Neutrino Mass and Discrete Dark Matter

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Based on 2406.17861, 2311.15997

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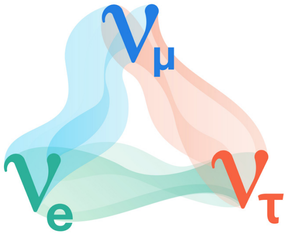
Neutrinos: the known and unknowns



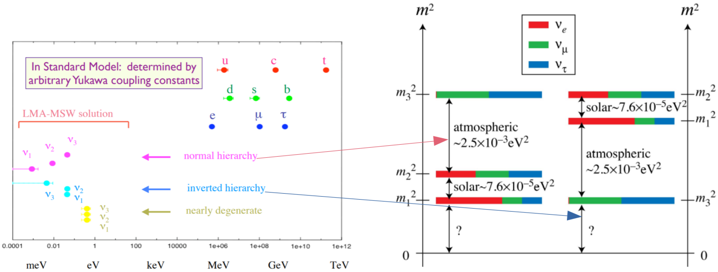
$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i}\nu_i \qquad\qquad (\alpha = e, \mu, \tau)$$

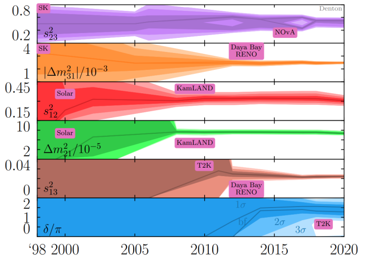
$$U_{PMNS} = \begin{bmatrix} C_{12}C_{13} & S_{12}C_{13} & S_{13}e^{-i\delta} \\ -S_{12}C_{23} - C_{12}S_{13}S_{23}e^{i\delta} & C_{12}C_{23} - S_{12}S_{13}S_{23}e^{i\delta} & C_{13}S_{23} \\ S_{12}S_{23} - C_{12}S_{13}C_{23}e^{i\delta} & C_{12}S_{23} - S_{12}S_{13}C_{23}e^{i\delta} & C_{13}C_{23} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha_{21}/2} & 0 \\ 0 & 0 & e^{i\alpha_{31}/2} \end{bmatrix}$$

here $C_{ij} = \cos\theta_{ij}$ and $S_{ij} = \sin\theta_{ij}$.

Three mixing angles: θ_{12}, θ_{23} and θ_{13}
Dirac CP-violating phase: $\delta_{\rm CP}$
Two mass squared differences: $\Delta m^2_\odot = m_2^2 - m_1^2$, $\Delta m^2_A = |m_3^2 - m_1^2|$

