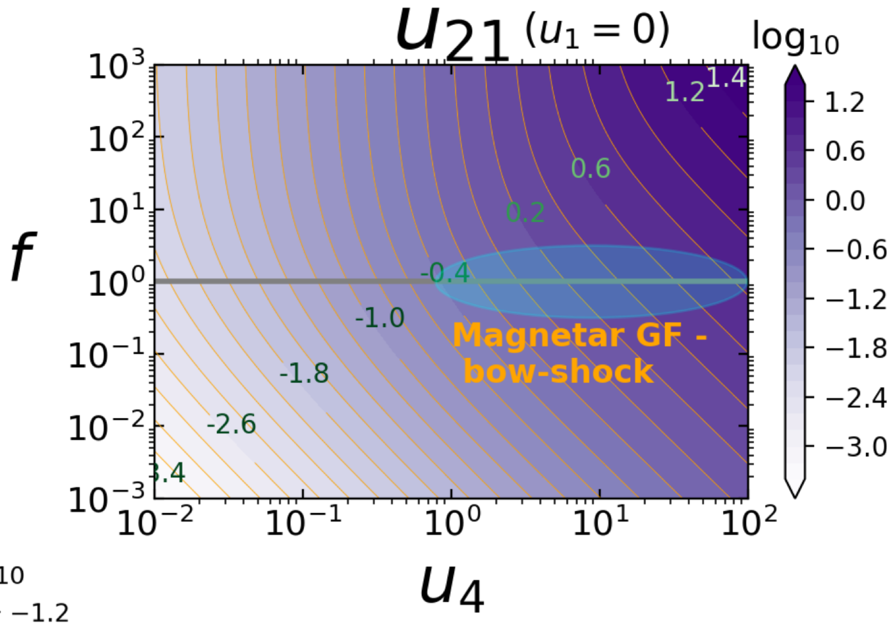
$$u = u_2 = u_3 = \Gamma_{21}\Gamma_1(\beta_{21} + \beta_1)$$

- Magnetar Giant Flare (GF): outflow & bow-shock shell



- SLSNe – Superluminous Supernovae: ejecta & external massive shell





$f = n'_4/n'_4$

$a_u = u_4/u_1 > 1$

u_{41} = pre-collision relative proper speed of the shells

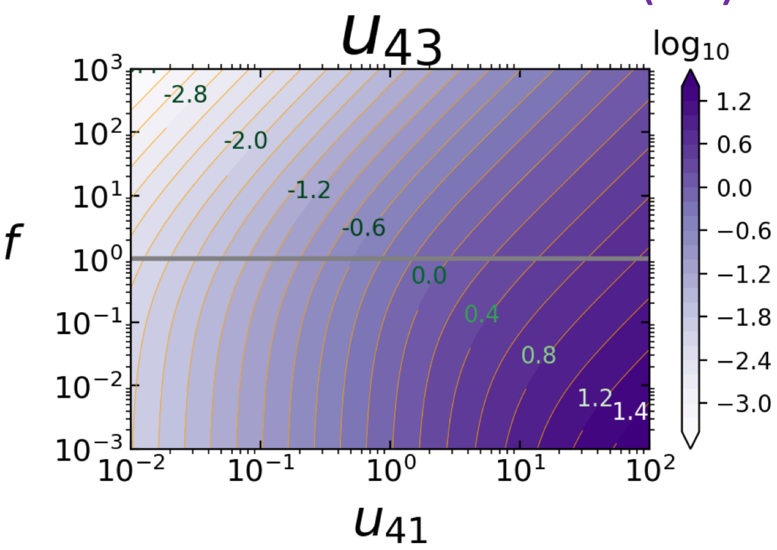
u_{21} = relative upstream to downstream proper speed across the forward shock

A Two Shell Collisions:

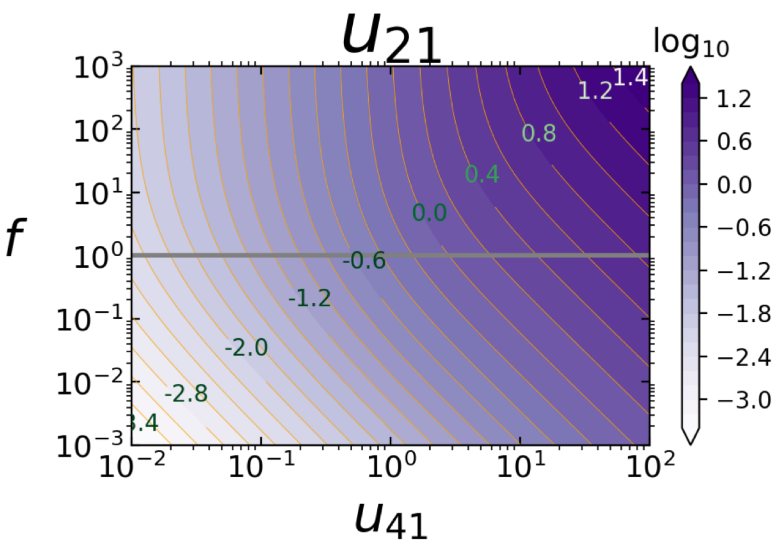
Mirror symmetry (for a given u_{41}): FS $\longleftrightarrow$ RS, $f \longleftrightarrow 1/f$

relative upstream to downstream proper speed

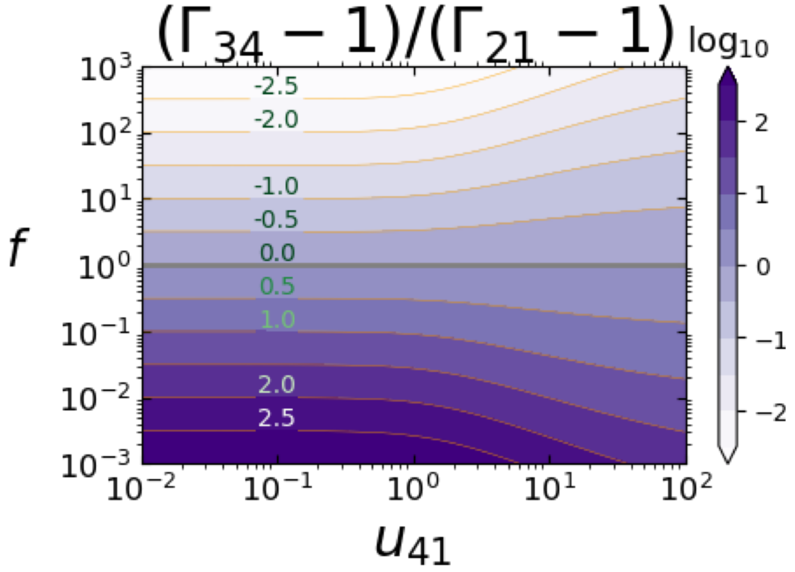
across reverse shock (RS)



across forward shock (FS)



Shock strength ratio



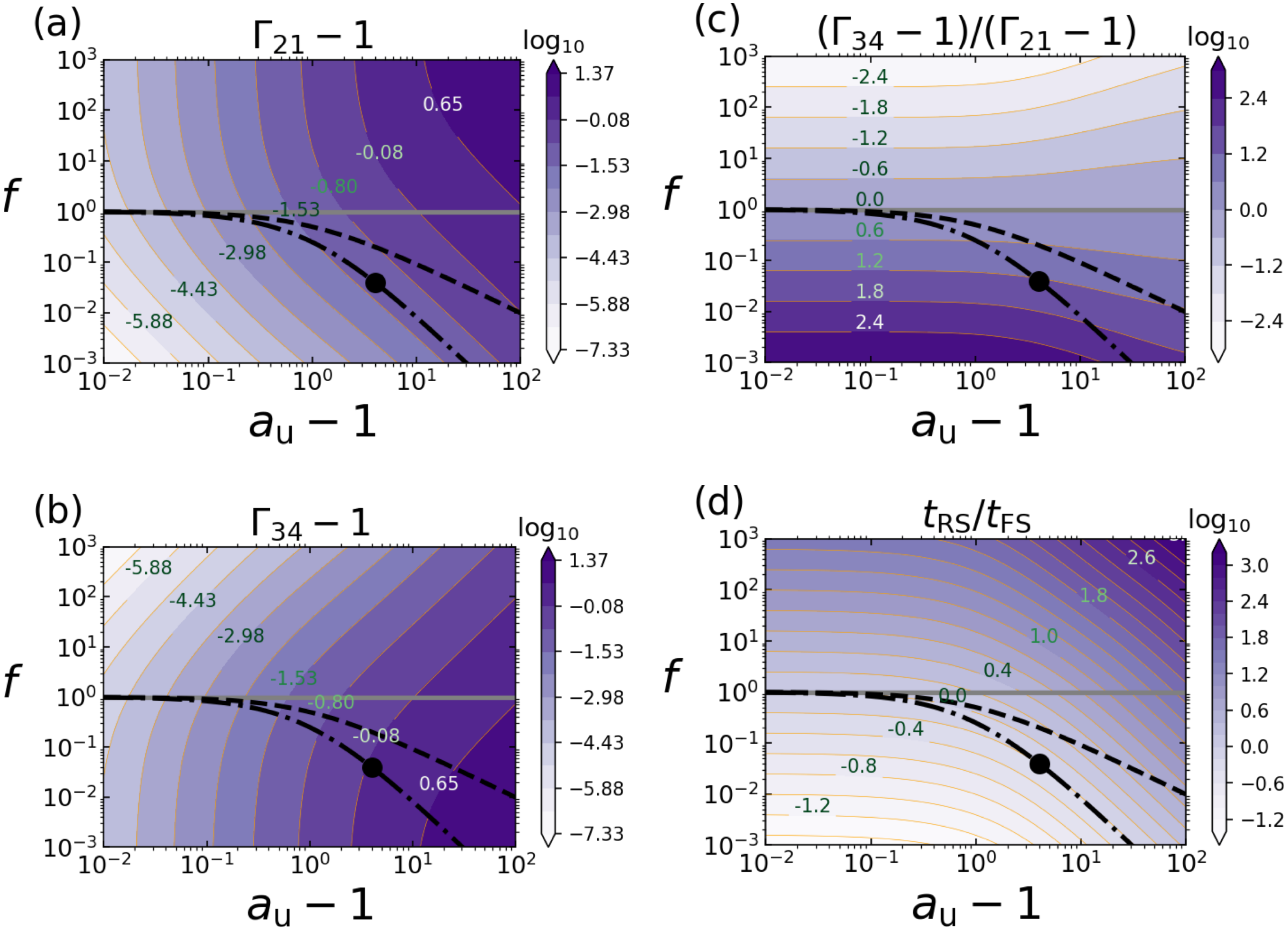
Ultra-Relativistic Shells:

Mirror symmetry: FS ⟷ RS, $f \longleftrightarrow 1/f$

$u_1 = 100$
$\left(u_{41} \approx \frac{a_u^2-1}{2a_u}\right)$

$\chi = \frac{\Delta_{1,0}}{\Delta_{4,0}} = 1$

Equal proper density
Equal mass
Equal energy



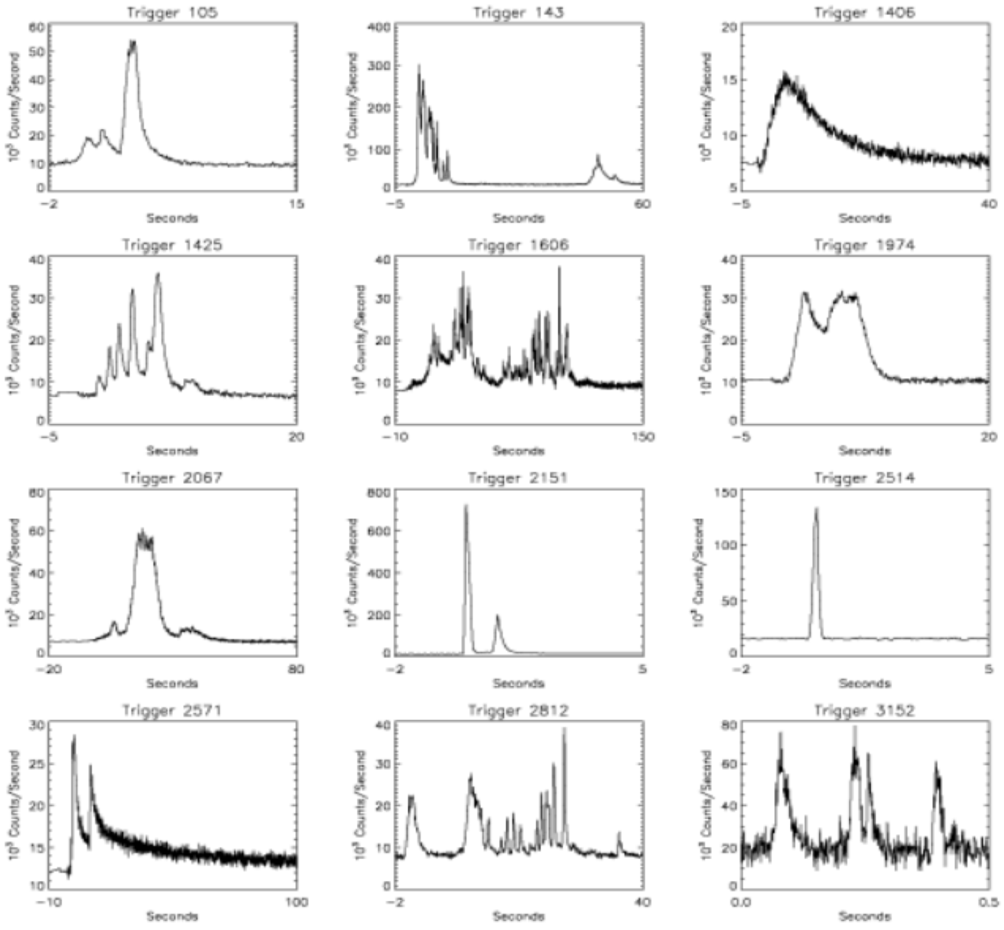
- For **equal proper density**: equal shock strength & FS crosses much earlier
- For **equal mass** shells: Reverse Shock is stronger but finishes crossing 2nd
- For **equal energy** shells: the RS is much stronger & finishes crossing first

Prompt GRB Emission from Internal Shocks:

(Rahaman, JG & Beniamini 2024a)

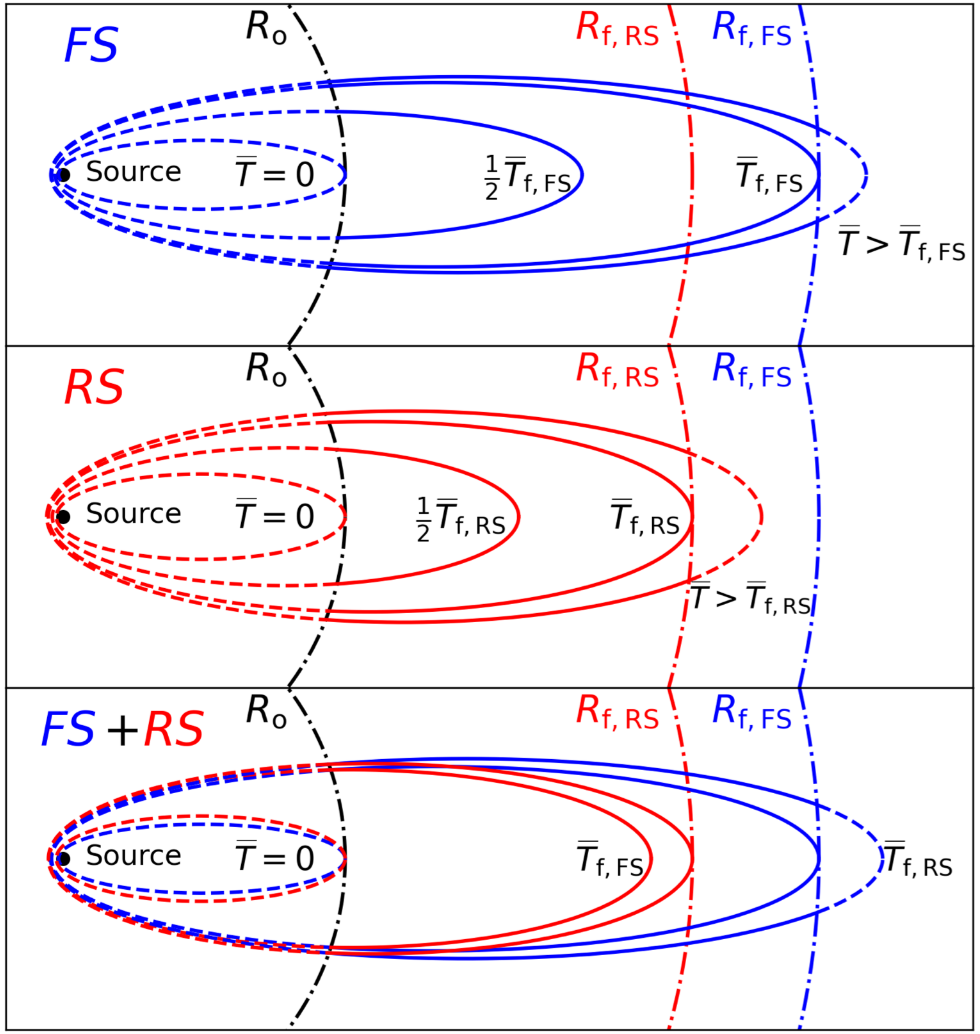
In our model:

