Infall through the Black Hole Horizon, Outflows from Naked Singularities

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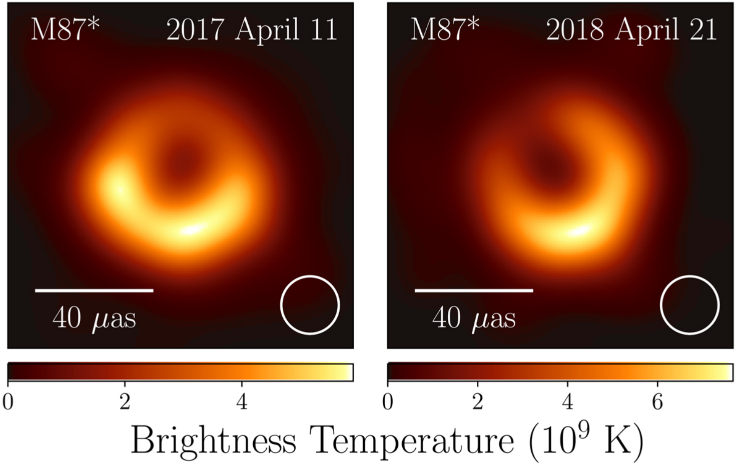
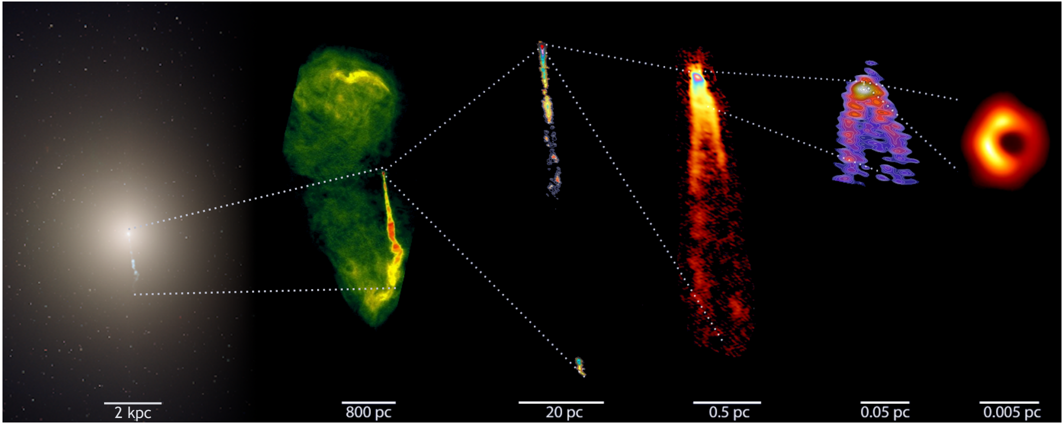
Shadow of a black hole



1. Event Horizon Telescope Collaboration *et al. Astronomy and Astrophysics* 681, A79 (Jan. 2024).

M87 central object



2. Gurvits, L. I. *et al. Acta Astronaut.* 196, 314. arXiv: 2204.09144 [astro-ph.IM] (2022).

No-hair theorem

No-hair theorem

Stationary black hole in General Relativity (coupled to Maxwell equations) can be completely characterized by:

- mass M,
- angular momentum a
- and its electric charge Q.

Reissner–Nordström metric

Reissner–Nordström metric can be expressed in Boyer-Lindquist as:

$$g = -f(r)dt \otimes dt + f^{\ 1}(r)dr \otimes dr + r^2(d\theta \otimes d\theta + \sin^2\theta d\phi \otimes d\phi)$$

where

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2}.$$

The RN metric:

- is spherically symmetric,
- describes gravitational field of charged and massive compact objects.