Neutrino parameters and the known unknowns: 'Big' Data

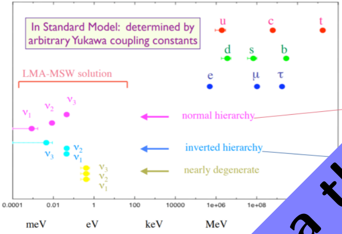


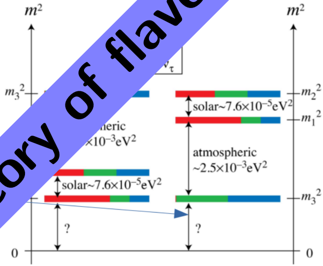


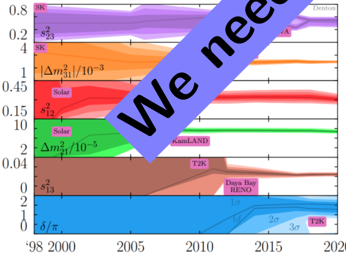
	Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.6$)	
	bfp $\pm1\sigma$	3σ range	bfp $\pm1\sigma$	3σ range
$\sin^2\theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \to 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \to 0.343$
$\theta_{12}/^\circ$	$33.44^{+0.77}_{-0.74}$	$31.27 \to 35.86$	$33.45^{+0.77}_{-0.74}$	$31.27 \to 35.87$
$\sin^2\theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \to 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \to 0.623$
$\theta_{23}/^\circ$	$49.2^{+1.0}_{-1.3}$	$39.5 \to 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \to 52.1$
$\sin^2\theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238^{+0.00064}_{-0.00062}$	$0.02053 \to 0.02434$
$\theta_{13}/^\circ$	$8.57^{+0.13}_{-0.12}$	$8.20 \to 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \to 8.98$
$\delta_{CP}/^\circ$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$
$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \to 8.04$
$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \to +2.599$	$-2.498^{+0.028}_{-0.029}$	$-2.584 \to -2.413$

Neutrino parameters and the known unknowns: 'Big' Data

We need a theory of flavor!!







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Flavor symmetries, why?

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$\Downarrow$

(Prior to 2012)

$s_{23} = 1/\sqrt{2}\ (\theta_{23} = 45^\circ)$ and $\theta_{13} = 0$

$\Downarrow$

$$U_0 = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}.$$

- $\theta_{12} = 45^\circ (s_{12} = 1/\sqrt{2})$ — Bimaximal Mixing
- $\theta_{12} = 35.26^\circ (s_{12} = 1/\sqrt{3})$ — Tribimaximal Mixing (TBM)
- $\theta_{12} = 31.7^\circ$ — Golden Ratio Mixing
- $\theta_{12} = 30^\circ (s_{12} = 1/2)$ — Hexagonal Mixing

$$U_0 = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \frac{\varphi}{\sqrt{2+\varphi}} & \frac{1}{\sqrt{2+\varphi}} & 0 \\ \frac{-1}{\sqrt{4+2\varphi}} & \frac{\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{4+2\varphi}} & \frac{-\varphi}{\sqrt{4+2\varphi}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \sqrt{\frac{3}{4}} & \frac{1}{2} & 0 \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} & \frac{\sqrt{3}}{2\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Fukugita, Tanimoto, Yanagida PRD98; Harrison Perkins, Scott PLB02; Dutta,Ramond NPB03; Rodejohann et. al. EPJC10

(GR: $\tan\theta_{12} = 1/\phi$ where $\phi = (1+\sqrt{5})/2$)

Flavor symmetries, why?

Simple example: $\mu - \tau$ permutation symmetry and TBM

$$m_\nu = U_0^\star \,\mathrm{diag}(m_1, m_2, m_3) U_0^\dagger ,$$

such a mixing matrices can easily diagonalize a $\mu - \tau$ symmetric (transformations $\nu_e \to \nu_e$, $\nu_\mu \to \nu_\tau$, $\nu_\tau \to \nu_\mu$ under which the neutrino mass term remains unchanged) neutrino mass matrix of the form

$$m_\nu = \begin{pmatrix} A & B & B \\ B & C & D \\ B & D & C \end{pmatrix} ,$$

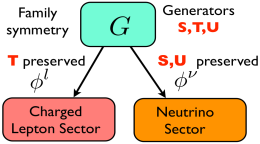
With $A + B = C + D$ this matrix yields tribimaximal mixing pattern where $s_{12} = 1/\sqrt{3}$ $i.e.$, $\theta_{12} = 35.26^\circ$

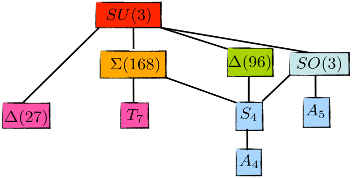
- Compatible Mixing Matrix :

$$U_{\mathrm{TB}} \simeq \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

General Framework: Symmetry based approach

- Fundamental symmetry in the lepton sector can easily explain the origin of neutrino mixing which is considerably different from quark mixing.
- Incidentally, both Abelian or non-Abelian family symmetries have potential to shade light on the Yukawa couplings.
- The Abelian symmetries (such as Froggatt-Nielsen symmetry) only points towards a hierarchical structure of the Yukawa couplings.
- Non-Abelian symmetries are more equipped to explain the non-hierarchical structures of the observed lepton mixing as observed by the oscillation experiments.