1 collision ⇒ 1 pulse in the lightcurve



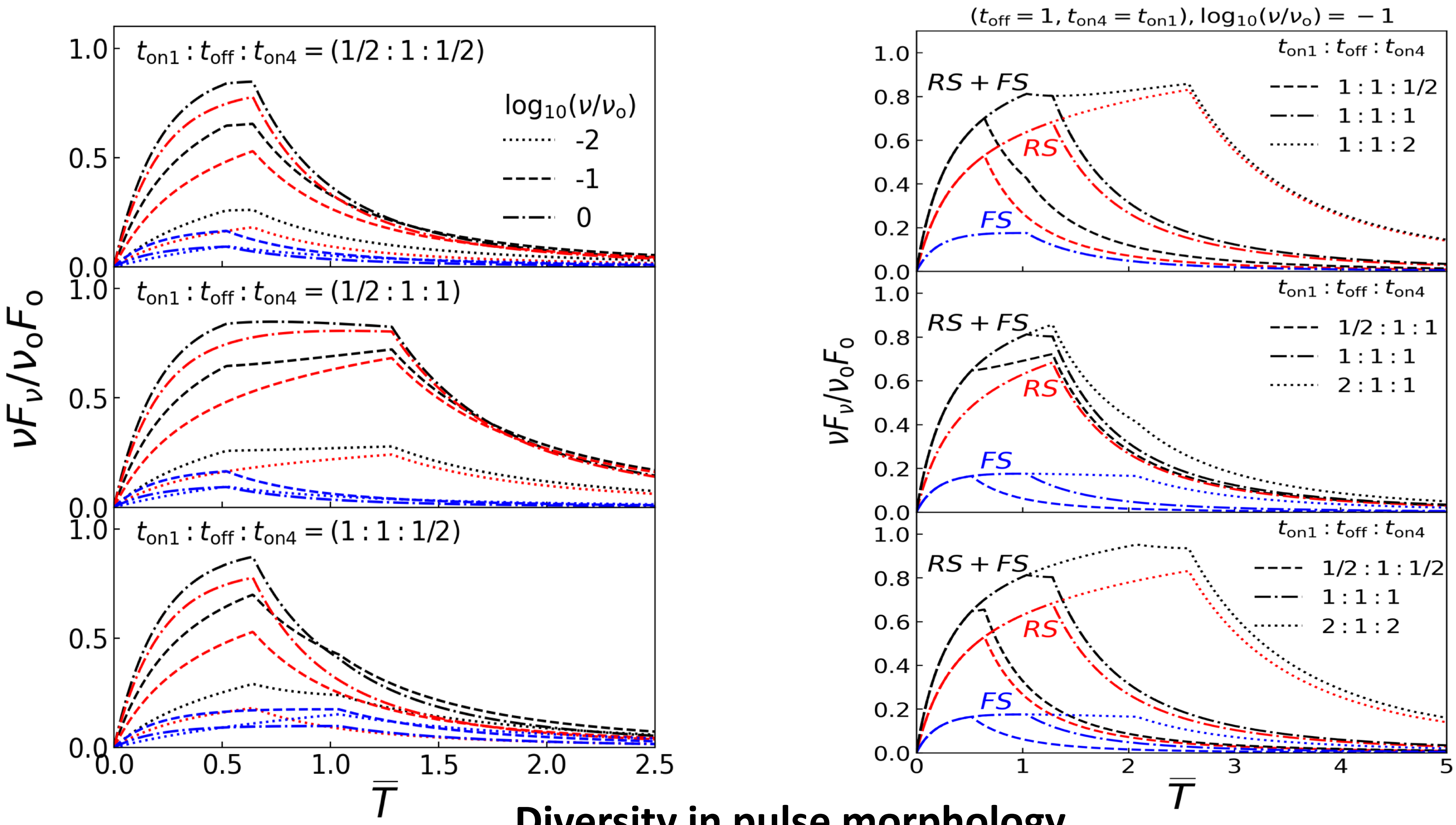
(Credit: J.T. Bonnell (NASA/GSFC))

Equal-Arrival-Time Surfaces (EATS)



Contribution of both EATS must be taken at a given observed time.

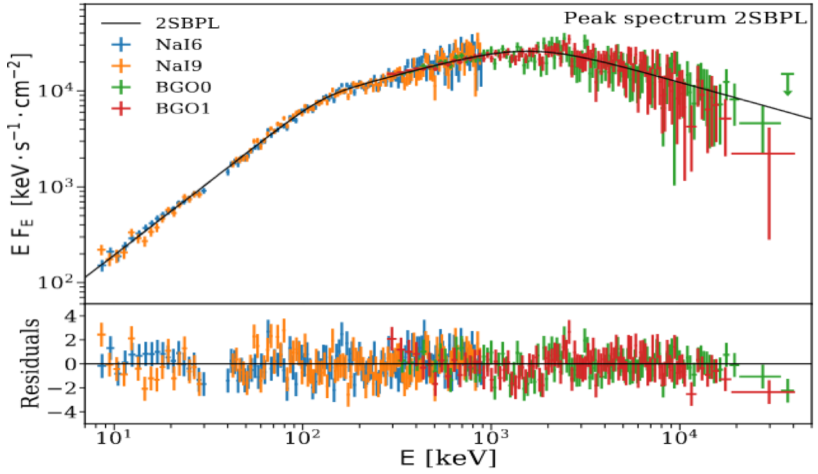
Lightcurves (single pulse; **Rahaman, JG & Beniamini 2024a**):



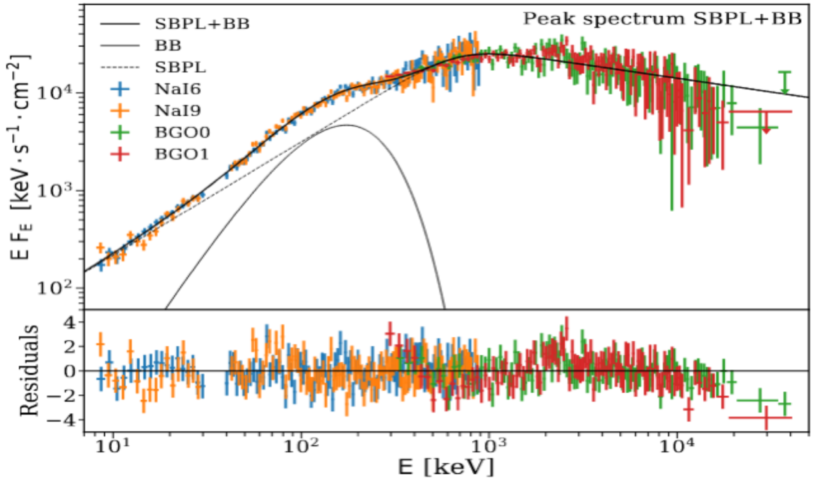
Diversity in pulse morphology

Spectra (Rahaman, JG & Beniamini 2024a):