Horizons around charged objects

An event horizon should occur when $g_{tt} = 0$ which for the Reissner–Nordström metric takes the form

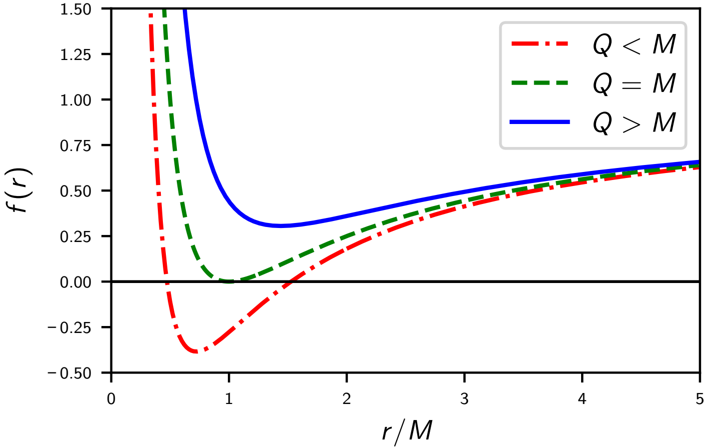
$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = 0$$

The solution of the above equation is given by,

$$r_H = M \pm \sqrt{M^2 - Q^2}.$$

When $Q < M$, we have a black hole solution with two coordinate singularities and two event horizons. For $Q = M$, we have the extreme black hole for which horizons coincide. When $Q > M$, there are no horizons at all so the metric describes a naked singularity.

Radial dependence of $-g_{tt}$



Trajectories of test particles

Due to symmetry of the RN metric with respect to rotations and time translations following quantities are conserved:

$$E \equiv -mu_t, \qquad L \equiv mu_\phi,$$

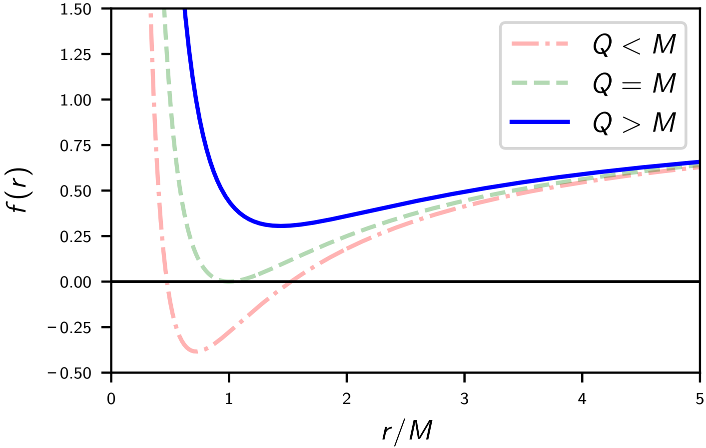
where u is the 4-velocity of the particle normalized to $u^2 = -1$.
Let us introduce

$$\ell := \frac{L}{E} = -\frac{u_\phi}{u_t}.$$

Trajectories of test particles solve the following equation of motion:

$$(u^r)^2 + V_{eff}(r) := (u^r)^2 + f(r)\left(1 + r^{-2}\frac{L^2}{m^2}\right) = \frac{E^2}{m^2}.$$

Repulsive gravity



Zero-gravity radius[3,4]

For $L = 0$ the effective potential V_{eff} takes the simple form:

$$V_{eff}(r) = f(r) = 1 - \frac{M}{r} + \frac{Q^2}{r^2}$$

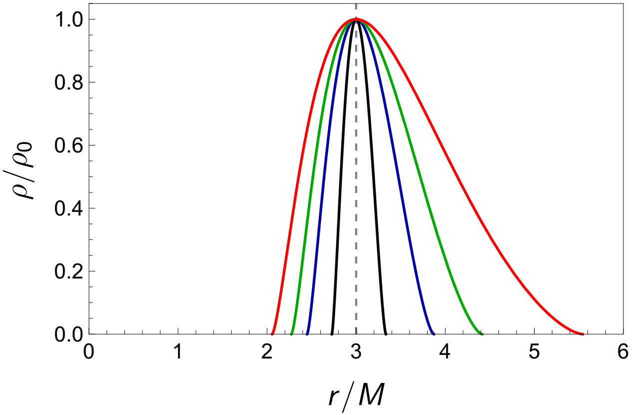
and has minimum at $r_0 := Q^2$, called zero-gravity radius.

- For $Q \leq M$, r_0 is below outer horizon, thus is unimportant for physics.
- For $Q > M$, there is no horizon and the zero-gravity sphere is visible for distant observers.
- According to the eom, a test particle can stay at rest ($u^r = 0$) at r_0.

