

Particle Astrophysics in Poland 2025

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Prospects of testing Dark Matter with gravitational lensing and gravitational waves

Marek Biesiada

Department of Astrophysics
National Centre for Nuclear Research
Warsaw, Poland



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Dark matter (satellite) halo mass deficit?

1. **Dark matter cores** of kpc size are preferred by observed circular velocities in dwarf/low-surface-brightness (LSB) galaxies, while simulations suggest cusps [Moore 1994; Burkert 1995, ...].

(core/cusp problem)

2. **Non-observation of very massive satellite halos** predicted by simulations in our Milky Way [M.Boylan-Kolchin et al. 2011, 2012] and others [Ferrero et al. 2011].

(too-big-to-fail problem)

3. Given the long lifetime of dwarfs, some globular/star clusters are expected to be destroyed, or sink to the center **if their host halos are cuspy** [J. Binney & S.Tremaine 2008, F. Contenta et al. 2017, P. Boldrini et al. 2018, ...].

(GC timing problem)

Self-interacting dark matter (SIDM)?

- Stronger self-scattering needed for (dwarf-sized) halos

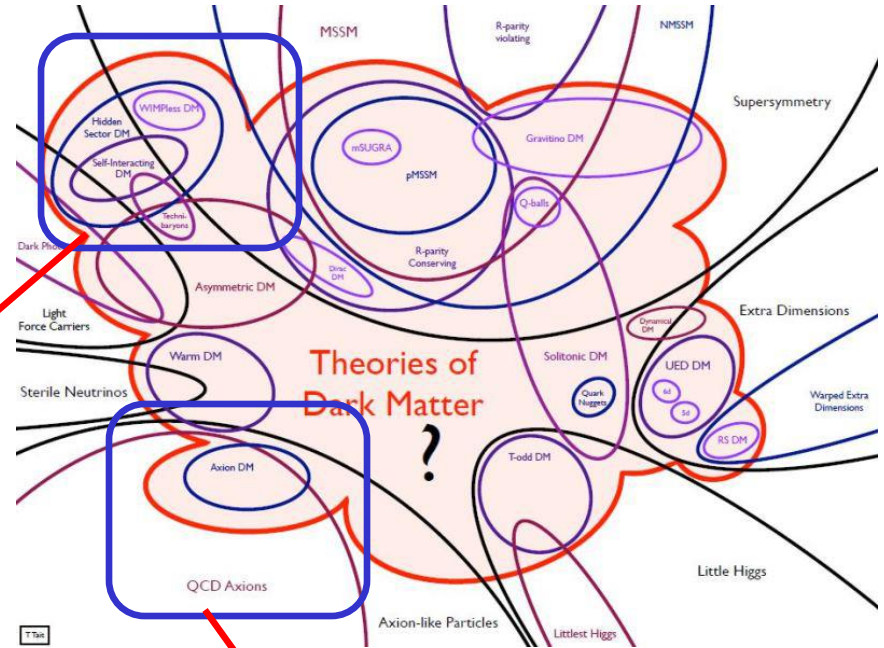
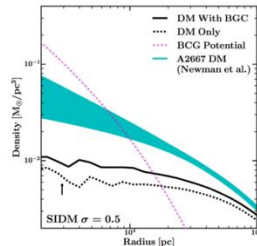
[O. D. Elbert et al. 2016, K. Bondarenko 2016, ...]