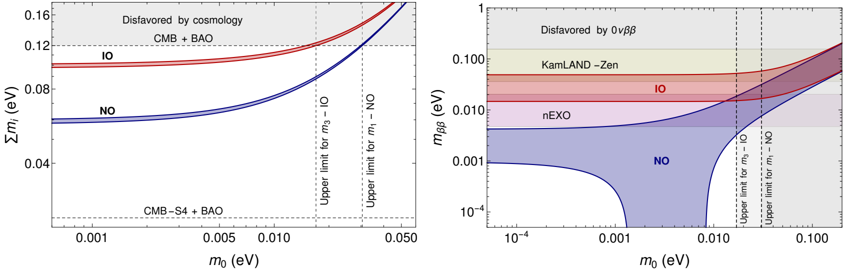
S. F. King 1301.1340

$\boxed{G_f \rightarrow G_e, G_\nu}$ typically, $G_e = Z_3$ and $G_\nu = Z_2 \times Z_2$.

Dirac or Majorana Particle??

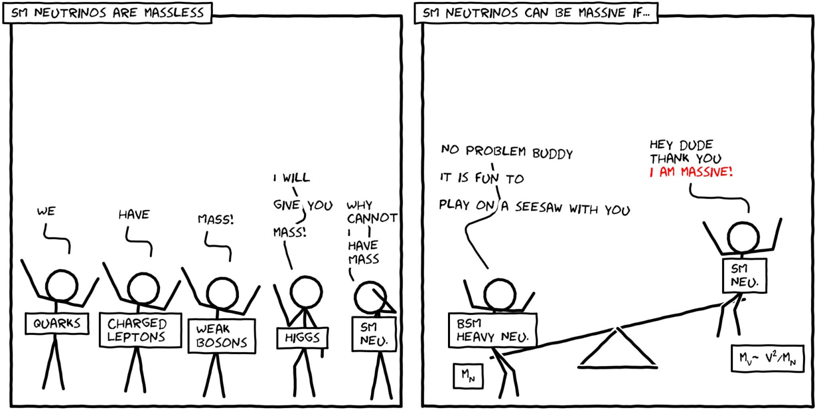
Neutrino Mass : Cosmology to $0\nu\beta\beta$

Gluza, Karmakar, Zieba et. al. Prog.Part.Nucl.Phys. 138 (2024) 104126, 2310.2068



- Absolute neutrino mass : $m_\nu^2 < 0.9$ eV^2 (The KATRIN Collaboration 2022)

Neutrino Mass Generation

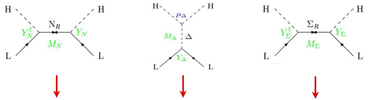


Cartoon by Sitian Qian

Neutrino Mass Generation

Seesaw frameworks

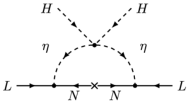
Right-handed singlet: (type-I seesaw) | Scalar triplet: (type-II seesaw) | Fermion triplet: (type-III seesaw)



$$m_\nu = Y_N^T \frac{1}{M_N} Y_N v^2 \qquad m_\nu = Y_\Delta \frac{\mu_\Delta}{M_\Delta^2} v^2 \qquad m_\nu = Y_\Sigma^T \frac{1}{M_\Sigma} Y_\Sigma v^2$$

- **Type-I Seesaw, Type-II Seesaw, Type-III Seesaw, etc.:** Minkowski 77; Gellman, Ramond, Slansky 80; Glashow, Yanagida 79; Mohapatra, Senjanovic 80; Lazarides, Shafi; Schechter, Valle 81; Schechter, Valle 80; Mohapatra, Senjanovic 81; Lazarides, Shafi, Wetterich 81; Mohapatra Valle 86; Foot, Lew, He, Joshi 89; Ma 98; Bajc, Senjanovic 07....