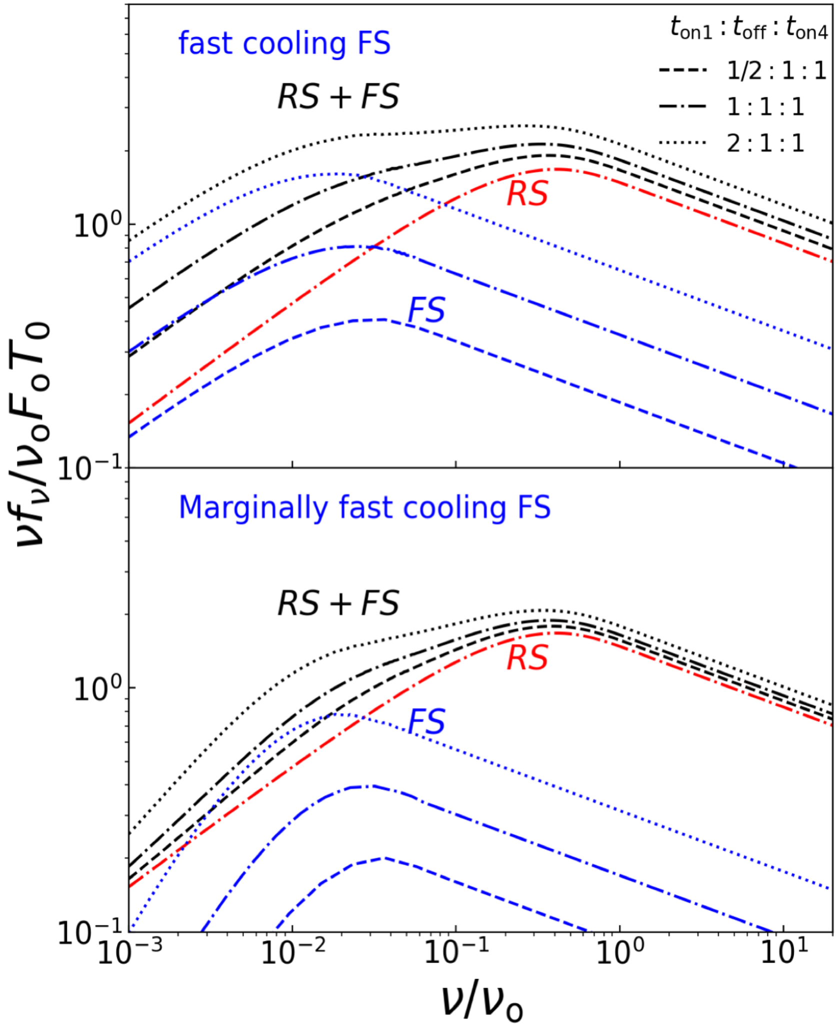
Example of Observations



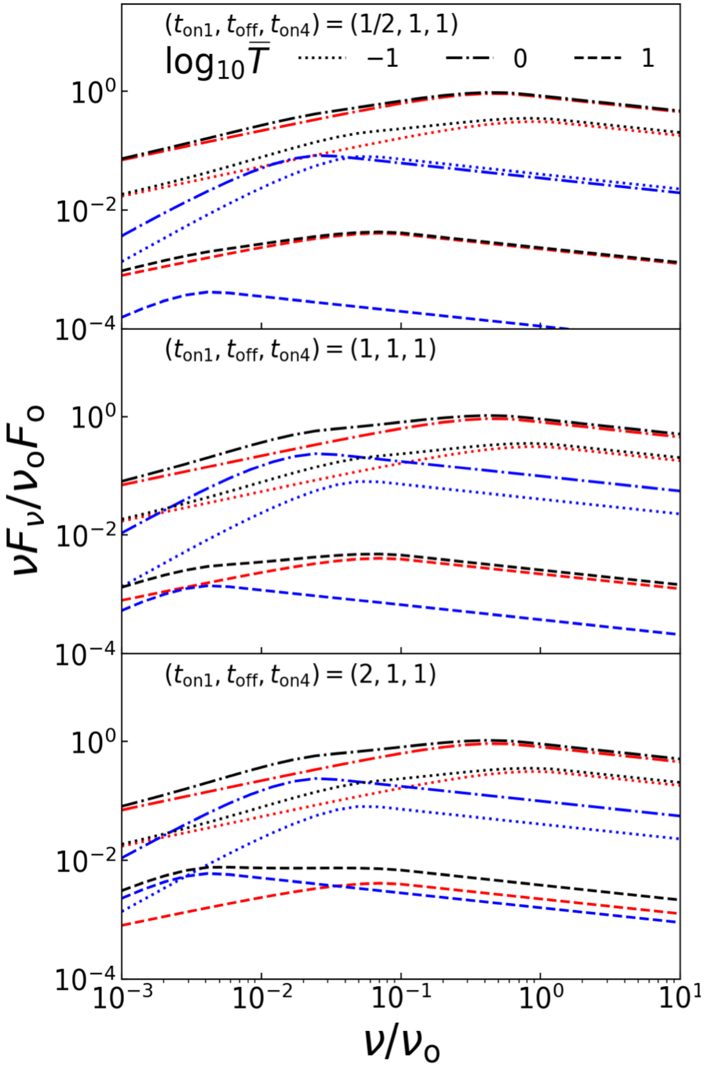
GRB 160625B Ravasio et al. 2018



Time-Integrated

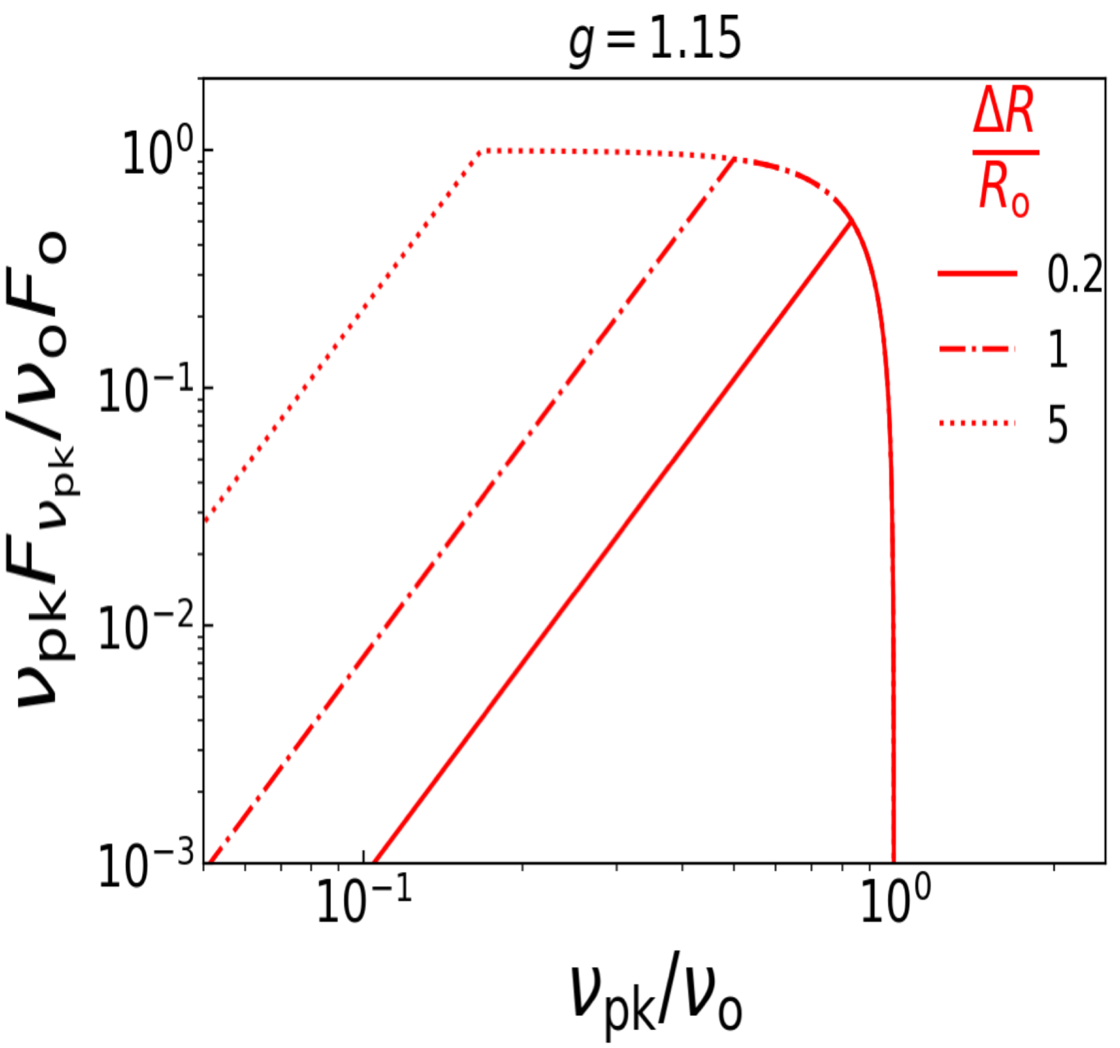


Time-resolved

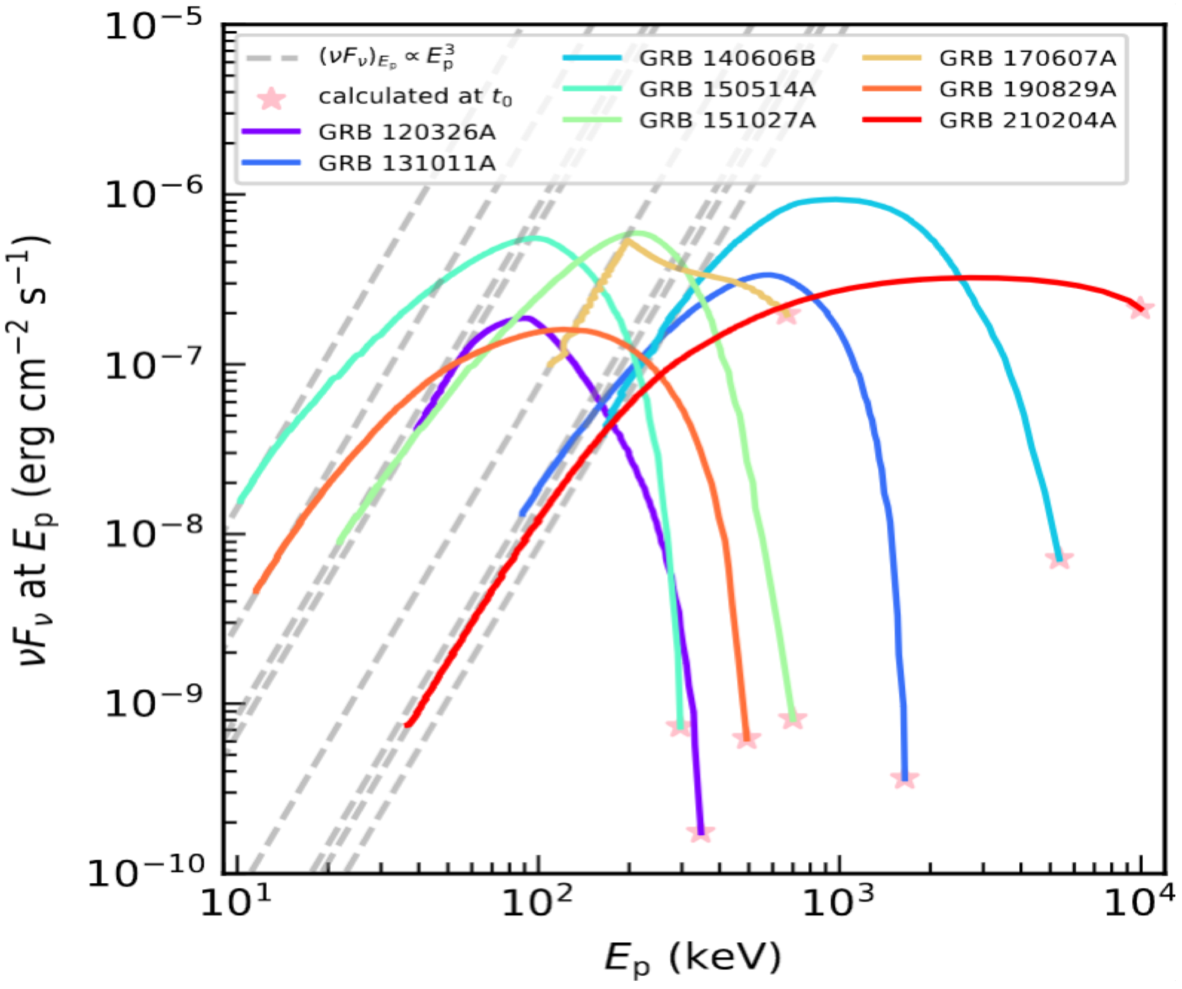


The weaker FS emission may mimic a "photospheric" quasi-thermal component or a low-E_{ph} spectral break – features that have been inferred in an increasing number of GRBs

Relating $F_\nu(t)$ to the colliding shells properties:



(Rahaman, JG & Beniamini 2024a)



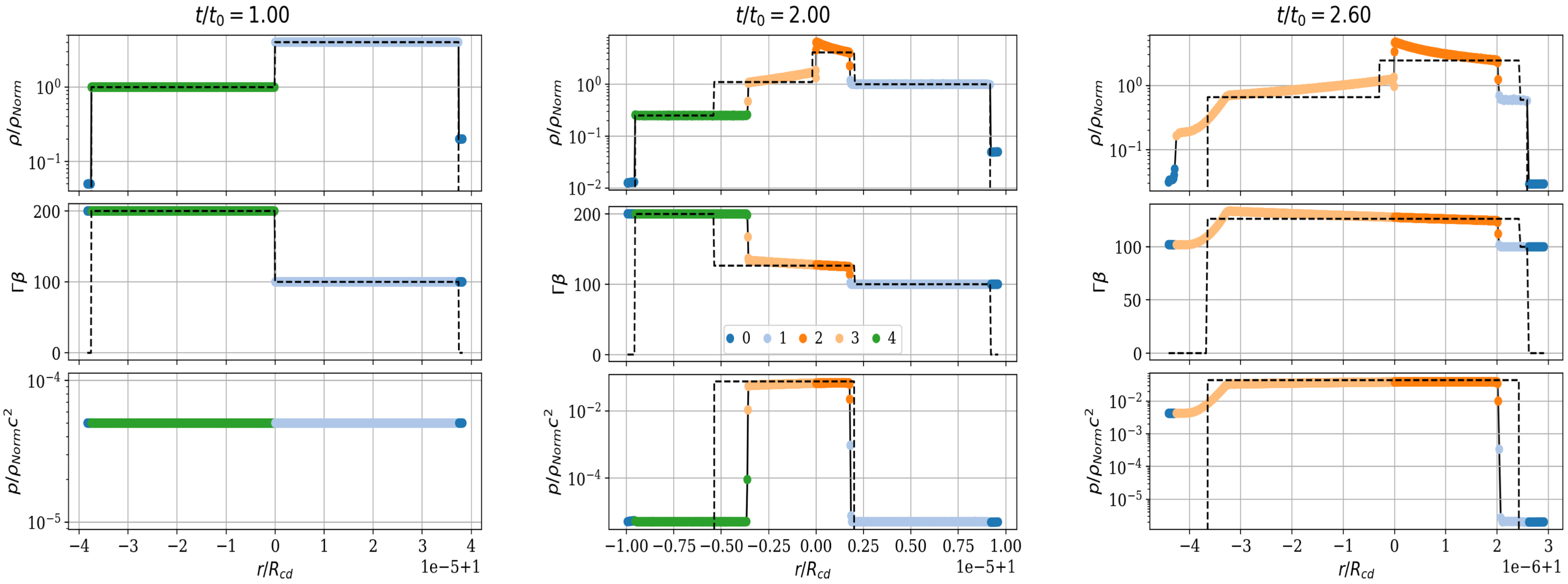
(Yan et al. 2023)

Numerical Simulations of Spherical Internal Shocks:

(Charlet, JG & Beniamini 2025)

Spherical effects cause the shock strengths to decrease with radius

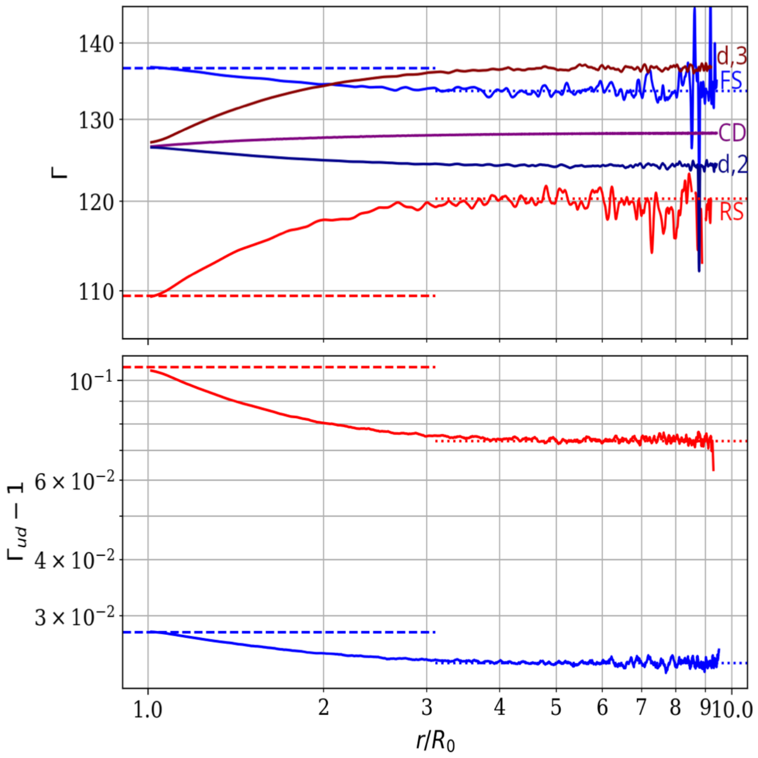
Initial Shell Collision | **With both shocks active** | **After reverse shock crossing**



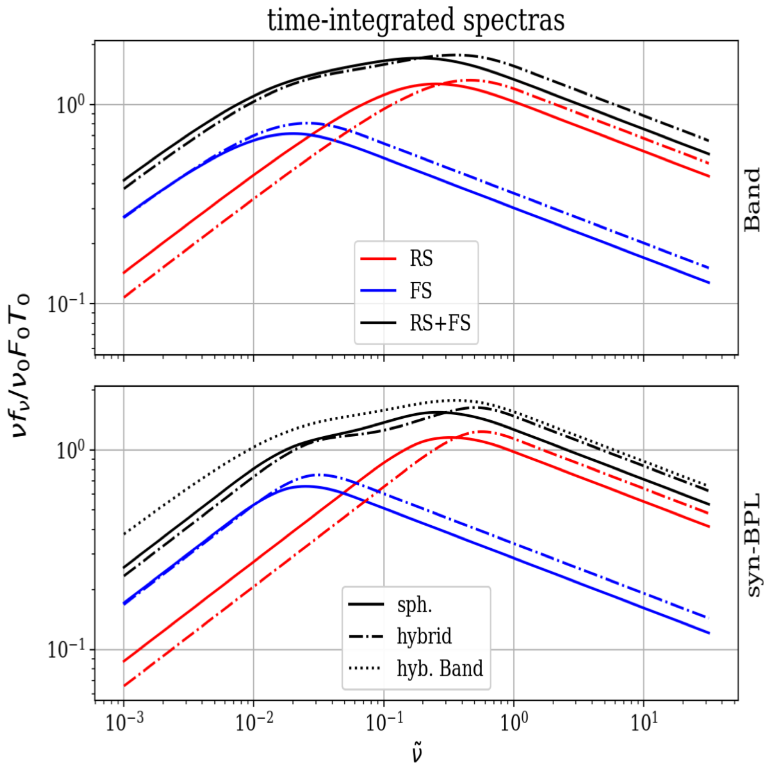
Numerical Simulations of Spherical Internal Shocks:
(Charlet, JG & Beniamini 2025)

Spherical effects cause the shock strengths to decrease with radius

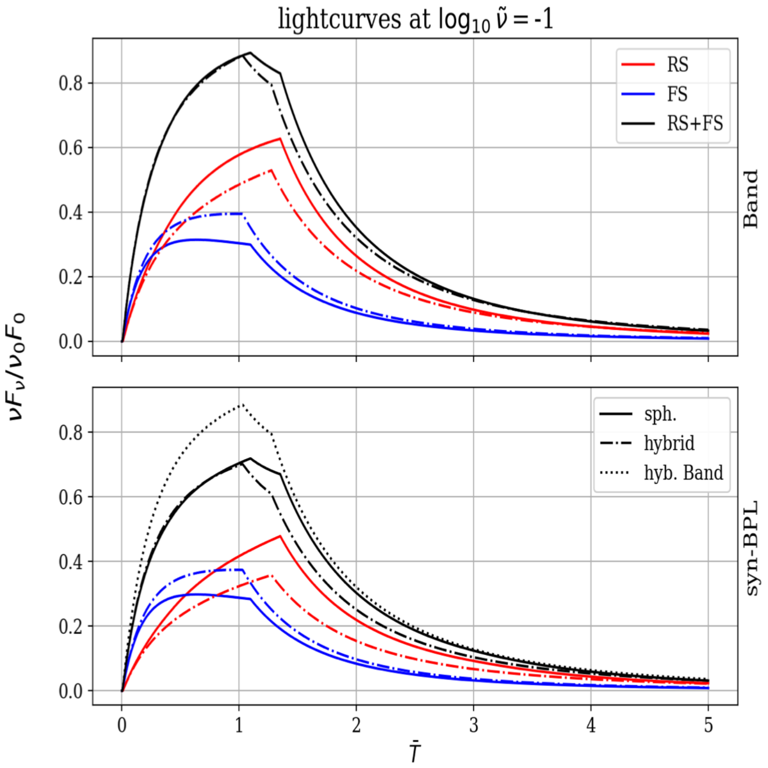
Lorentz Factors & Shock Strengths



Time integrated Spectrum



Lightcurves / Pulse Shapes

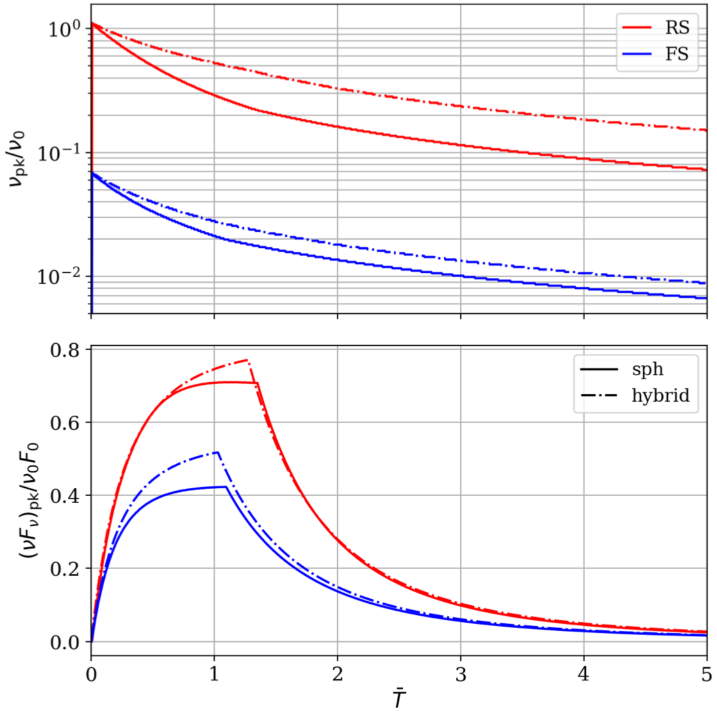


Numerical Simulations of Spherical Internal Shocks:

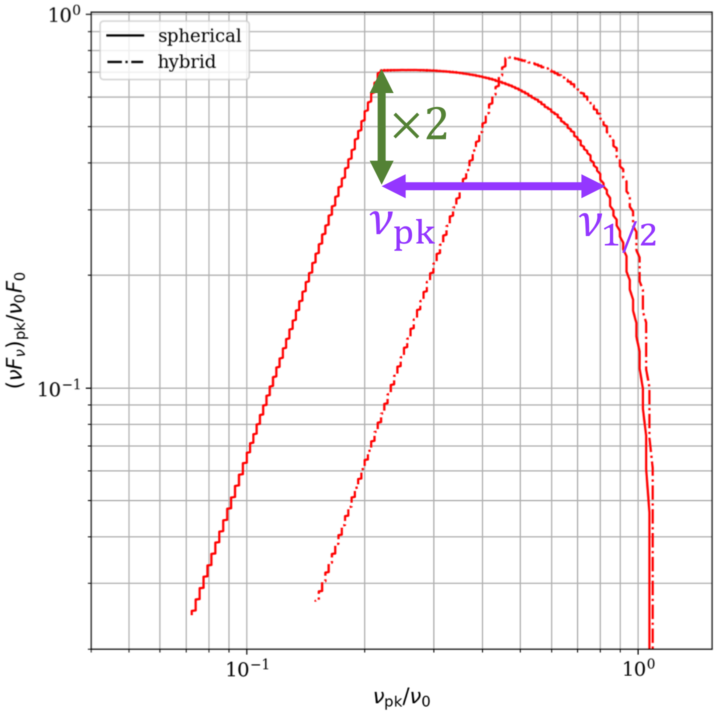
(Charlet, JG & Beniamini 2025)