😊 $\frac{\sigma_{SI}}{m_{DM}} \sim 0.5 - 10 \text{ cm}^2/\text{g}$ at dwarf scales of DM velocity $\sim 10 \text{ km/s}$

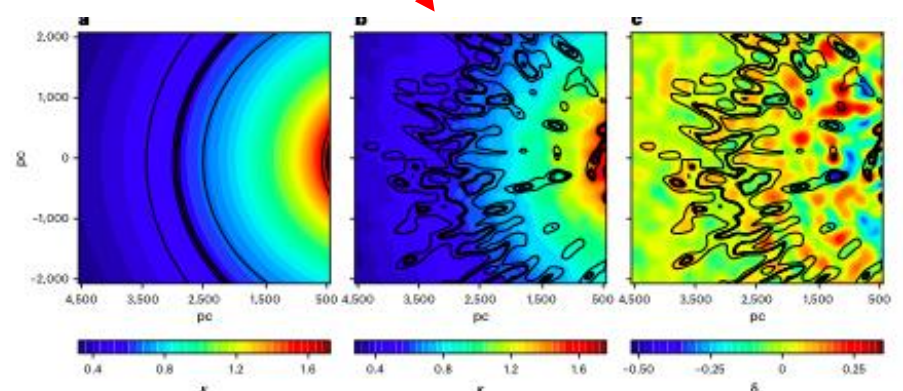
- Weaker self-scattering favored by cluster merging/halo profiles etc.

[O. D. Elbert et al. 2016, K. Bondarenko 2016, ...]

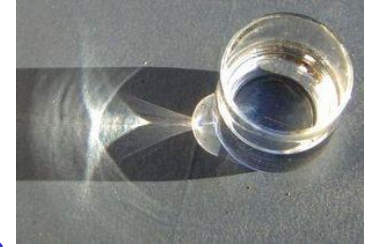
😞 $\frac{\sigma_{SI}}{m_{DM}} \leq 0.2 - 1 \text{ cm}^2/\text{g}$ at cluster scales of DM velocity $\sim 1000 \text{ km/s}$



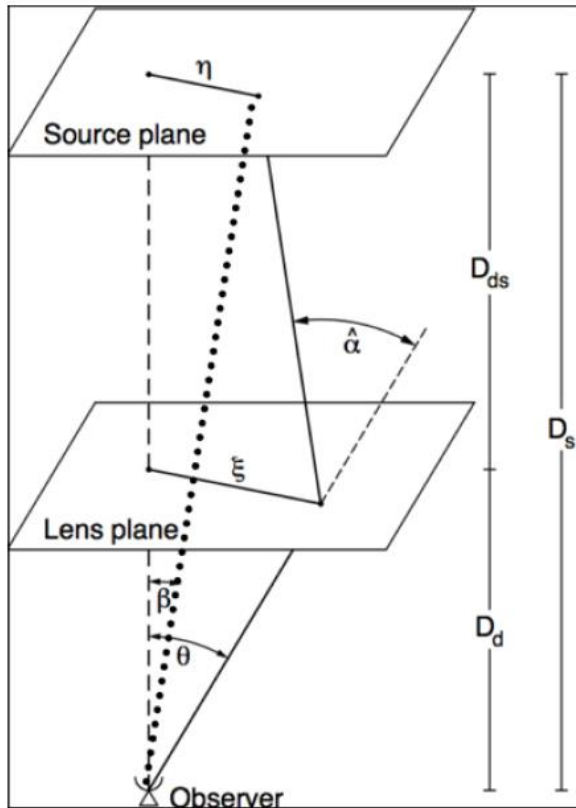
$$\lambda_{\text{dm}} = 150 \left(\frac{10^{-22} \text{ eV}}{m_{\text{pl}}} \right) \left(\frac{M_{\text{h}}}{10^{12} M_{\odot}} \right)^{-1/3} \text{ pc}$$