Radiative neutrino mass



- **Radiative models, started in 80s:** Zee 80, Cheng, Li 80; Zee 86; Babu 88; Babu, Ma, Valle, 02; Ma 06;
- **For a review of radiative models:** Cai, Herrero-Garcia, Schmidt, Vicente, Volkas 17;

Hybrid Scenarios **In this talk we explore this less explored possibility**

Neutrinos and Dark Matter (DM)

Are they connected?

Caldwell, Mohapatra 1993; Peltoniemi, Valle 1993, Asaka, Blanchet, Shaposhnikov 2005; Boehm 2008; Kubo, Ma, Suematsu 2006; Hambye, Kannike, Ma, Raidal 2007; Lindner, Schmidt, Schwetz 2011; Borah, Adhikari 2012; Restrepo, Zapata, Yaguna 2013; Huang, Deppisch 2014; Escudero, Rius, Sanz 2016; Borah, Karmakar, Nanda 2018;..many more..

Stability of DM

- Many DM candidates[†] require a stabilizing symmetry

$$\tau_{\rm DM} > \tau_{\rm U} \sim 10^{18}\ \text{sec}$$

ensuring its stability over cosmological timescales, preventing it from decaying into Standard Model particle

- In many particle DM models: **stability assumed by hand**

ad hoc symmetries come into play

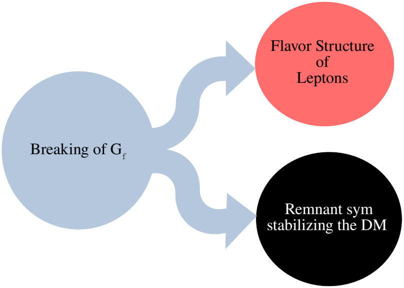
Examples: Z_2 symmetry (e.g., in Supersymmetry: R-parity), Global $U(1)$ symmetry (e.g., Axions), Discrete symmetries in various BSM models (Multi-component DMs)

[†] more on DM candidates : Sebastian Trojanowski at 12:00

Stability of DM

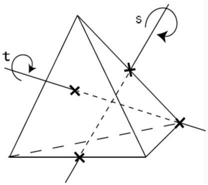
Flavor symmetry, which helps us **explain observed neutrino mixing**, may also be responsible for ensuring the **stability of DM**.

Stability of DM from breaking of discrete symmetry



$\boxed{G_f \longrightarrow \mathcal{Z}_2}$; $G_f = A_4, S_4, A_4 \otimes CP, S_4 \otimes CP, A_4 \otimes Z_N \ldots$

- We consider A_4 : Even permutation of 4 objects, invariant group of a tetrahedron
- Smallest group with 3 dim. irreps (unify three flavors of leptons)



Flavor Symmetry and Hybrid Neutrino Mass: Why?

- Ratio of solar to atmospheric mass difference :

$$r = \frac{\Delta m^2_{\rm SOL}}{\Delta m^2_{\rm ATM}} \simeq \frac{7.41 \times 10^{-5}\ \mathrm{eV}^2}{2.51 \times 10^{-3}\ \mathrm{eV}^2} \simeq 3 \times 10^{-2}$$

- Two different mass scales that might originate from **entirely separate mechanisms !!**

- Neutrino mass with **Scoto Seesaw** scenario: Phys.Rev.D 110 (2024) 3, 035012, 2311.15997

Greek word **'skótos'** → **'darkness'**

Fields	$e_R,\ \mu_R,\ \tau_R$	L_α	H	N_R	f	η	ϕ_s	ϕ_a	ϕ_T	ξ
A_4	$1,\ 1'',\ 1'$	3	1	1	1	1	3	3	3	$1''$

$$\mathcal{L} = \frac{y_N}{\Lambda}(\bar{L}\phi_s)\tilde{H}N_R + \frac{1}{2}M_N\bar{N}_R^c N_R + \frac{y_s}{\Lambda^2}(\bar{L}\phi_a)\xi i\sigma_2\eta^* f + \frac{1}{2}M_f\bar{f}^c f + h.c.,$$

$$M_\nu = -\frac{v^2}{M_N}Y_N^i Y_N^j + \mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)M_f Y_f^i Y_f^j$$