3. Vieira, R. S. S. & Kluźniak, W. *Mon. Not. Roy. Astron. Soc.* 523, 4615–4623. arXiv: 2304.05932 [astro-ph.HE] (2023).

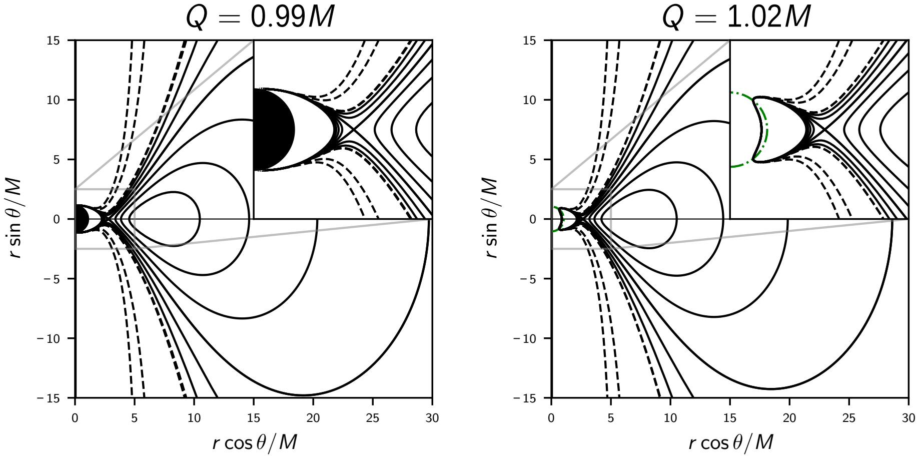
4. Semerák, O. *et al. Monthly Notices of the Royal Astronomical Society* 308, 691–704. ISSN: 0035-8711 (Sept. 1999).

Levitating atmosphere around naked singularity



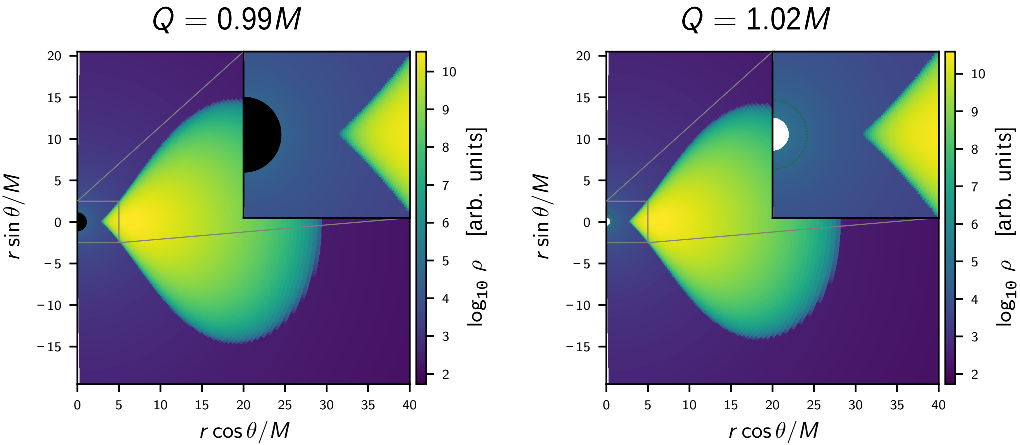
3. Vieira, R. S. S. & Kluźniak, W. *Mon. Not. Roy. Astron. Soc.* 523, 4615–4623. arXiv: 2304.05932 [astro-ph.HE] (2023).

