Conversions in two-component dark sectors: A phase space level analysis

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NCBJ, Warszawa

based on arXiv:2502.08725 (hep-ph)
with Dr. Andrzej Hryczuk

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PAiP-2025 conference
Particle Astrophysics
in Poland

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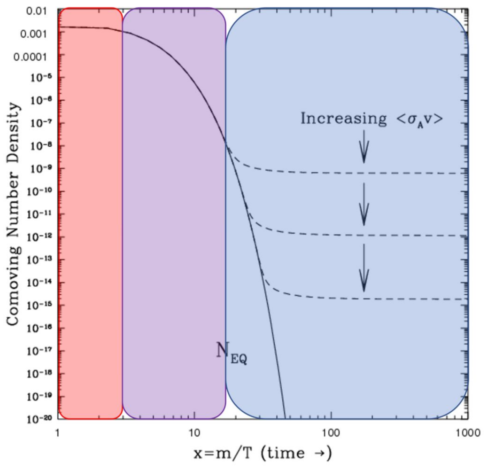
February 20th, 2025

Outline

- Dark sector can be multicomponent
- Two-component Coy DM: A DM explanation to the Galactic Centre excess
- Results of phase space level analysis
- Summary

Multicomponent Dark Sectors

- Canonical Weakly Interacting Massive Particle (WIMP) Dark Matter (DM)



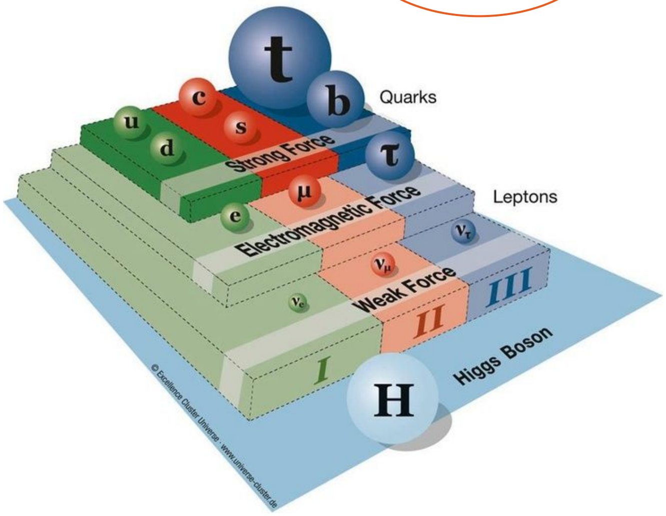
Multicomponent Dark Sectors

- Canonical Weakly Interacting Massive Particle (WIMP) Dark Matter (DM)
- The structure of SM as also well motivated beyond Standard Model theories suggestive of a multicopmonent dark sector
- Dark Matter (DM) being the lightest stable particle/s of an extended (dark) sector

Standard Model

portal interactions

Dark Matter



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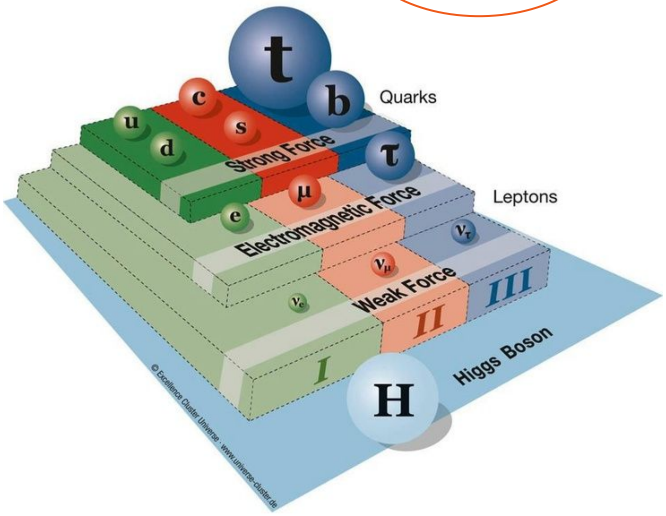
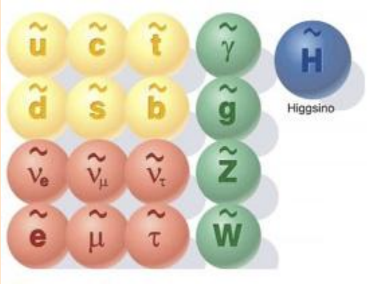


SUSY models,



Extra-dimensional models,

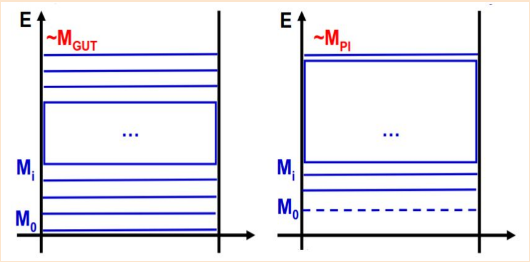


Figure from slides of Greg Landsberg

Mirror DM, Little Higgs DM…

Multicomponent Dark Sectors

- There can then exist many more processes (for change in number density as well as temperature)