- Flavon fields get VEVs along $\langle\phi_s\rangle = (0, -v_s, v_s),\ \langle\phi_a\rangle = (2v_a, v_a, 0)$ **A_4 breaking ⇒ flavor structure**

$$Y_N = (Y_N^e, Y_N^\mu, Y_N^\tau)^T = (0, y_N\frac{v_s}{\Lambda}, -y_N\frac{v_s}{\Lambda})^T, ;\ Y_F = (Y_F^e, Y_F^\mu, Y_F^\tau)^T = (y_s\frac{v_\xi}{\Lambda}\frac{v_a}{\Lambda}, y_s\frac{v_\xi}{\Lambda}\frac{2v_a}{\Lambda}, 0)^T$$

- Light neutrino mass matrix :

$$M_\nu = \begin{pmatrix} b & 2b & 0 \\ 2b & -a+4b & a \\ 0 & a & -a \end{pmatrix},\ a = y_N^2\frac{v^2}{M_N}\frac{v_s^2}{\Lambda^2},\ b = y_s^2\frac{v_\xi^2}{\Lambda^2}\frac{v_a^2}{\Lambda^2}\mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)M_f = \kappa^2\mathcal{F}(m_{\eta_R}, m_{\eta_I}, M_f)M_f$$

Flavor Symmetry and Hybrid Neutrino Mass : Why?

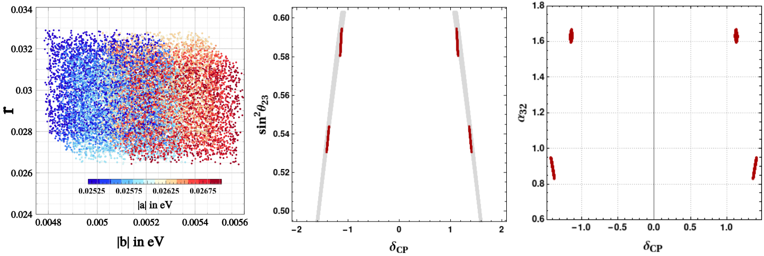
- Diagonalizing matrix:

$$U_\nu = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{\cos\theta}{\sqrt{3}} & \frac{e^{-i\psi}\sin\theta}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} + \frac{e^{i\psi}\sin\theta}{\sqrt{2}} & -\frac{\cos\theta}{\sqrt{2}} + \frac{e^{-i\psi}\sin\theta}{\sqrt{3}} \\ -\frac{1}{\sqrt{6}} & \frac{\cos\theta}{\sqrt{3}} - \frac{e^{i\psi}\sin\theta}{\sqrt{2}} & \frac{\cos\theta}{\sqrt{2}} + \frac{e^{-i\psi}\sin\theta}{\sqrt{3}} \end{pmatrix} U_m$$

- Mass Eigenvalues :

$$\begin{aligned} \widetilde{m}_1 &= 0, \\ \widetilde{m}_2 &= \frac{1}{2}\Big(-2a + 5b - \sqrt{4a^2 + 4ab + 25b^2}\Big), \\ \widetilde{m}_3 &= \frac{1}{2}\Big(-2a + 5b + \sqrt{4a^2 + 4ab + 25b^2}\Big). \end{aligned}$$

- Ratio of the solar to atmospheric mass-squared differences: $r \sim \frac{m_2^2}{m_3^2}$