Spherical effects cause the shock strengths to decrease with radius

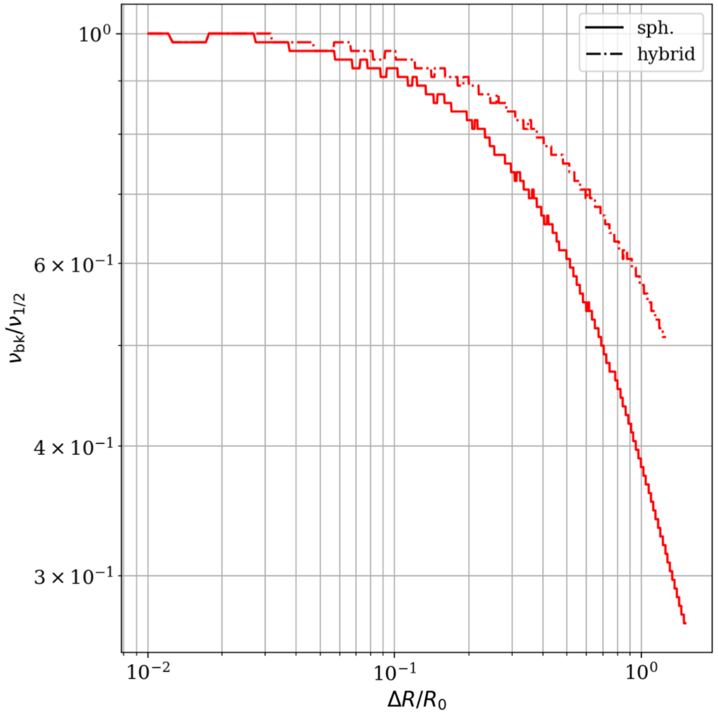
Evolution of Peak Frequencies & Flux



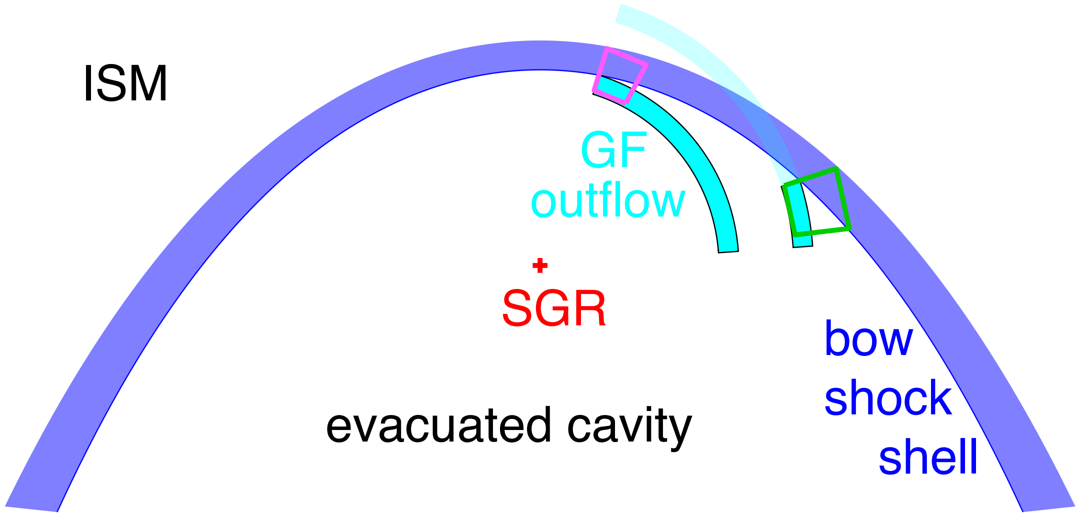
Peak Flux vs. Frequency: spherical vs. hybrid models



Lightcurves / Pulse Shapes



An Oblique Two Shell Collision in 2D: Motivation & Reduction



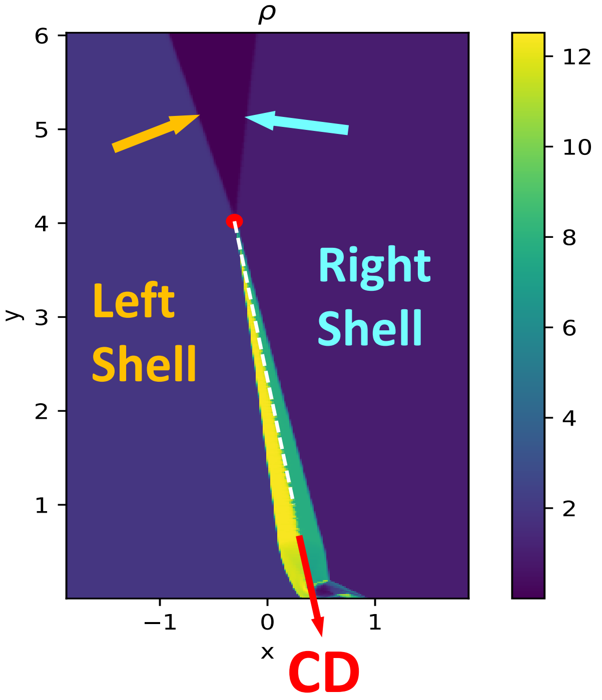
2-shell collision ⟶ 2 × 1 shell - wall collision

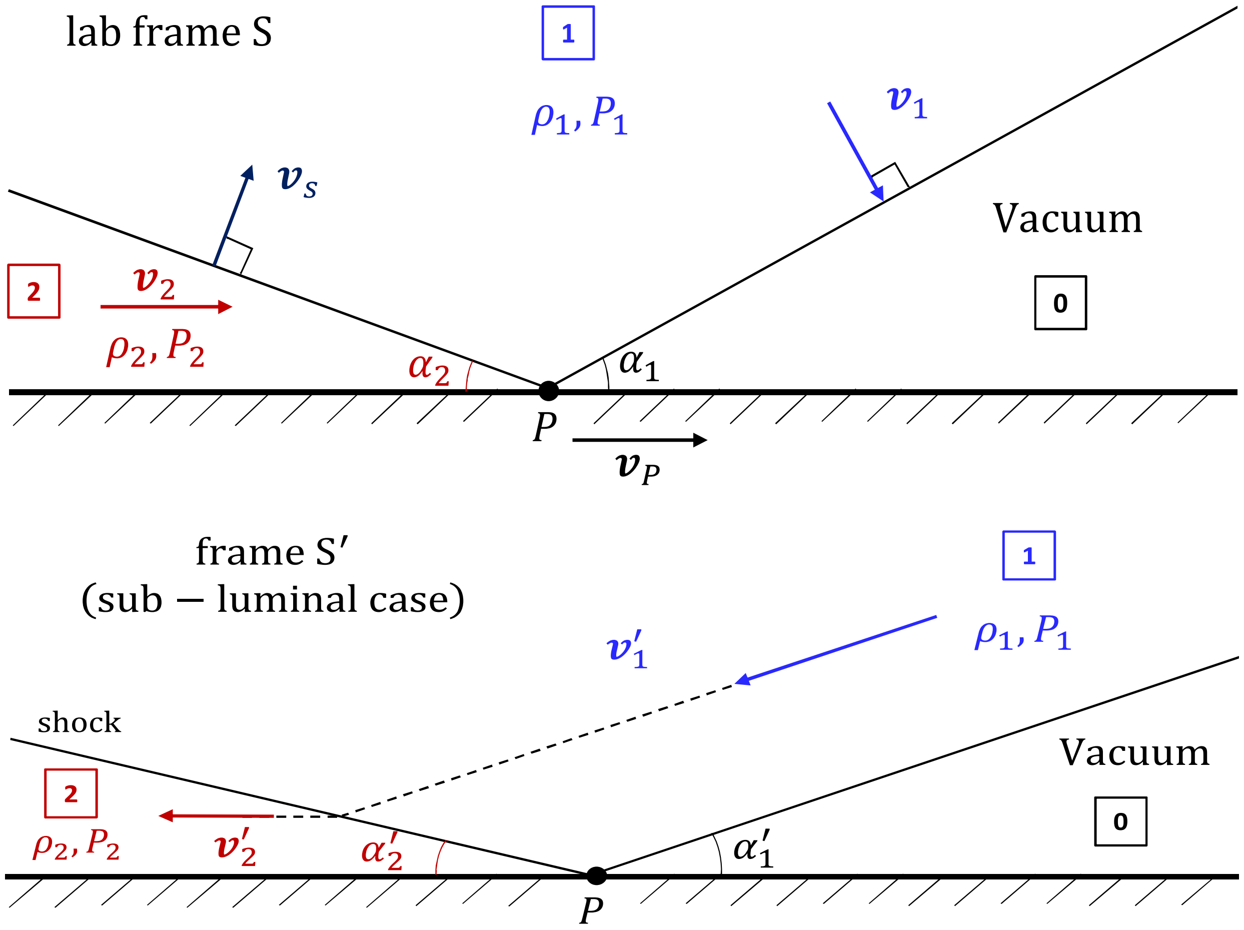
(the **CD** that forms may be regarded as a **wall**)

Motivation: Magnetar (SGR) Giant Flares (GF)

Radio nebula powered by the SGR 1806–20 27 Dec. 2004 GF (u ~ 1)

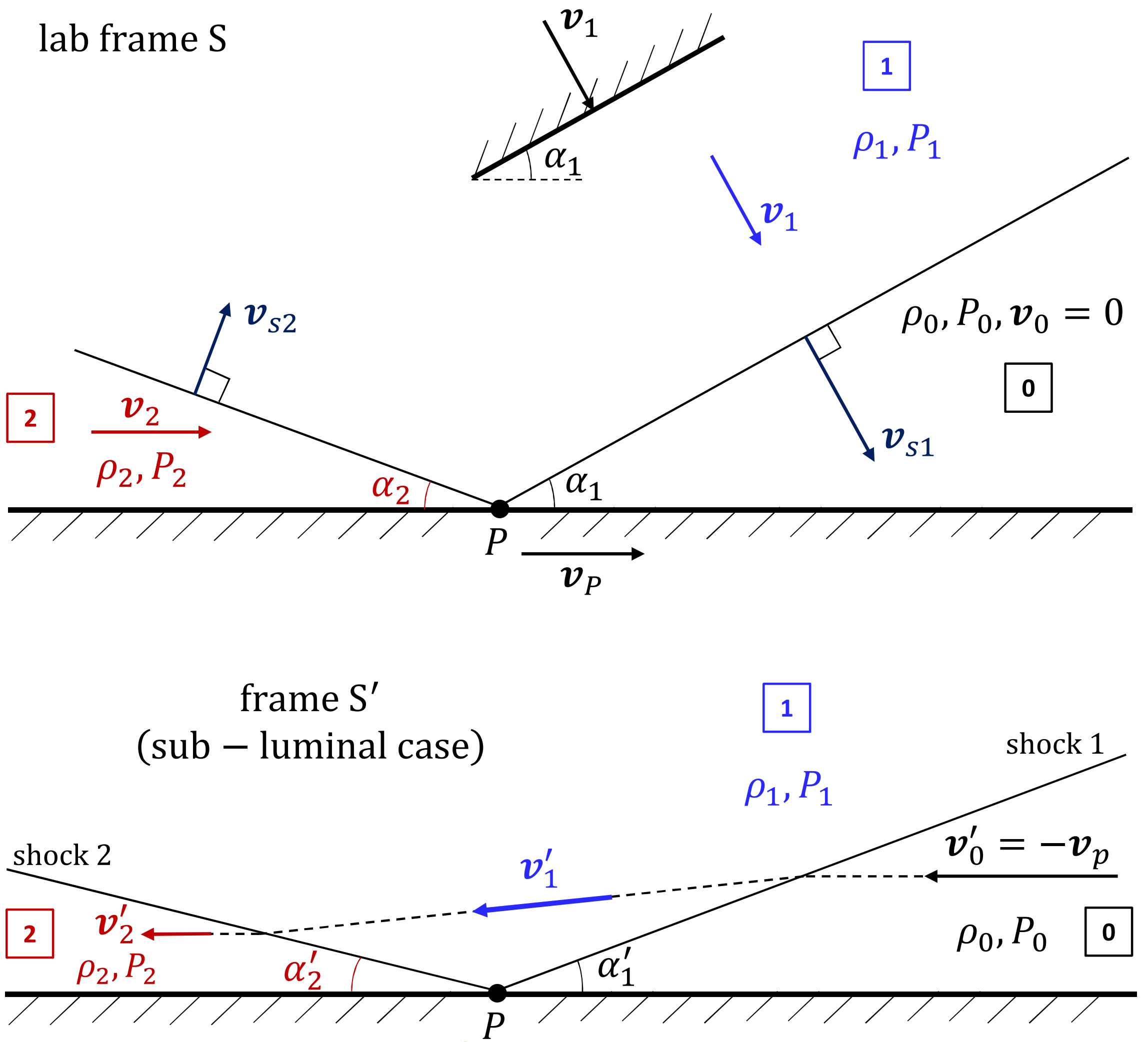
The 15 April 2020, magnetar GF from the Sculptor galaxy, D = 3.5 Mpc; u ~ 100)





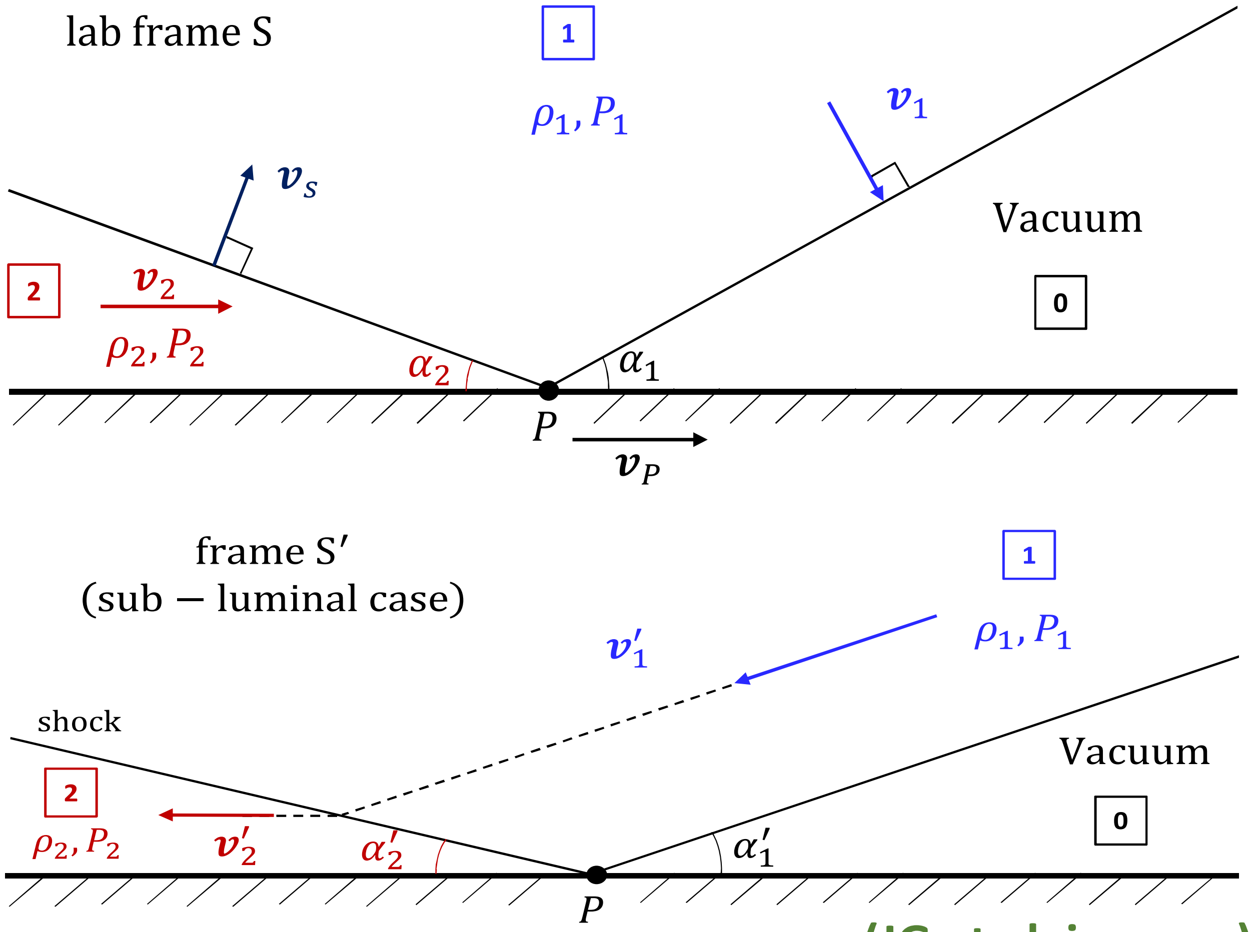
Analogous to Shock reflection (a classic problem):

Shock Reflection



(JG & Rabinovich 2024)