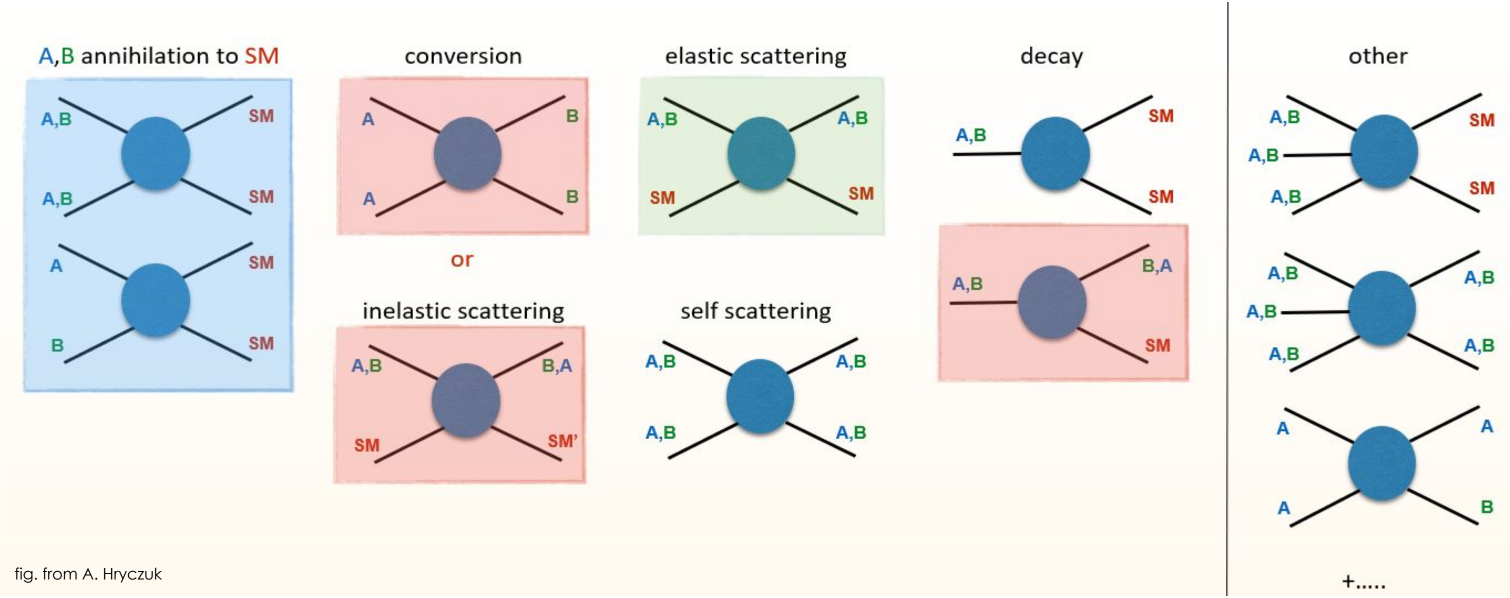


fig. from A. Hryczuk

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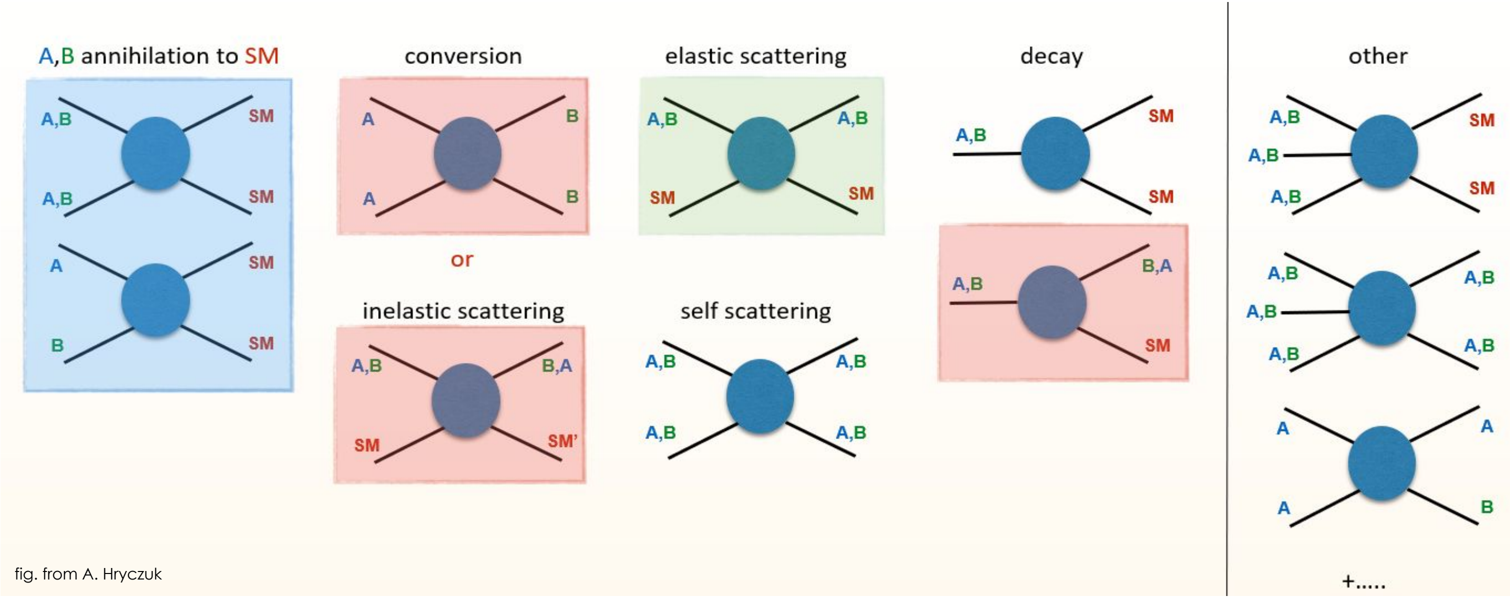

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- Can expect to generate non-thermal shapes of the phase space distributions of the dark sector particles

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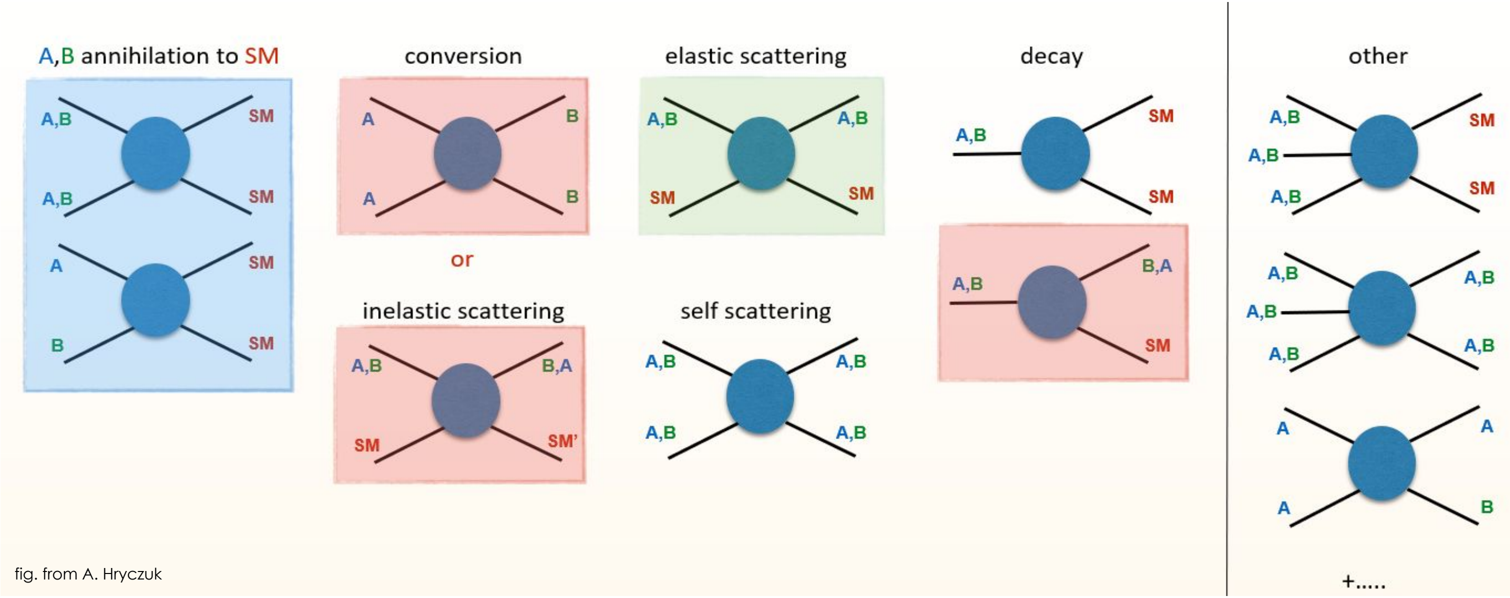

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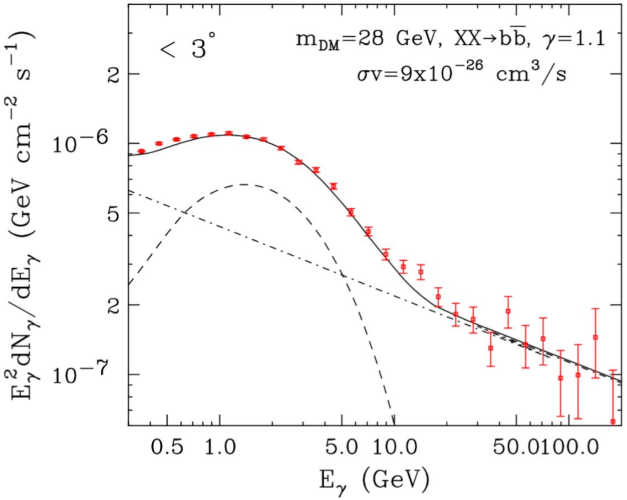
- Computationally more challenging (unless special circumstances allow for reduction of coupled equations)
- During chemical decoupling of DM, maintenance of kinetic equilibrium is not guaranteed
- Can expect to generate non-thermal shapes of the phase space distributions of the dark sector particles
- We carry out a closer study by way of example with a DM model of *phenomenological interest* with suppressed elastic scatterings by construction: the **Coy DM** to explain the observed Galactic Centre Excess

Galactic Centre Excess: Coy Dark Matter

- Fermi-LAT observes an excess in the spatially extended γ-rays from the Galactic Centre with a spectrum that peaks at a few GeV. Leading explanations:
  - DM annihilation
  - Millisecond Pulsar (MSP)

