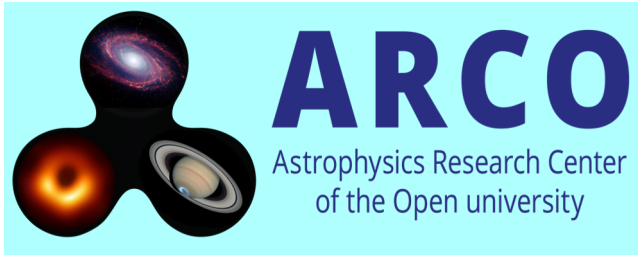


Colliding Relativistic Shells: New Insights



Open University of Israel

**Jonathan
Granot**



& George Washington University

Collaborators: S.M. Rahaman, P. Beniamini, M. Rabinovich, P. Bera, A. Charlet
Rahaman, Granot & Beniamini 2024a, b (MNRAS, 258, L45; MNRAS, 258, 160)
Granot & Rabinovich 2024 (Physics of Fluids, 36, 016142)
Bera, Granot, Rabinovich & Beniamini 2024 (Physics of Fluids, 36, 016141)
Charlet, Granot & Beniamini 2025 (submitted)

Relativistic Fluids around Compact Objects, Warsaw, Poland, 5 May 2025

Relativistic Bulk Collisions in Astrophysics:

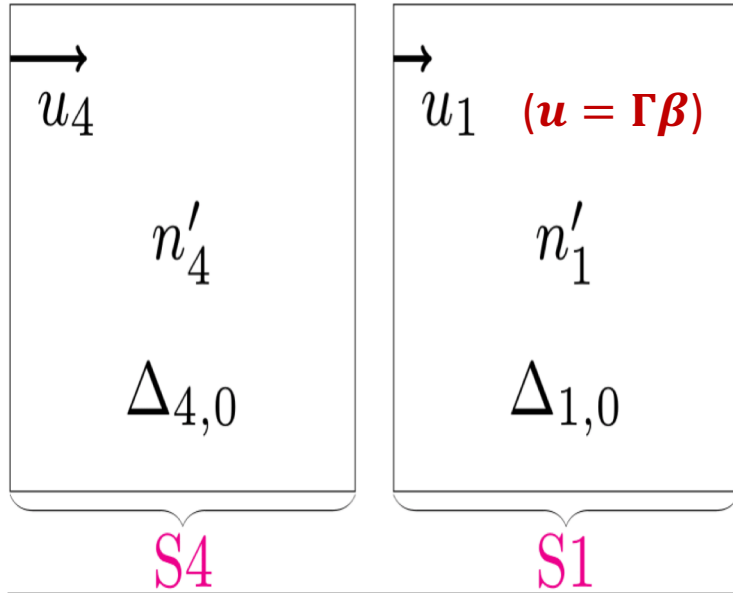
a fast shell overtakes and collides with a slower shell

- Blazars: internal shocks (Rees 1978) $u = \Gamma\beta \sim 10 - 30$
- Gamma-Ray Bursts (GRBs): internal shocks (Rees & Meszaros 1994) $u \sim 10^2 - 10^3$
- Magnetar Giant Flare (GF): outflow & external shell (JG et al. 2006) $u \sim 1 - 10^2$
- Superluminous Supernovae (SLSNe): ejecta & external massive shell
(Smith & McCray 2007) $u \sim 10^{-2}$
- Fast Radio Bursts (FRBs): GF outflow + MWN or another GF outflow
(Lyubarsky 2014; Metzger+ 2017; Beloborodov 2017) $u \sim 10^2 - 10^4$

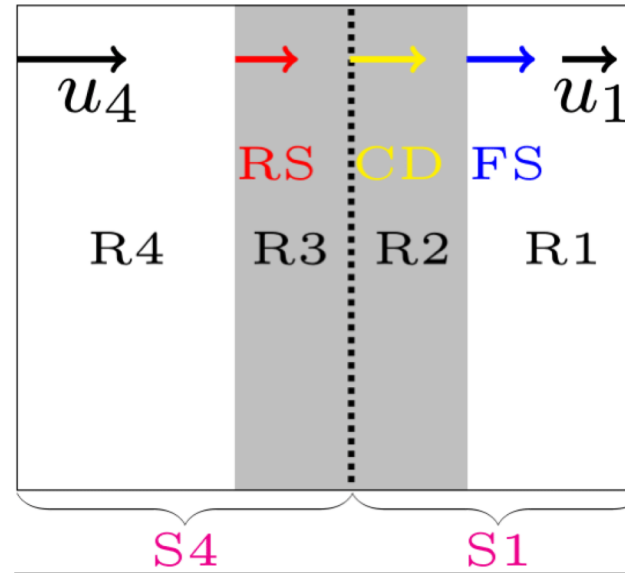
A Two Shell Collisions: (Rahaman, JG & Beniamini 2024b)

(cold & uniform shells, in 1D planar symmetry)

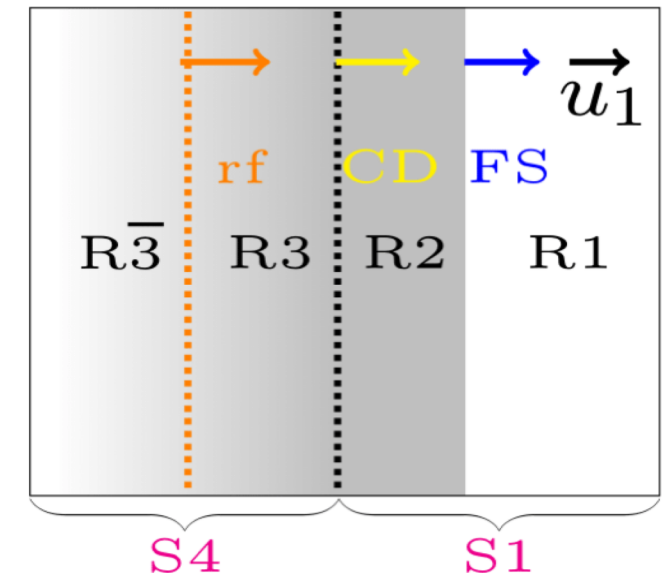
Pre-collision



Post collision



Post reverse shock crossing (for $t_{RS} < t_{FS}$)



Proper density ratio: $f = n'_4/n'_1$

Proper speed ratio: $a_u = u_4/u_1 > 1$

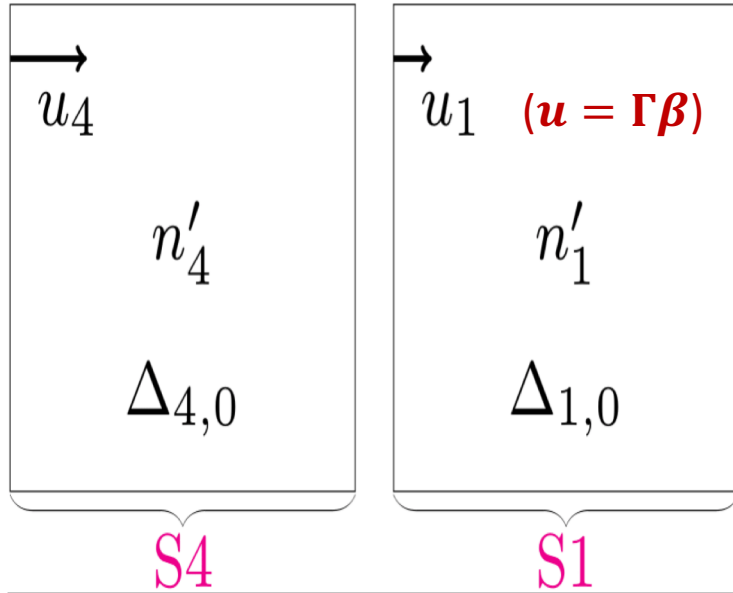
Radial width ratio: $\chi = \frac{\Delta_{1,0}}{\Delta_{4,0}} \sim 1$

Collision radius: $R_0 = \frac{\beta_1\beta_4 ct_{\text{off}}}{\beta_4 - \beta_1}$

A Two Shell Collisions: (Rahaman, JG & Beniamini 2024b)

(cold & uniform shells, in 1D planar symmetry)

Pre-collision



- 10 unknowns ($p_2, n_2, e_2, \beta_2, p_3, n_3, e_3, \beta_3, \beta_{FS}, \beta_{RS}$)
- 10 constraints (2 EoS - TM, 2 CD, 3 FS, 3 RS)
- First Solve in the rest frame of Shell 1 :

$$u_{21} = u_{31} = u_{41} \sqrt{\frac{2f^{3/2}\Gamma_{41} - f(1+f)}{2f(u_{41}^2 + \Gamma_{41}^2) - 1 - f^2}}$$
- Then transform to the lab frame

Proper density ratio: $f = n'_4/n'_1$

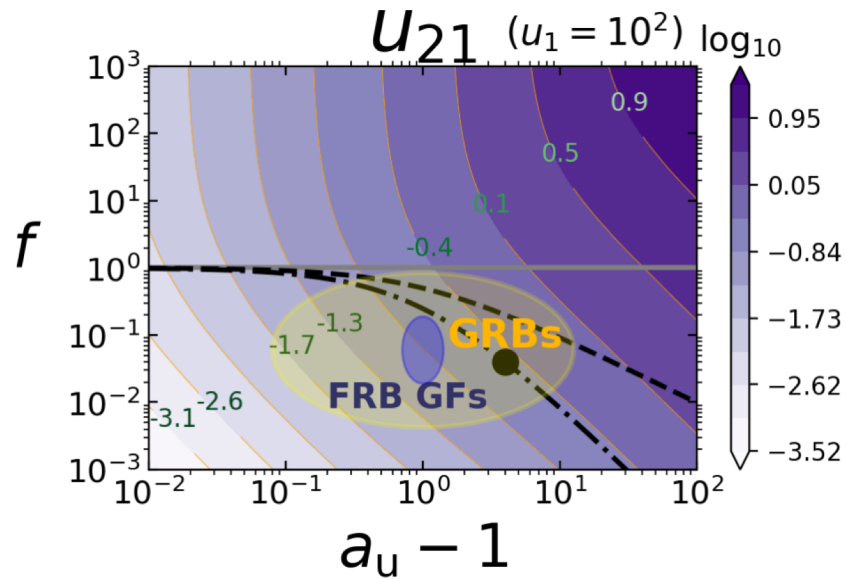
Proper speed ratio: $a_u = u_4/u_1 > 1$

Radial width ratio: $\chi = \frac{\Delta_{1,0}}{\Delta_{4,0}} \sim 1$

Collision radius: $R_0 = \frac{\beta_1\beta_4 c t_{\text{off}}}{\beta_4 - \beta_1}$

A Two Shell Collisions:

- Gamma-Ray Bursts (GRBs): internal shocks
- Fast Radio Bursts (FRBs): GF outflow + MWN or GF outflow

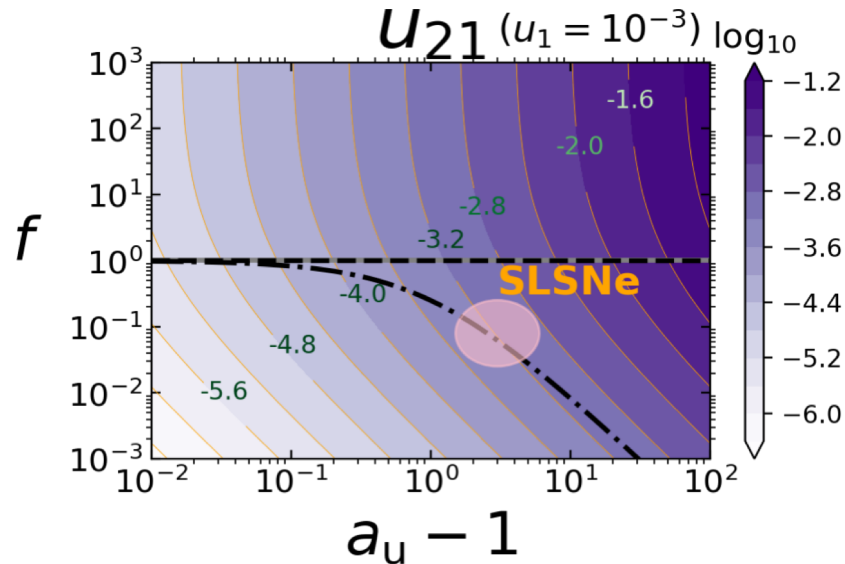


$$u_{41} = \left(a_u \Gamma_1 - \sqrt{1 + a_u^2 u_1^2} \right) u_1$$

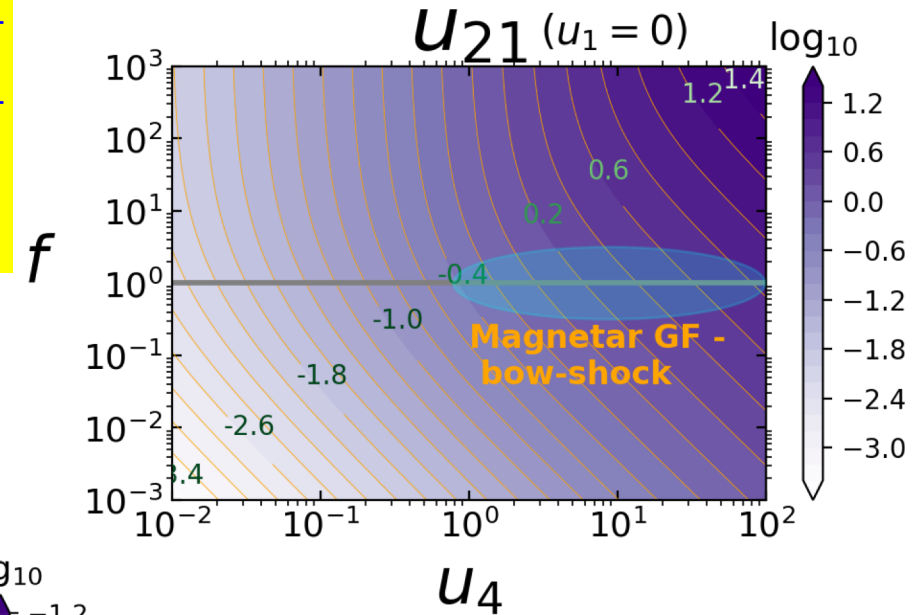
$$u_{21} = u_{41} \sqrt{\frac{2f^{3/2} \Gamma_{41} - f(1+f)}{2f(u_{41}^2 + \Gamma_{41}^2) - 1 - f^2}}$$

$$u = u_2 = u_3 = \Gamma_{21} \Gamma_1 (\beta_{21} + \beta_1)$$

- SLSNe – Superluminous Supernovae: ejecta & external massive shell



- Magnetar Giant Flare (GF): outflow & bow-shock shell



U_{41} = pre-collision relative proper speed of the shells
 U_{21} = relative upstream to downstream proper speed across the forward shock

$$f = n'_4 / n'_4$$

$$a_u = u_4 / u_1 > 1$$

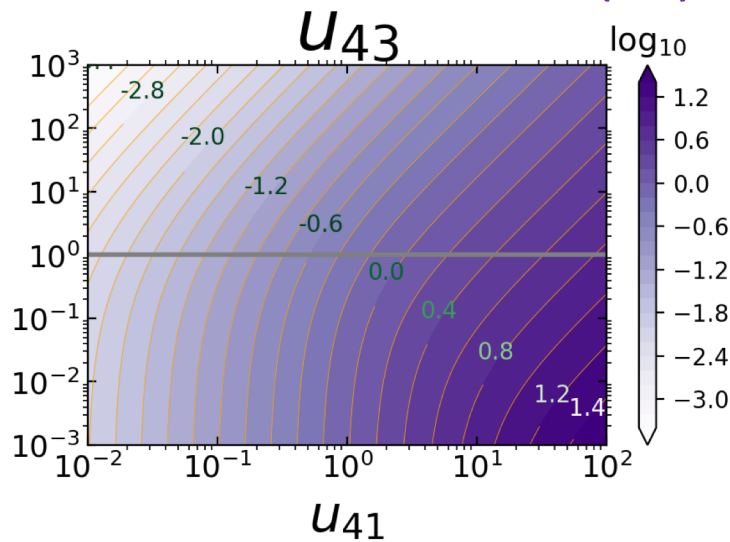
A Two Shell Collisions:

Mirror symmetry (for a given u_{41}): FS $\leftrightarrow$ RS, $f \leftrightarrow 1/f$

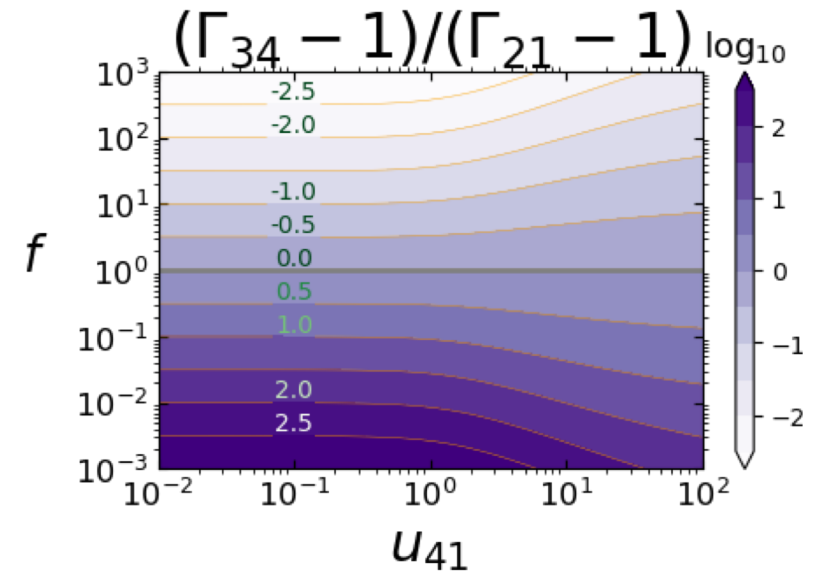
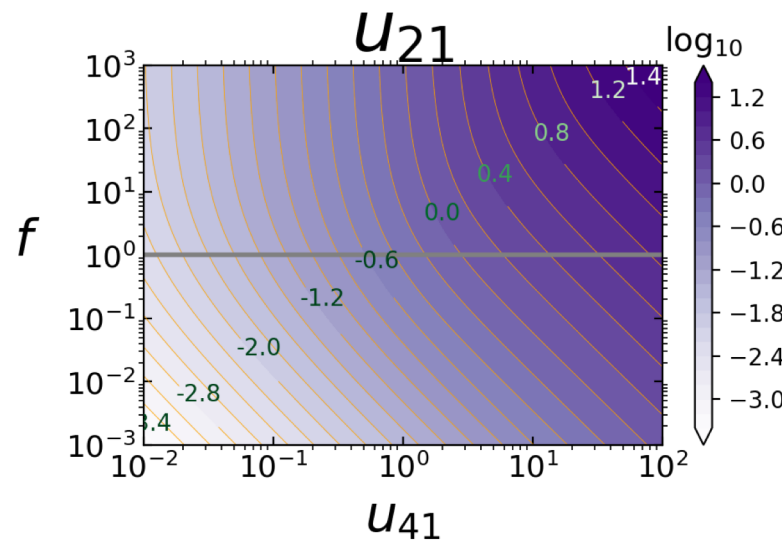
relative upstream to downstream proper speed