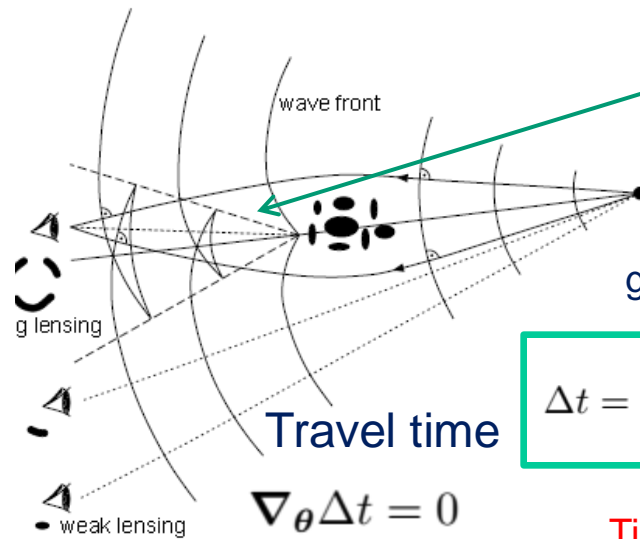
Gravitational lensing



Light rays formalism



Wavefront formalism (Fermat principle)



caustics

$$\phi(\boldsymbol{\theta}) = \frac{D_{ls}}{D_l D_s} \frac{2}{c^2} \int \Phi(D_l \boldsymbol{\theta}, z) dz$$

Newtonian potential at lens plane

geometrical term

$$\Delta t = \frac{1+z_l}{c} \frac{D_{ol} D_{os}}{D_{ls}} \left[\frac{(\boldsymbol{\theta} - \boldsymbol{\beta})^2}{2} - \phi(\boldsymbol{\theta}) \right]$$

Travel time

$$\nabla_{\boldsymbol{\theta}} \Delta t = 0$$

Time delay distance

Fermat potential

Lens equation