Fit to Galactic Centre Excess (GCE) from DM annihilation:



Goodenough, Hooper arXiv:0910.2998

Rebecca K. Leane et al., arXiv:2203.06859

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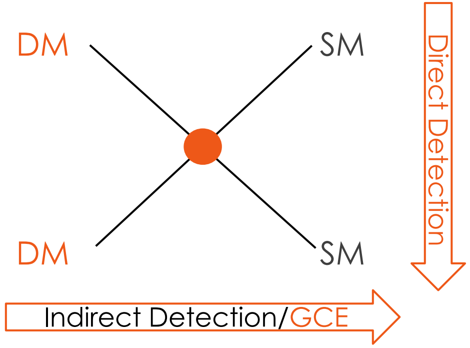

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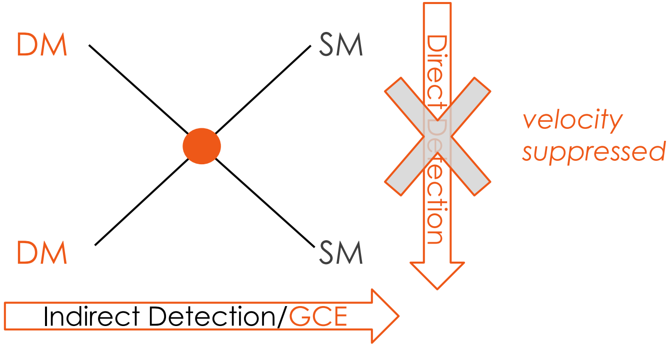

- Coy DM: fermionic DM with pseudoscalar mediator and coupling with SM proportional to Yukawa couplings of the SM fermions (Minimal Flavor Violation)

$$\mathcal{L} \supset -i\lambda_\chi s\,\bar{\chi}\gamma^5\chi - i\lambda_y \sum_{f\in\mathcal{SM}} y_f s\,\bar{f}\gamma^5 f$$

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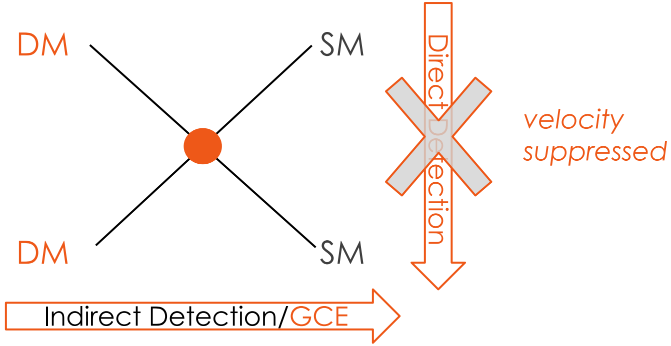

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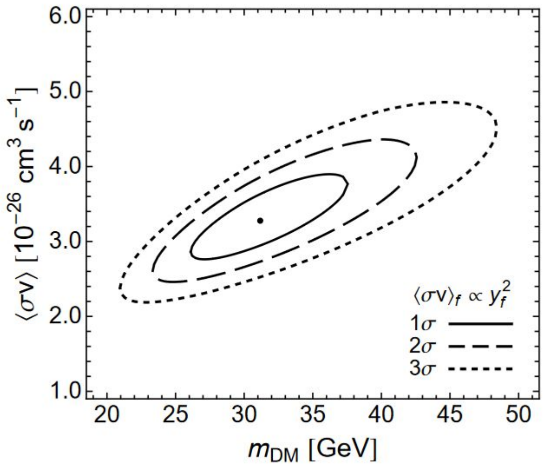

Boehm et al 2014

- Coy DM: fermionic DM (χ) with pseudoscalar mediator (s) and coupling with SM proportional to Yukawa couplings of the SM fermions (Minimal Flavor Violation)

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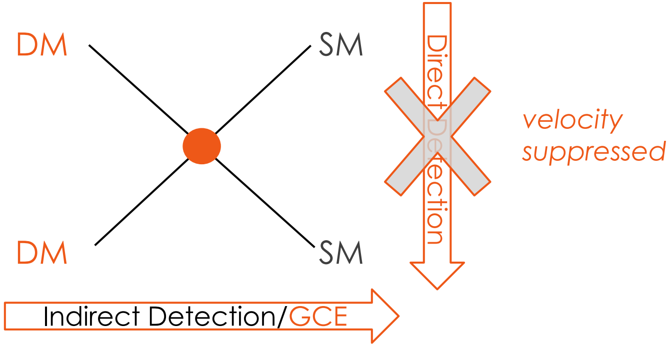

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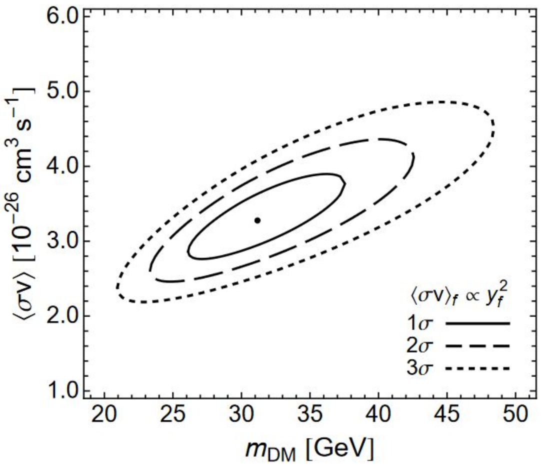
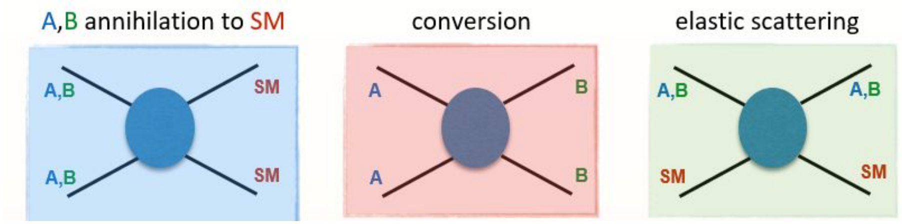


Boehm et al 2014

- Minimally extended coy DM: Two fermions (χ_1, χ_2) with pseudoscalar mediator (s) and coupling with SM proportional to Yukawa couplings of the SM fermions (Minimal Flavor Violation)

$$\mathcal{L} \supset -i\lambda_{\chi_1} s\,\bar{\chi}_1\gamma^5\chi_1 - i\lambda_{\chi_2} s\,\bar{\chi}_2\gamma^5\chi_2 - i\lambda_y \sum_{f\in SM} y_f s\,\bar{f}\gamma^5 f$$

Results: Double Coy Dark Matter



We develop a code to solve for this multicomponent DM at phase space level: extending the publicly available code DRAKE

$$(\partial_t - p_i H \partial_{p_i}) f_i(p_i,t) = \underbrace{\hat{C}_{\chi_i,SM\to\chi_i,SM}(p_i,t)}_{Elastic\ scattering} + \underbrace{\hat{C}_{\chi_i,\chi_i\to SM,SM}(p_i,t)}_{Annihilations} + \underbrace{\sum_{i\neq j}\hat{C}_{\chi_i,\chi_i\to\chi_j,\chi_j}(p_i,t)}_{Conversions}$$

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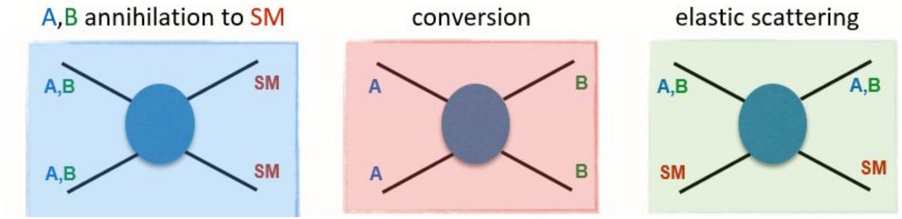