Shock strength ratio

across reverse shock (RS)



across forward shock (FS)



Ultra-Relativistic Shells:

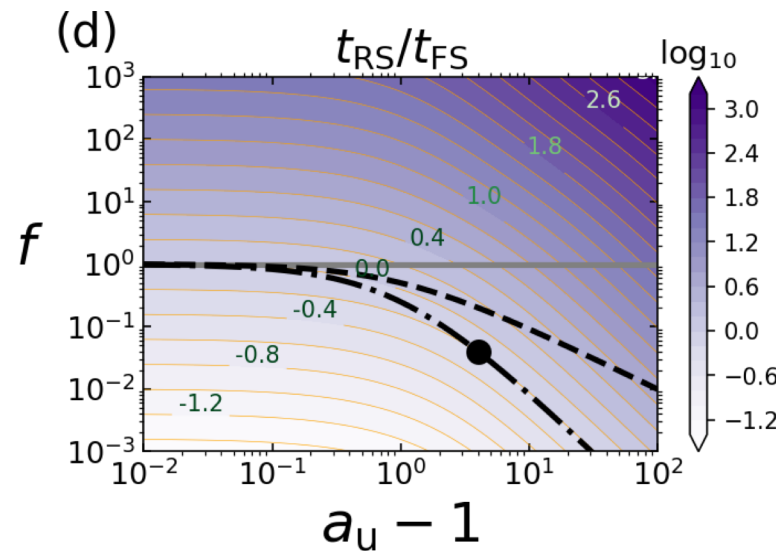
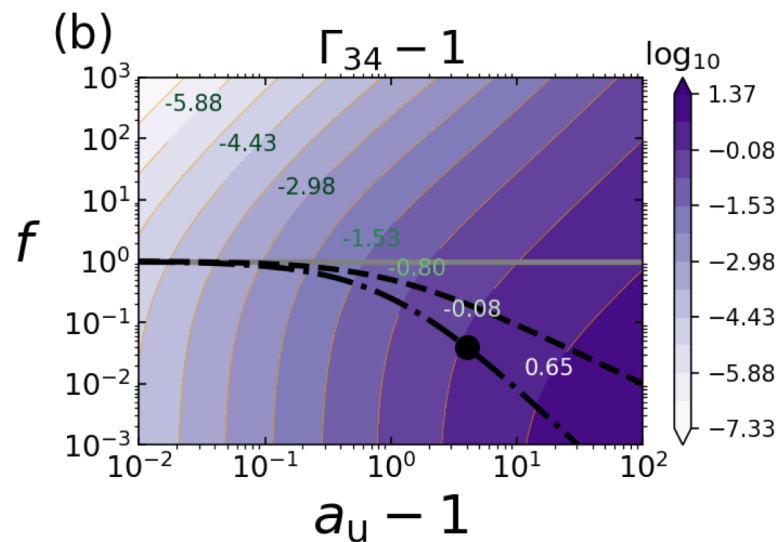
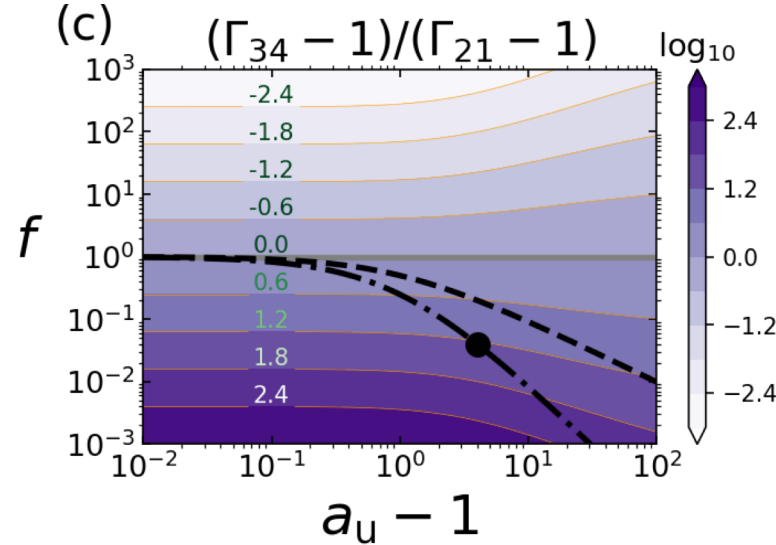
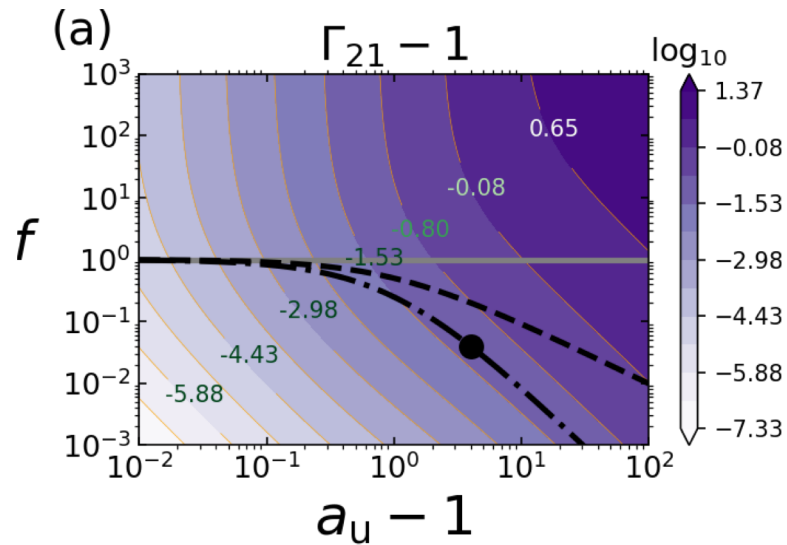
Mirror symmetry: FS $\leftrightarrow$ RS, $f \leftrightarrow 1/f$

$$u_1 = 100$$

$$(u_{41} \approx \frac{a_u^2 - 1}{2a_u})$$

$$\chi = \frac{\Delta_{1,0}}{\Delta_{4,0}} = 1$$

Equal proper density
Equal mass
Equal energy



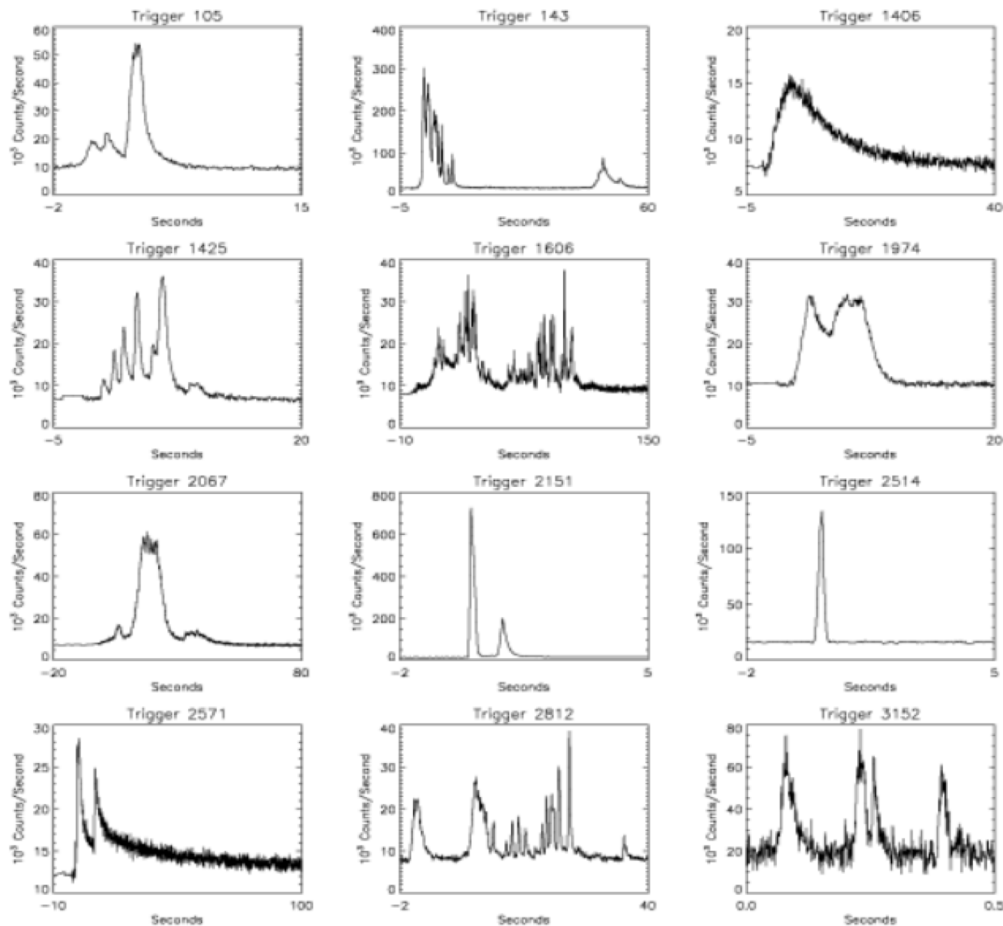
- For **equal proper density**: equal shock strength & FS crosses much earlier
- For **equal mass** shells: Reverse Shock is stronger but finishes crossing 2nd
- For **equal energy** shells: the RS is much stronger & finishes crossing first

Prompt GRB Emission from Internal Shocks:

(Rahaman, JG & Beniamini 2024a)

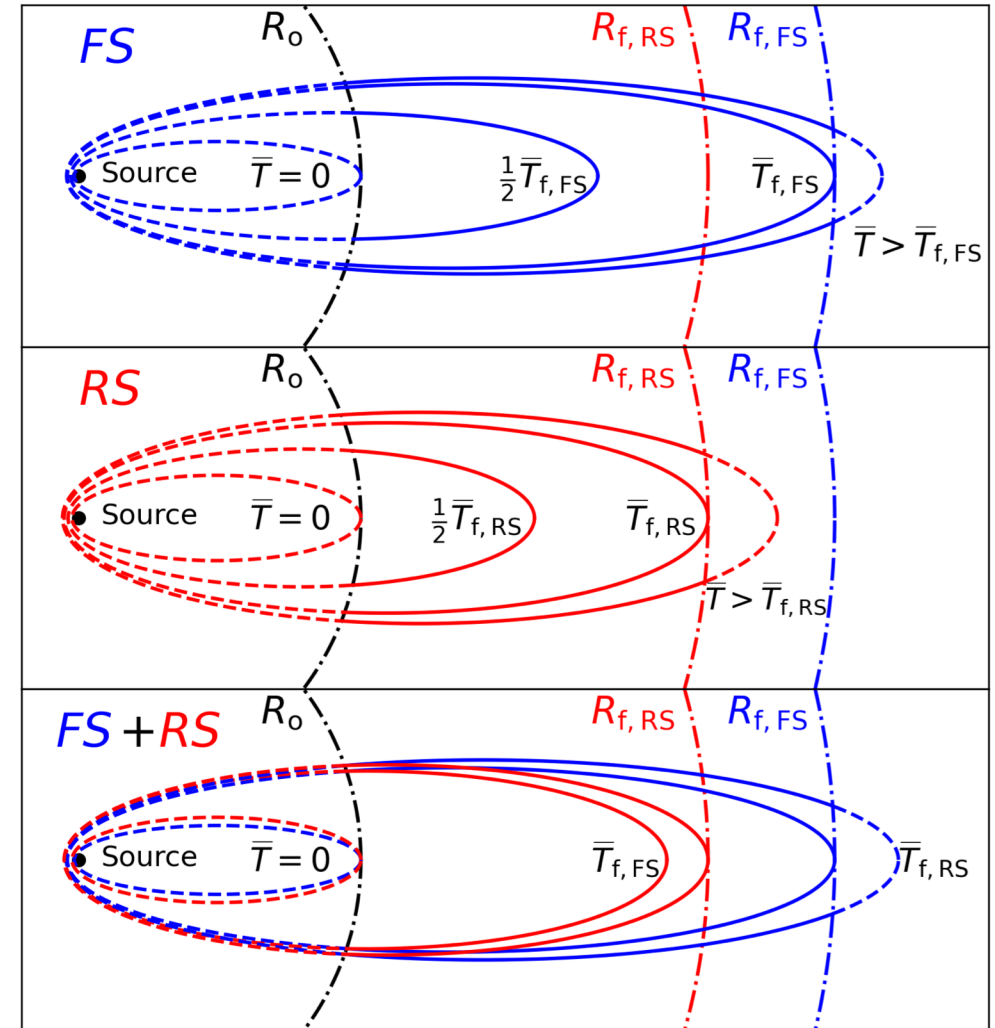
In our model:

1 collision $\Rightarrow$ 1 pulse in the lightcurve



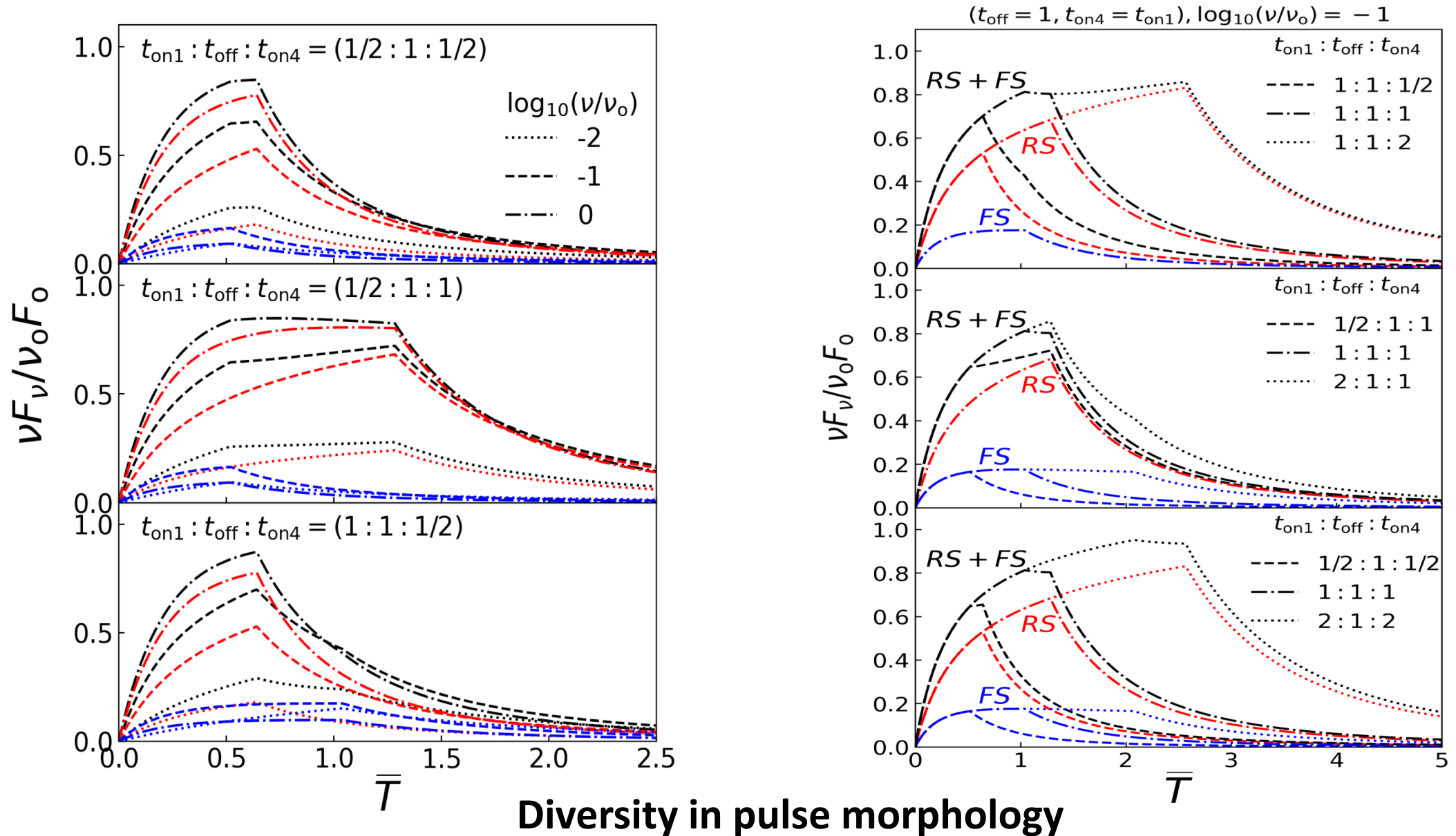
(Credit: J.T. Bonnell (NASA/GSFC))

Equal-Arrival-Time Surfaces (EATS)



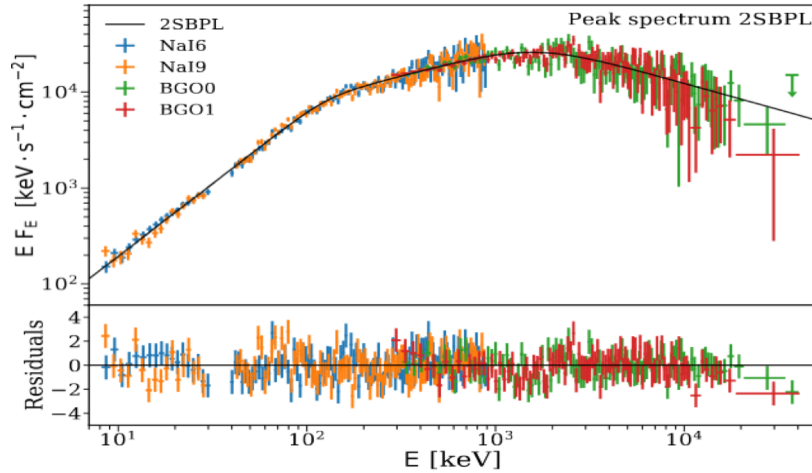
Contribution of both EATS must be taken at a given observed time.