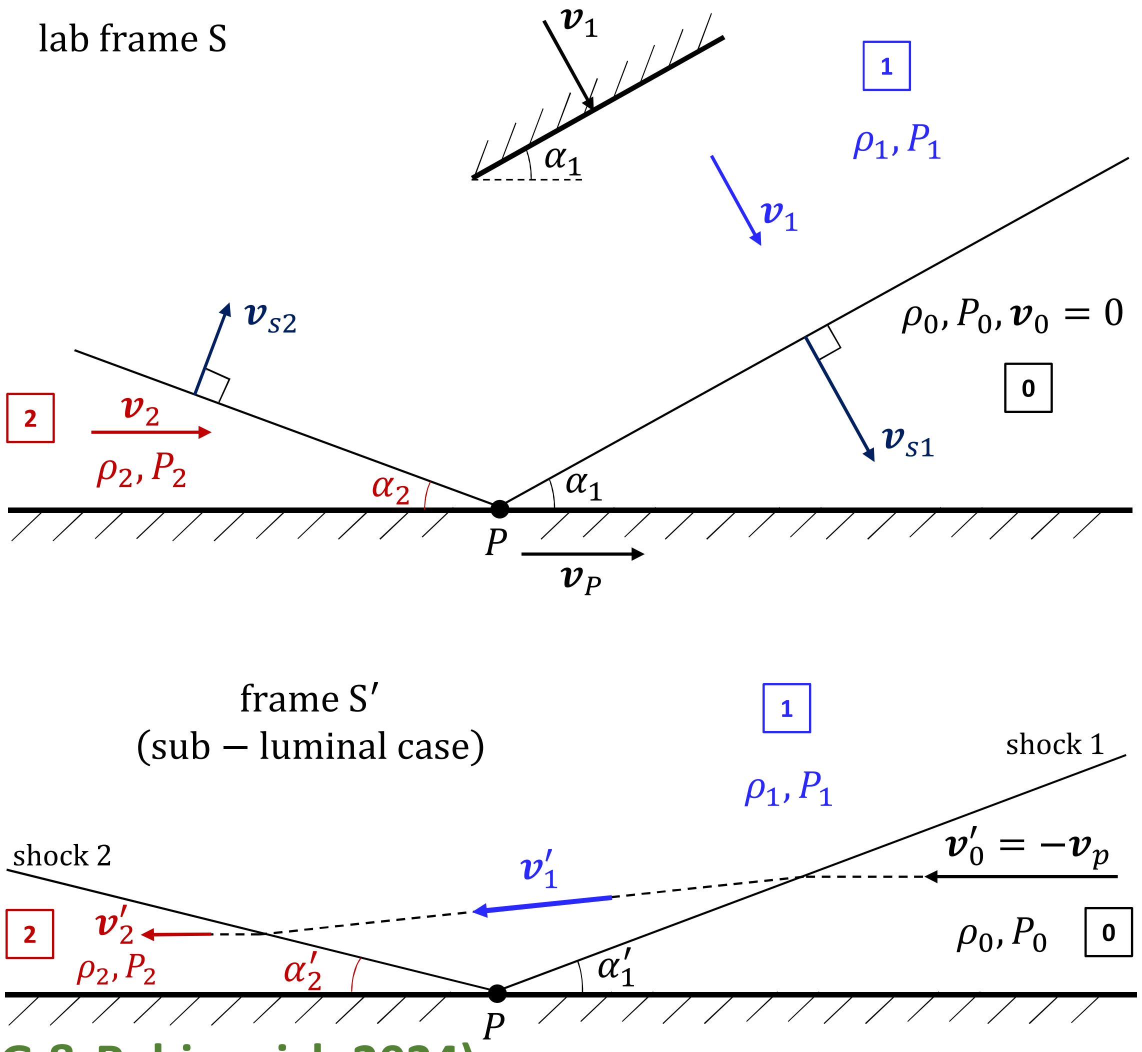
Single Shell - Wall Collision



(JG et al. in prep.)

Shock reflection (a classic problem):

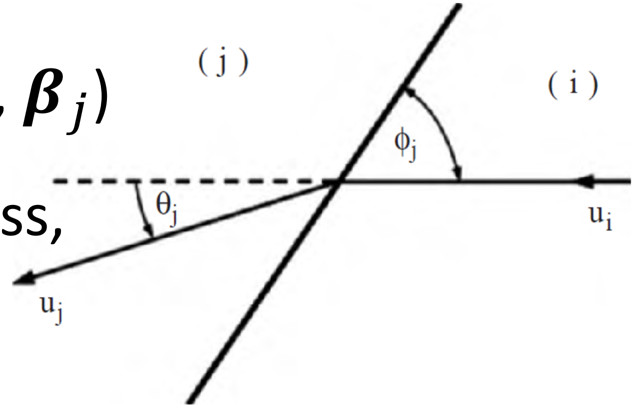
Shock Reflection



(JG & Rabinovich 2024)

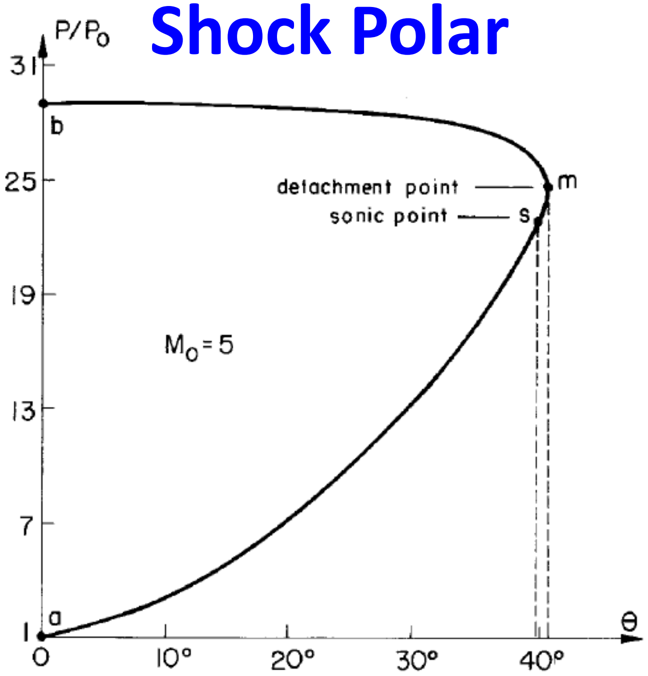
Oblique Shock

- 5 unknowns $(p_j, n_j, e_j, \boldsymbol{\beta}_j)$
- 5 constraints (EoS, mass, energy, momentum × 2)

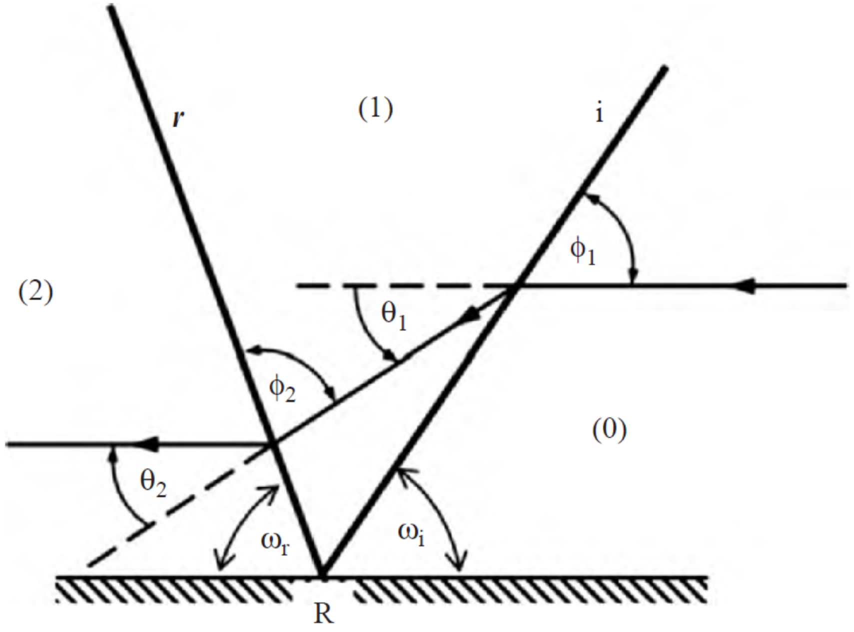


Shock Polar

There is a maximum deflection angle $\theta_{\max}$



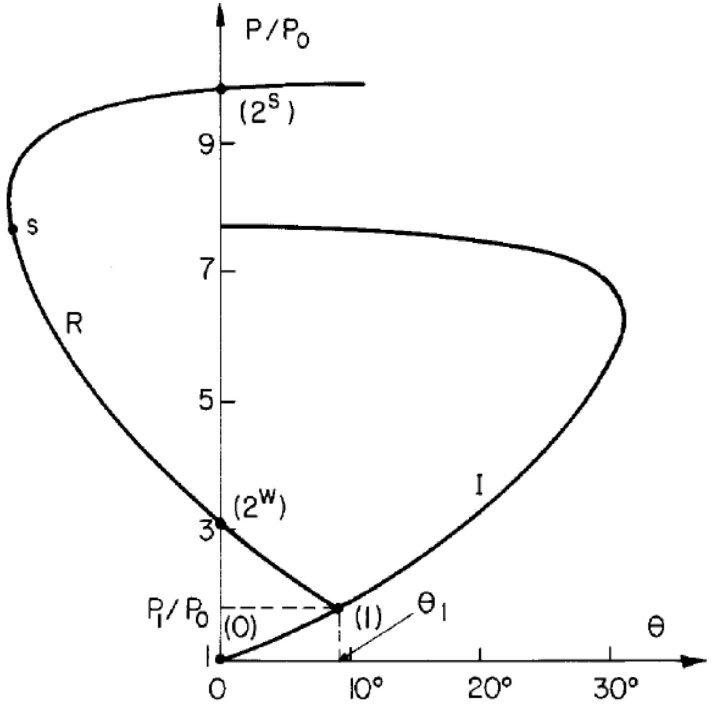
Newtonian Shock reflection: studied in the steady state frame



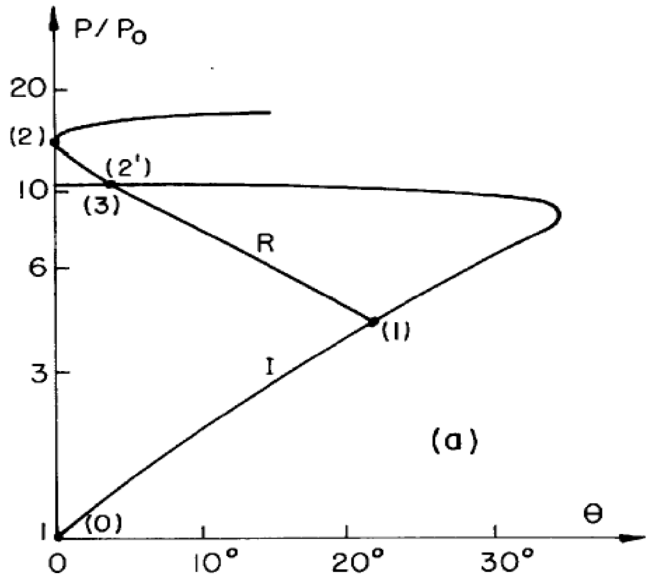
Regular Reflection: Shocks attach to the wall

- 10 unknowns $(p_1, n_1, e_1, \boldsymbol{\beta}_1, p_2, n_2, e_2, \boldsymbol{\beta}_2)$
- 10 constraints (2 EoS, 2 shocks × [mass, energy, momentum × 2] conserved)

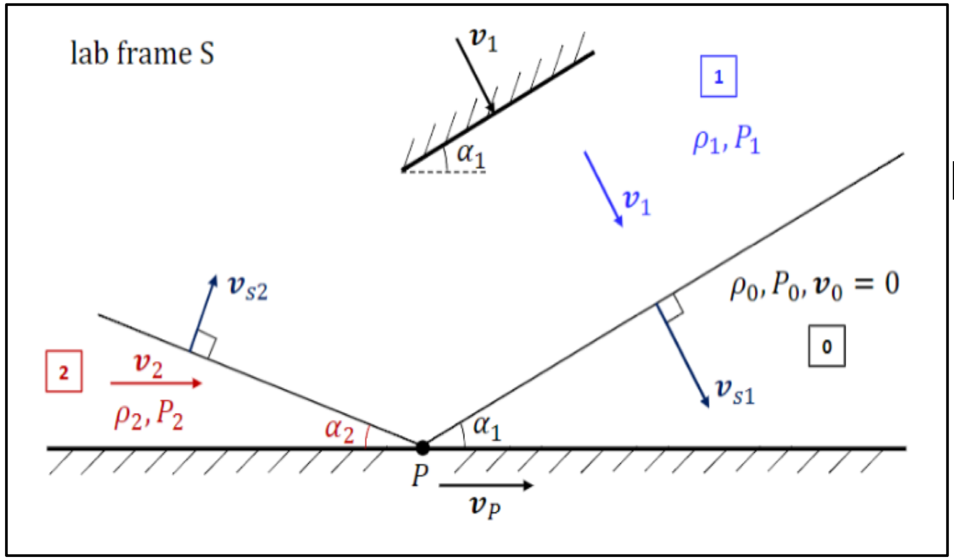
- **There are 2 solutions**: Weak Shock (W) & Strong Shock (S)

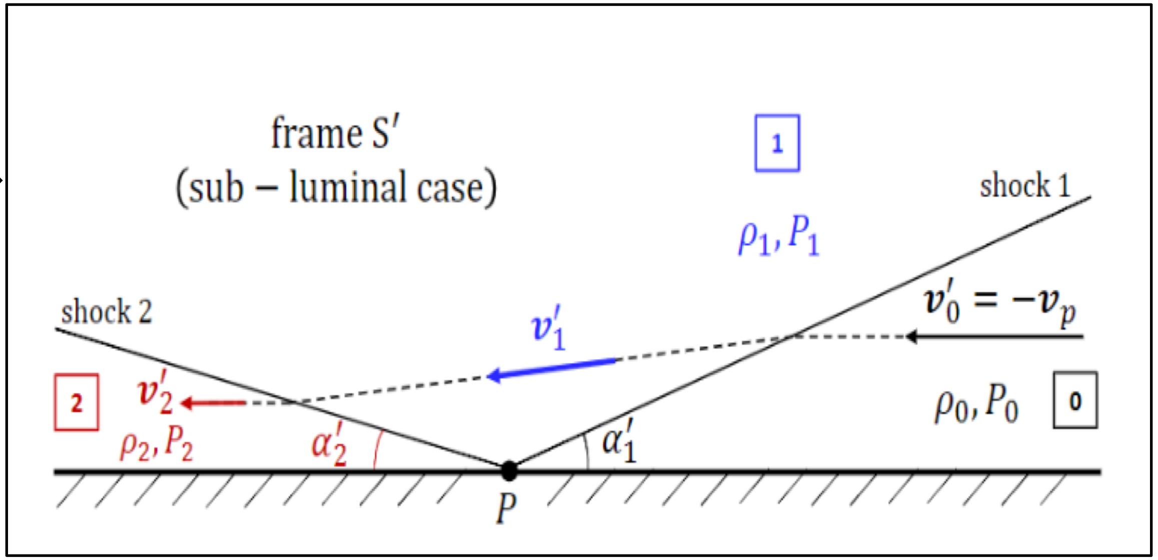


Detachment Criterion: (close to sonic criterion)



Relativistic Shock reflection (JG & Rabinovich 2024; Bera et al. 2024):





$$v_p = \frac{v_{s1}}{\sin \alpha_1} = \frac{v_{s2}}{\sin \alpha_2}$$

If $v_P > c$ (the super-luminal regime/region) then the usual method of working in the rest-frame of point P is inapplicable

A. Incident shock 1, driven by a piston, is the well known 1D shock tube problem

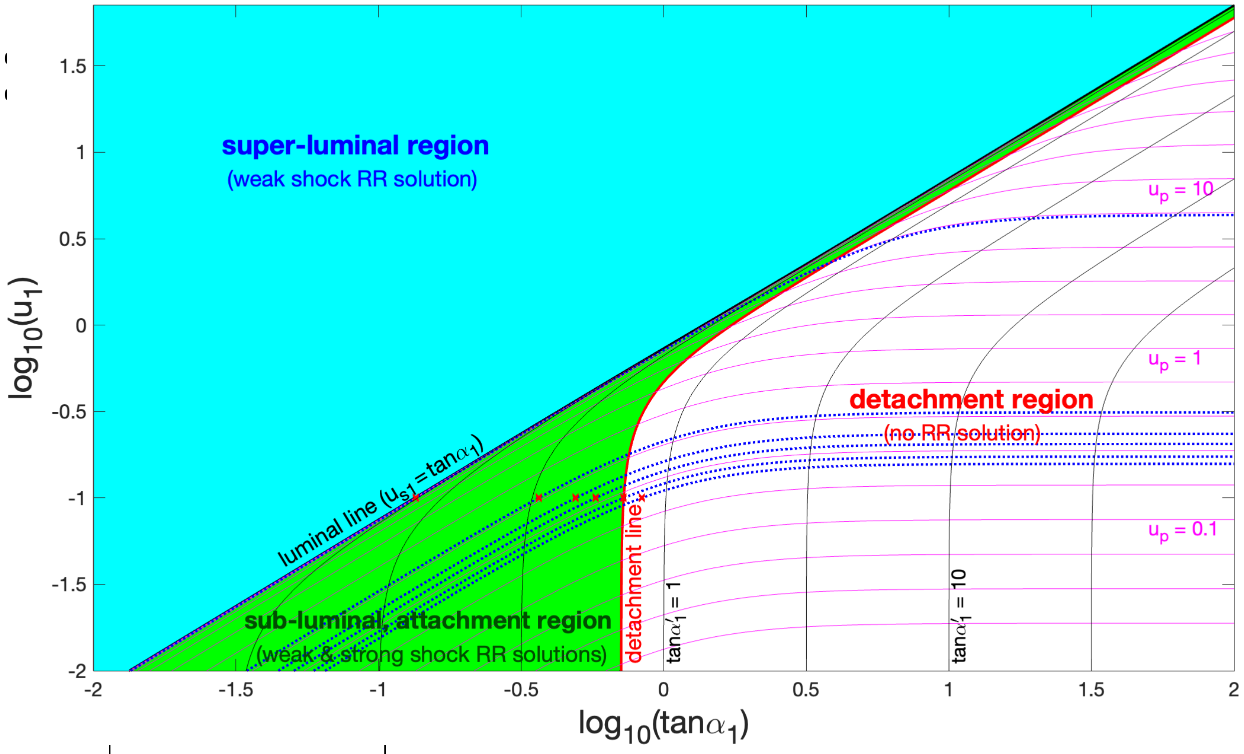
B. For reflected shock 2 & regular reflection we use integral conservation laws