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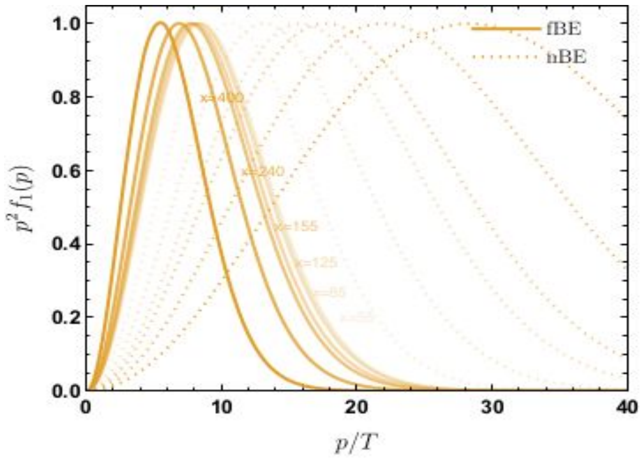

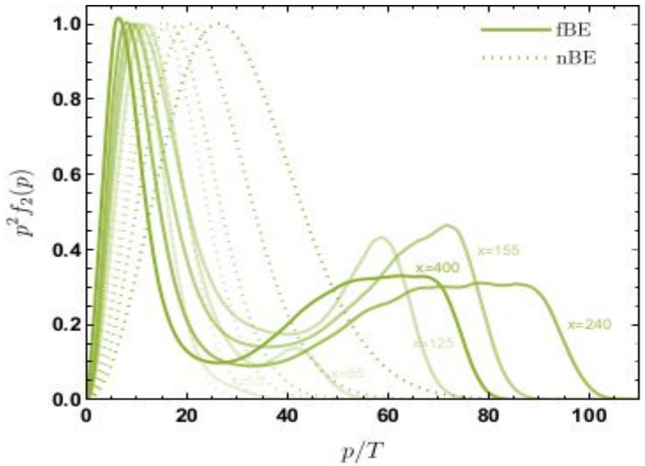

$$M_{\chi_1} = 44\,\mathrm{GeV},\ M_{\chi_2} = 38\,\mathrm{GeV}$$
$$M_s = 80\,\mathrm{GeV}$$
$$\lambda_{\chi_1} = 0.023,\ \lambda_{\chi_2} = 0.39,\ \lambda_y = 0.3$$

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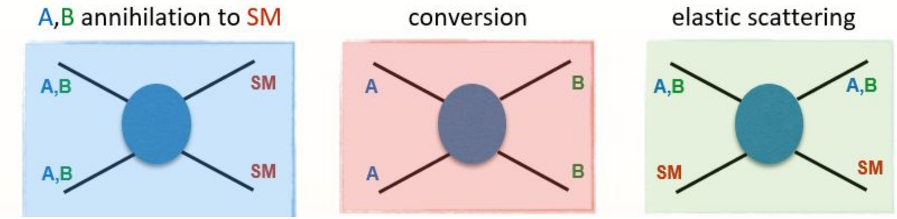

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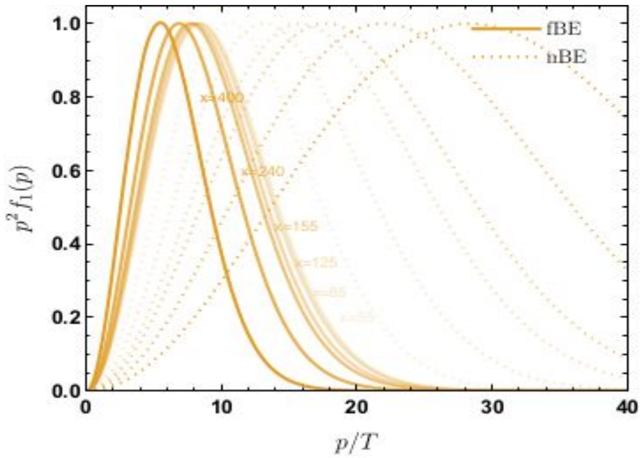

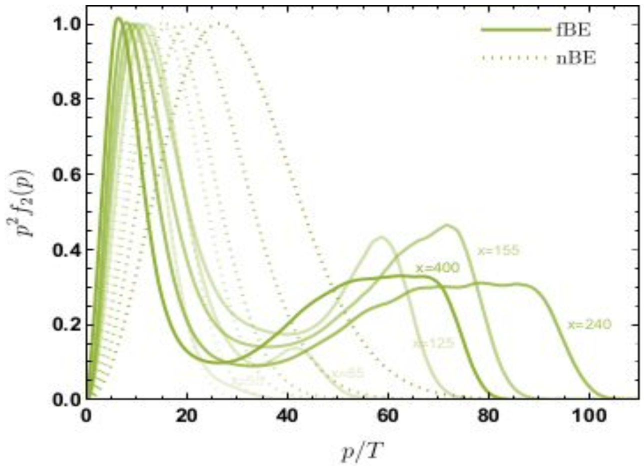

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- Conversions $\chi_1 \to \chi_2$ with momentum ≥ 22 GeV

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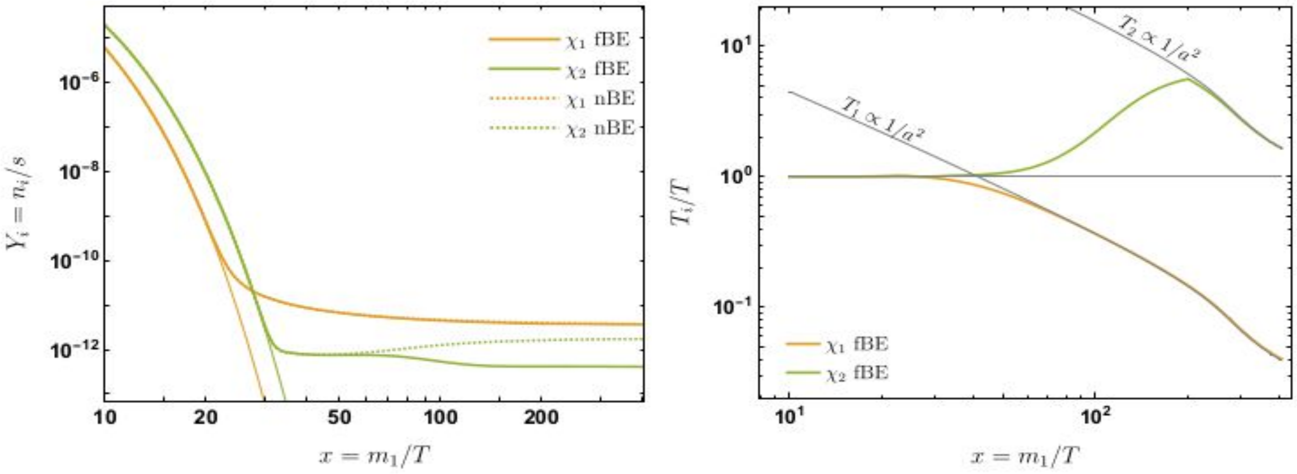