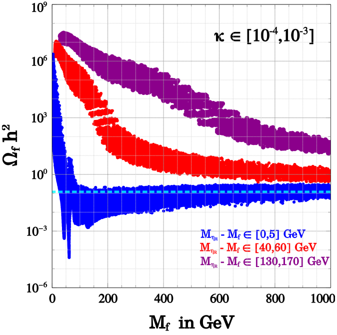


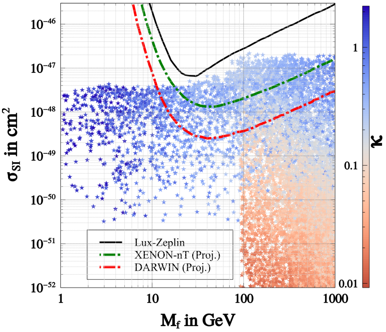
Discrete Dark Matter phenomenology

- 2 viable DM candidates ⇒ the **lightest neutral scalar**
  ⇒ the **singlet fermion** .

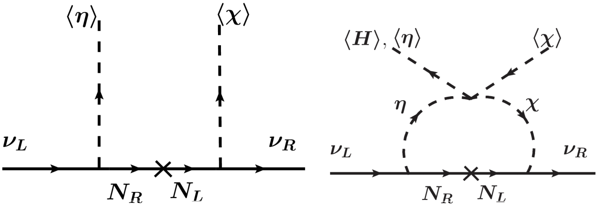
$$A_4 \longrightarrow \mathcal{Z}_2$$

DMs (η, f) are odd under $\mathcal{Z}_2$





Discrete Dark Matter: Dirac Neutrinos



Fields	ℓ_e	ℓ_μ	ℓ_τ	e_R	μ_R	τ_R	$N_{L,R}$	$\nu_{R_{e,\mu,\tau}}$	η	χ	H
A_4	1	$1''$	$1'$	1	$1''$	$1'$	3	$1\ ,\ 1'\ ,1''$	3	3	1
Z_2	1	1	1	1	1	1	1	-1	1	-1	1
$U(1)_L$	1	1	1	1	1	1	1	1	0	0	0

$$A_4 \otimes Z_2 \otimes U(1)_L \to \mathcal{Z}_2$$

- VEV the alignment of scalars leaves a remnant $\mathcal{Z}_2 \subset A_4$ unbroken after spontaneous symmetry breaking

$$\mathcal{Z}_2 :\ \ \eta_{2,3} \to -\eta_{2,3};\quad \chi_{2,3} \to -\chi_{2,3};\quad N_{2,3} \to -N_{2,3}$$

2406.17861, to be published in PRD

Conclusion

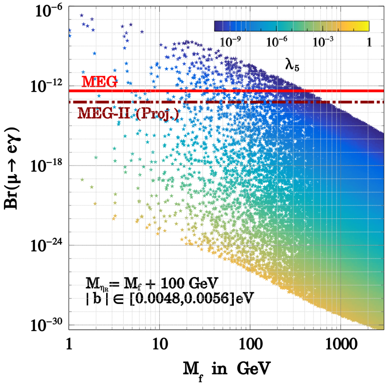
- Is there any guiding principle behind the observed pattern of lepton mixing?
- (Discrete) flavor symmetry is one such potential candidate.
- Tiny neutrino mass may originate from hybrid scoto-seesaw scenarios.
- It explains the hierarchy of the mass scales involved in neutrino oscillation
- Discrete DM scenarios: explain neutrino mixing angles and CP phases involved.
- Discrete DM scenarios: Provides stable DMs.
- Rich phenomenology: $h \to \gamma\gamma$, LFV decays, Collider prospect of BSM states, leptogenesis

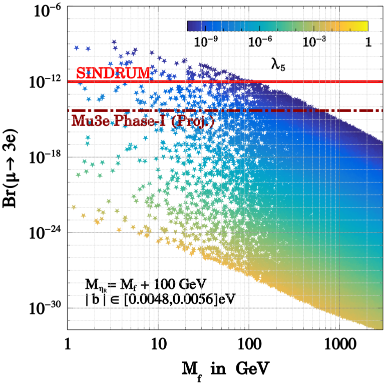
In a nutshell, the symmetry responsible leptonic flavor structure can provide stable DMs

Thank you for your attention!!

