

Infall through the Black Hole Horizon, Outflows from Naked Singularities

Tomasz Krajewski Institute of Fundamental Technological Research Supported by NCN grants 2019/33/B/ST9/01564, 2023/51/B/ST9/00943.

Shadow of a black hole



1. Event Horizon Telescope Collaboration et al. Astronomy and Astrophysics 681, A79 (Jan. 2024).

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M87 central object



2. Gurvits, L. I. et al. Acta Astronaut. 196, 314. arXiv: 2204.09144 [astro-ph.IM] (2022).

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No-hair theorem

No-hair theorem

Stationary black hole in General Relativity (coupled to Maxwell equations) can be completely characterized by:

- mass *M*,
- angular momentum a
- and its electric charge Q.

Reissner–Nordström metric

Reissner-Nordström metric can be expressed in Boyer-Lindquist as:

$$g=-f(r)dt\otimes dt+f^{-1}(r)dr\otimes dr+r^2(d heta\otimes d heta+\sin^2 heta d\phi\otimes d\phi)$$

where

$$f(r)=1-\frac{2M}{r}+\frac{Q^2}{r^2}.$$

The RN metric:

- is spherically symmetric,
- describes gravitational field of charged and massive compact objects.

Horizons around charged objects

An event horizon should occur when $g_{tt} = 0$ which for the Reissner–Nordström metric takes the form

$$f(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} = 0$$

The solution of the above equation is given by,

$$r_{H\pm}=M\pm\sqrt{M^2-Q^2}$$

When Q < M, we have a black hole solution with two coordinate singularities and two event horizons. For Q = M, we have the extreme black hole for which horizons coincide. When Q > M, there are no horizons at all so the metric describes a naked singularity. Radial dependence of $-g_{tt}$



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Trajectories of test particles

Due to symmetry of the RN metric with respect to rotations and time translations following quantities are conserved:

$$E \equiv -mu_t, \qquad \qquad L \equiv mu_{\phi},$$

where u is the 4-velocity of the particle normalized to $u^2 = -1$. Let us introduce

$$\ell := rac{L}{E} = -rac{u_{\phi}}{u_t}.$$

Trajectories of test particles solve the following equation of motion:

$$(u^r)^2 + V_{eff}(r) := (u^r)^2 + f(r)\left(1 + r^{-2}\frac{L^2}{m^2}\right) = \frac{E^2}{m^2}.$$

Repulsive gravity



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Zero-gravity radius^{3,4}

For L = 0 the effective potential V_{eff} takes the simple form:

$$V_{eff}(r)=f(r)=1-rac{M}{r}+rac{Q^2}{r^2}$$

and has minimum at $r_0 := Q^2$, called zero-gravity radius.

- For $Q \leq M$, r_0 is below outer horizon, thus is unimportant for physics.
- For Q > M, there is no horizon and the zero-gravity sphere is visible for distant observers.
- According to the eom, a test particle can stay at rest $(u^r = 0)$ at r_0 .
- 3. Vieira, R. S. S. & Kluźniak, W. *Mon. Not. Roy. Astron. Soc.* **523**, 4615–4623. arXiv: 2304.05932 [astro-ph.HE] (2023).
- 4. Semerák, O. *et al. Monthly Notices of the Royal Astronomical Society* **308**, 691–704. ISSN: 0035-8711 (Sept. 1999).

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Levitating atmosphere around naked singularity



3. Vieira, R. S. S. & Kluźniak, W. *Mon. Not. Roy. Astron. Soc.* **523**, 4615–4623. arXiv: 2304.05932 [astro-ph.HE] (2023).

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5. Mishra, R. et al. Phys. Rev. D 110, 124030 (2024).

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Black hole vs naked singularity Q = 0.99M

Q = 1.02M



6. Kluźniak, W. & Krajewski, T. Phys. Rev. Lett. 133, 241401. arXiv: 2408.08359 [astro-ph.HE] (2024).

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Inner torus



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Summary

- 1. General relativity predicts that black holes can not only have mass and spin, but also electric charge.
- 2. When the charge or spin of the object is large enough the horizons disappear and the object is the naked singularity.
- 3. Recent hydrodynamical simulations have revealed that naked singularities, unlike black holes, are prone to produce strong outflows.
- 4. A possible application of these calculations is to the active galactic nuclei (AGNs) with their powerful jets and outflows.

Summary

- 1. General relativity predicts that black holes can not only have mass and spin, but also electric charge.
- 2. When the charge or spin of the object is large enough the horizons disappear and the object is the naked singularity.
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Thank you for your attention.

Orbits around charged objects

• Radius of last stable circular orbit (LSCO) in the Reissner–Nordström metric is a solution of the following equation:⁷

$$Mr^3 - 6M^2r^2 + 9MQ^2r - 4Q^4 = 0.$$

 Radius of marginally bound orbit (MBO) in the Reissner-Nordström metric is a solution of the following equation:⁸

$$Mr^3 - 4M^2r^2 + 4MQ^2r - Q^4 = 0.$$

- 7. Pugliese, D. et al. Phys. Rev. D 83, 024021. arXiv: 1012.5411 [astro-ph.HE] (2011).
- 8. Beheshti, S. & Gasperin, E. Phys. Rev. D 94, 024015. arXiv: 1512.08707 [gr-qc] (2016).

Stability of orbits



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