$$\hat{\alpha}(\boldsymbol{\theta}) D_{ls} + \beta D_s = \boldsymbol{\theta} D_s$$

$$\boldsymbol{\theta} - \boldsymbol{\beta} - \nabla_{\boldsymbol{\theta}} \phi = 0$$

$$\theta_E = 4\pi \frac{\sigma_{ap}^2}{c^2} \frac{D_{ls}}{D_s} \left(\frac{\theta_E}{\theta_{ap}} \right)^{2-\gamma} f(\gamma)$$

Einstein radius

- Observables:
- * image positions and shape distortions
 - * time delay between images
 - * flux ratios magnification ratios

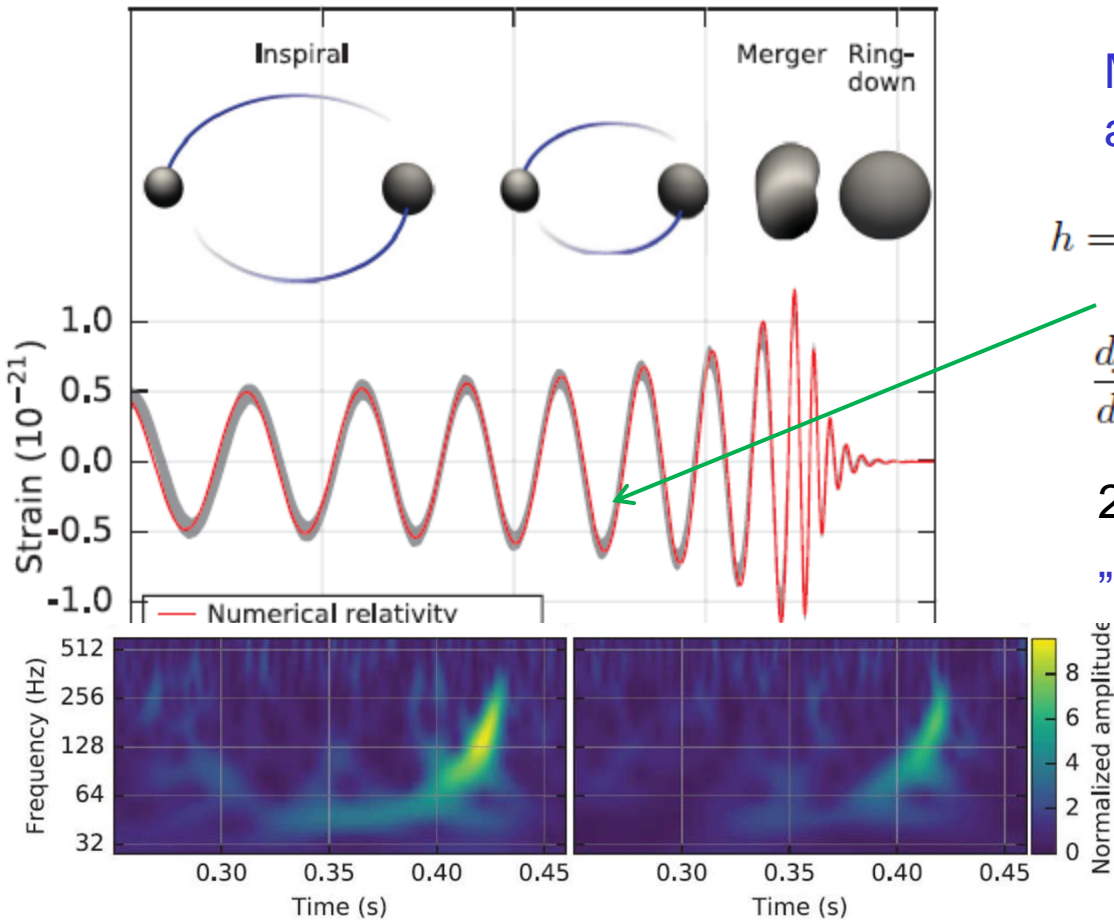
$$A(\boldsymbol{\theta}) = \frac{\partial \beta}{\partial \boldsymbol{\theta}} \quad \mu(\boldsymbol{\theta}) = \frac{1}{\det A(\boldsymbol{\theta})}$$