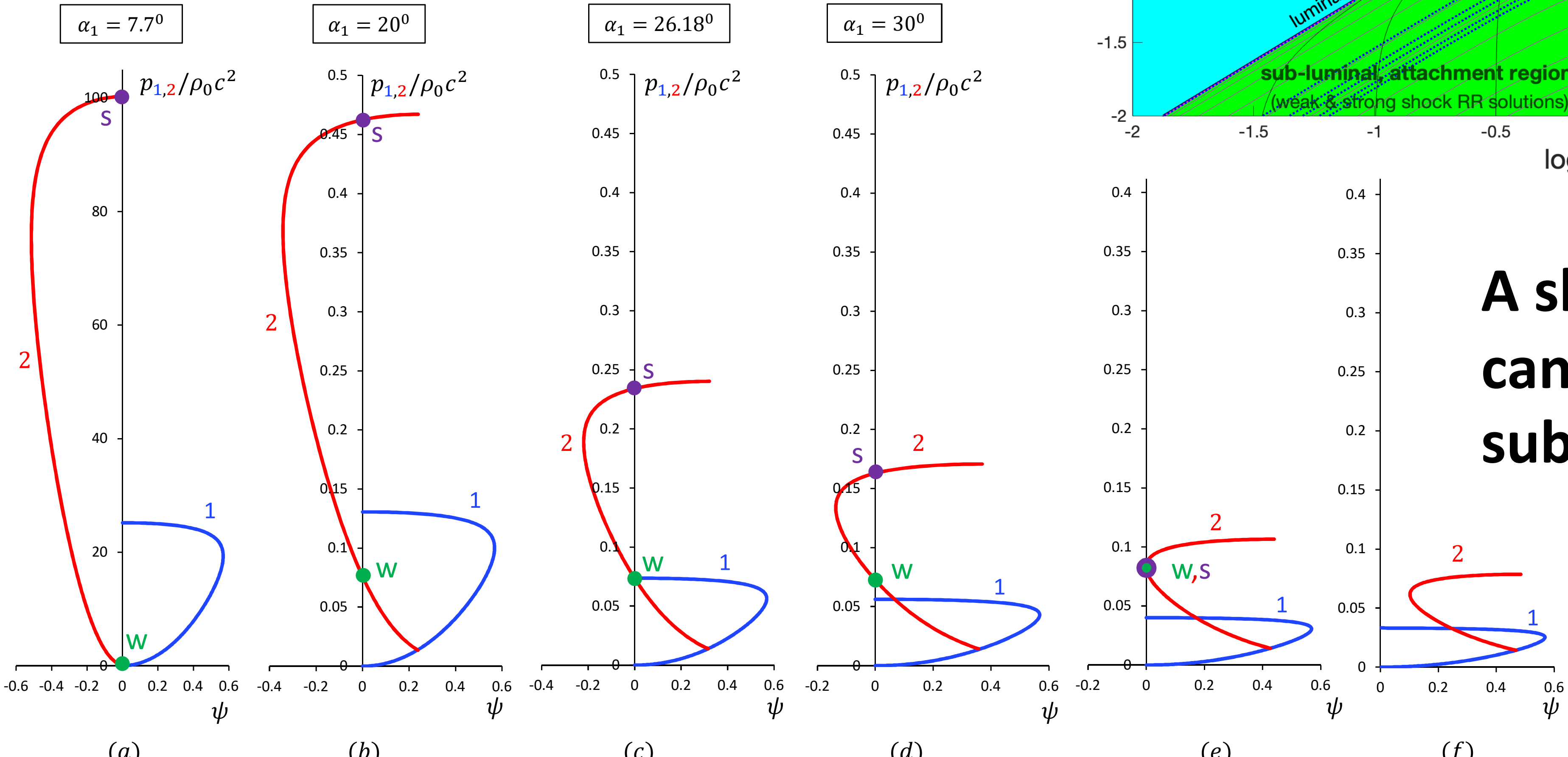
Relativistic Shock reflection:

(JG & Rabinovich 2024)

Shock 2 polar: u_1 = 0.1

Shock 1 polar: u_P = 6.12, 0.419, 0.314, 0.274, 0.231, 0.210

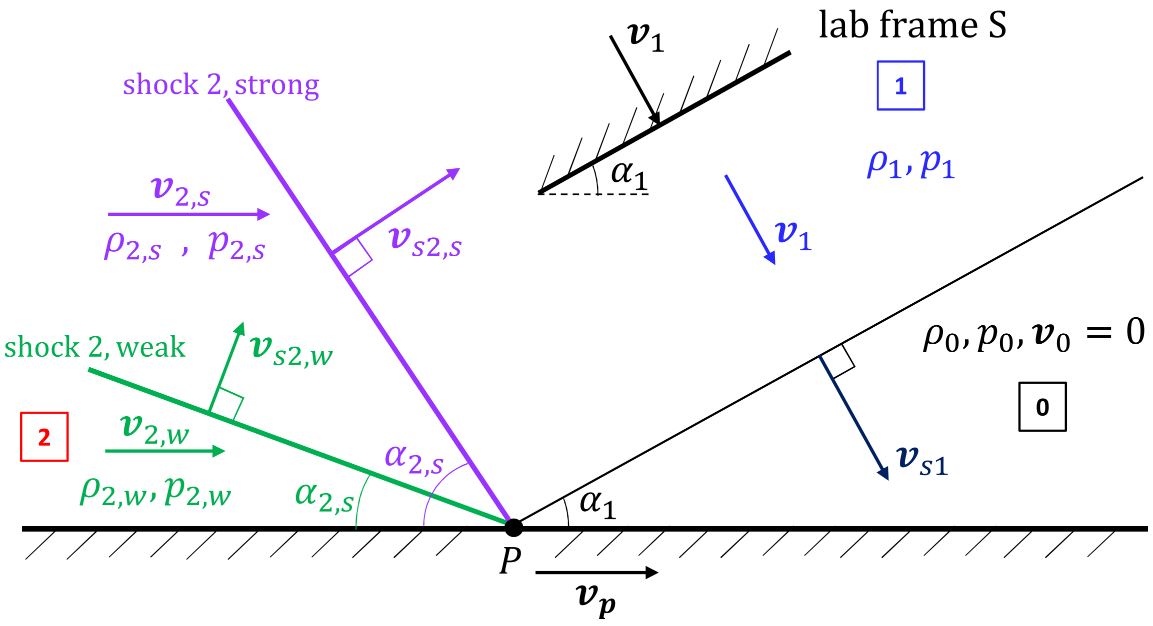


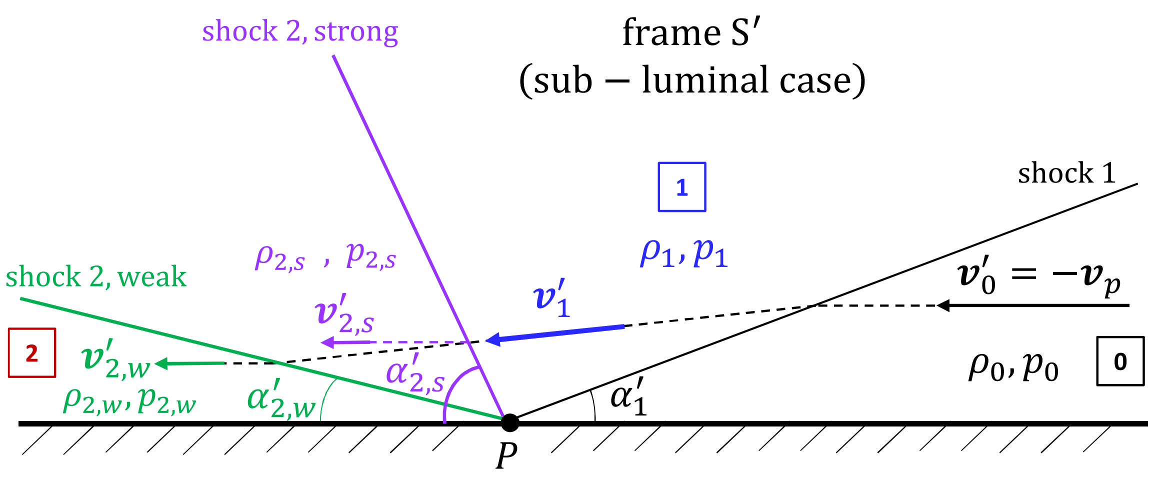


A shock polar analysis can only be done in the sub-luminal region

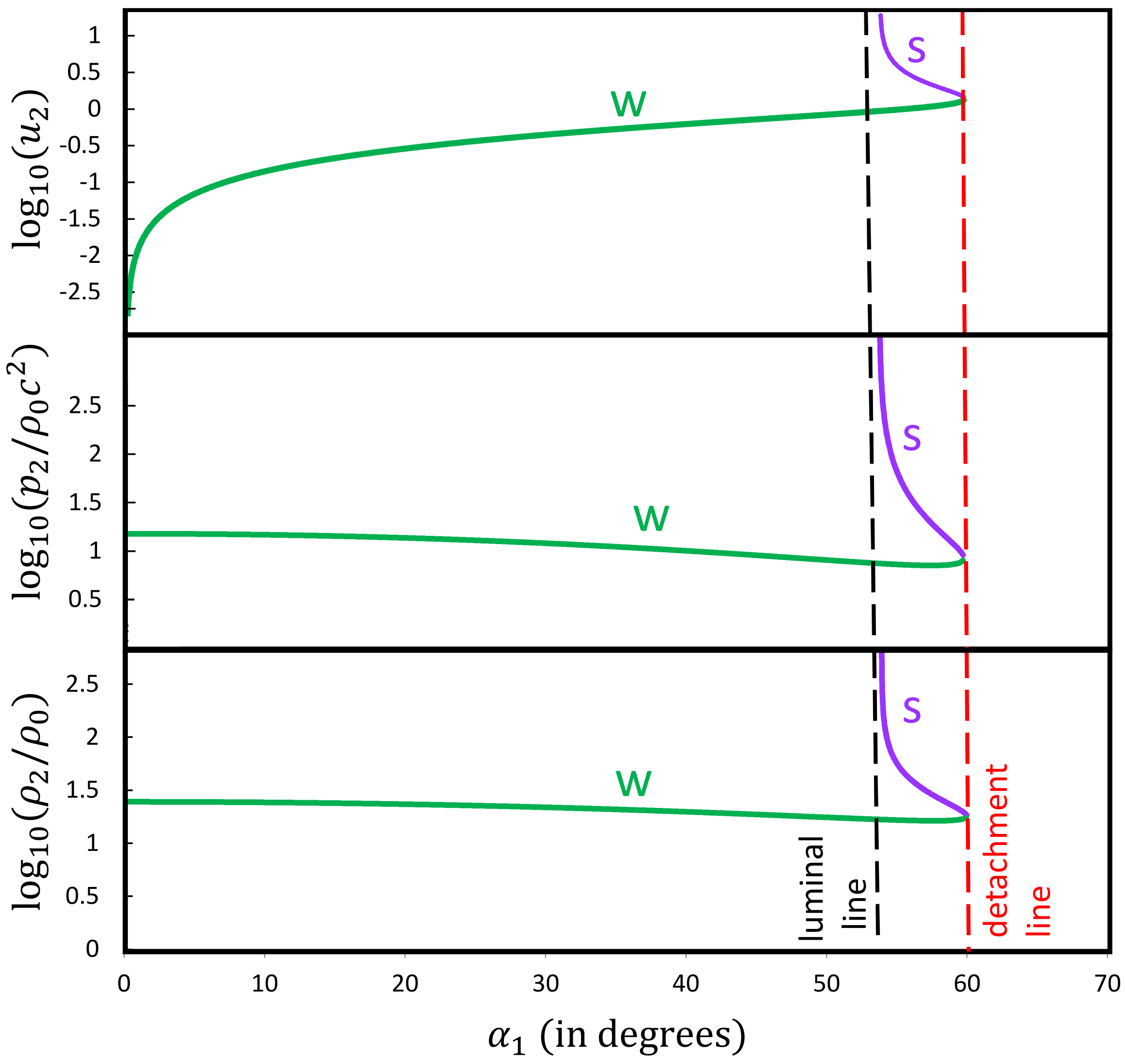
Relativistic Shock reflection:

(JG & Rabinovich 2024)



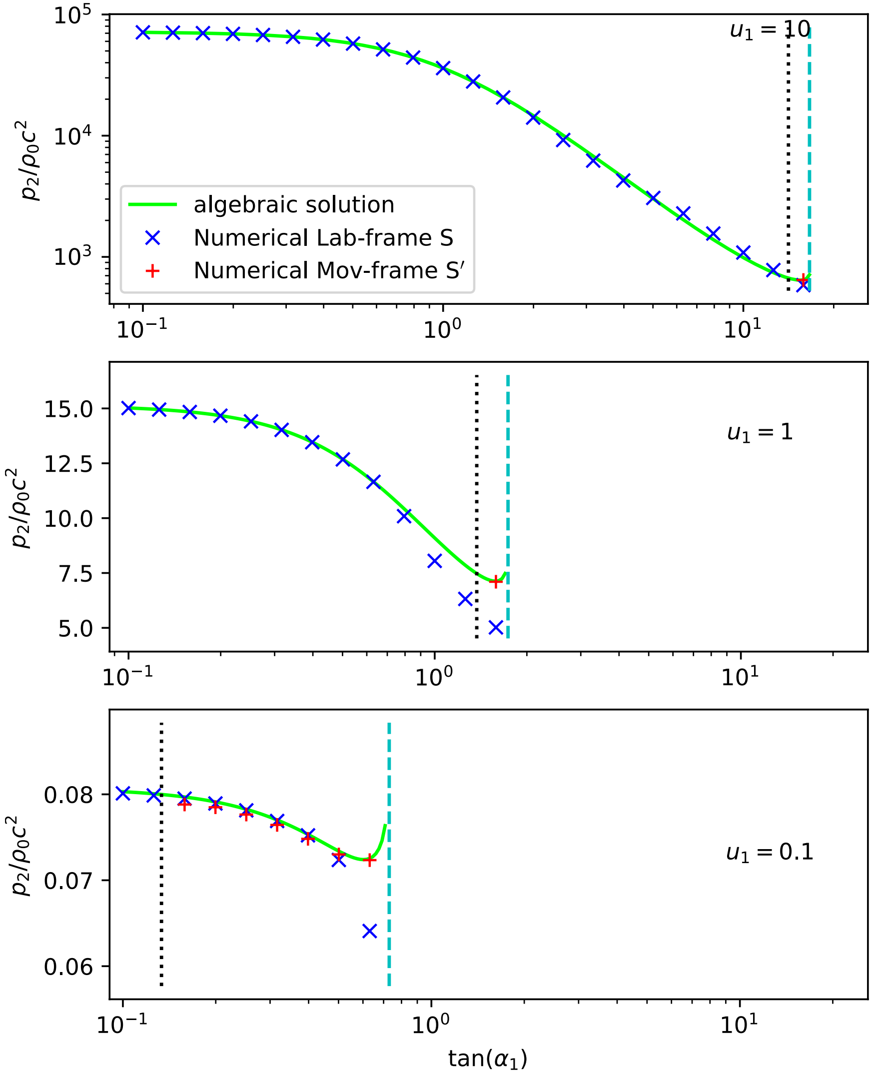


The Strong Shock solution cannot cross the luminal line as it diverges

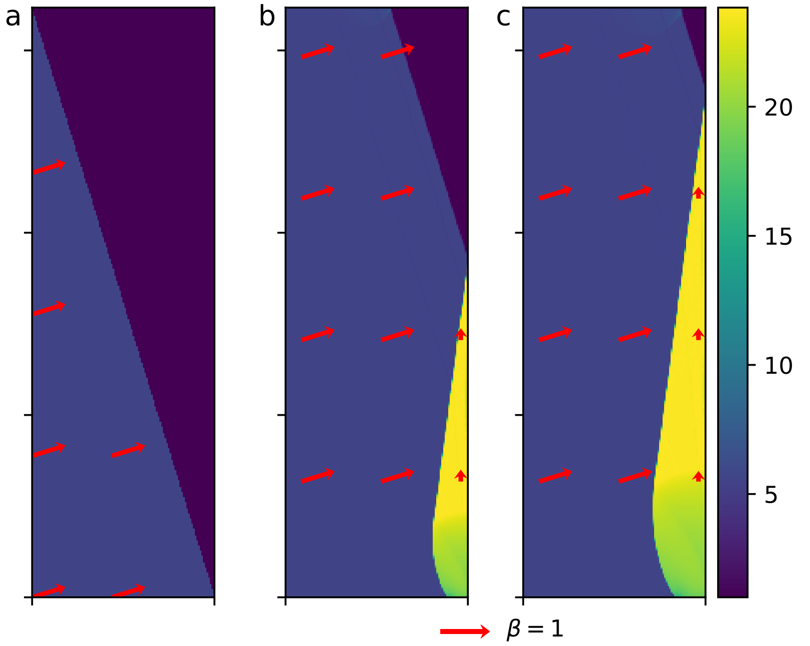


Relativistic Shock reflection (Bera et al. 2024):