Equilibrium tori



5. Mishra, R. *et al. Phys. Rev. D* 110, 124030 (2024).

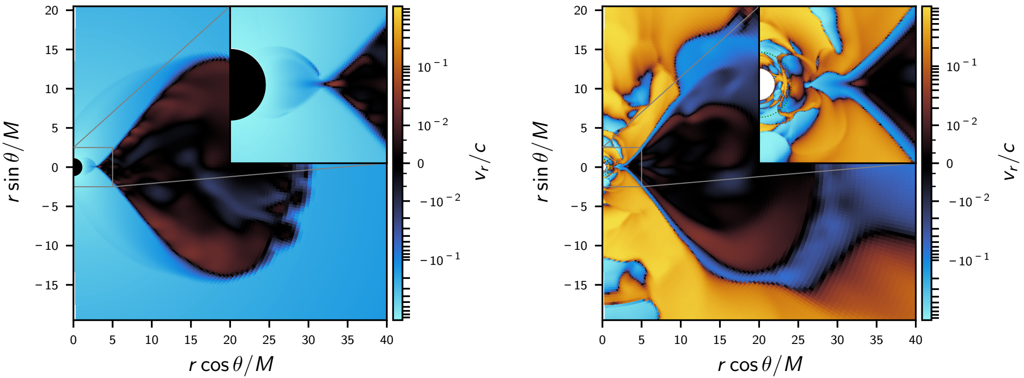
Initial setup



6. Kluźniak, W. & Krajewski, T. *Phys. Rev. Lett.* 133, 241401. arXiv: 2408.08359 [astro-ph.HE] (2024).

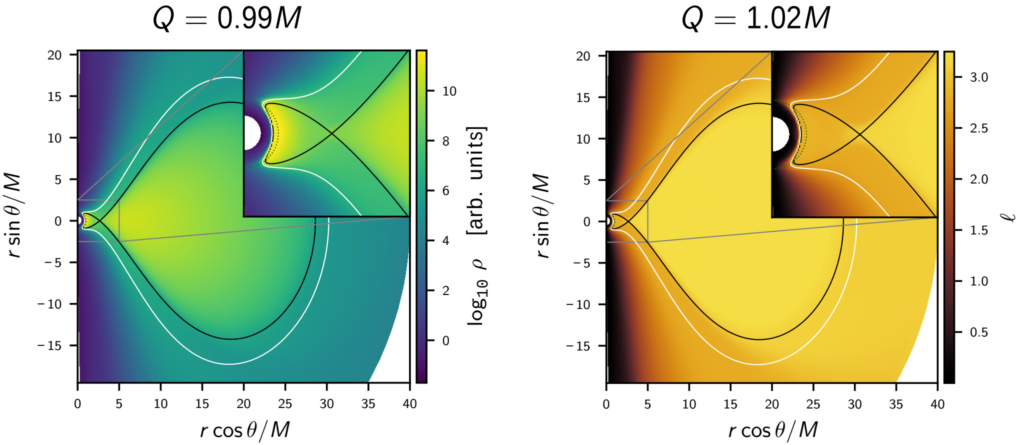
Black hole vs naked singularity

$Q = 0.99M$ $\qquad$ $Q = 1.02M$



6. Kluźniak, W. & Krajewski, T. *Phys. Rev. Lett.* 133, 241401. arXiv: 2408.08359 [astro-ph.HE] (2024).

Inner torus



$Q = 0.99M$
$Q = 1.02M$
$r \sin\theta/M$
$r \cos\theta/M$
$\log_{10}\rho$ [arb. units]
ℓ
20
15
10
5
0
−5
−10
−15
0
5
10
15
20
25
30
35
40
0
2
4
6
8
10
0.5
1.0
1.5
2.0
2.5
3.0

Summary

1. General relativity predicts that black holes can not only have mass and spin, but also electric charge.
2. When the charge or spin of the object is large enough the horizons disappear and the object is the naked singularity.
3. Recent hydrodynamical simulations have revealed that naked singularities, unlike black holes, are prone to produce strong outflows.
4. A possible application of these calculations is to the active galactic nuclei (AGNs) with their powerful jets and outflows.

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Thank you for your attention.

Orbits around charged objects

- Radius of last stable circular orbit (LSCO) in the Reissner–Nordström metric is a solution of the following equation:[7]

$$Mr^3 - 6M^2r^2 + 9MQ^2r - 4Q^4 = 0.$$

- Radius of marginally bound orbit (MBO) in the Reissner-Nordström metric is a solution of the following equation:[8]

$$Mr^3 - 4M^2r^2 + 4MQ^2r - Q^4 = 0.$$

7. Pugliese, D. *et al. Phys. Rev. D* 83, 024021. arXiv: 1012.5411 [astro-ph.HE] (2011).
8. Beheshti, S. & Gasperin, E. *Phys. Rev. D* 94, 024015. arXiv: 1512.08707 [gr-qc] (2016).

Stability of orbits