Lightcurves (single pulse; Rahaman, JG & Beniamini 2024a):

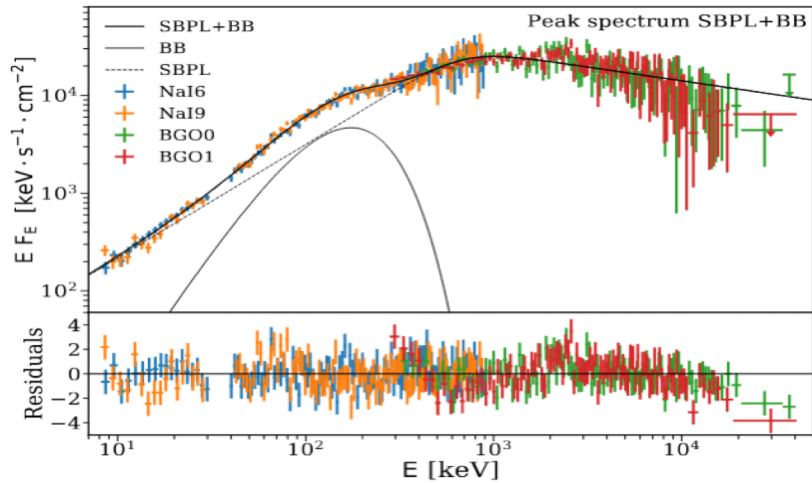


Spectra (Rahaman, JG & Beniamini 2024a):

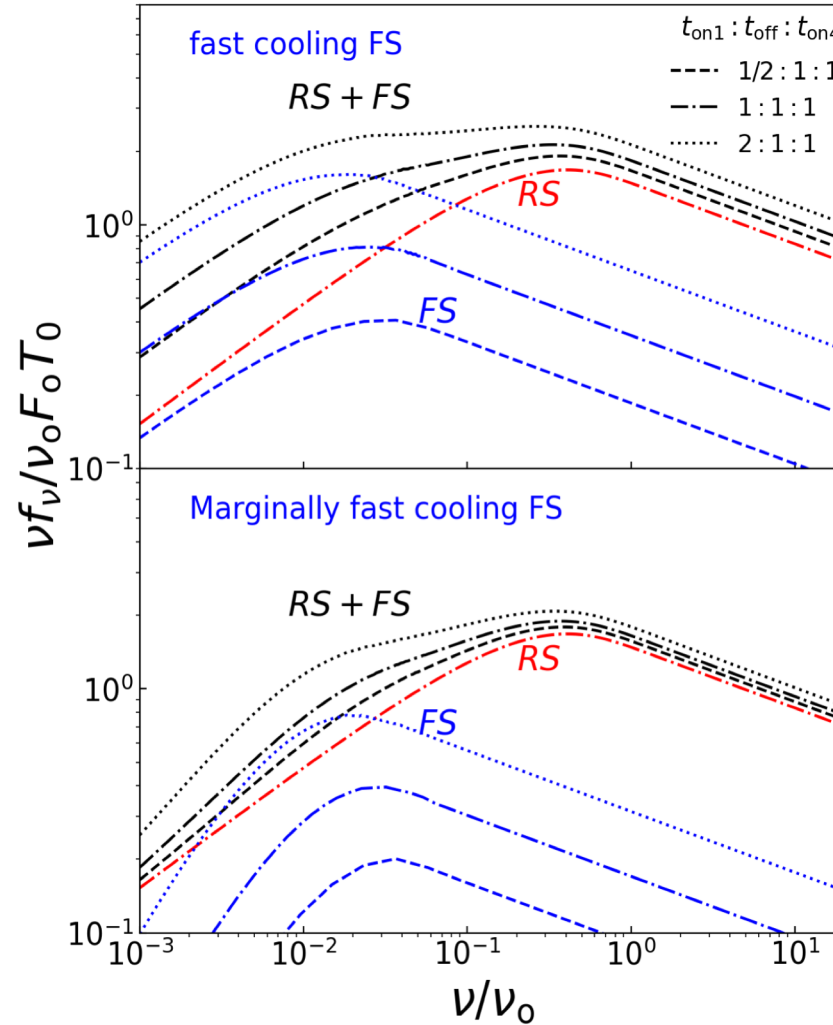
Example of Observations



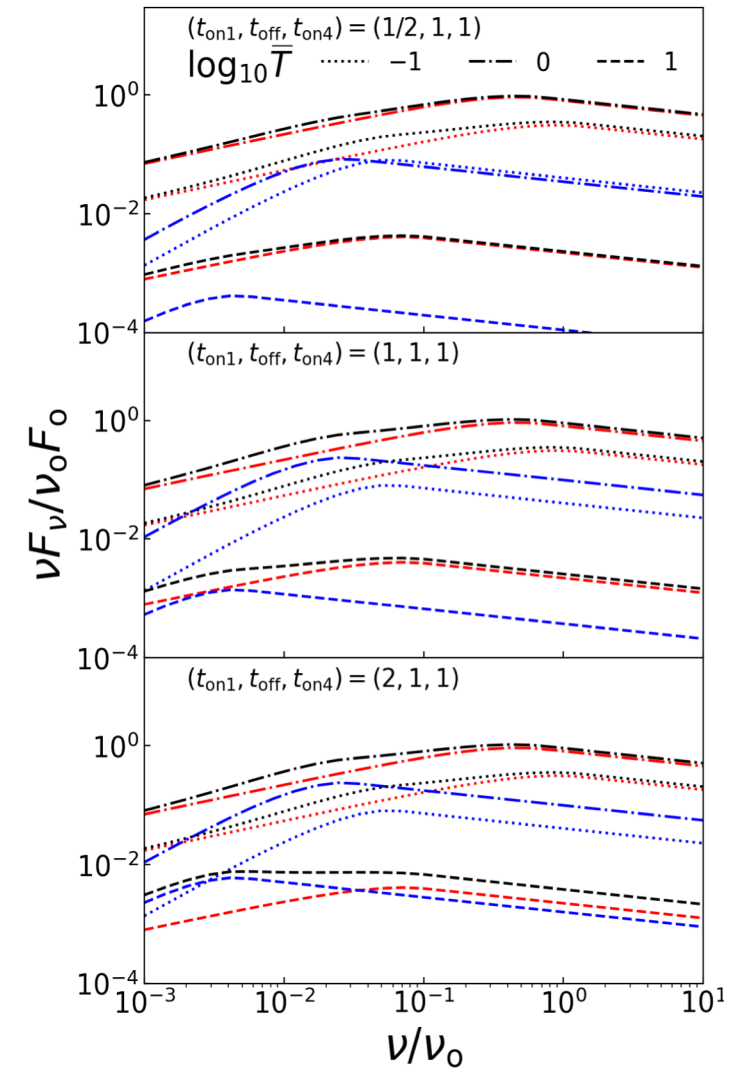
GRB 160625B Ravasio et al. 2018



Time-Integrated

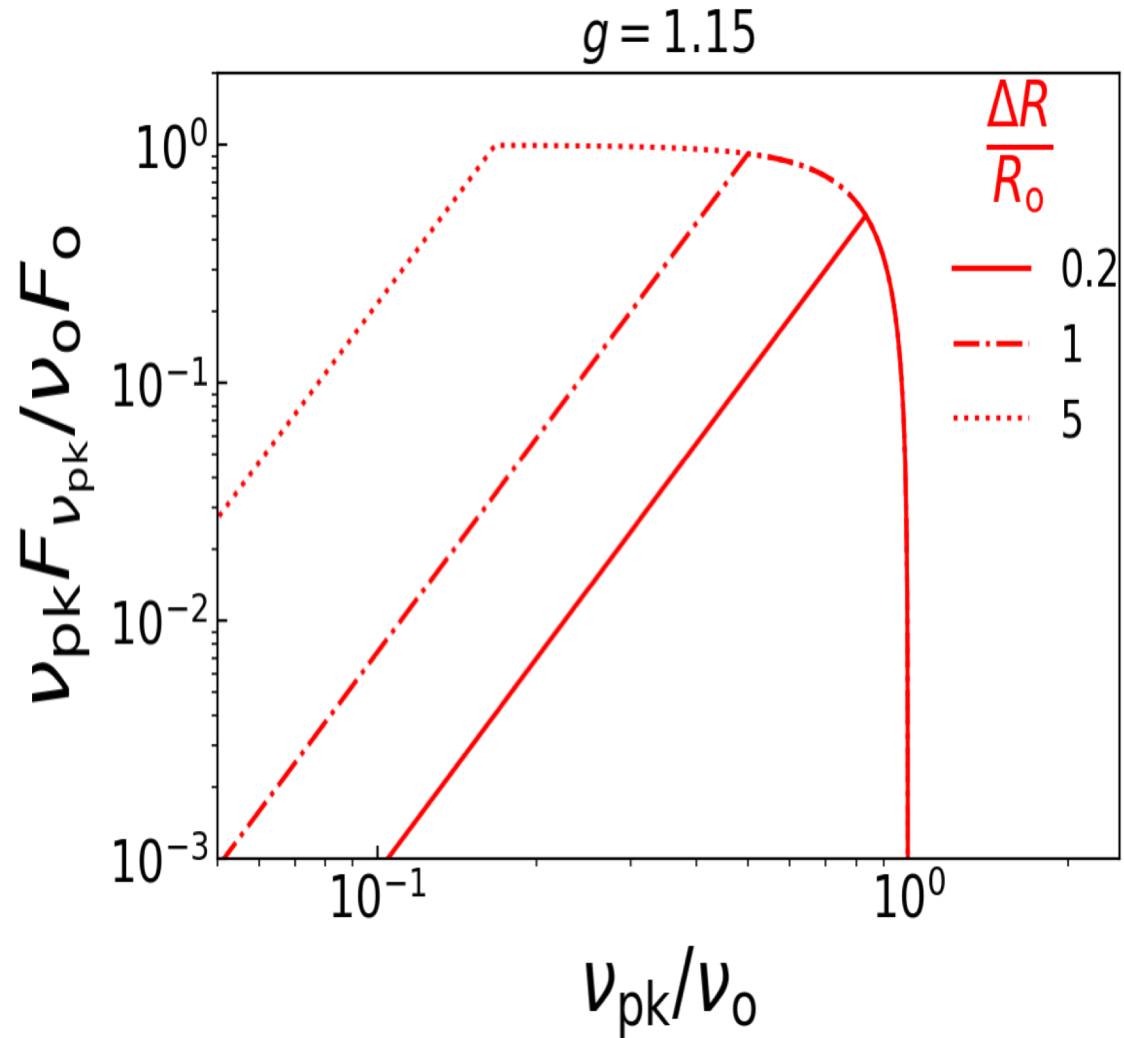


Time-resolved

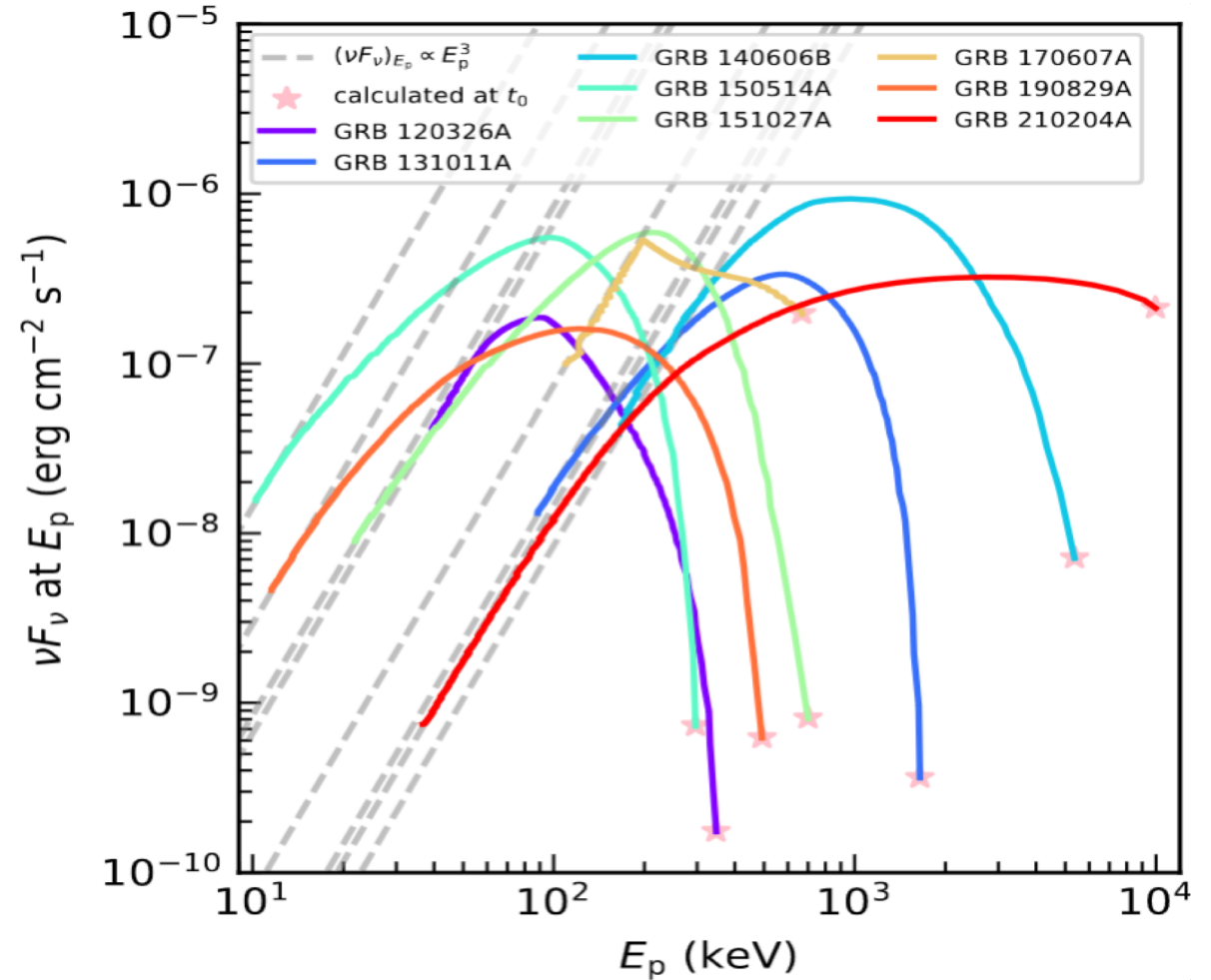


The weaker FS emission may mimic a “photospheric” quasi-thermal component or a **low- E_{ph} spectral break** – **features that have been inferred in an increasing number of GRBs**

Relating $F_\nu(t)$ to the colliding shells properties:



(Rahaman, JG & Beniamini 2024a)



(Yan et al. 2023)

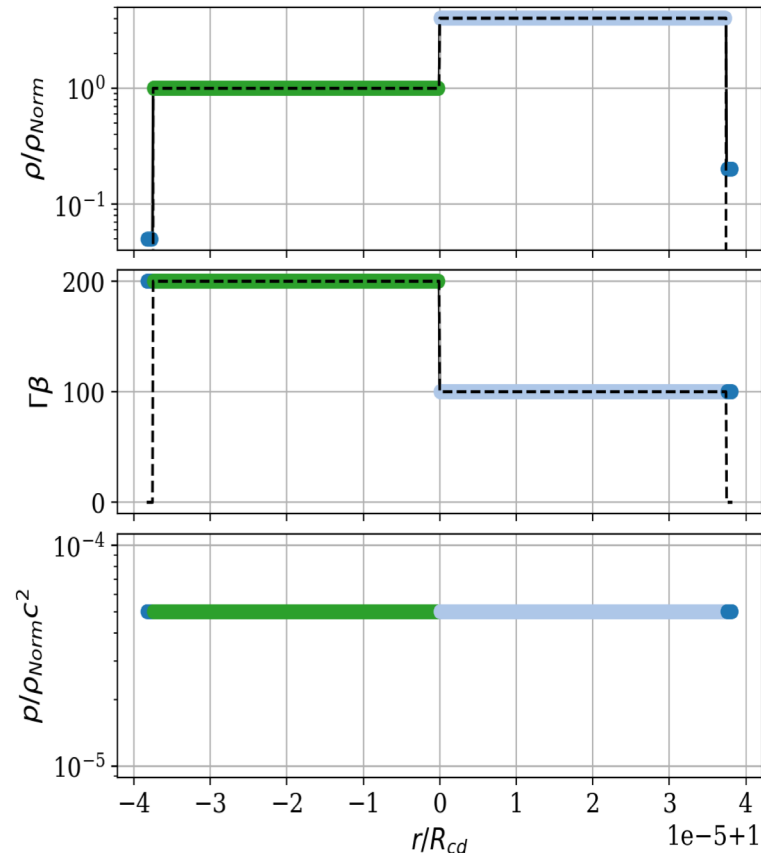
Numerical Simulations of Spherical Internal Shocks:

(Charlet, JG & Beniamini 2025)