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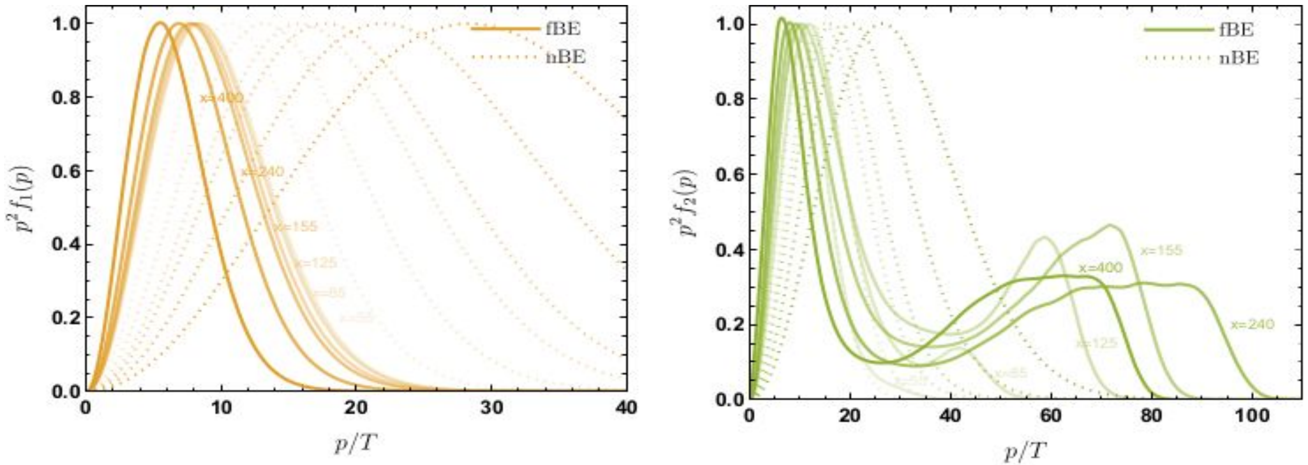

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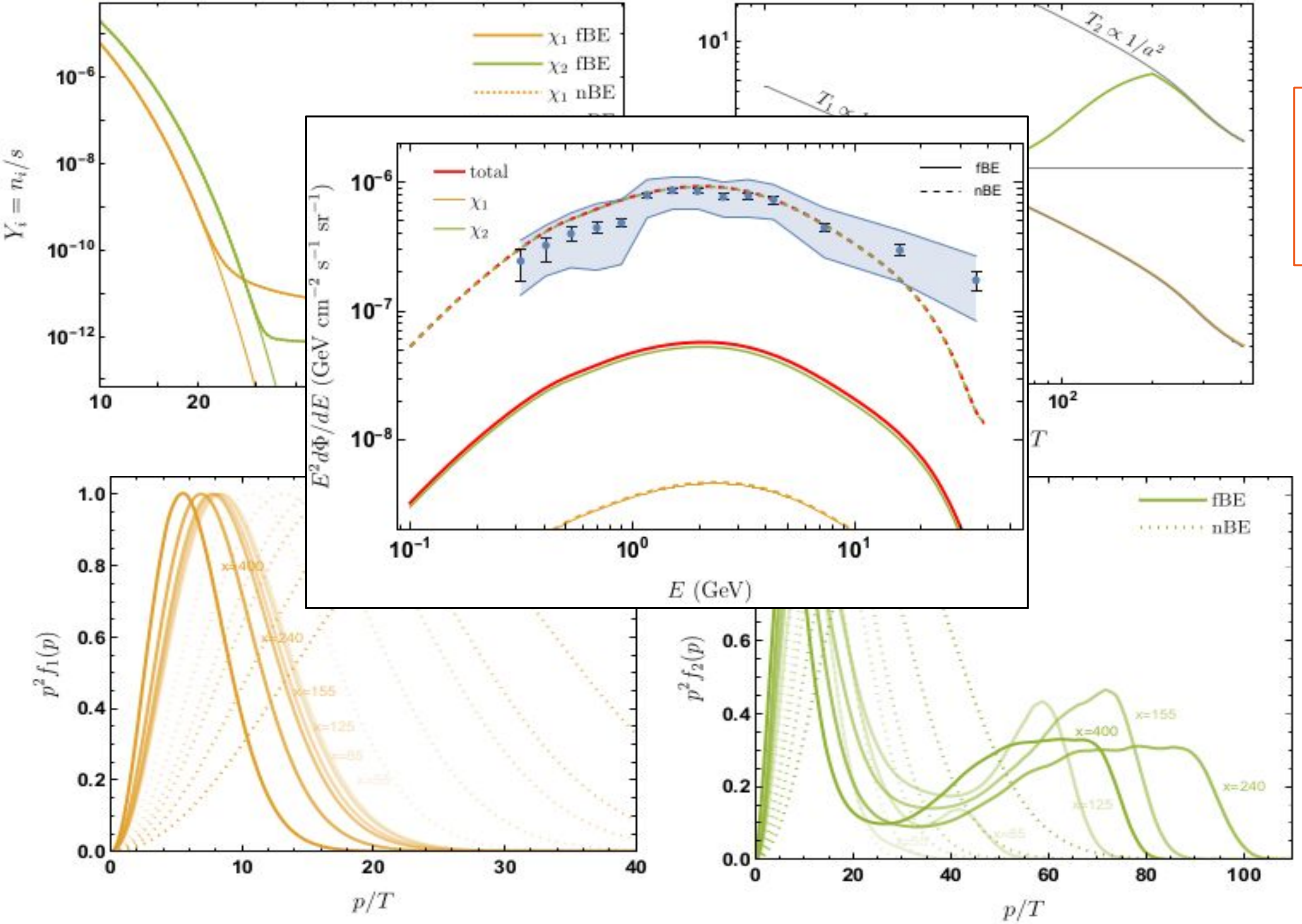

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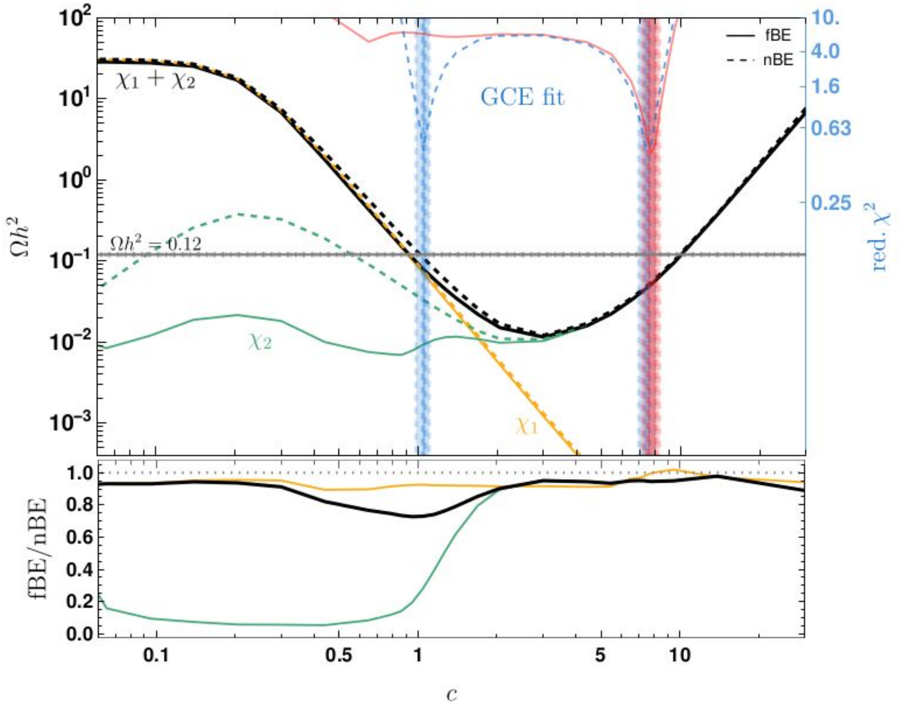
- Changing conversion strength ‘c’ keeping annihilation strength constant

$$\lambda_y \to \lambda_y/c,\ \ \lambda_{\chi_1} \to \lambda_{\chi_1}c,\ \ \lambda_{\chi_2} \to \lambda_{\chi_2}c \quad \Longrightarrow \quad \begin{array}{c}\sigma_{\chi_i,\chi_i\leftrightarrow \mathrm{SM,SM}} \propto \lambda_y^2\lambda_{\chi_i}^2 \propto \text{constant} \\ \sigma_{\chi_1,\chi_1\leftrightarrow\chi_2\chi_2} \propto \lambda_{\chi_1}^2\lambda_{\chi_2}^2 \propto c^4\end{array}$$