FSS_1 phenomenology: Lepton Flavor Violation

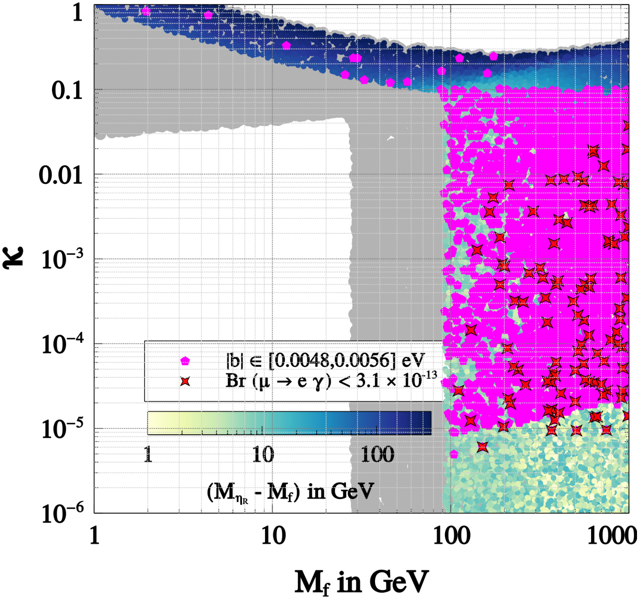
Decay Modes	Scotogenic contribution	Seesaw Contribution	Remarks
$\mu \to e\gamma$	✓	✗	$Y_N^e = 0$
$\tau \to e\gamma$	✗	✗	$Y_F^\tau = 0, Y_N^e = 0$
$\tau \to \mu\gamma$	✗	✓	$Y_F^\tau = 0$
$\mu \to 3e$	✓	✗	$Y_N^e = 0$
$\tau \to 3e$	✗	✗	$Y_F^\tau = 0, Y_N^e = 0$
$\tau \to 3\mu$	✗	✓	$Y_F^\tau = 0$





λ_5 is the coupling for the interaction $(H^\dagger \eta)(H^\dagger \eta)$

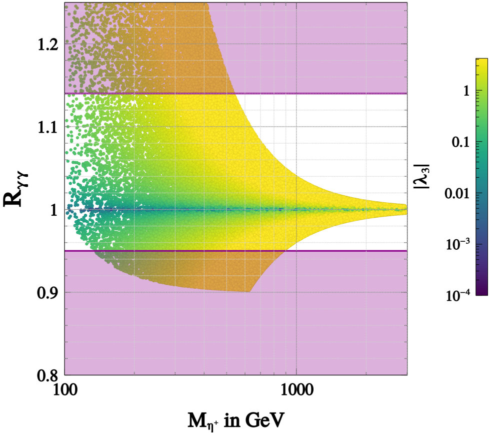
FSS_1 phenomenology: Summary



FSS$_1$ phenomenology: $h \to \gamma\gamma$

$$R_{\gamma\gamma} = \frac{\left[\sigma(gg \to h) \times \mathrm{Br}(h \to \gamma\gamma)\right]_{\mathrm{FSS}_1}}{\left[\sigma(gg \to h) \times \mathrm{Br}(h \to \gamma\gamma)\right]_{\mathrm{SM}}}$$

$$= \frac{\Gamma^h_{\mathrm{SM}}}{\Gamma^h_{\mathrm{FSS}_1}} \frac{\Gamma(h \to \gamma\gamma)_{\mathrm{FSS}_1}}{\Gamma(h \to \gamma\gamma)_{\mathrm{SM}}}.$$



λ_3 is the coupling for the interaction $(H^\dagger H)(\eta^\dagger \eta)$