$\alpha = \nabla_{\boldsymbol{\theta}} \phi$ magnification

The idea of „standard sirens”

B. Schutz 1986

B.Schutz, A. Królak 1987



Measure the strain $h(t)$
and frequency drift df/dt

$$h = \frac{4\pi^{2/3}(GM)^{5/3}}{c^4 D} f(t)^{2/3} \cos \left[\int_0^t f(t') dt' \right]$$

$$\frac{df}{dt} = \frac{96\pi^{8/3}}{5} \left(\frac{GM}{c^3} \right)^{5/3} f^{11/3}$$

2 equations for 2 unknowns:
„chirp mass” M & distance D

$$M = \frac{c^3}{G} \left(\frac{5}{96\pi^{8/3}} \frac{df}{dt} \right)^{3/5} f^{-11/5}$$

$$D = \frac{4c}{\pi^2 h} \frac{df(t)}{dt} f^{-3} \cos \left(\int_0^t f(t') dt' \right)$$

for cosmological sources

$$M_c \rightarrow (1+z)M_c \quad \begin{aligned} t &\rightarrow (1+z)t \\ f &\rightarrow \frac{1}{1+z}f \\ \frac{df}{dt} &\rightarrow \frac{1}{(1+z)^2} \frac{df}{dt} \end{aligned}$$

The distance inferred is the **luminosity distance**

$$D_L = (1+z) D \quad 4$$

Plane GW Traveling Through Homogeneous Matter

- **Fluid:**

- GW shears the fluid, (rate of shear) = $\sigma_{jk} = \frac{1}{2}\dot{h}_{jk}^{\text{GW}}$
- no resistance to shear, so no action back on wave
- Viscosity $\eta \sim \rho v s = (\text{density})(\text{mean speed of particles})(\text{mean free path})$
produces stress $T_{jk} = -2\eta\sigma_{jk} = -\eta\dot{h}_{jk}^{\text{GW}}$ **NOTE:** s must be $< \lambda$
- Linearized Einstein field equation: $\square h_{jk}^{\text{GW}} = -16\pi(T_{jk})^{\text{TT}} = 16\pi\eta\dot{h}_{jk}^{\text{GW}}$
- Wave attenuates: $h_{jk}^{\text{GW}} \sim \exp(-z/\ell_{\text{att}})$ where $\ell_{\text{att}} = \frac{1}{8\pi\eta} = \frac{1}{8\pi\rho v s}$