Spherical effects cause the shock strengths to decrease with radius

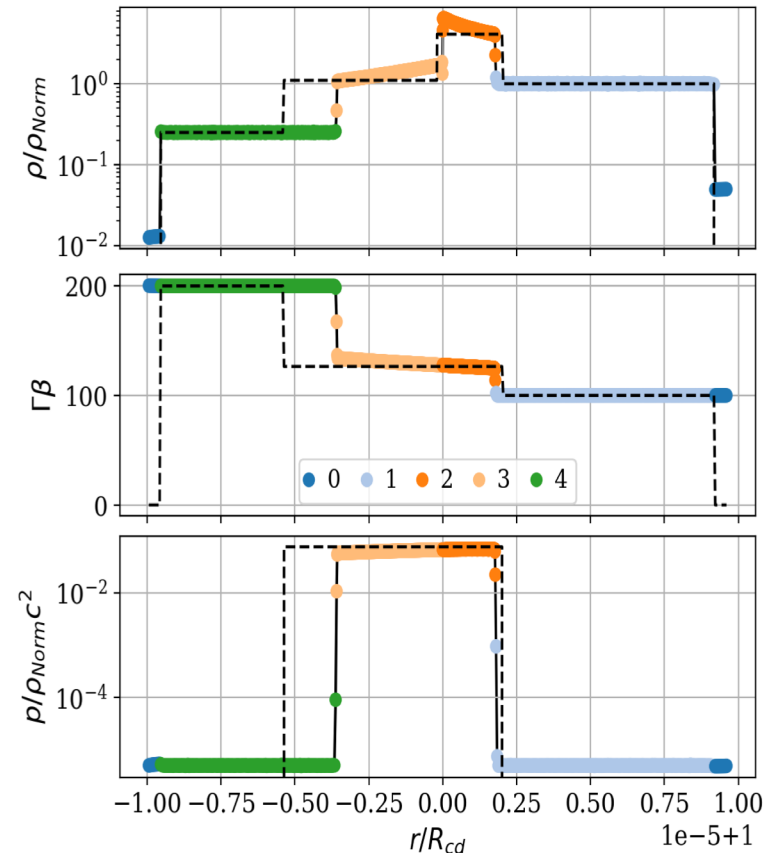
Initial Shell Collision

$t/t_0 = 1.00$



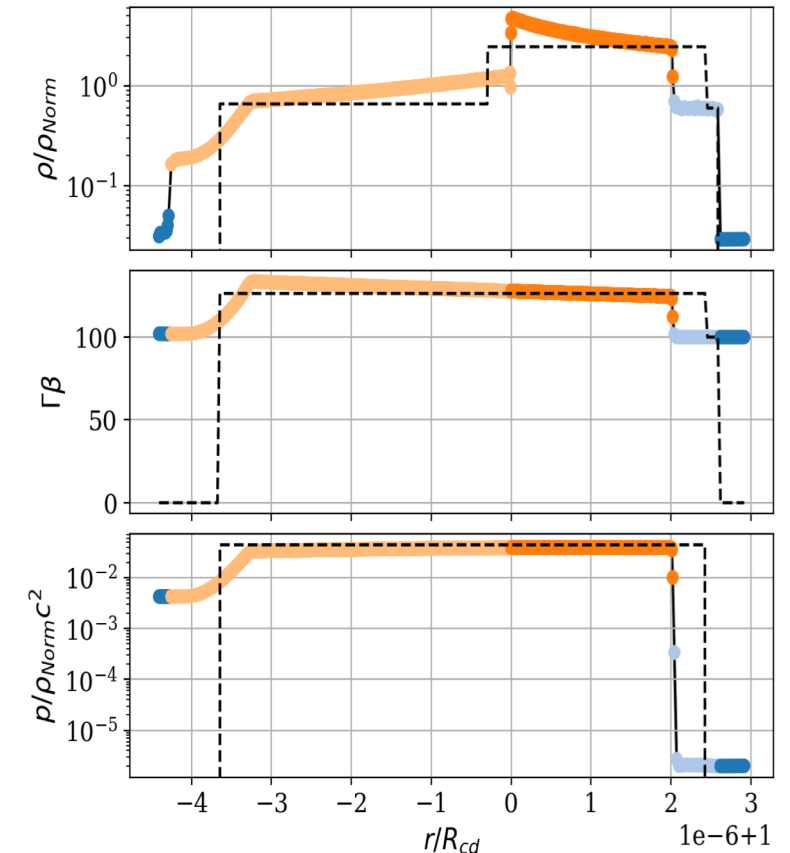
With both shocks active

$t/t_0 = 2.00$



After reverse shock crossing

$t/t_0 = 2.60$

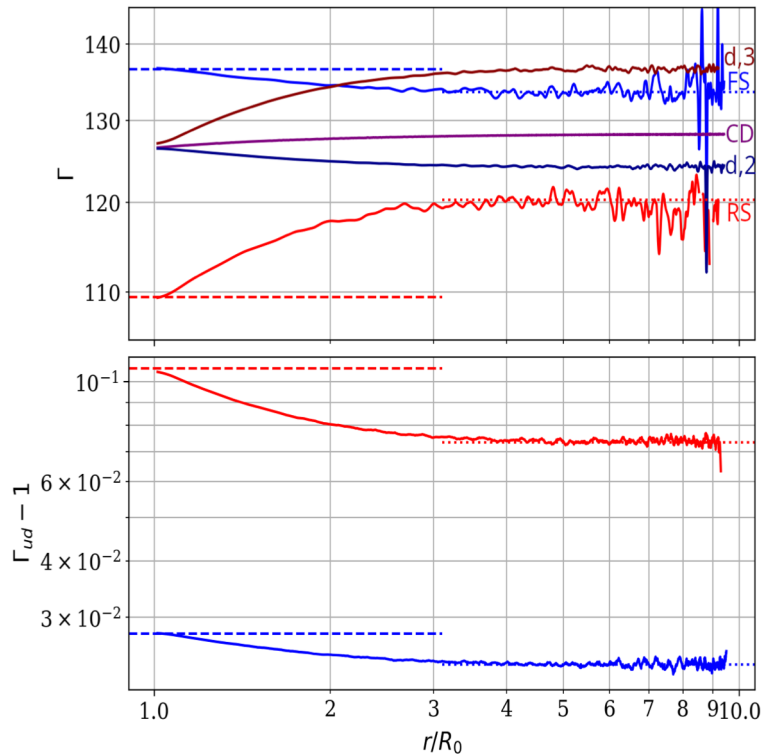


Numerical Simulations of Spherical Internal Shocks:

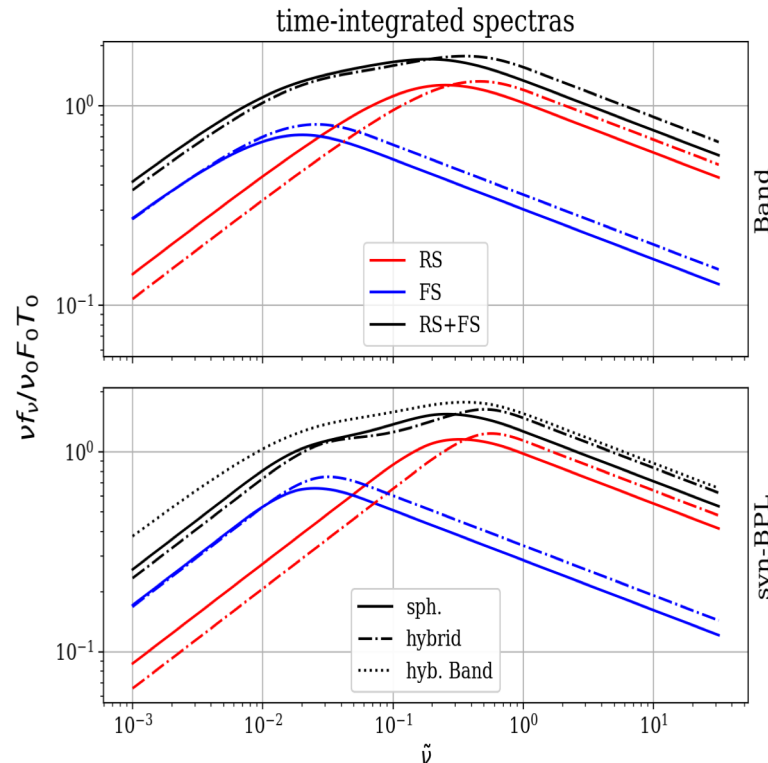
(Charlet, JG & Beniamini 2025)

Spherical effects cause the shock strengths to decrease with radius

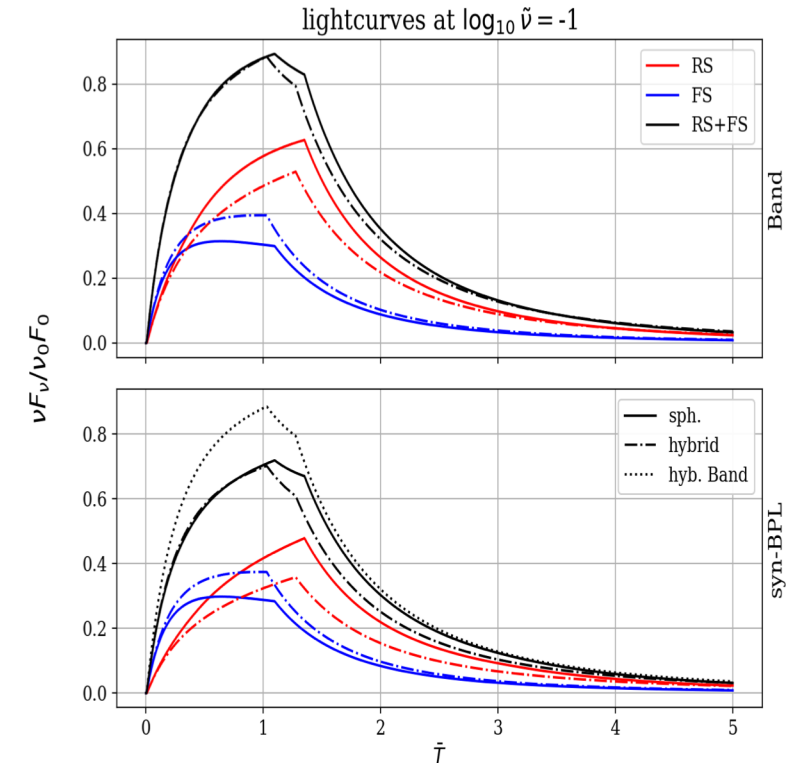
Lorentz Factors & Shock Strengths



Time integrated Spectrum



Lightcurves / Pulse Shapes

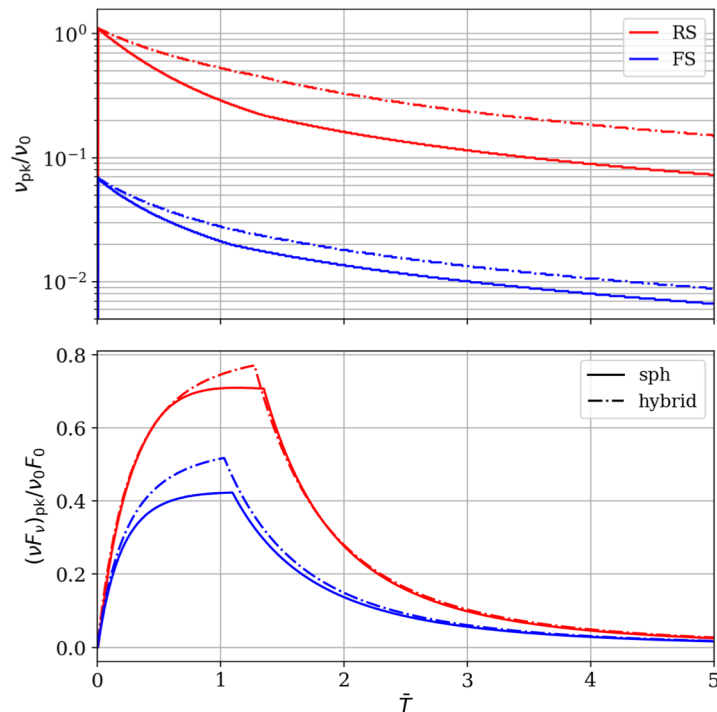


Numerical Simulations of Spherical Internal Shocks:

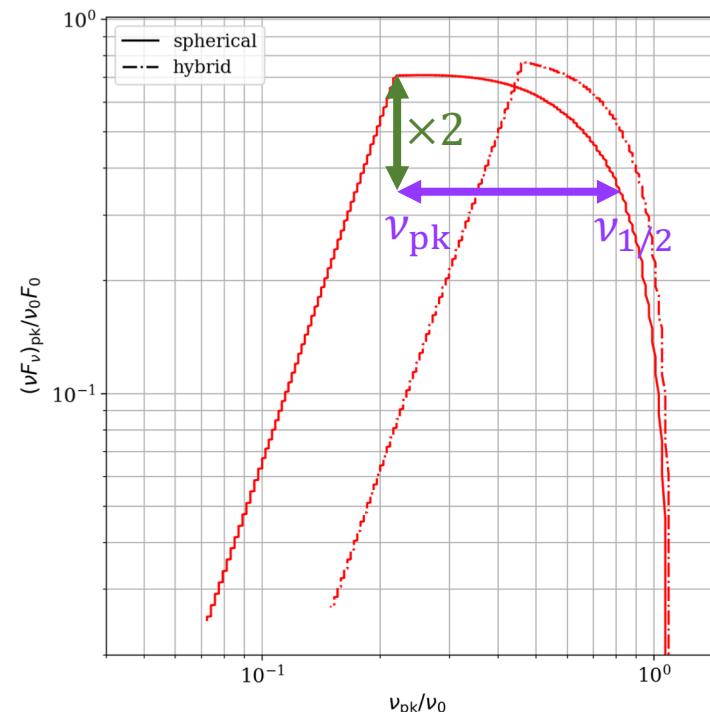
(Charlet, JG & Beniamini 2025)

Spherical effects cause the shock strengths to decrease with radius

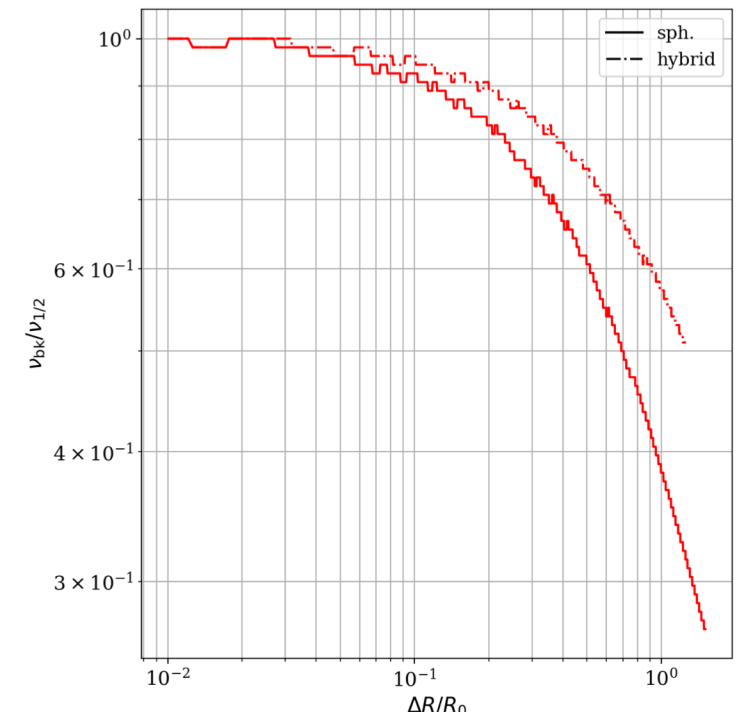
Evolution of Peak Frequencies & Flux



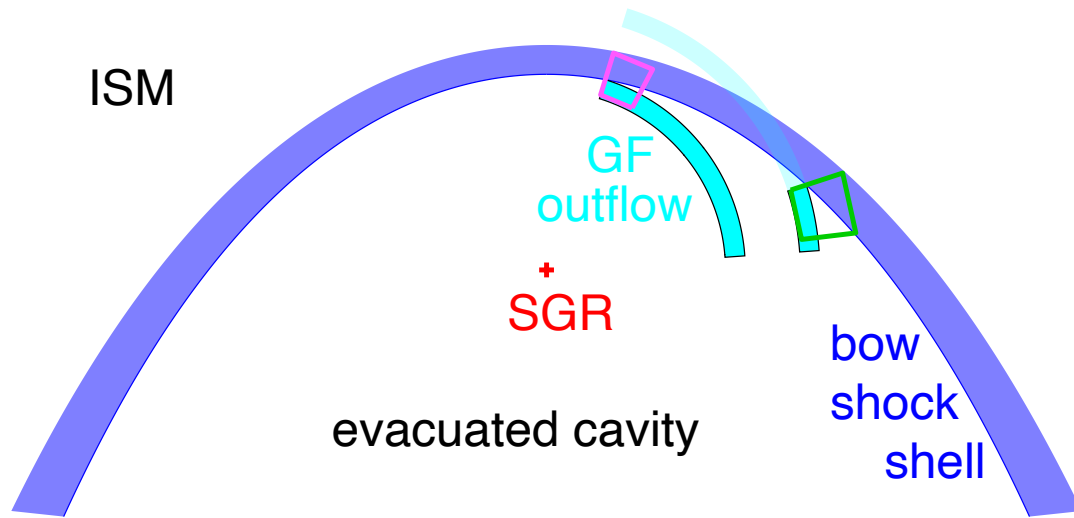
Peak Flux vs. Frequency: spherical vs. hybrid models



Lightcurves / Pulse Shapes

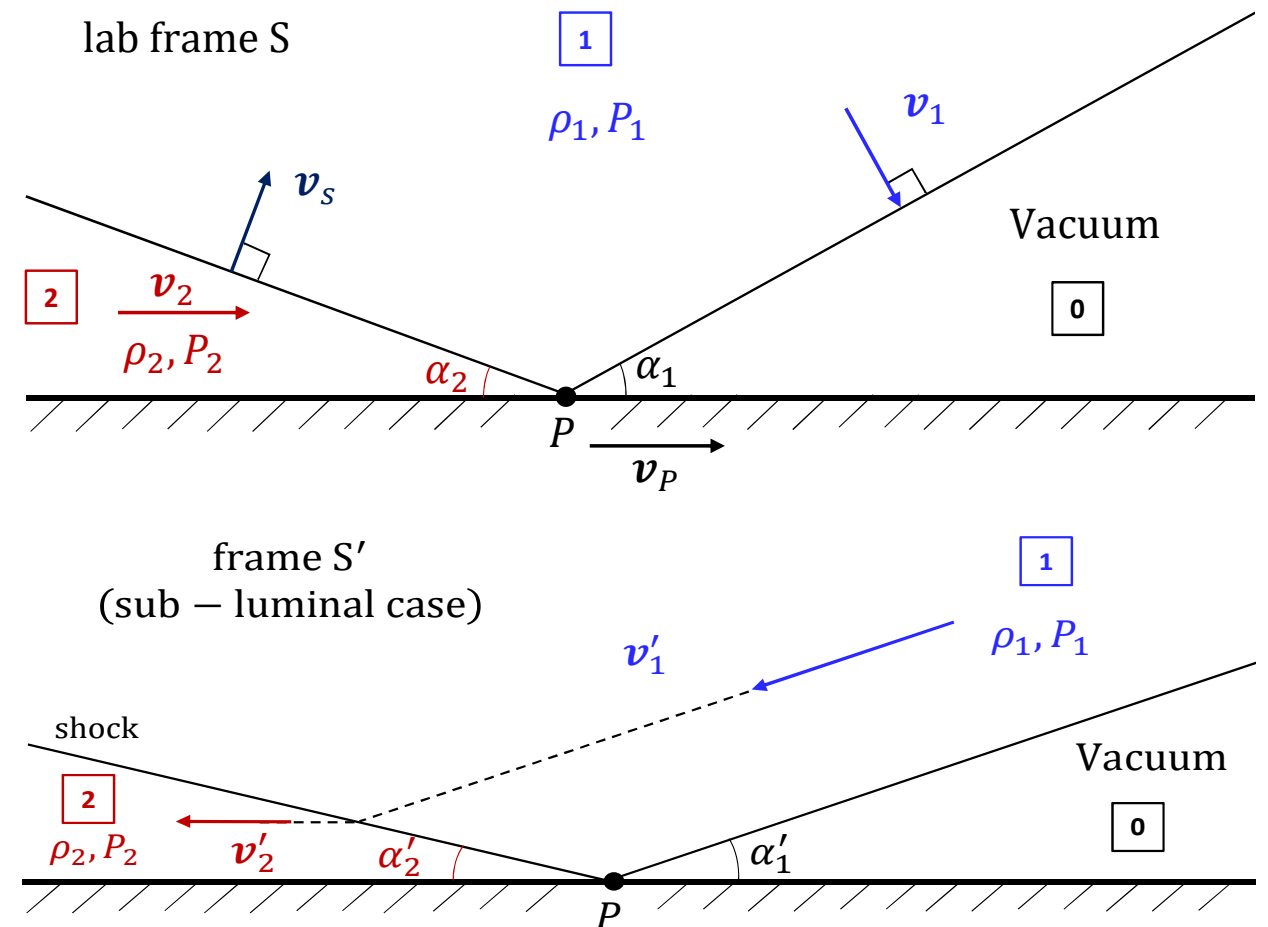
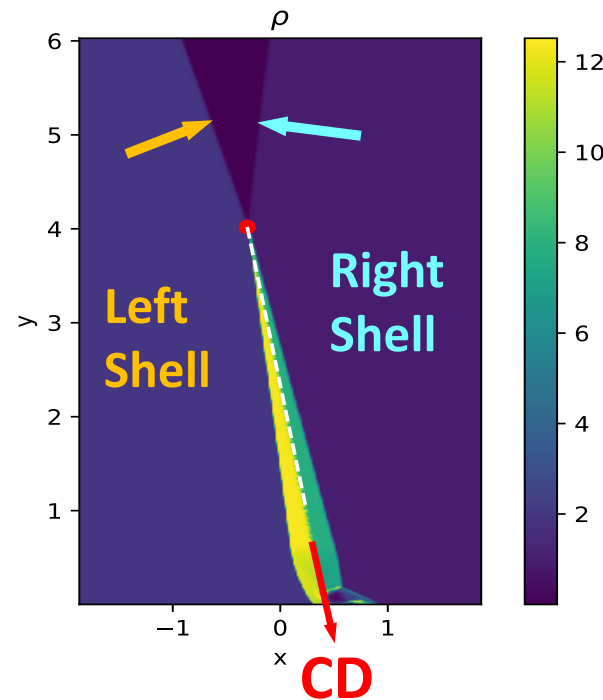


