Eccentric Inspiral-Merger-Ringdown Models for Binary Black Holes with Gauge-invariant Eccentricity

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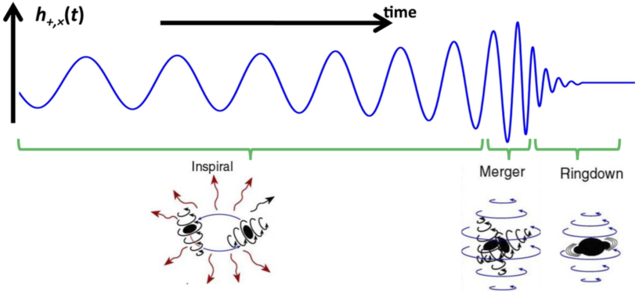
Why Eccentricity?

- Binary evolution phases : **Inspiral** (perturbative methods), **Merger** (numerical relativity) and **Ringdown** (black hole perturbation theory)



- Residual eccentricity can be a unique tool to identify binaries formed in dynamical environments – Lack of accurate eccentric models at present.

Gauge-invariant eccentricity

$$e_{\rm gw} = \cos(\psi/3) - \sqrt{3}\sin(\psi/3)\,,$$

$$\psi = \arctan\left(\frac{1 - e_{\omega_{22}}^2}{2e_{\omega_{22}}}\right)\,,$$

$$e_{\omega_{22}} = \frac{\sqrt{\omega_{22}^{\rm p}} - \sqrt{\omega_{22}^{\rm a}}}{\sqrt{\omega_{22}^{\rm p}} + \sqrt{\omega_{22}^{\rm a}}}\,,$$

- Eccentricity definition based on waveform quantities, not on orbital elements.
- Reduces to Newtonian definition of eccentricity at 0PN order.

PN-NR Comparison and Hybrids

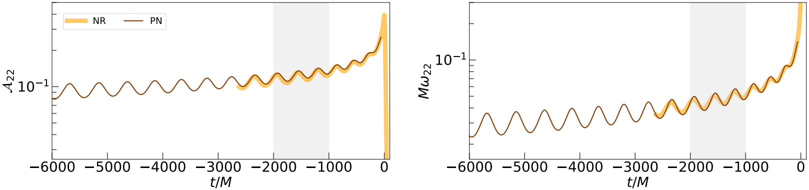


Figure: PN-NR amplitude and frequency comparison for $\ell = 2, m = 2$ spherical harmonic mode.

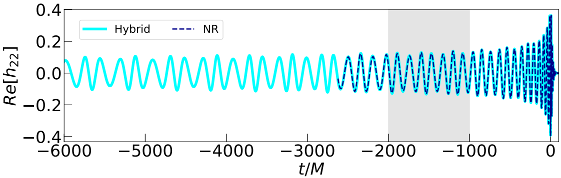


Figure: PN-NR eccentric hybrid waveform containing NR data.

Dominant mode model

- Hybrids are used as targets for calibration purposes.

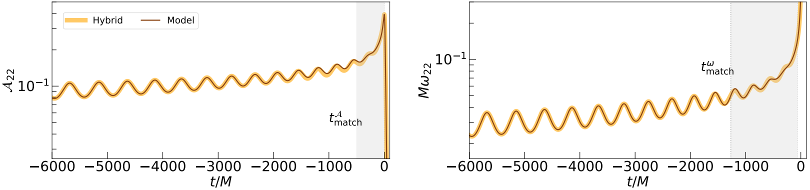


Figure: A dominant ($\ell = 2, m = 2$) mode model reconstructed by matching an eccentric inspiral (EccentricTD) with a quasi-circular waveform (SEOBNRv4) for merger-ringdown phase. For comparison, the target hybrid is also shown here.