In the fluid with shear viscosity

$$h_{\alpha,\text{visc}} = h_{\alpha} e^{-\beta D/2}$$

attenuated wave leads
to biased luminosity distance

$$D_{\text{L,eff}}(z, \beta) = D_{\text{L}}(z) e^{\beta D(z)/2}$$



Measuring the viscosity of dark matter with strongly lensed gravitational waves

Shuo Cao,^{1★} Jingzhao Qi,² Marek Biesiada,^{1,3★} Tonghua Liu,¹ Jin Li⁴ and Zong-Hong Zhu^{1★}

¹Department of Astronomy, Beijing Normal University, Beijing 100875, China

²Department of Physics, College of Sciences, Northeastern University, Shenyang 110004, China

³National Centre for Nuclear Research, Pasteura 7, PL-02-093 Warsaw, Poland

⁴Department of physics, Chongqing University, 400044 Chongqing, China

LETTER TO THE EDITOR

Direct measurement of the distribution of dark matter with strongly lensed gravitational waves

Shuo Cao^{1,2}, Jingzhao Qi³, Zhoujian Cao^{1,2}, Marek Biesiada⁴, Wei Cheng⁵, and Zong-Hong Zhu^{1,2}

$$h_{\alpha, \text{visc}} = h_{\alpha} e^{-\beta D/2} \quad \text{attenuated wave leads to biased luminosity distance} \quad D_{L, \text{eff}}(z, \beta) = D_L(z) e^{\beta D(z)/2}$$

lensed transients (GW, SNIa) signal leads to very precise determination of „time delay distance”

$$\Delta t_{i,j} = \frac{D_{\Delta t}(1+z_l)}{c} \Delta \phi_{i,j} \quad D_{\Delta t}(z_l, z_s) \equiv \frac{D_A(z_l) D_A(z_s)}{D_A(z_l, z_s)}$$

To be determined by unlensed standard sirens

$$D_{\Delta t} = \frac{D_L(z_l) D_L(z_s)}{(1+z_l)^2 D_L(z_s) - (1+z_s)(1+z_l) D_L(z_l)}$$

objective function fitted
for beta lensed

$$\chi^2 = \sum_{i=1}^i \frac{(D_{\Delta t,i}^{lens}(z_{l,i}, z_{s,i}) - D_{\Delta t,i}^{unlens}(z_{l,i}, z_{s,i}; \beta))^2}{\sigma_{i,lens}^2 + \sigma_{i,unlens}^2}$$

relation with DM physical parameters

$$\frac{\langle \sigma_\chi \rangle}{m_\chi} = \frac{6.3\pi G \langle v \rangle}{c^3 \beta}$$

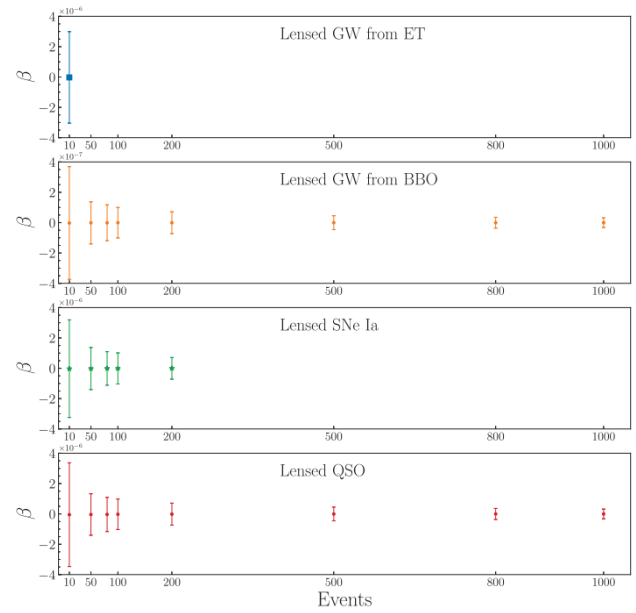


Table 1. Summary of the constraints obtained from different observations. Type I and II, respectively, correspond to the two cases of self-interacting DM in galaxies and galaxy clusters.