An Oblique Two Shell Collision in 2D: Motivation & Reduction



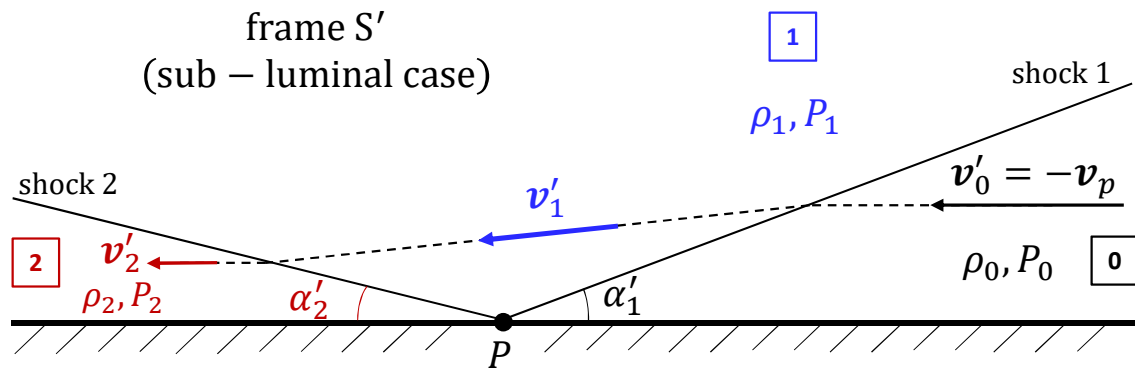
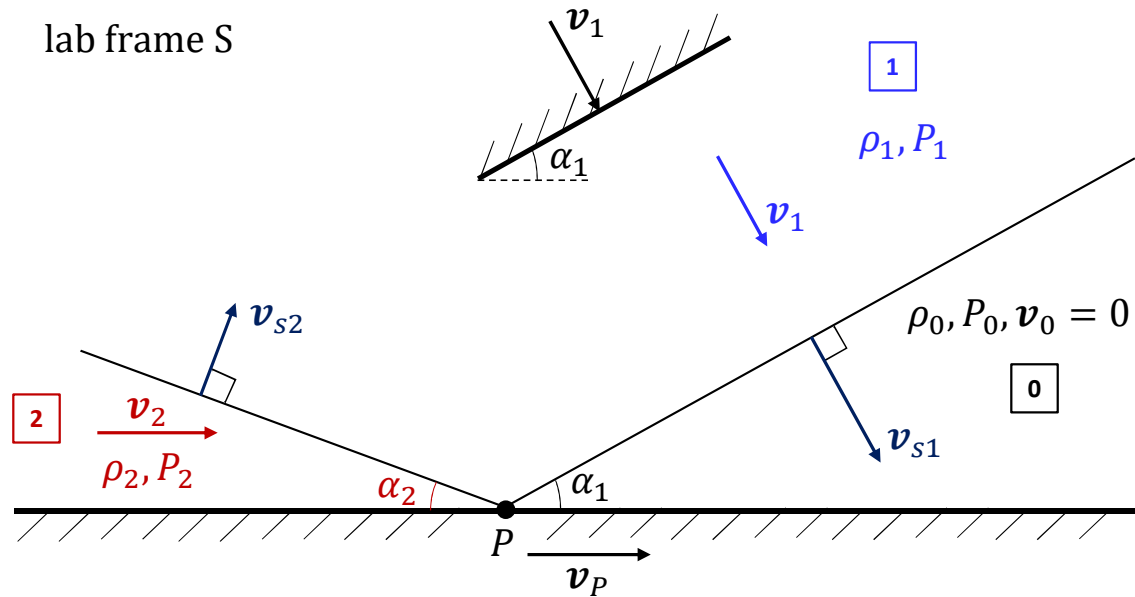
2-shell collision $\rightarrow 2 \times 1$ shell - wall collision
(the **CD** that forms may be regarded as a **wall**)

Motivation: Magnetar (SGR) Giant Flares (GF)
Radio nebula powered by the SGR 1806–20
27 Dec. 2004 GF ($u \sim 1$)
The 15 April 2020, magnetar GF from the Sculptor galaxy,
D = 3.5 Mpc; $u \sim 100$)



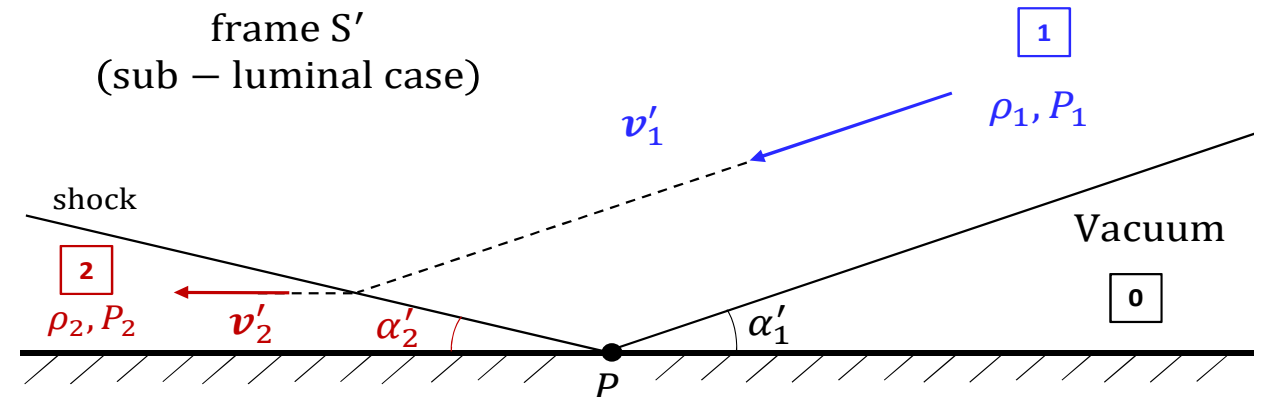
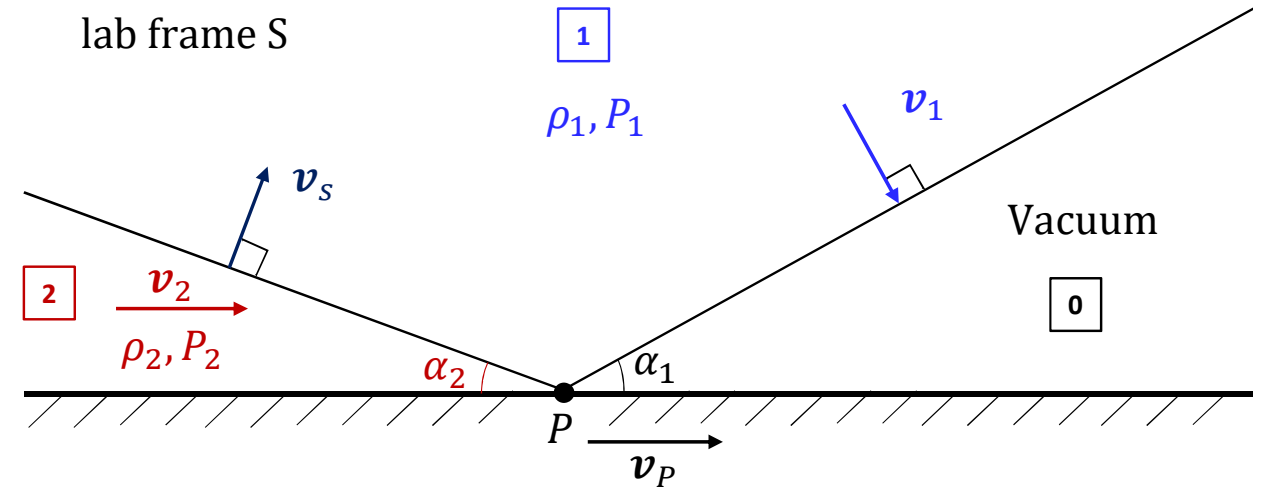
Analogous to Shock reflection (a classic problem):

Shock Reflection



(JG & Rabinovich 2024)

Single Shell - Wall Collision

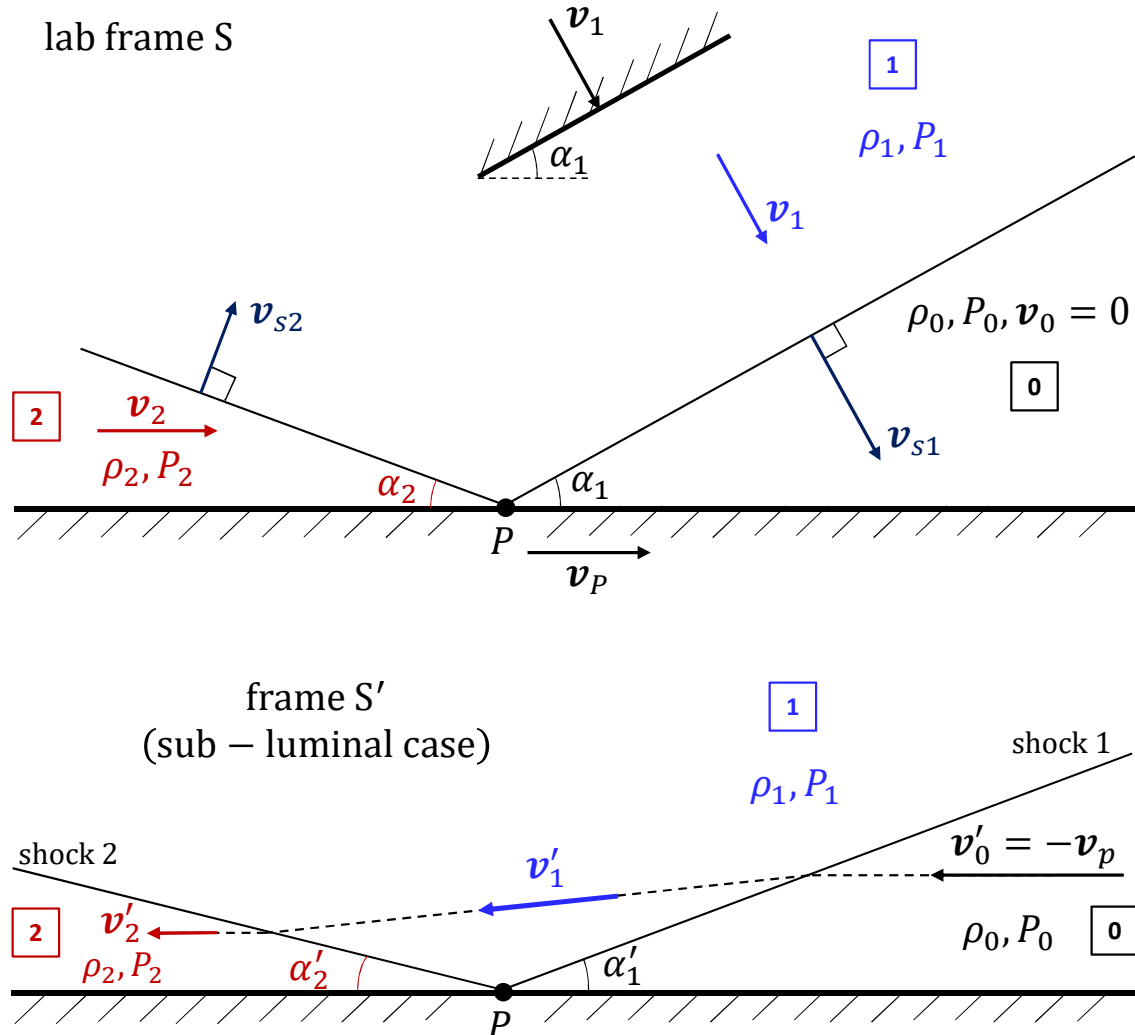


(JG et al. in prep.)

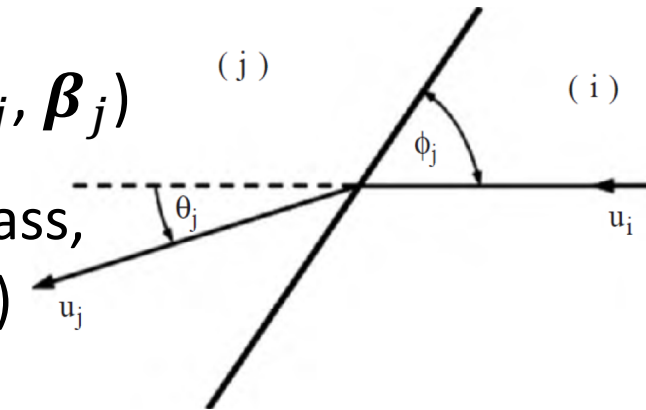
Shock reflection (a classic problem):

Oblique Shock

Shock Reflection

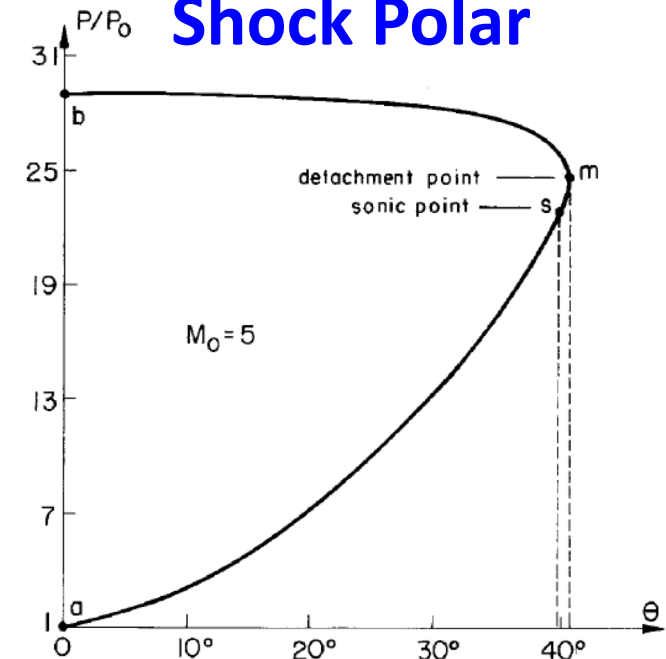


- 5 unknowns (p_j, n_j, e_j, β_j)
- 5 constraints (EoS, mass, energy, momentum $\times 2$)

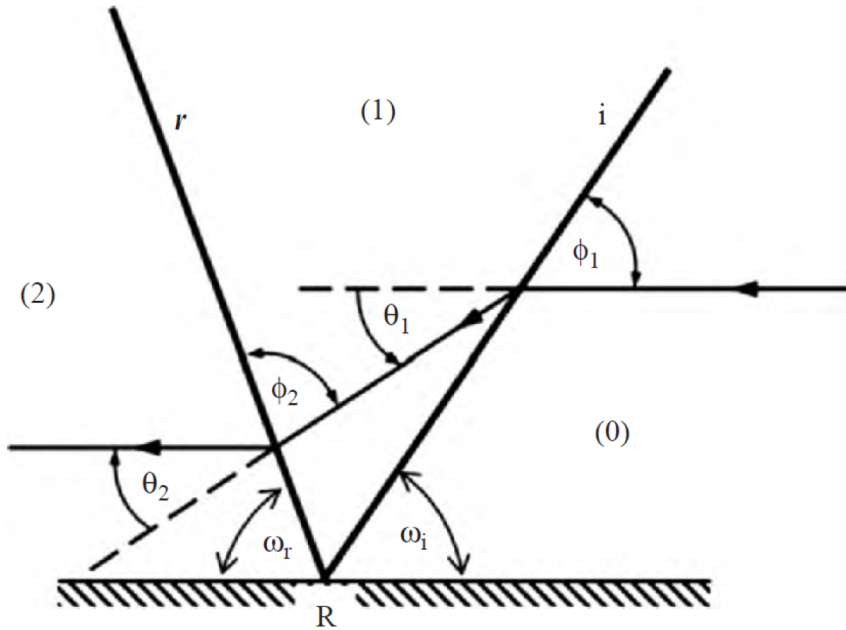


There is a maximum deflection angle $\theta_{\max}$

Shock Polar



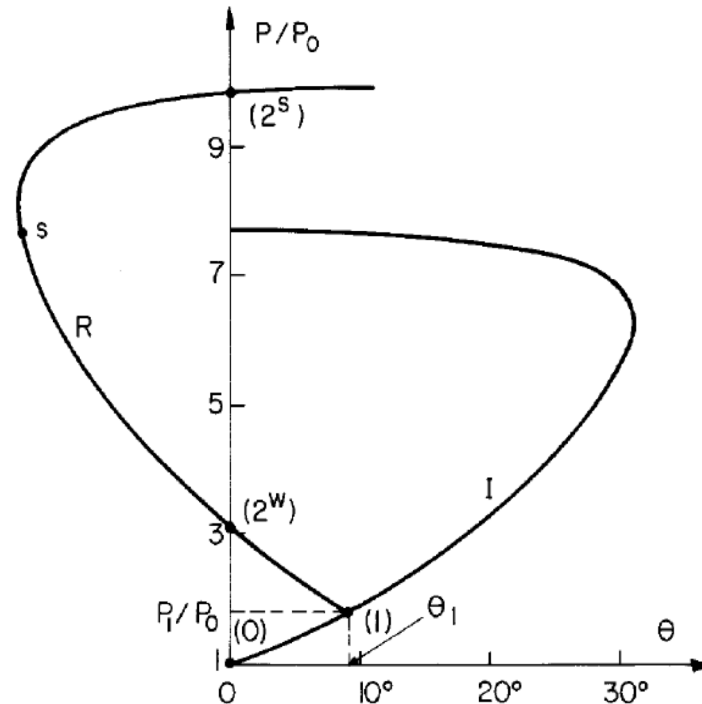
Newtonian Shock reflection: studied in the steady state frame