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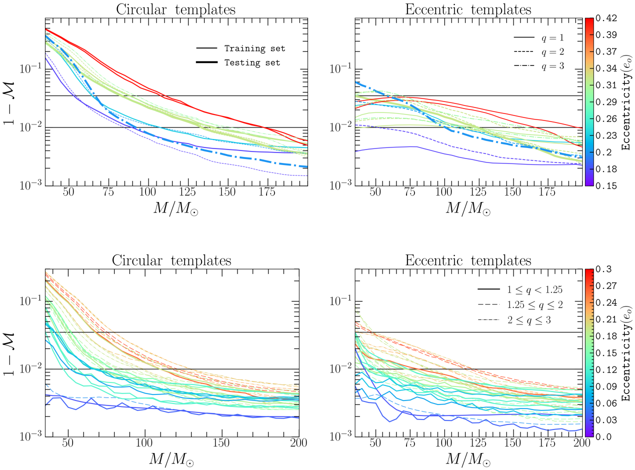
$$\mathcal{A}_{22}^{\rm model}(t) \;\equiv\; \tau_a(t)\,\mathcal{A}_{22}^{\rm IMR}(t) \;+\; (1-\tau_a(t))\,\mathcal{A}_{22}^{\rm inspiral}(t)$$

Mismatch plots



- 1st row: Mismatch against hybrids. 2nd row: Mismatch against TEOBResumS-Dali.

Conclusions

- We constructed a set of 20 long eccentric hybrids (including dominant and higher modes) containing the accurate numerical relativity data.
- We developed a dominant mode model which performs better than state-of-the-art quasicircular templates in capturing eccentricities in the range $0 \leq e_0 \leq 0.3$.

References:
1) A. Chattaraj, T. RoyChowdhury, Divyajyoti, C. K. Mishra, and A. Gupta. 2022, Phys. Rev. D, 106:124008.
2) P. Manna, T. RoyChowdhury, and C. K. Mishra. An improved IMR model for BBHs on elliptical orbits. 2024, arXiv: 2409.10672.

Conclusions

Thank you!

Credit: https://gigazine.net/gsc_news/

Backup Slides

$$\tau_a(t) \equiv \begin{cases} 0 & \text{if } t < t_i \\ \frac{t-t_i}{t_f-t_i} & \text{if } t_i \leq t < t_f \\ 1 & \text{if } t_f \leq t. \end{cases}$$

Count	Simulation ID	q	x_0	e_0	l_0	N_{orb}
	Training Set					
1	HYB:SXS:BBH:1355	1	0.0389	0.173	2.455	63.0
2	HYB:SXS:BBH:1356	1	0.0375	0.230	1.717	65.5
3	HYB:SXS:BBH:1358	1	0.0340	0.322	1.215	69.5
4	HYB:SXS:BBH:1359	1	0.0347	0.317	1.131	67.0
5	HYB:SXS:BBH:1360	1	0.0317	0.416	0.796	64.0
6	HYB:SXS:BBH:1361	1	0.0313	0.416	0.796	66.0
7	HYB:SXS:BBH:1364	2	0.0391	0.172	2.681	69.0
8	HYB:SXS:BBH:1365	2	0.0376	0.209	2.262	72.5
9	HYB:SXS:BBH:1366	2	0.0344	0.320	1.299	74.0
10	HYB:SXS:BBH:1367	2	0.0346	0.320	1.299	73.5
11	HYB:SXS:BBH:1368	2	0.0338	0.324	1.382	77.5
12	HYB:SXS:BBH:1372	3	0.0344	0.300	1.789	90.0
13	HYB:SXS:BBH:1373	3	0.0344	0.300	1.789	89.0
	Testing Set					
14	HYB:SXS:BBH:1357	1	0.0344	0.322	1.215	67.5
15	HYB:SXS:BBH:1362	1	0.0328	0.483	0.464	48.5
16	HYB:SXS:BBH:1363	1	0.0308	0.505	0.590	51.5
17	HYB:SXS:BBH:1369	2	0.0329	0.478	0.545	52.5
18	HYB:SXS:BBH:1370	2	0.0291	0.508	0.628	63.0
19	HYB:SXS:BBH:1371	3	0.0380	0.204	2.621	82.5
20	HYB:SXS:BBH:1374	3	0.0290	0.495	0.832	77.5