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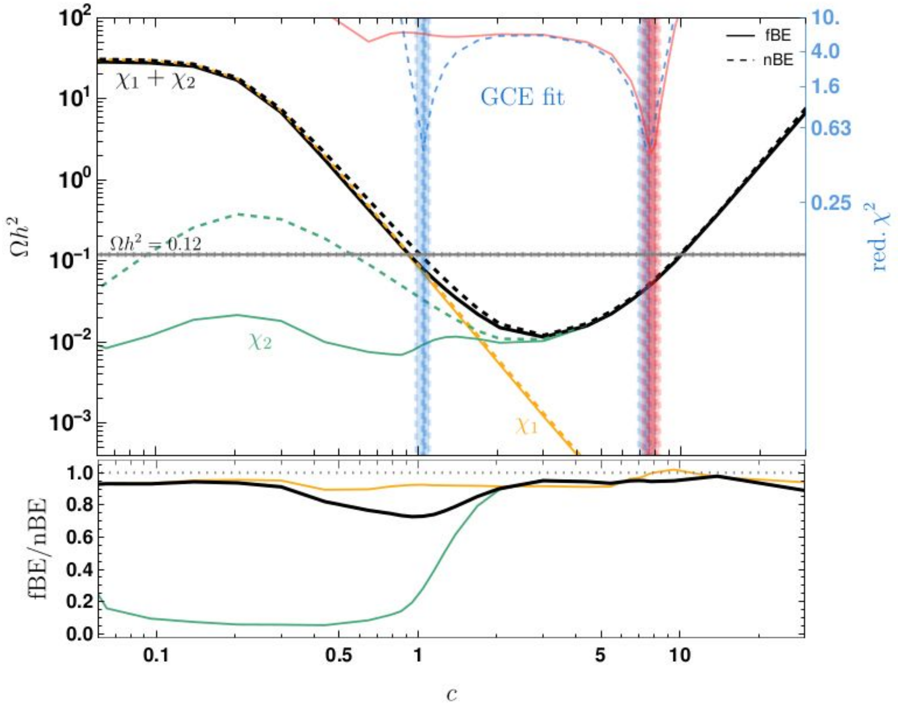

Modification of the abundance of *subdominant component* completely changes the preferred region for the GCE fit

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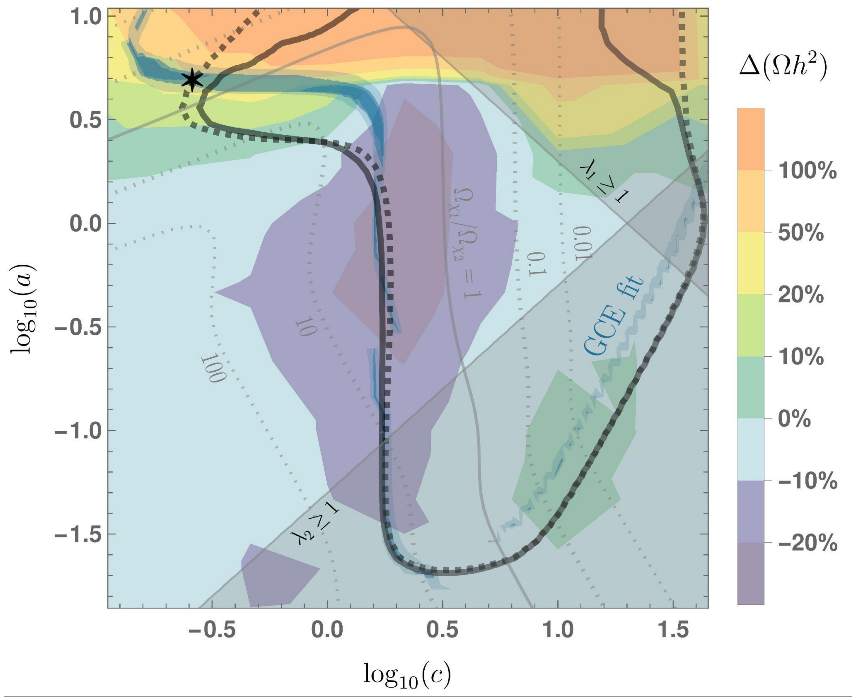

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$$\frac{(\Omega h^2)_{fBE} - (\Omega h^2)_{nBE}}{(\Omega h^2)_{nBE}} \times 100$$ is valued -20% to 100%

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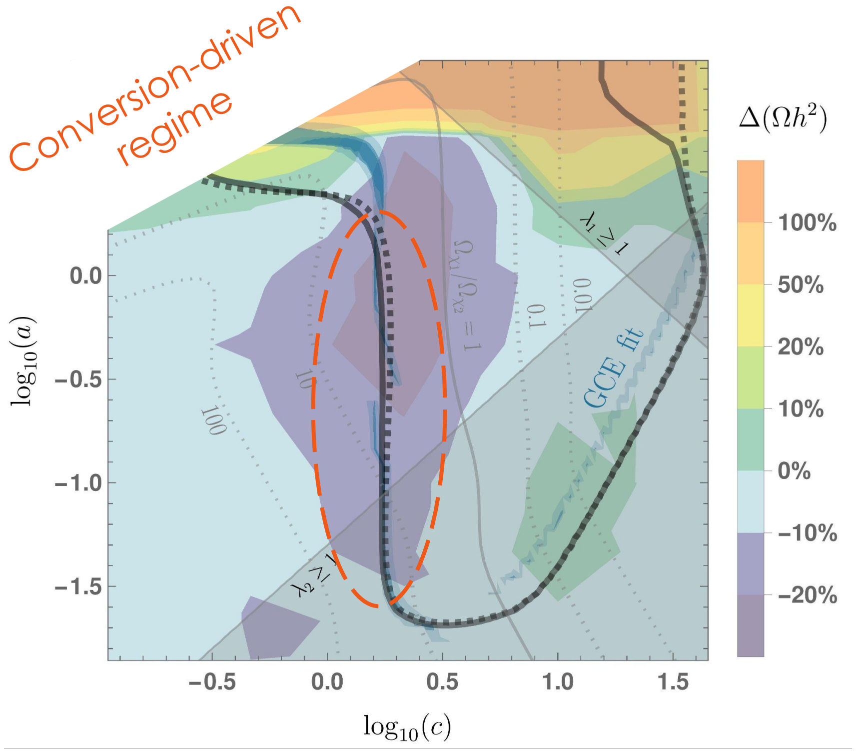

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Summary

- The sector containing DM can *in general* be richly populated with multiple particles.

- The kinetic equilibrium of DM with SM cannot be guaranteed requiring a solution of the full Boltzmann equation (fBE) at the phase-space level for a precise determination of the relic abundance.

- We study the double coy DM as an example, to quantify the effect the conversion processes on the departure from equilibrium and the evolution of phase space densities, finding:
  - Departure from kinetic equilibrium causes deviation in total relic abundance in the range from around -20% to 50% in most of the interesting parameter space
  - Much larger effect for the DM constituents *separately*: O(few)
  - Subdominant component can affect the present-day γ-ray flux in a significant way: completely changing the preferred region for the GCE fit
  - Extension of Coy DM allows for richer phenomenology: conversion-driven GCE explanation

- We develop a numerical framework of including conversions in a generic two-component DM model at phase space level to be included in next public release of Mathematica based code DRAKE (Dark matter Relic Abundance out-of-Kinetic Equilibrium)

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Thank you!