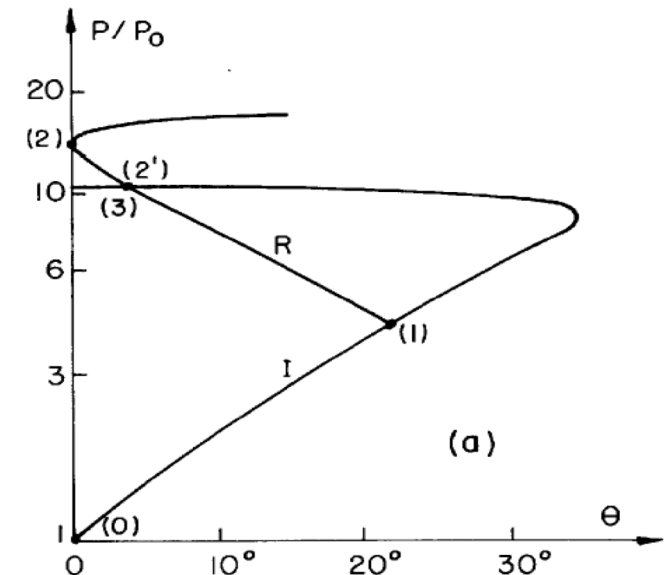
Regular Reflection: Shocks attach to the wall

- 10 unknowns ($p_1, n_1, e_1, \beta_1, p_2, n_2, e_2, \beta_2$)
- 10 constraints (2 EoS, 2 shocks $\times$ [mass, energy, momentum $\times$ 2] conserved)

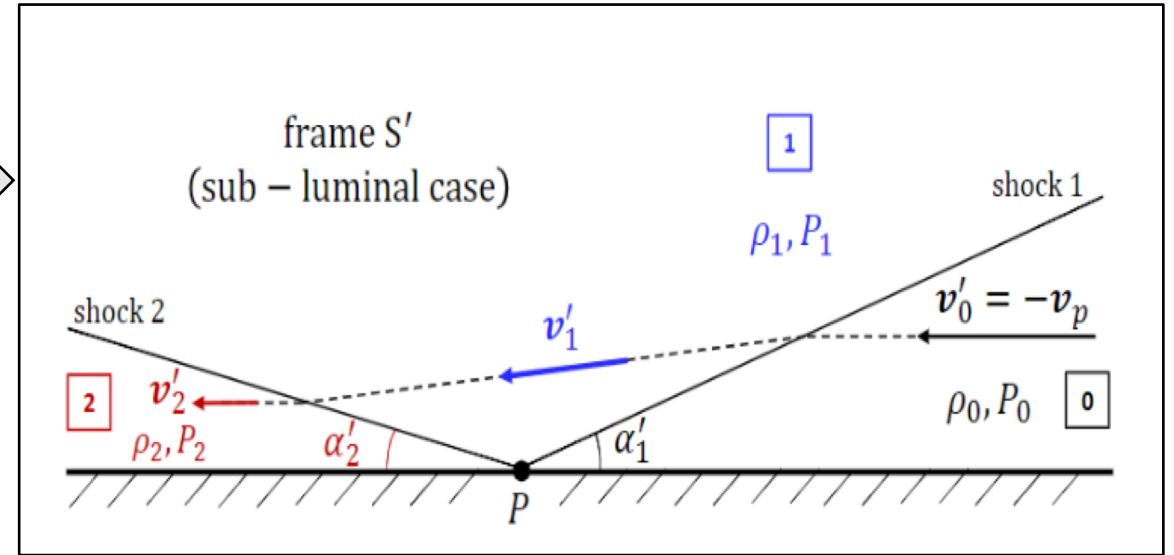
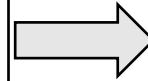
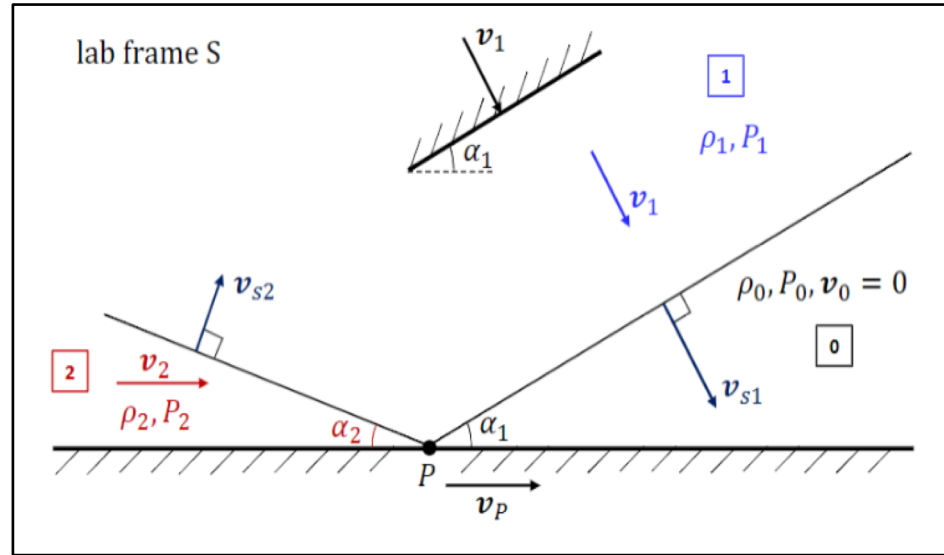
- **There are 2 solutions:**
Weak Shock (W) & Strong Shock (S)



Detachment Criterion: (close to sonic criterion)



Relativistic Shock reflection (JG & Rabinovich 2024; Bera et al. 2024):



$$v_p = \frac{v_{s1}}{\sin \alpha_1} = \frac{v_{s2}}{\sin \alpha_2}$$

If $v_p > c$ (the **super-luminal regime/region**) then the usual method of working in the rest-frame of point P is inapplicable

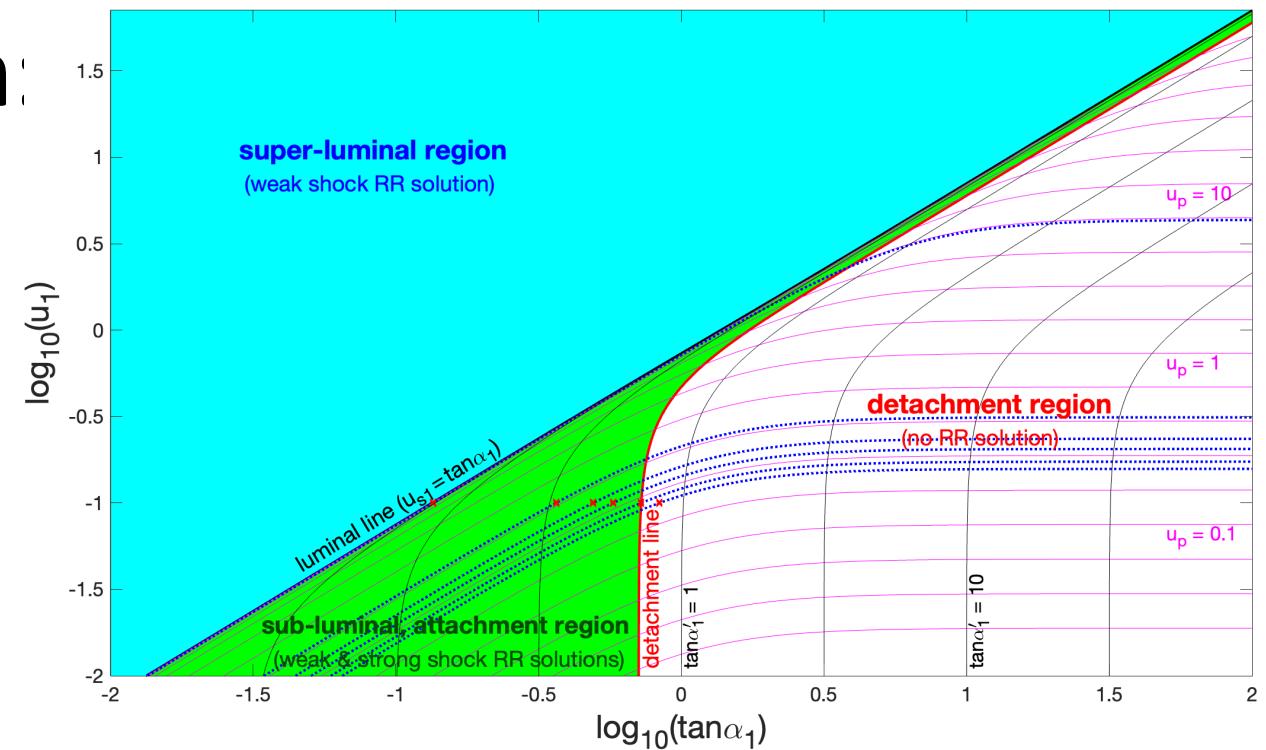
- A. Incident shock 1, driven by a piston, is the well known 1D shock tube problem
- B. For reflected shock 2 & regular reflection we use integral conservation laws**

Relativistic Shock reflection

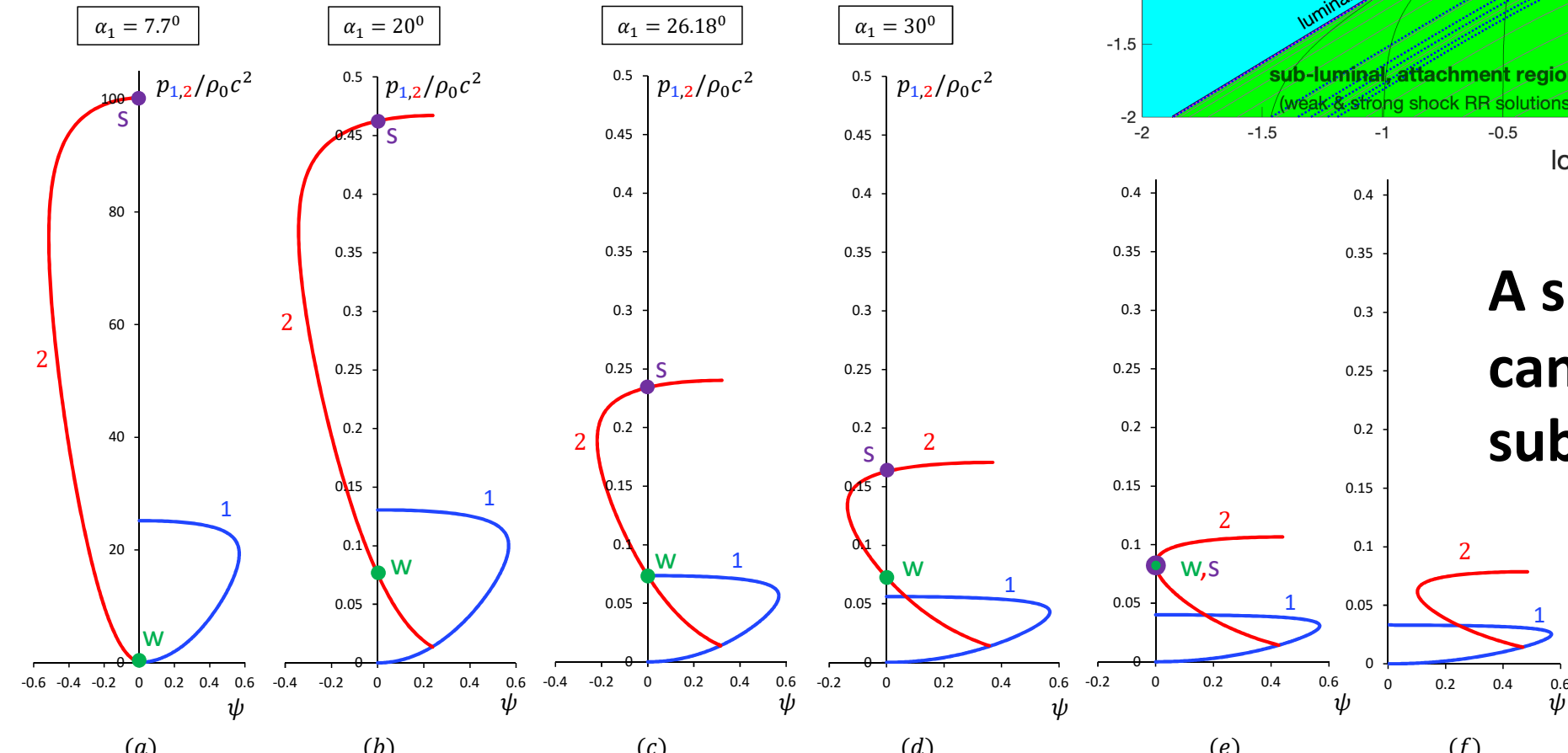
(JG & Rabinovich 2024)

Shock 2 polar: $u_1 = 0.1$

Shock 1 polar: $u_p = 6.12, 0.419, 0.314, 0.274, 0.231, 0.210$

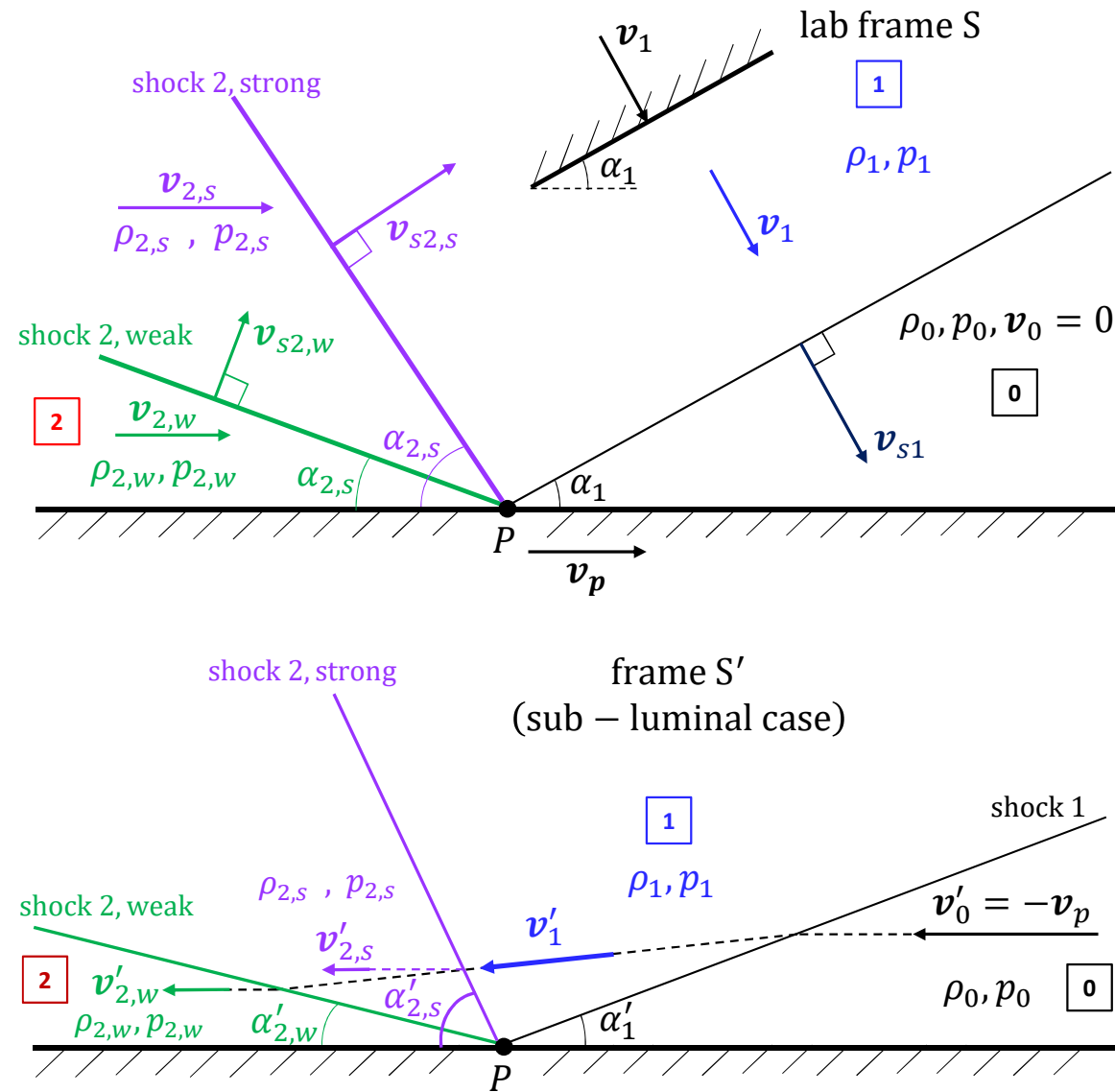


**A shock polar analysis
can only be done in the
sub-luminal region**

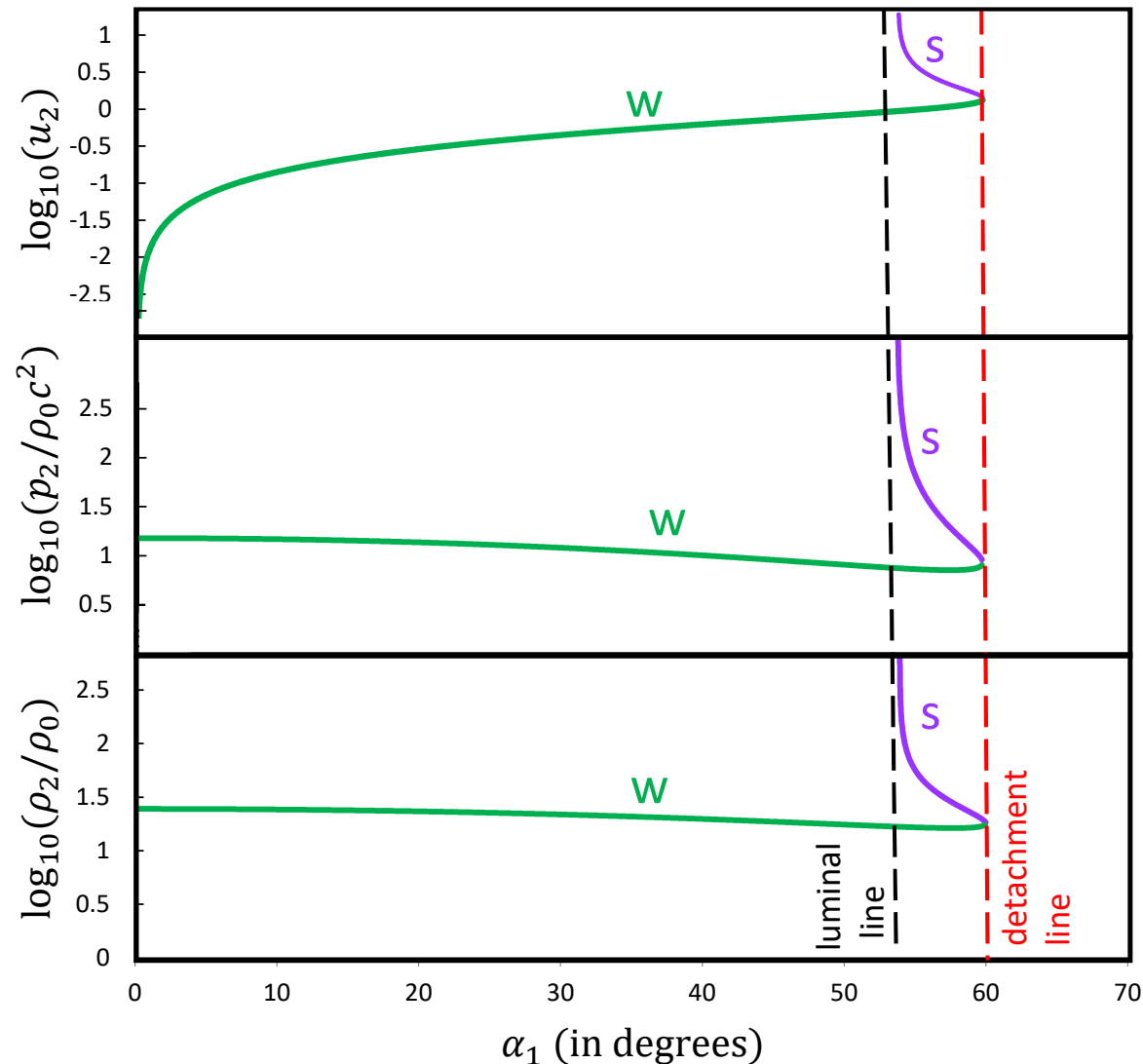


Relativistic Shock reflection:

(JG & Rabinovich 2024)

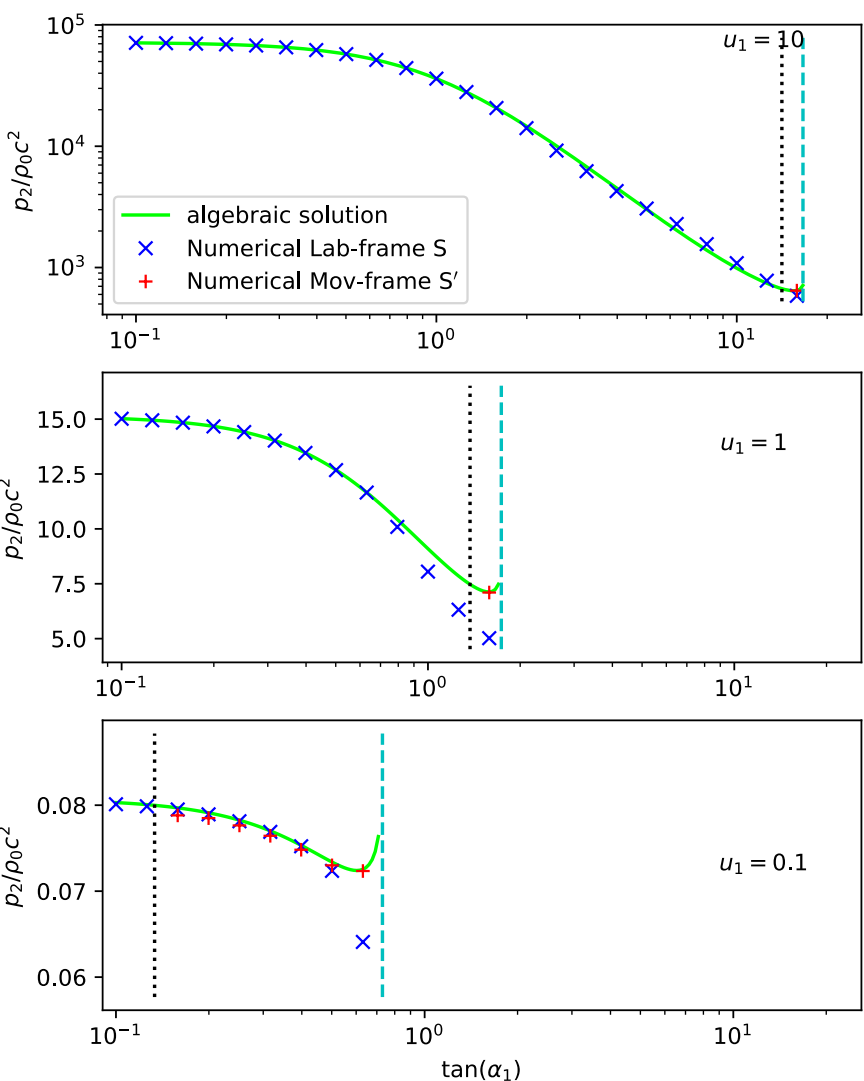


The **Strong Shock** solution cannot cross the luminal line as it diverges

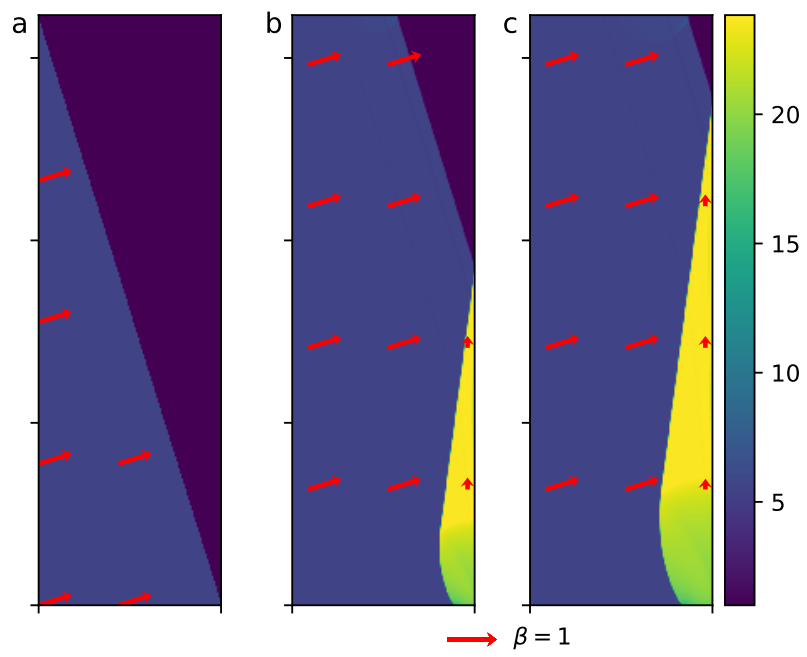


Relativistic Shock reflection (Bera et al. 2024):

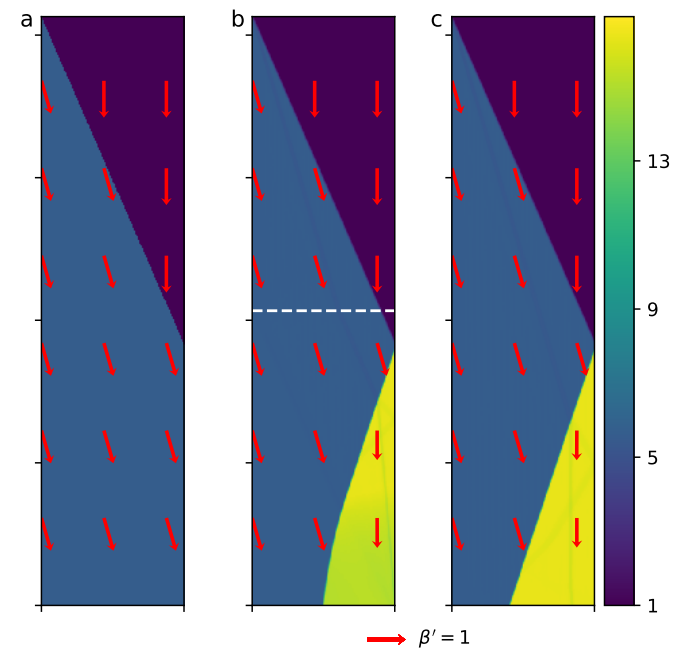
Reproduces the analytic
weak shock solution



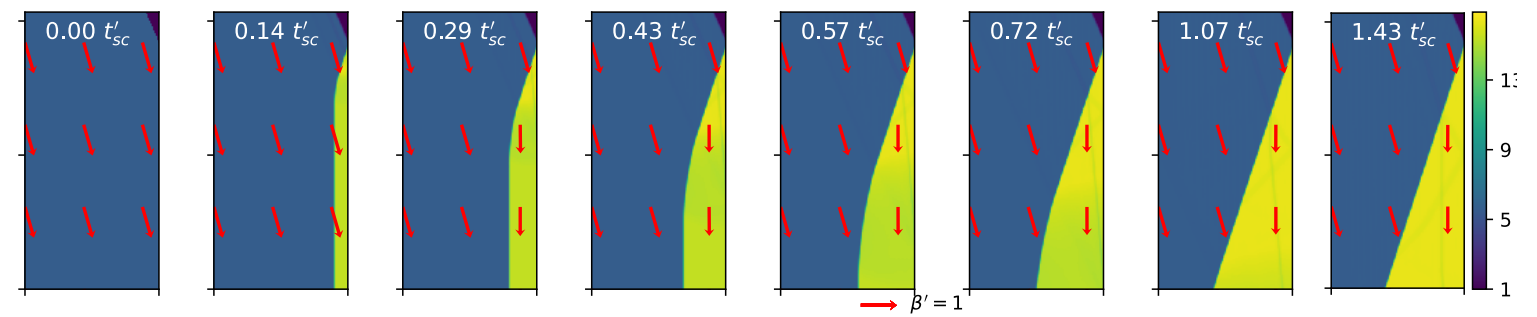
Lab frame (density)



Steady frame (density)

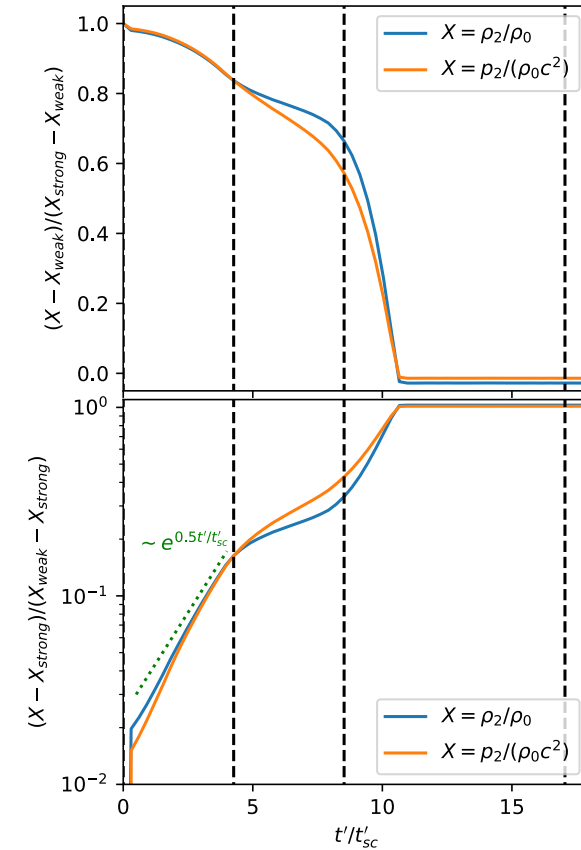
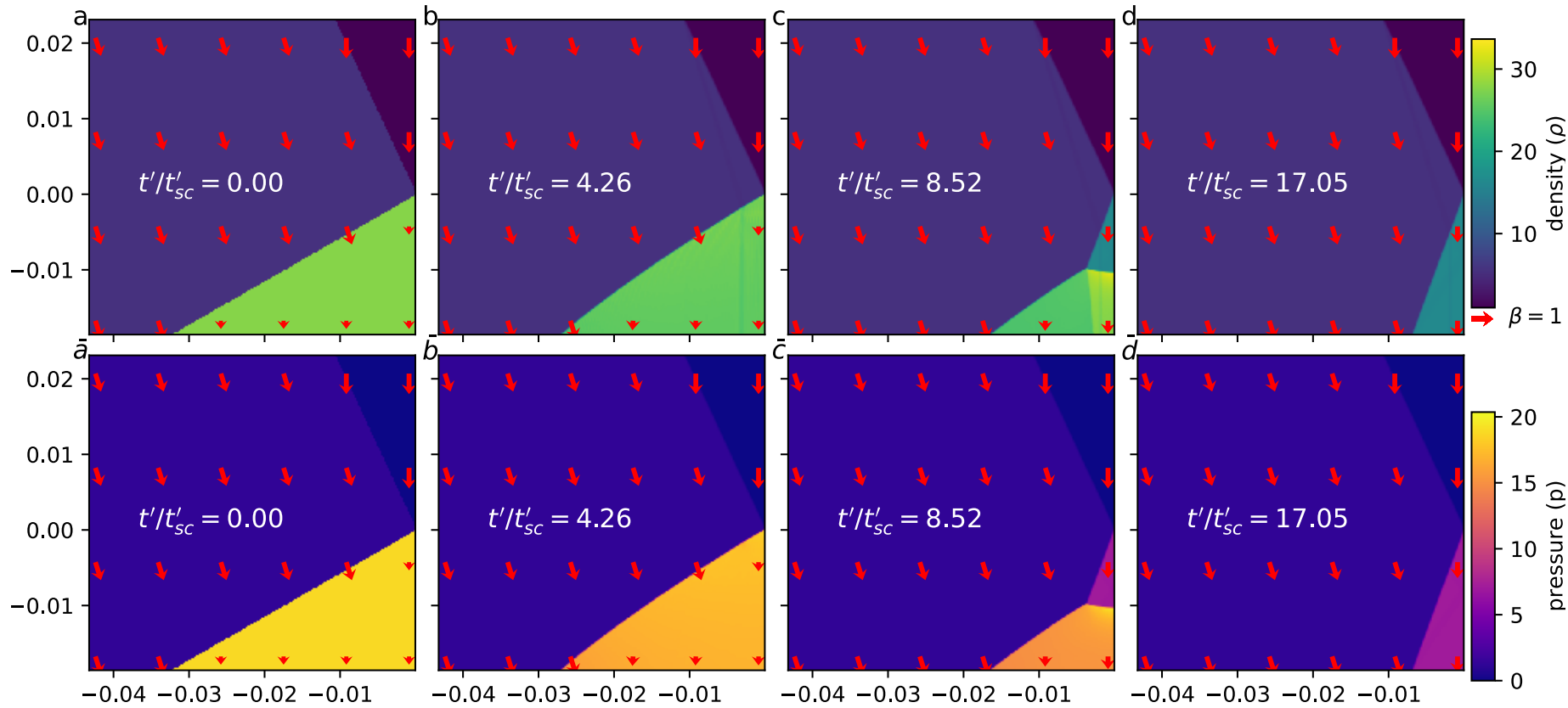


Gradually achieving steady state (density):



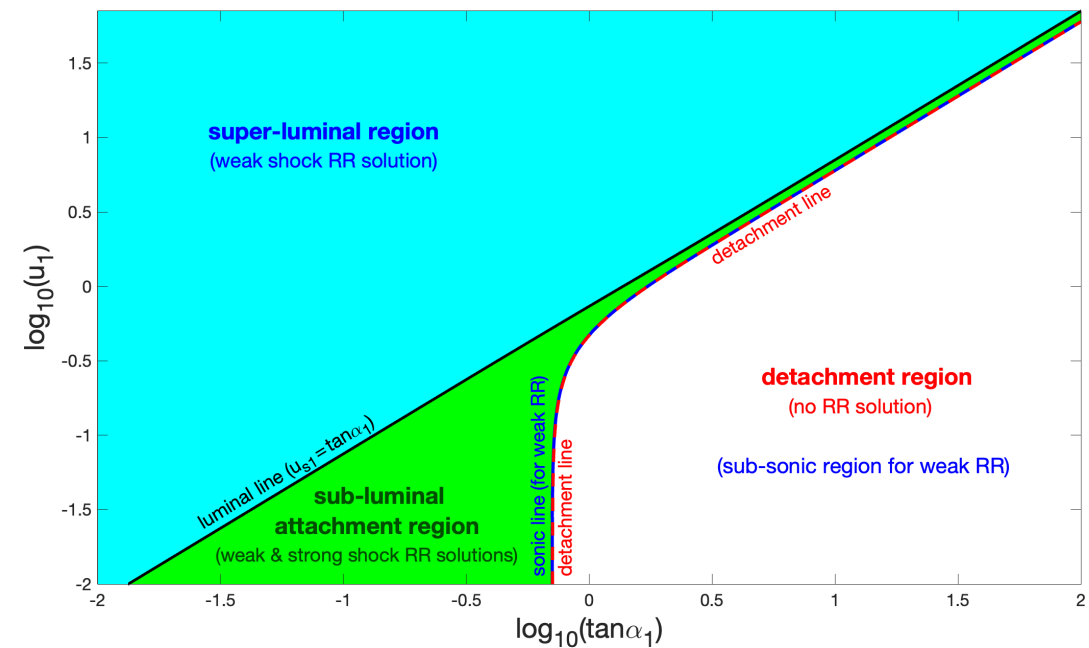
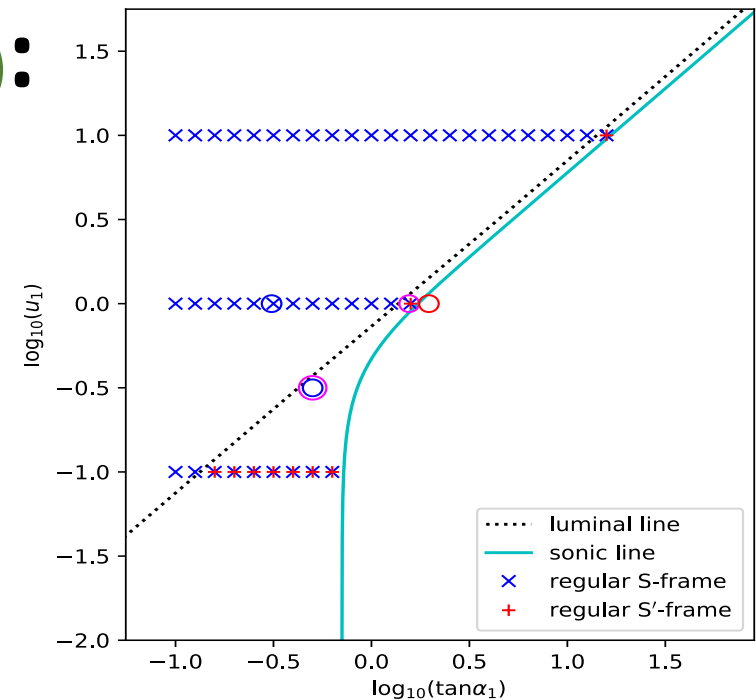
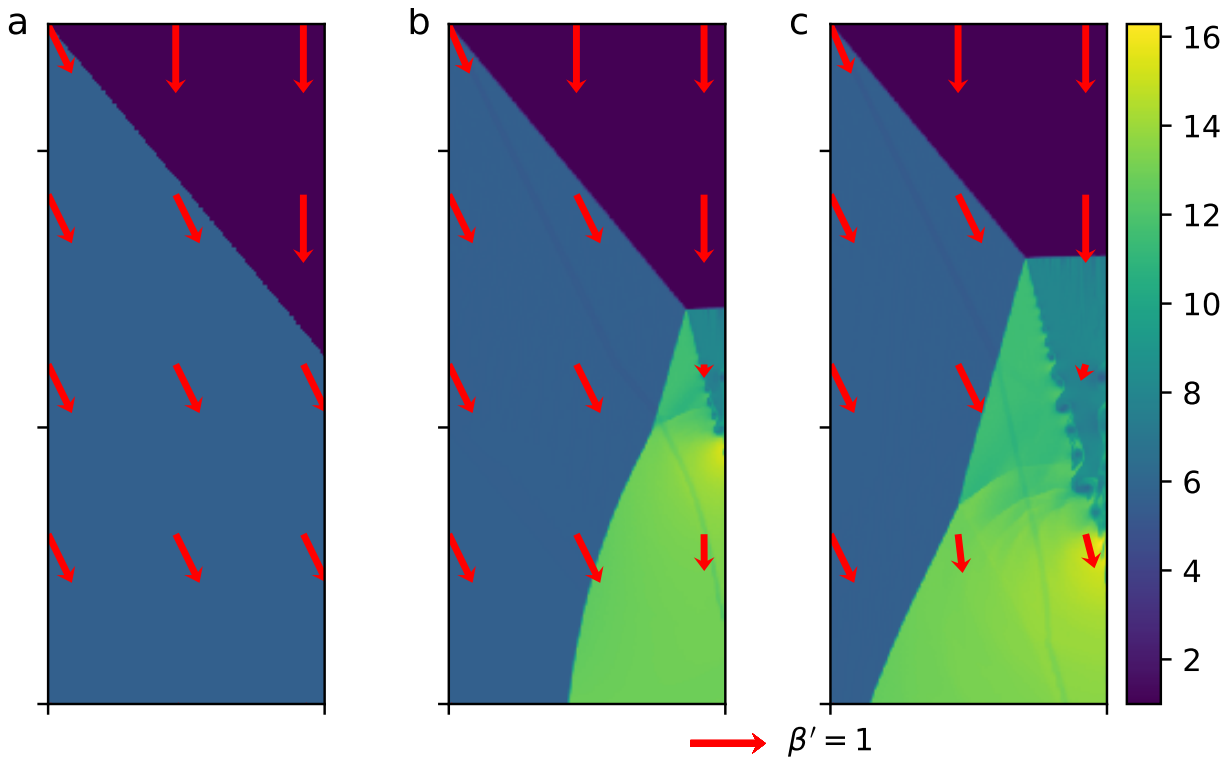
Relativistic Shock reflection (Bera et al. 2024):