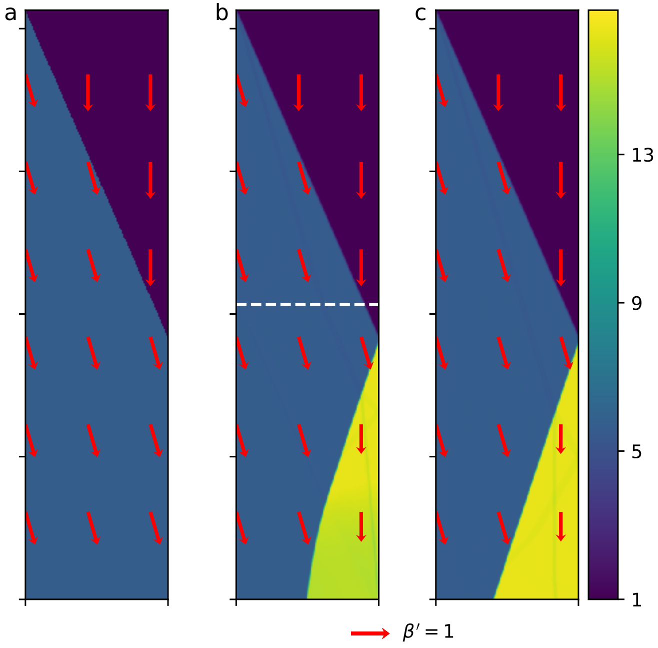
Reproduces the analytic **weak shock** solution



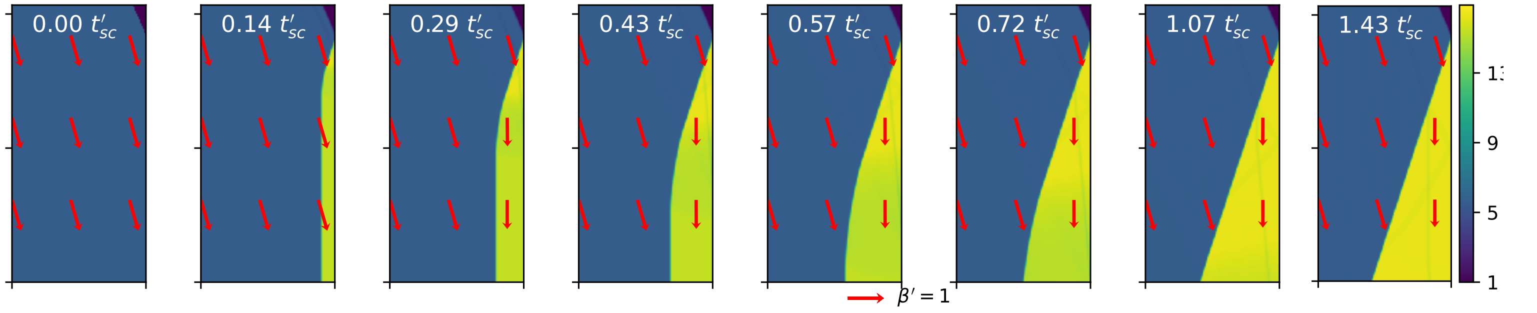
Lab frame (density)



Steady frame (density)

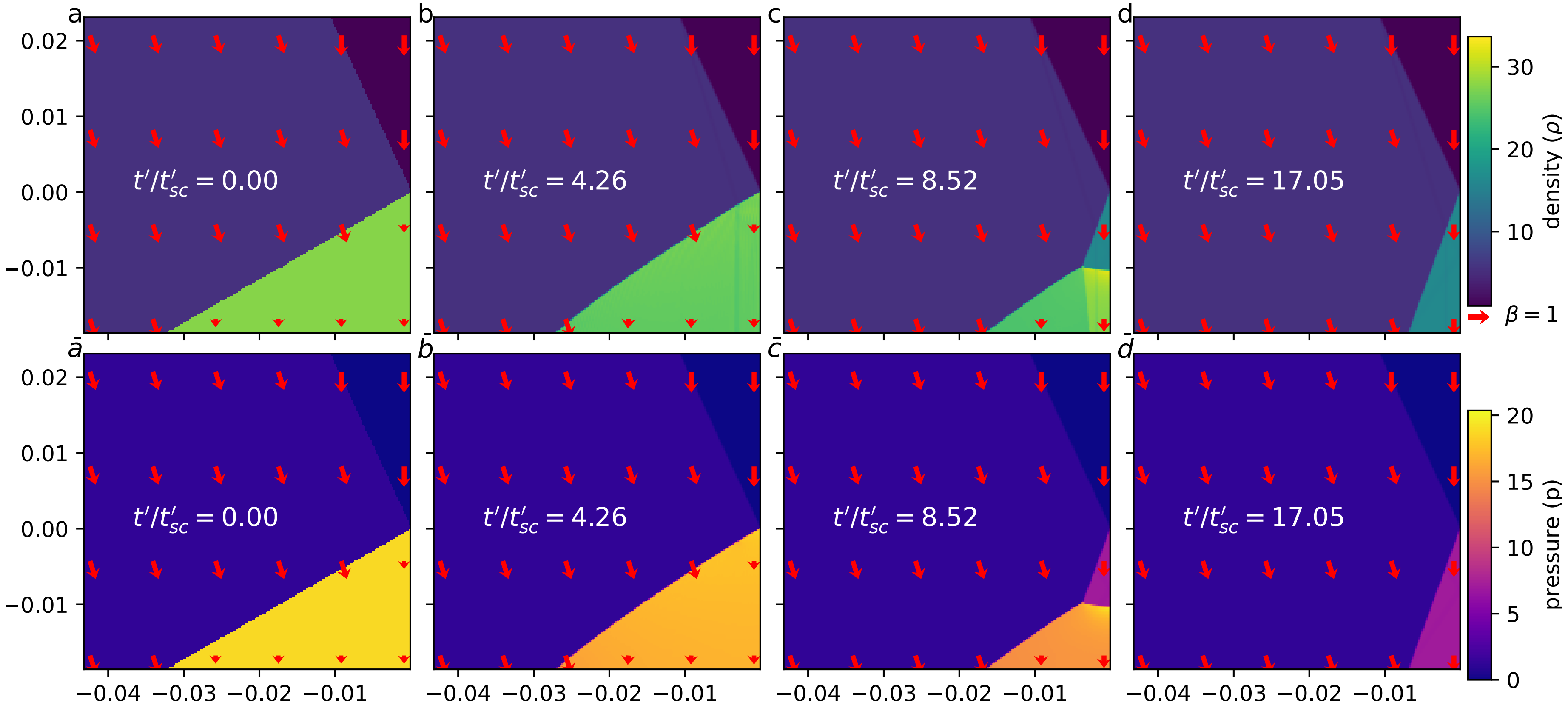


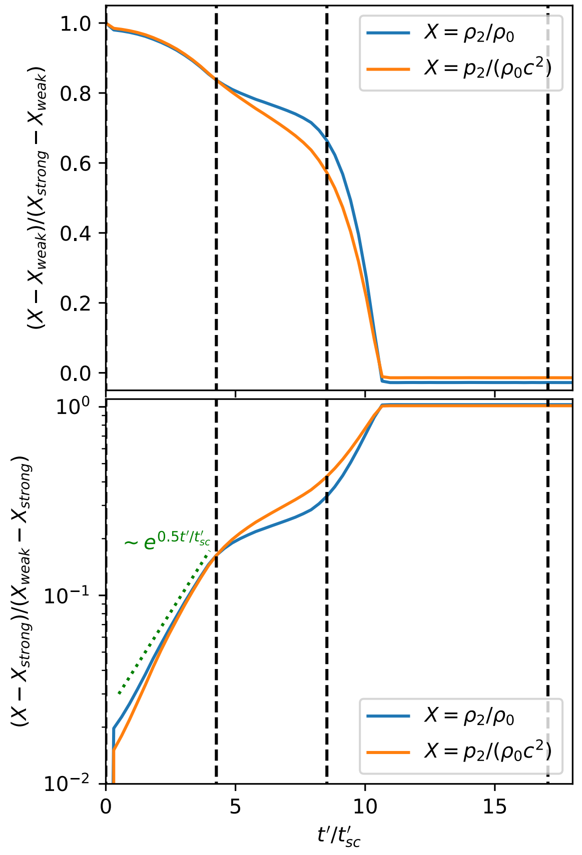
Gradually achieving steady state (density):



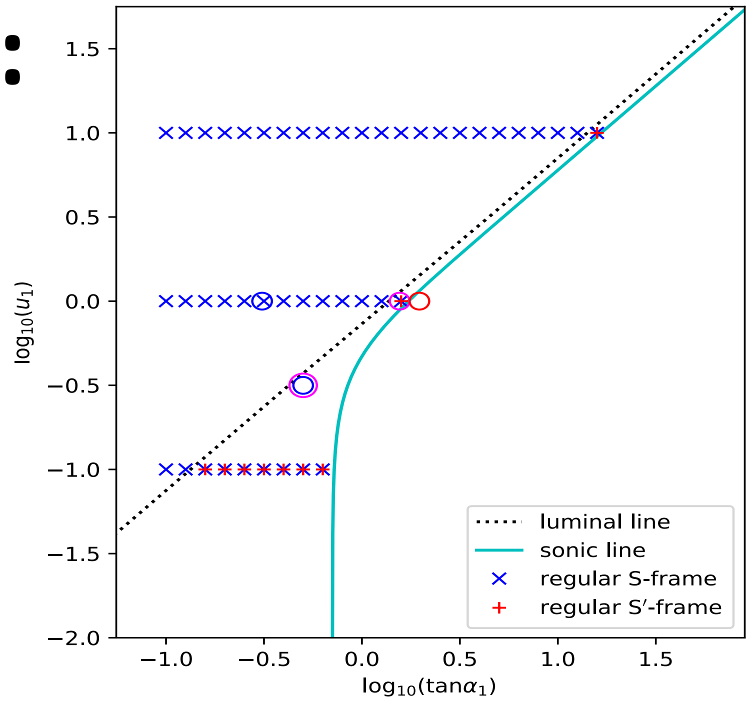
Relativistic Shock reflection (Bera et al. 2024):

Strong Shock solution is found to be unstable & transitions to Weak Shock solution

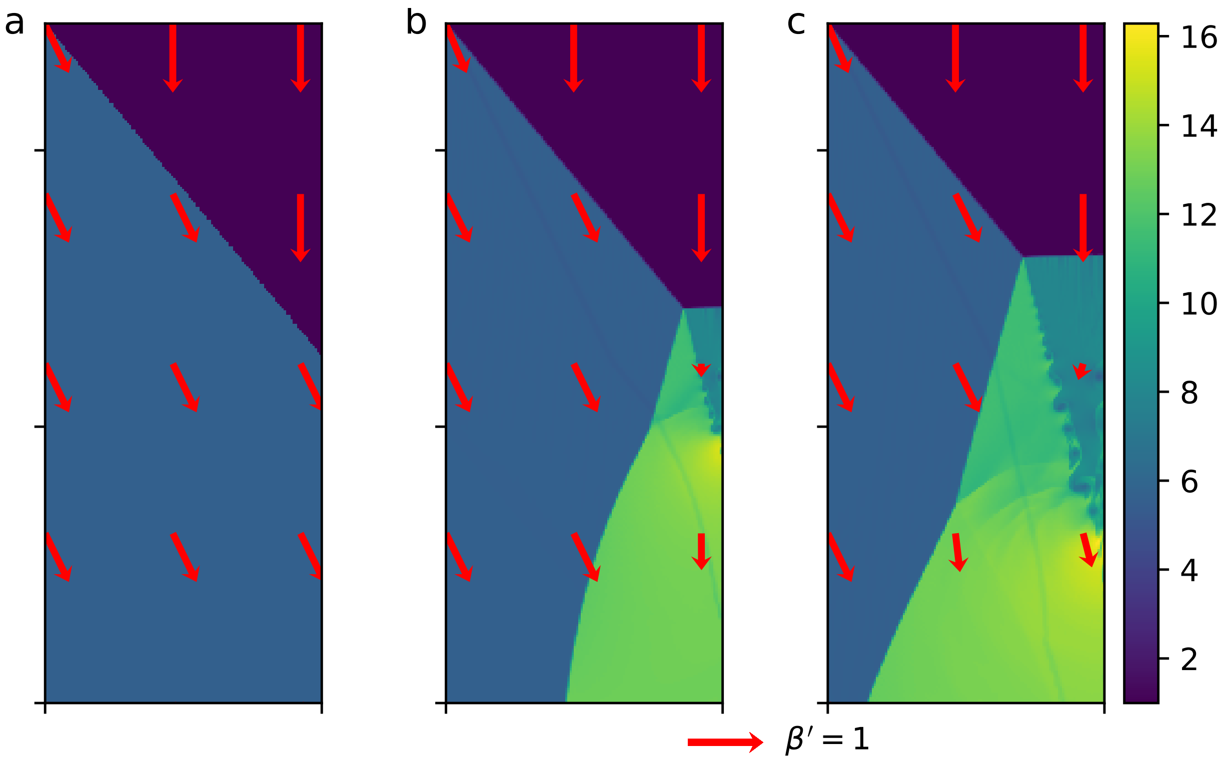


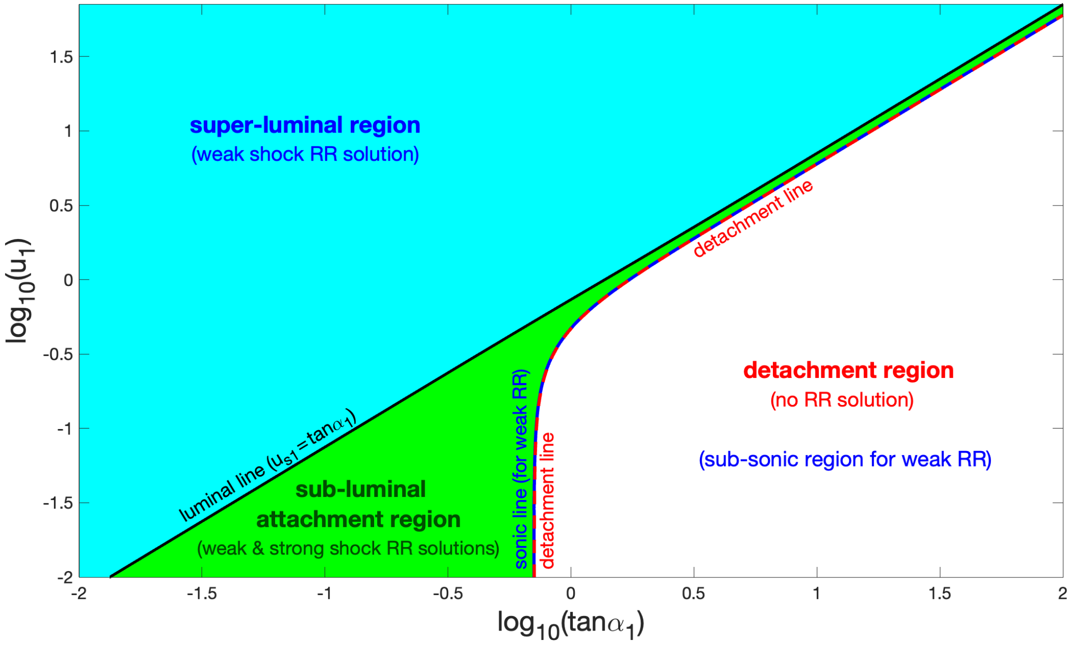


Relativistic Shock reflection (Bera et al. 2024):