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Back-up slides

Boltzmann equation at the phase space level

Solving the DM distribution function at the full phase space level:

$$\partial_t f_{DM} - Hp\partial_p f_{DM} = C_{el}[f_{DM}] + C_{ann}[f_{DM}] \qquad \text{where, } f_{DM} \equiv f_{DM}(p,T).$$

CAN proceed fully numerically but it is time and CPU costly, due to the multidimensional integrations in the collision operators:

$$C_{el}[f_{DM}] = \int d\Pi \, |M|^2_{DM,SM\to DM,SM} \left(\underbrace{f_{DM}(p_1)f_{eq}(p_3)}_{\text{easier}} - \underbrace{\boldsymbol{f_{DM}(p_2)}f_{eq}(p_4)}_{\text{harder}}\right)$$

$$C_{ann}[f_{DM}] = \int d\Pi \, |M|^2_{DM,DM\to SM,SM} \left(\underbrace{f_{DM}(p_1)\boldsymbol{f_{DM}(p_2)}}_{\text{harder}} - \underbrace{f_{eq}(p_3)f_{eq}(p_4)}_{\text{easier}}\right)$$

Typically the average momentum transferred during the scattering events is small

$$\delta^{(3)}(\vec{p_3} + \vec{p_4} - \vec{p_1} - \vec{p_2}) \approx \sum_n \left(\frac{1}{n!}\left(\vec{q}.\vec{\nabla}_{p_3}\right)^n \delta^{(3)}(\vec{p_3} - \vec{p_1})\right)$$

$$C_{el}[f_{DM}] = C_2 + C_4 + C_6 + C_8 + \cdots$$

$$C_{el}[f_{DM}] \simeq C_{FP} = \frac{1}{2E_1}\gamma(f_{eq})\,\widehat{FP}(p_1).\boldsymbol{f_{DM}(p_1)}$$

no integration on f_{DM}

$$\frac{\Delta\vec{p}}{\vec{p}} < 1, \frac{p_1}{E_1} < 1$$

Fokker Planck approximation

Bringmann, Hoffman '06
Gondolo, Hisano, Kadota '12
Binder, Bringmann, Gustafsson, Hryczuk '17, '21

----DRAKE: publicly available code for solving at this full phase space level

$$d\Pi \equiv d\pi_{p_2}\,d\pi_{p_3}\,d\pi_{p_4}\;\delta^{(4)}(p_1 + p_3 - p_2 - p_4)$$

The Fokker Planck approximation

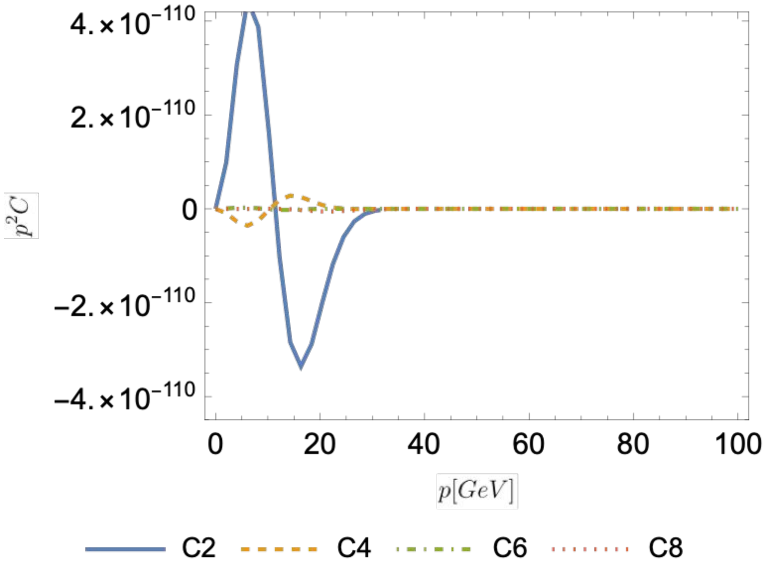
$$C_{el}[f_{DM}] = \underbrace{C_2 + C_4}_{} + C_6 + C_8 + \cdots$$

$$C_{FP} = \frac{1}{2E_1}\gamma(f_{eq})\,\widehat{FP}(p_1).\boldsymbol{f_{DM}(p_1)}$$

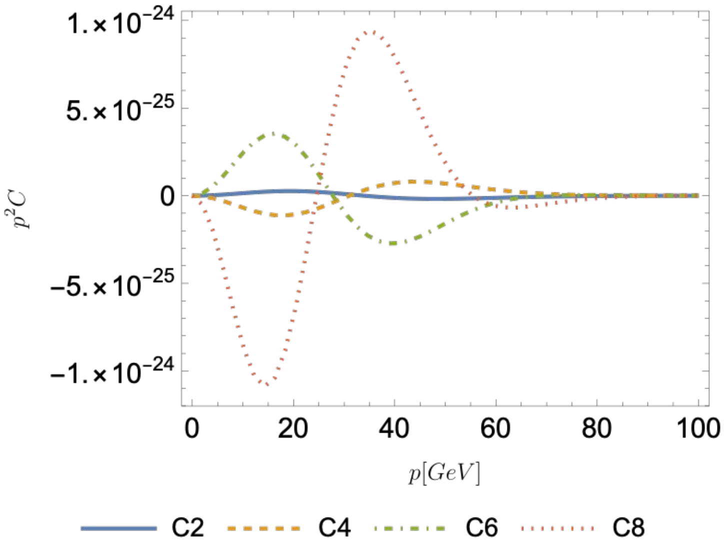
Has all the nice features:
- ✓ no integration on f_{DM}
- ✓ number conserving
- ✓ 0 on equilibrium distribution

$$x \equiv \frac{m_{DM}}{T} = \frac{m_\chi}{T}$$

$$\frac{\Delta\vec{p}}{p} < 1, \frac{p_1}{E_1} < 1$$



$m_\chi = 100$ GeV, $m = 1$ GeV, $x = 250$



$m_\chi = 100$ GeV, $m = 100$ GeV, $x = 25$

When does the Fokker Planck approx. work?

- Arrived at by *dropping higher order terms* in $\Delta\vec{p}/\vec{p}$ and p_1/E_1.
- Very good "approximation" (O(1%)) while the conditions of the expansion hold true.

Q: How to know when the FP approximation works?

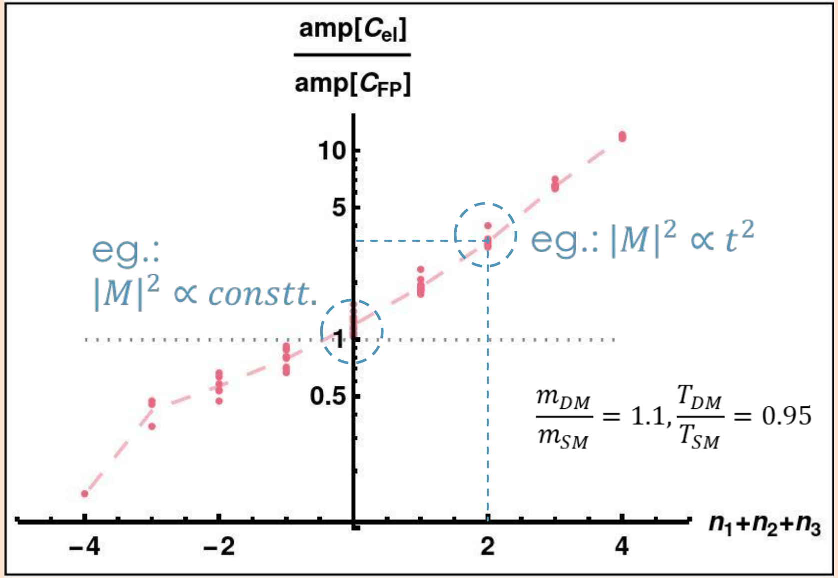
$$|M|^2 \longrightarrow \underbrace{t^{n_1}}_{\propto \text{ transfer momentum}} \underbrace{(s-(m_{DM}+m_{SM})^2)^{n_2}}_{\propto \text{ relative velocity}} \underbrace{(u-(m_{DM}-m_{SM})^2)^{n_3}}_{\propto \text{ velocities}}$$

With an efficiently implemented fully numerical[1] solver for the Boltzmann equation into DRAKE, we find that The Fokker Planck approximation works well for:

1. Scattering particle with masses significantly smaller than DM mass (small reduced mass ⇒ small momentum transfer

   &
2. DM temperatures close to the SM temperature (eg.: near kinetic decoupling)

   &
3. Scattering amplitudes that aren't strongly dependent on momentum transfer (the dropped higher order terms are more relevant for an amplitude sensitive to said dropped quantity)



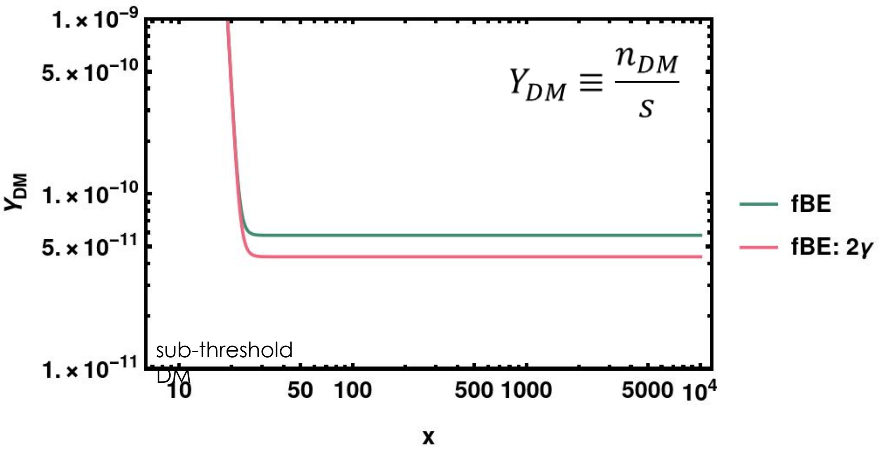
[1] Ala-Mattinen, Kainulainen '19
Hryczuk, Laletin '20
Aboubrahim, Klasen, Wiggering '23
Beauchesne, Chiang '24;

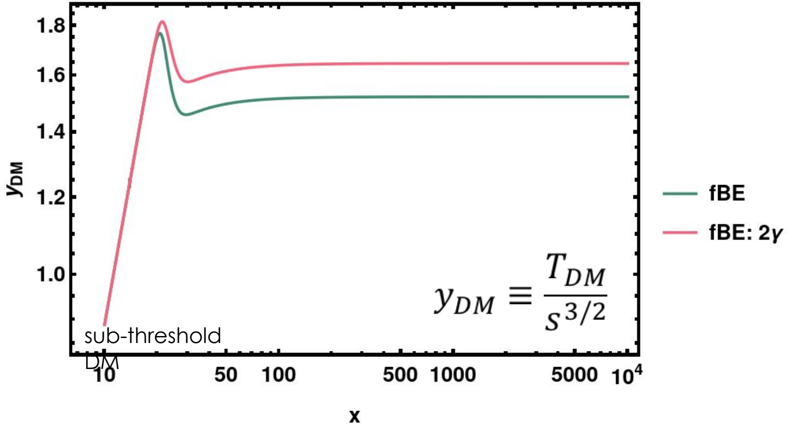
Improvement on Fokker Planck: Relic density

$$\partial_t f_{DM} - Hp\partial_p f_{DM} = C_{el}[f_{DM}] + C_{ann}[f_{DM}]$$

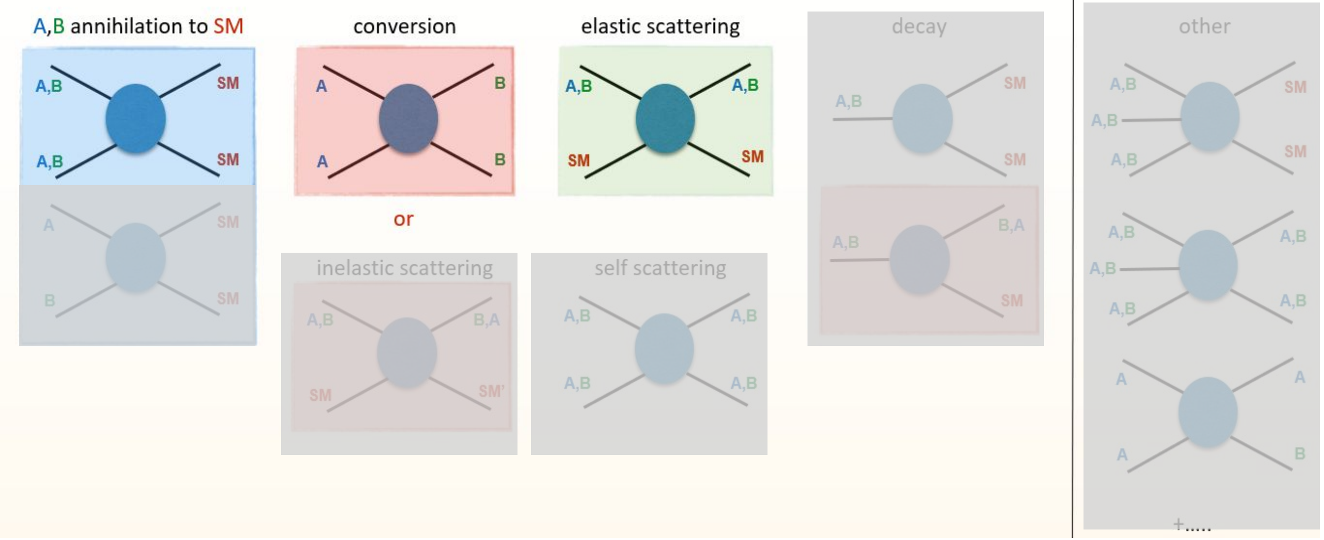
$$C_{el}[f_{DM}] \simeq C_{FP} = \frac{1}{2E_1}\gamma(f_{eq})\,\widehat{FP}(p_1).\boldsymbol{f_{DM}(p_1)}$$

An overall factor 2 at the level of collision operator ⟹ 25% change in DM relic density





Coy Dark Matter: 2-component



- Code to solve at Yield level: micrOMEGAs 6.0: N-component DM
- We develop a code to solve for this multicomponent DM at phase space level: extending the publicly available code DRAKE

$$(\partial_t - p_i H \partial_{p_i}) f_i(p_i,t) = \underbrace{\hat{C}_{\chi_i,SM\rightarrow\chi_i,SM}(p_i,t)}_{Elastic\ scattering} + \underbrace{\hat{C}_{\chi_i\chi_i\rightarrow SM,SM}(p_i,t)}_{Annihilations} + \underbrace{\sum_{i\neq j}\hat{C}_{\chi_i\chi_i\rightarrow\chi_j\chi_j}(p_i,t)}_{Conversions}$$

Collision operators:

$$C_{el}[f_{DM}] = \int d\Pi\, |M|^2_{DM,SM\rightarrow DM,SM} \left(f_{DM;A,B}(p_1) f_{eq}(p_3) - f_{DM;A,B}(p_2) f_{eq}(p_4)\right)$$

$$C_{ann}[f_{DM}] = \int d\Pi\, |M|^2_{DM,DM\rightarrow SM,SM} \left(f_{DM;A,B}(p_1) f_{DM;A,B}(p_2) - f_{eq}(p_3) f_{eq}(p_4)\right)$$

$$C_{conv}[f_{DM}] = \int d\Pi\, |M|^2_{A,A\rightarrow B,B} \left(f_{DM,A}(p_1) f_{DM,A}(p_2) - f_{DM,B}(p_3) f_{DM,B}(p_4)\right)$$

$$\mathcal{L} \supset -i\lambda_1 a\, \bar{\chi}_1\gamma^5\chi_1 - i\lambda_2 a\, \bar{\chi}_2\gamma^5\chi_2 - i\lambda_y \sum_{f\in SM} y_f\, a\, \bar{f}\gamma^5 f$$