Strong Shock solution is found to be unstable & transitions to **Weak Shock** solution



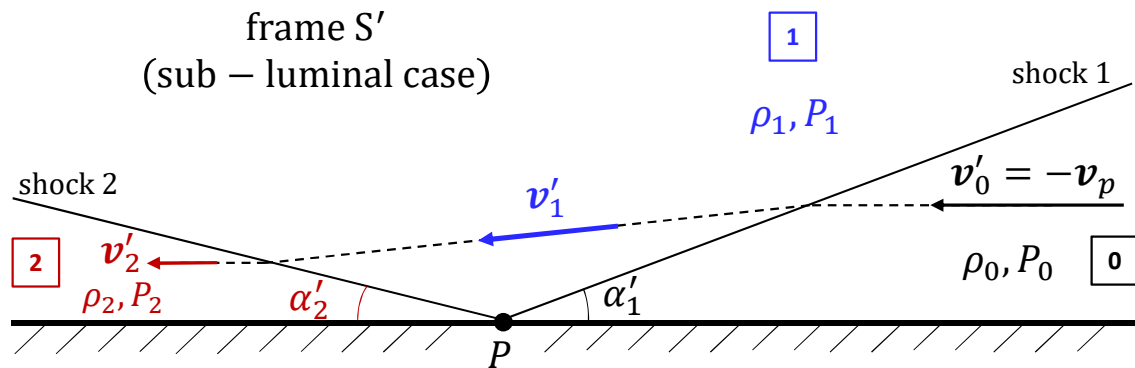
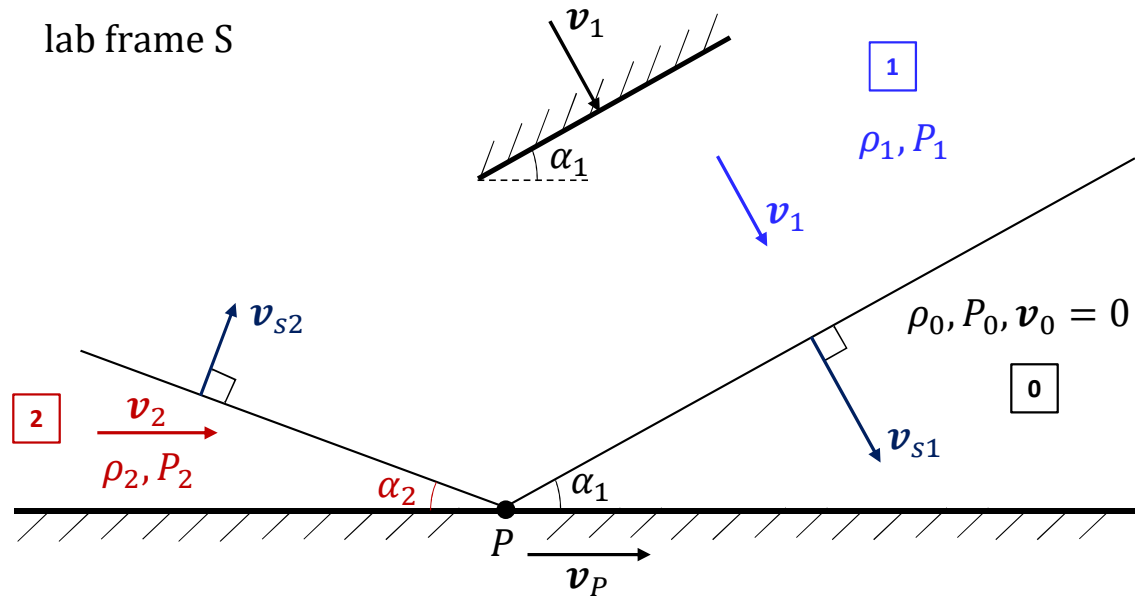
Relativistic Shock reflection (Bera et al. 2024):

In the **detachment region**: **Mach reflection**

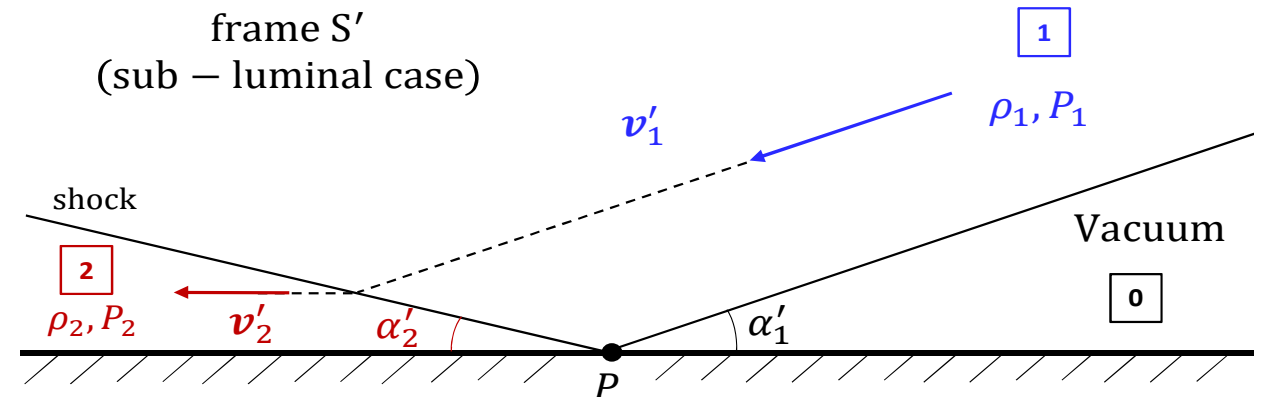
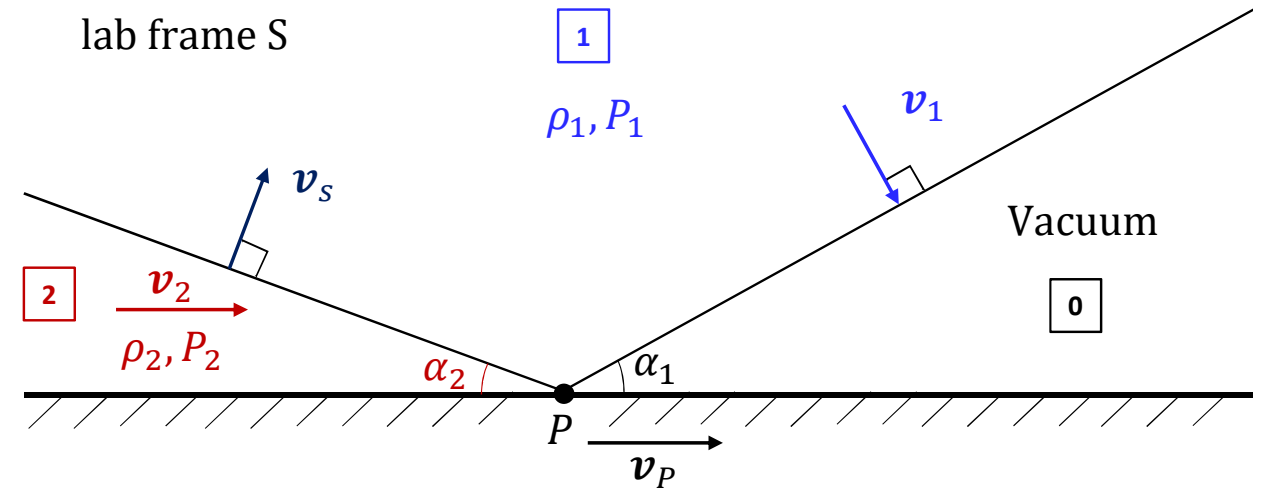


Single Shell – Wall Collision (JG et al. in prep.):

Shock Reflection



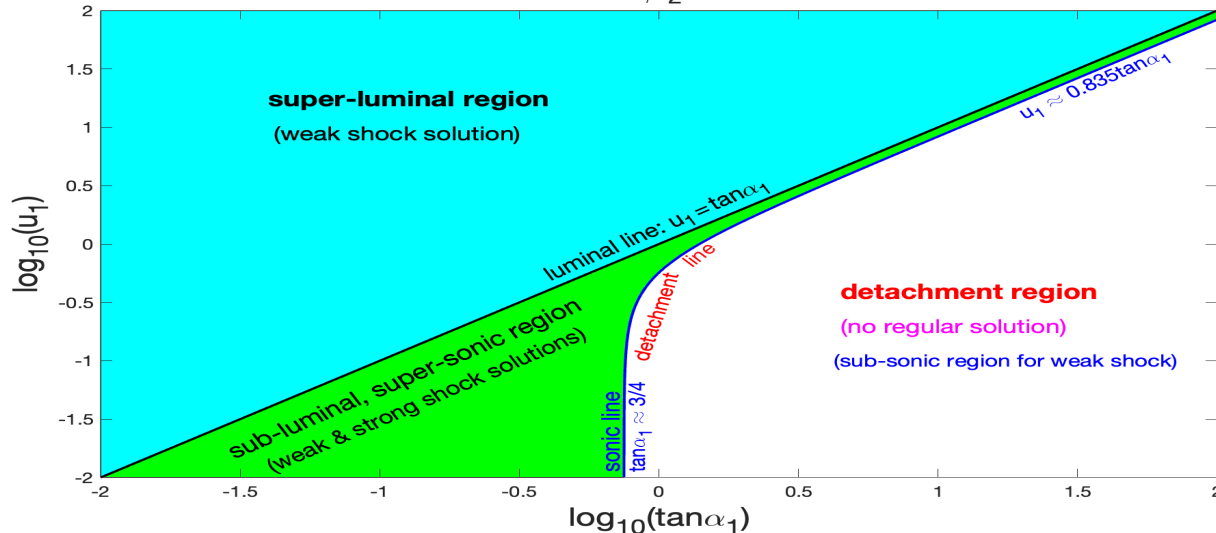
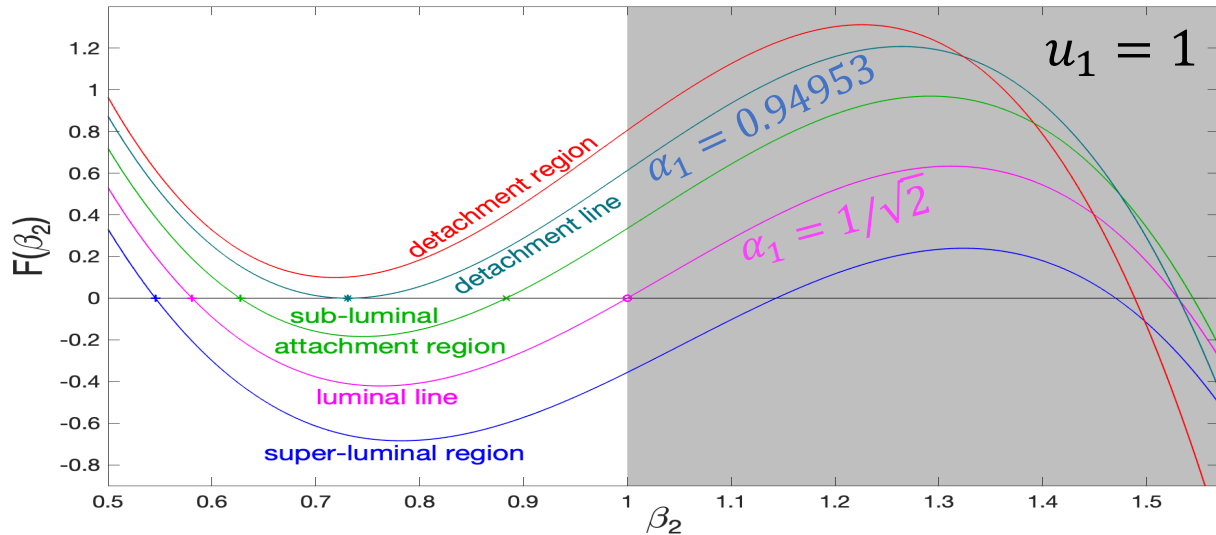
Single Shell - Wall Collision



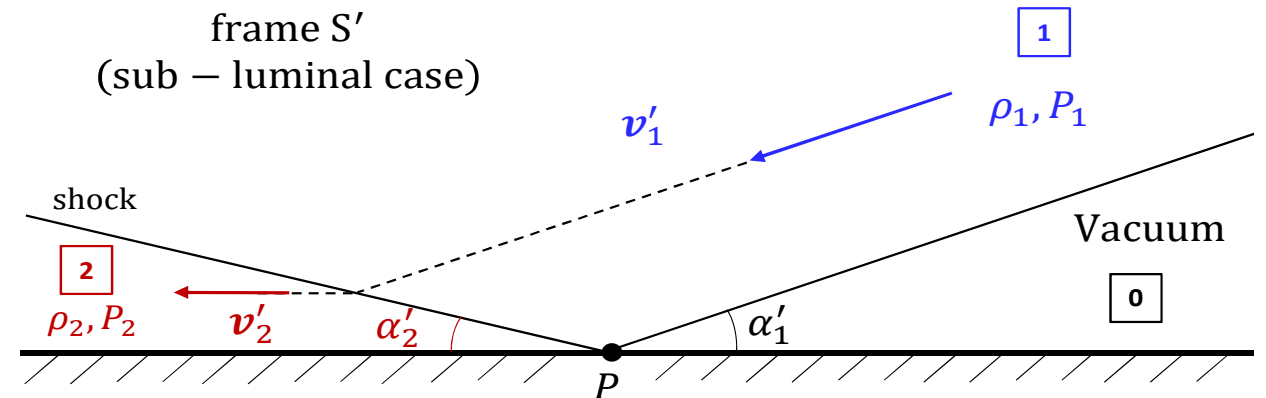
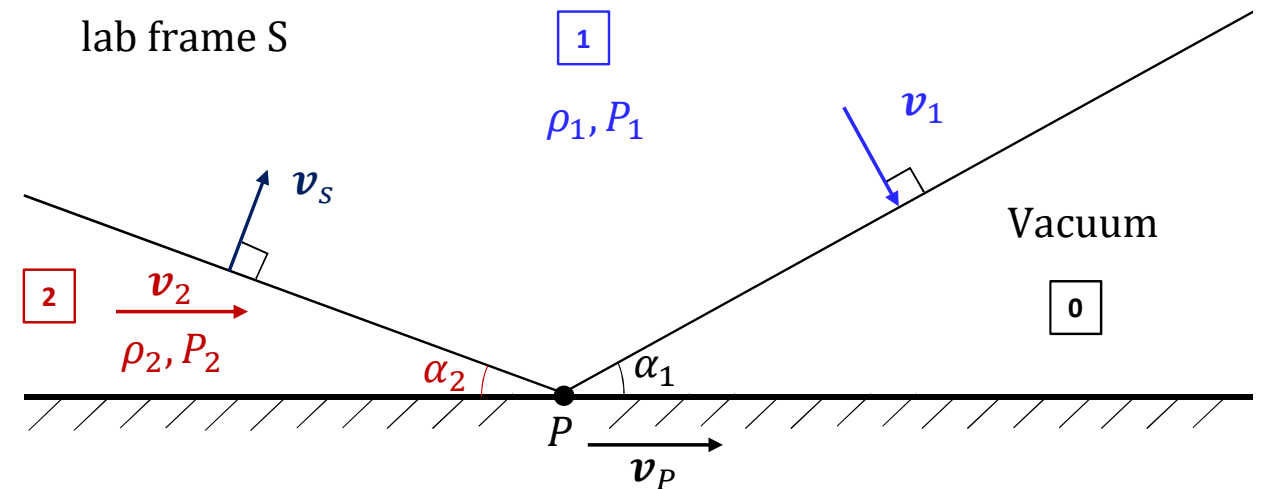
Single Shell – Wall Collision (JG et al. in prep.):

Fully analytic regular solution:

$$F(\beta_2) = A\beta_2^3 + B\beta_2^2 + C\beta_2 + D = 0$$



Single Shell - Wall Collision



Conclusions: Colliding Relativistic Shells

- Such collisions are expected in GRBs, blazars, magnetar GFs, SLSNe, FRBs,...
- One collision produces two shocks: forward shock (FS) & reverse shock (RS)
- The reverse shock is typically stronger, dominating νF_ν near E_{peak}
- The forward shock peaks below E_{peak} & might mimic photospheric emission
- RS + FS produce diverse pulse shapes & spectra (also time-integrated)
- Fits to data can help constrain important physical parameters of the outflow
- Oblique collisions are analogous to the classical shock reflection problem
- Both have analytic “regular” solutions for initially cold fluids
- There is a super-luminal regime where only the weak shock solution exists
- Between luminal & detachment lines an unstable strong shock solution exists
- Oblique collisions modify β of $\Gamma \gg 1$ shells, affecting their beamed emission

The End