Data	$\Delta\beta$	$\Delta(\sigma_\chi/m_\chi)(I)$	$\Delta(\sigma_\chi/m_\chi)(II)$
GW (lensed; ET) + GW (unlensed)	10^{-6} Mpc^{-1}	$10^{-4} \text{ cm}^2 \text{ g}^{-1}$	$10^{-3} \text{ cm}^2 \text{ g}^{-1}$
GW (lensed; BBO) + GW (unlensed)	10^{-8} Mpc^{-1}	$10^{-6} \text{ cm}^2 \text{ g}^{-1}$	$10^{-5} \text{ cm}^2 \text{ g}^{-1}$
QSO (lensed; LSST) + GW (unlensed)	10^{-7} Mpc^{-1}	$10^{-5} \text{ cm}^2 \text{ g}^{-1}$	$10^{-4} \text{ cm}^2 \text{ g}^{-1}$
SNe Ia (lensed; LSST) + GW (unlensed)	10^{-6} Mpc^{-1}	$10^{-4} \text{ cm}^2 \text{ g}^{-1}$	$10^{-3} \text{ cm}^2 \text{ g}^{-1}$

$$1 \text{ cm}^2 \text{ g}^{-1} = 1.8 \text{ barn GeV}^{-1}$$

it would be able to DM viscosity and differentiate
cluster and small scale scenarios

Einstein rings modulated by wavelike dark matter from anomalies in gravitationally lensed images

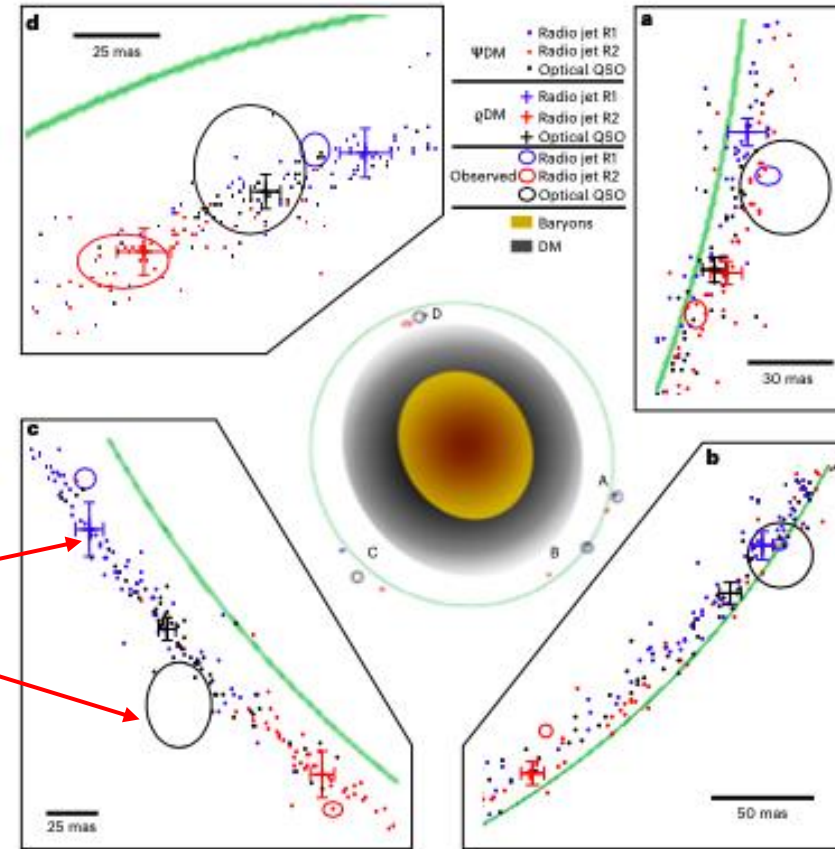
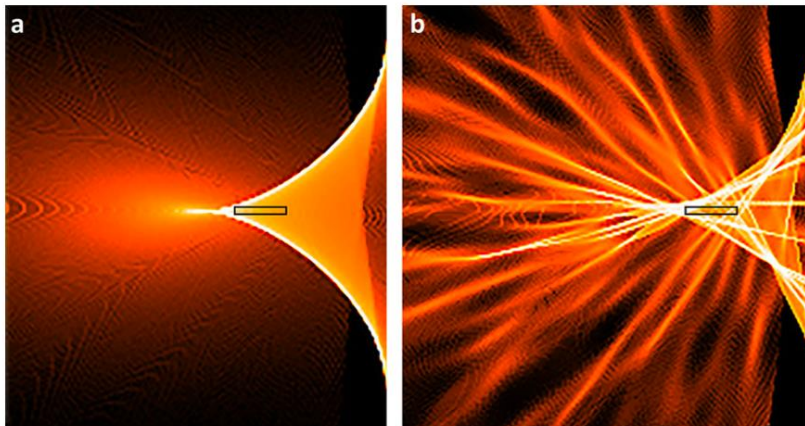
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Alfred Amruth¹, Tom Broadhurst^{2,3,4}, Jeremy Lim¹,
 Masamune Oguri^{5,6,7}, George F. Smoot^{8,9,10}, Jose M. Diego¹¹, Enoch Leung¹²,
 Razieh Emami¹³, Juno Li¹, Tzihong Chiueh^{14,15,16}, Hsi-Yu Schive^{14,15,16,17},
 Michael C. H. Yeung¹ & Sung Kei Li¹