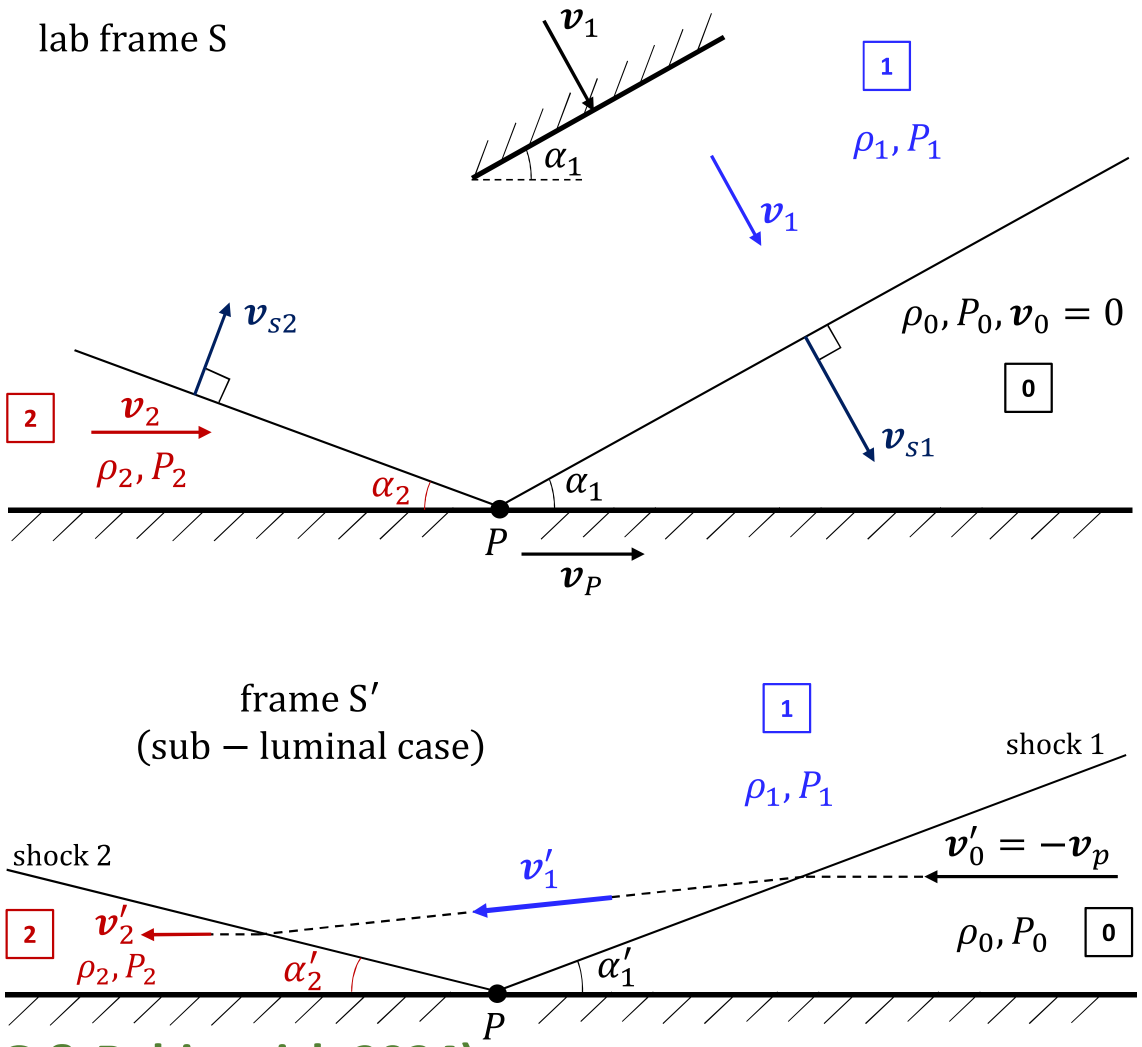
In the detachment region: Mach reflection



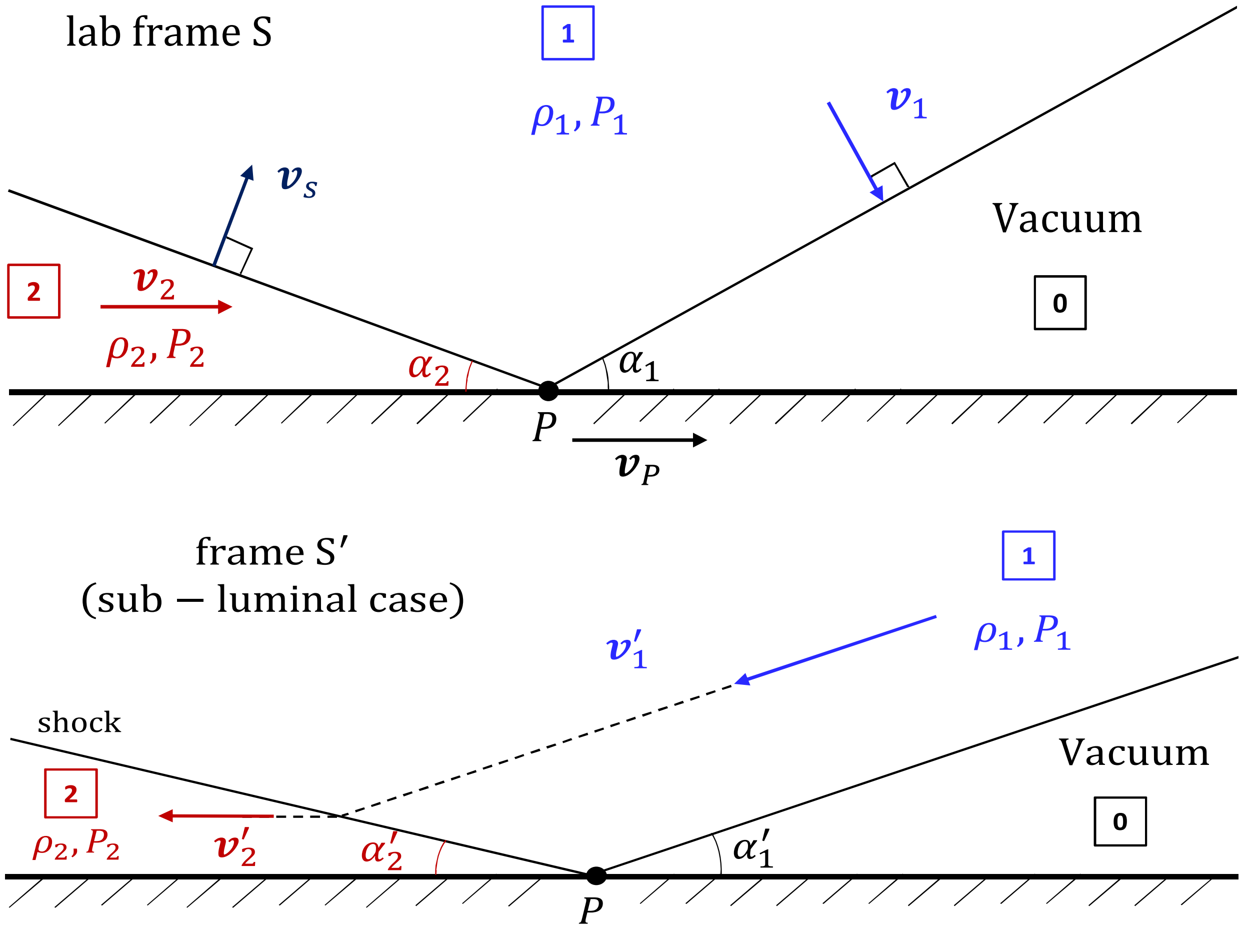


Single Shell – Wall Collision (JG et al. in prep.):

Shock Reflection



Single Shell - Wall Collision

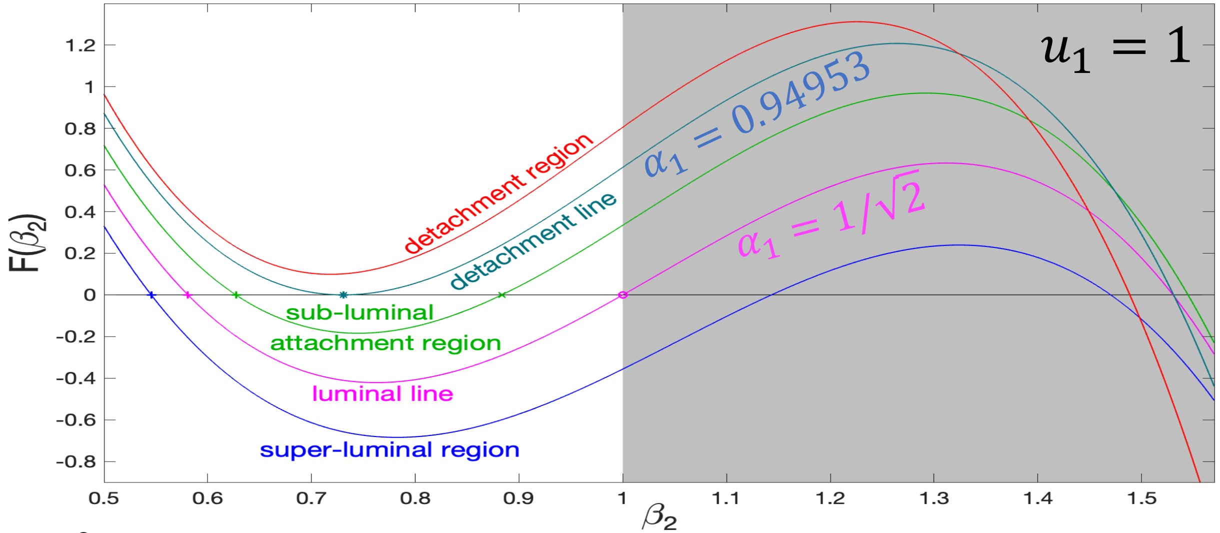


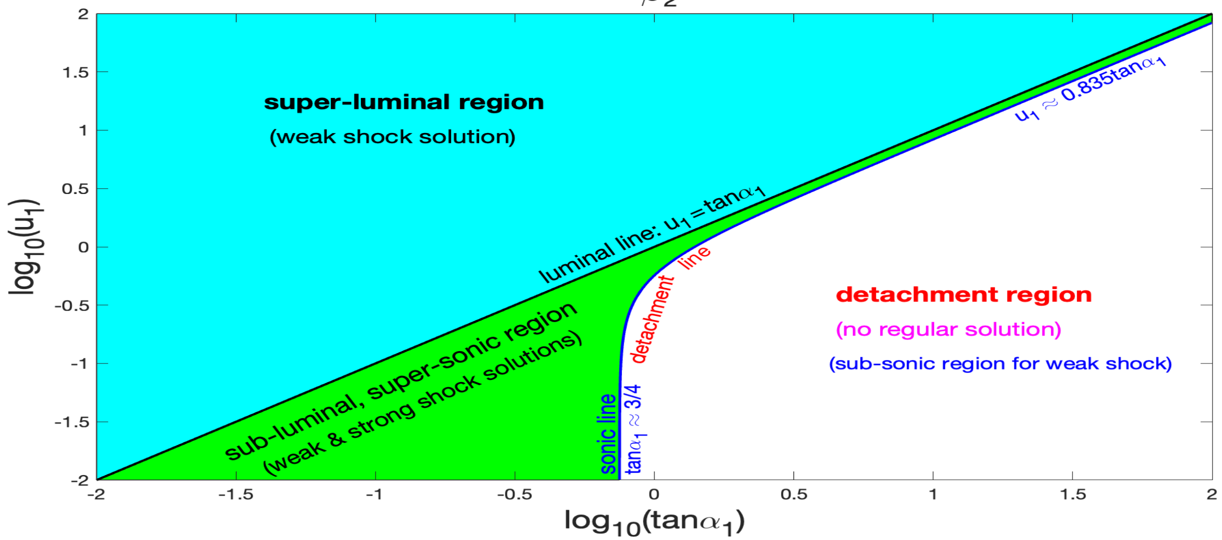
(JG & Rabinovich 2024)

Single Shell – Wall Collision (JG et al. in prep.):

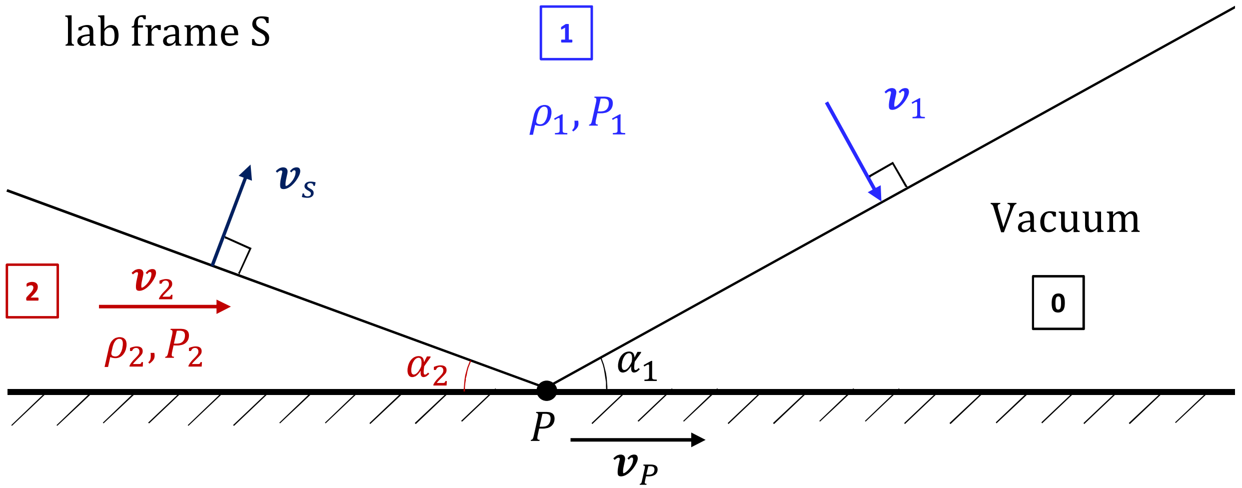
Fully analytic regular solution:

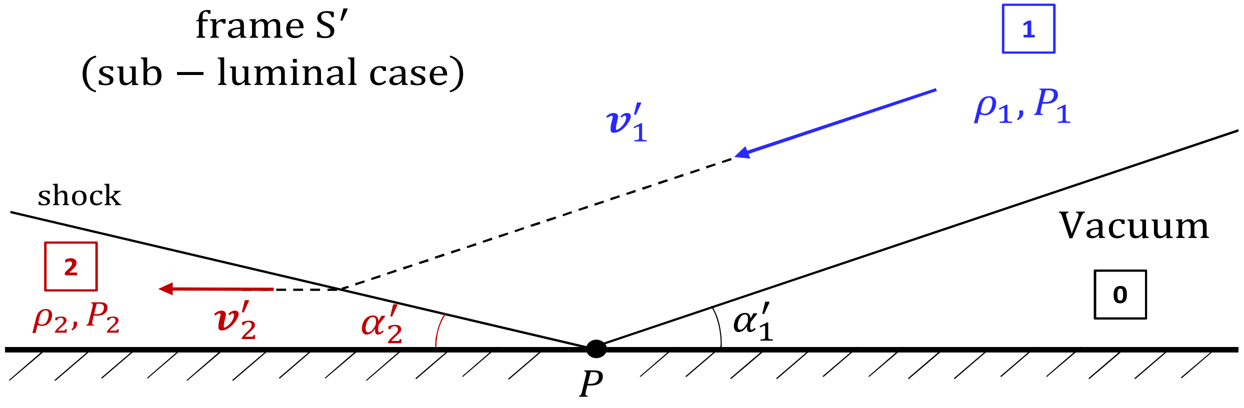
$$F(\beta_2) = A\beta_2^3 + B\beta_2^2 + C\beta_2 + D = 0$$





Single Shell - Wall Collision





Conclusions: Colliding Relativistic Shells

- Such collisions are expected in GRBs, blazars, magnetar GFs, SLSNe, FRBs,…
- One collision produces two shocks: forward shock (FS) & reverse shock (RS)
- The reverse shock is typically stronger, dominating νF_ν near $E_{\rm peak}$
- The forward shock peaks below $E_{\rm peak}$ & might mimic photospheric emission
- RS + FS produce diverse pulse shapes & spectra (also time-integrated)
- Fits to data can help constrain important physical parameters of the outflow
- Oblique collisions are analogous to the classical shock reflection problem
- Both have analytic "regular" solutions for initially cold fluids
- There is a super-luminal regime where only the weak shock solution exists
- Between luminal & detachment lines an unstable strong shock solution exists
- Oblique collisions modify $\boldsymbol{\beta}$ of $\Gamma \gg 1$ shells, affecting their beamed emission

The End