Positions predicted by best fit smooth potential
 Positions observed



Lens HS 0810+2554 observed by VLBI

QSO by HST; radio. Jets by VLBI

$z_s = 1.51$ $z_l = 0.89$

Time delay anomalies of fuzzy gravitational lenses

Jianxiang Liu¹, Zijun Gao¹, Marek Biesiada², and Kai Liao^{1,*}

¹*School of Physics and Technology, Wuhan University, Wuhan 430072, China*

²*National Centre for Nuclear Research, Pasteura 7, PL-02-093 Warsaw, Poland*

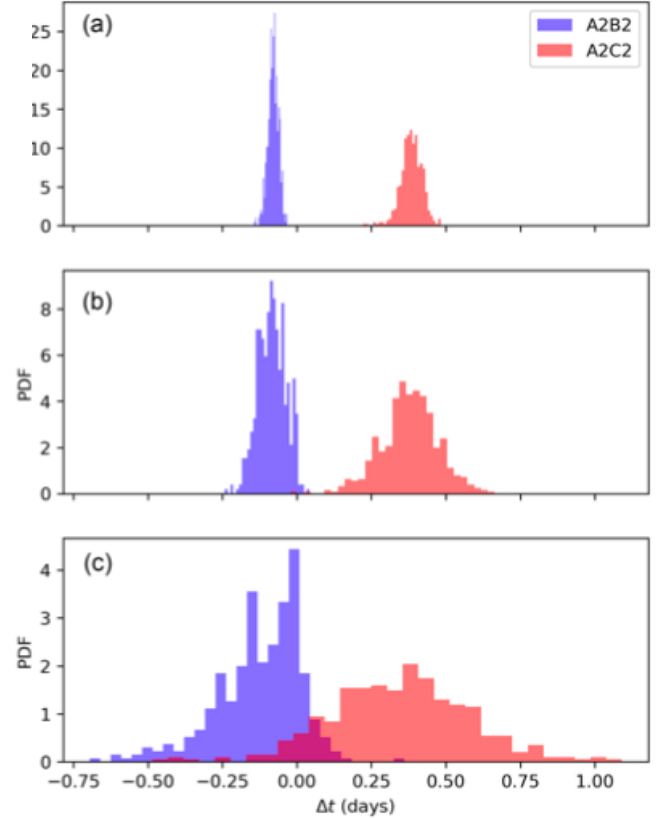
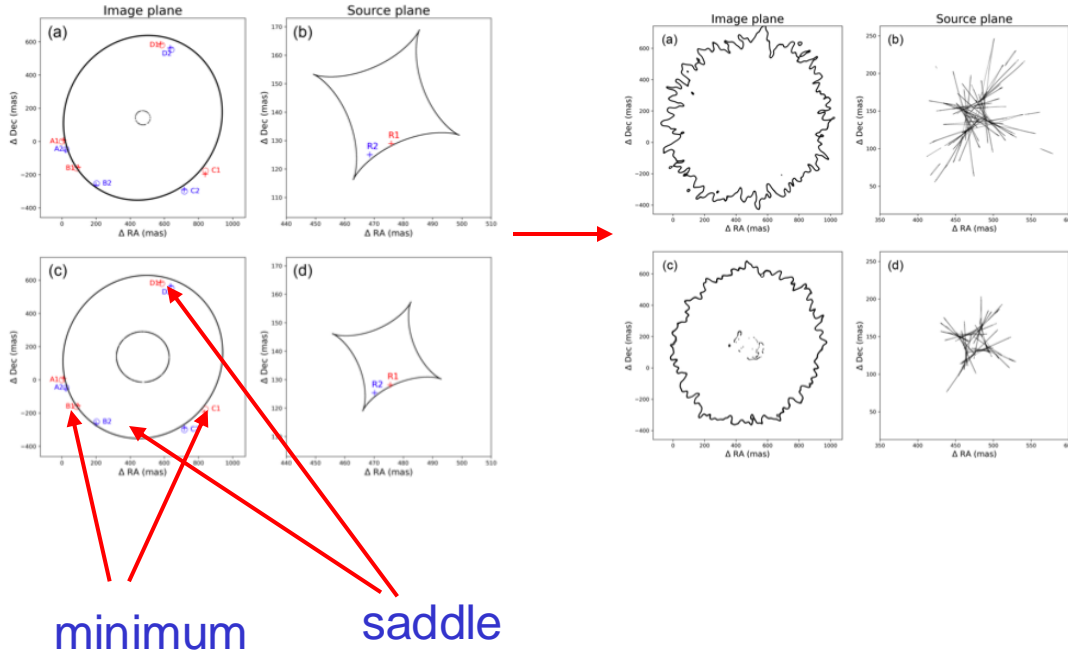


FIG. 7. Time delays anomalies of A2B2 and A2C2. A2 and C2 are minimum points; B2 is the saddle point. Strength of the ψ DM perturbation is $d = 0.3$ with different de Broglie wavelength: $\lambda_{dB} = 60$ pc panel (a), $\lambda_{dB} = 120$ pc panel (b) and $\lambda_{dB} = 240$ pc panel (c).

Conclusions

- Vera Rubin Observatory (LSST survey) will become a game changer in strong lensing
 - 10 000 strong lensing systems including 1000 quasar lenses.
- Great opportunity from lensed transients and multiwavelength studies
 - strongly lensed SN Ia (and other transient events)
 - anomalies in time delays and DM substructure, fuzzy DM
 - K.Liao, ... ,M.B., et al. ApJ 867:69, 2018
 - multiwavelength (optical, IR, radio) study of SL systems
 - lensed GRBs, FRBs
- New generation of ground-based and space-borne GW detectors (ET, CE, DECIGO, LISA) will considerably enhance the statistics of GW events – lensed signals will be detected
- Opportunity to explore the fundamental questions in Physics, like the nature of DM.

Thank you !