Probing the hadron-quark phase transition in twin stars with f-modes

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Particle Astrophysics in Poland
February 21st, 2025

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CREDO
THE QUEST FOR THE UNEXPECTED

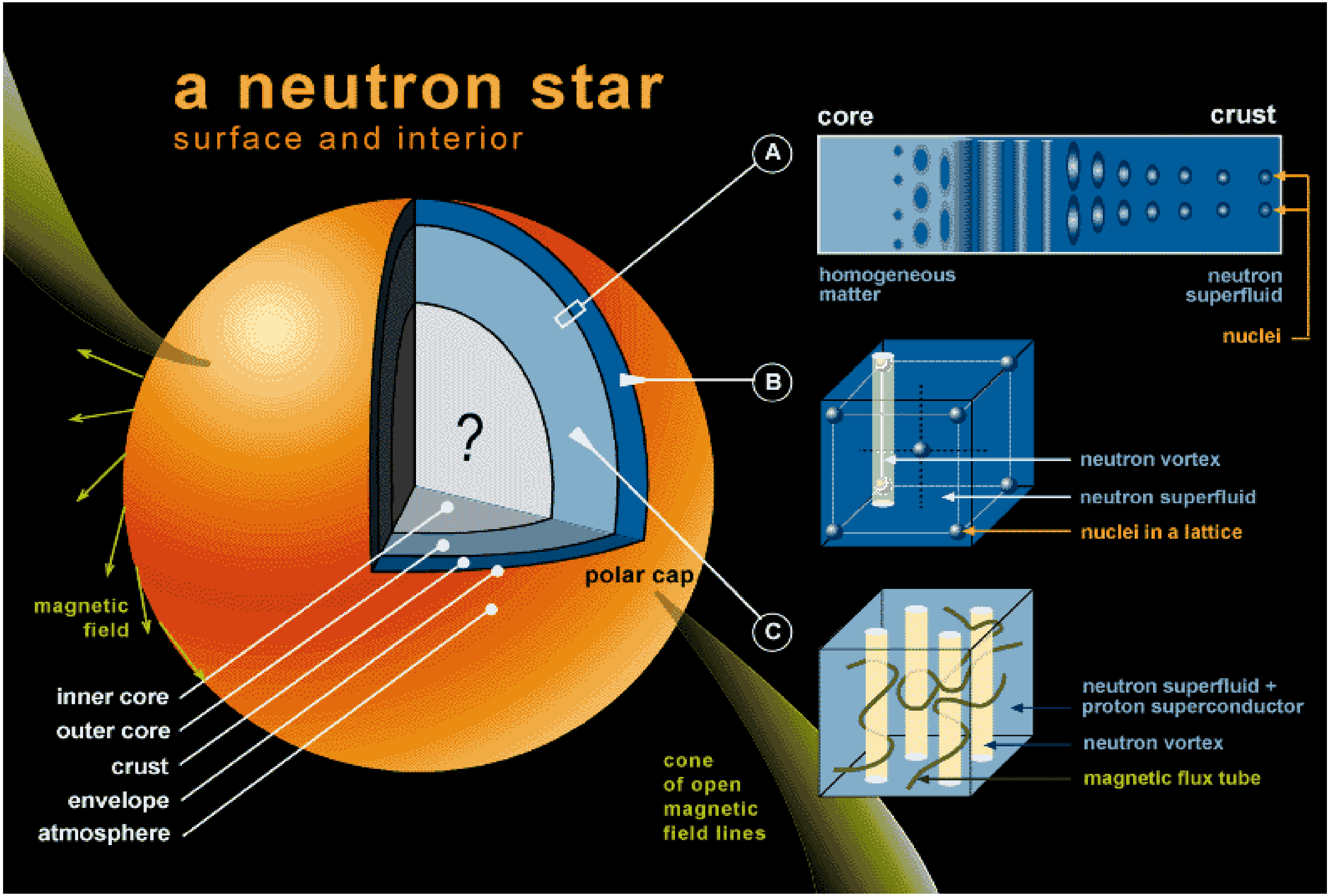
ET EINSTEIN TELESCOPE

compstar

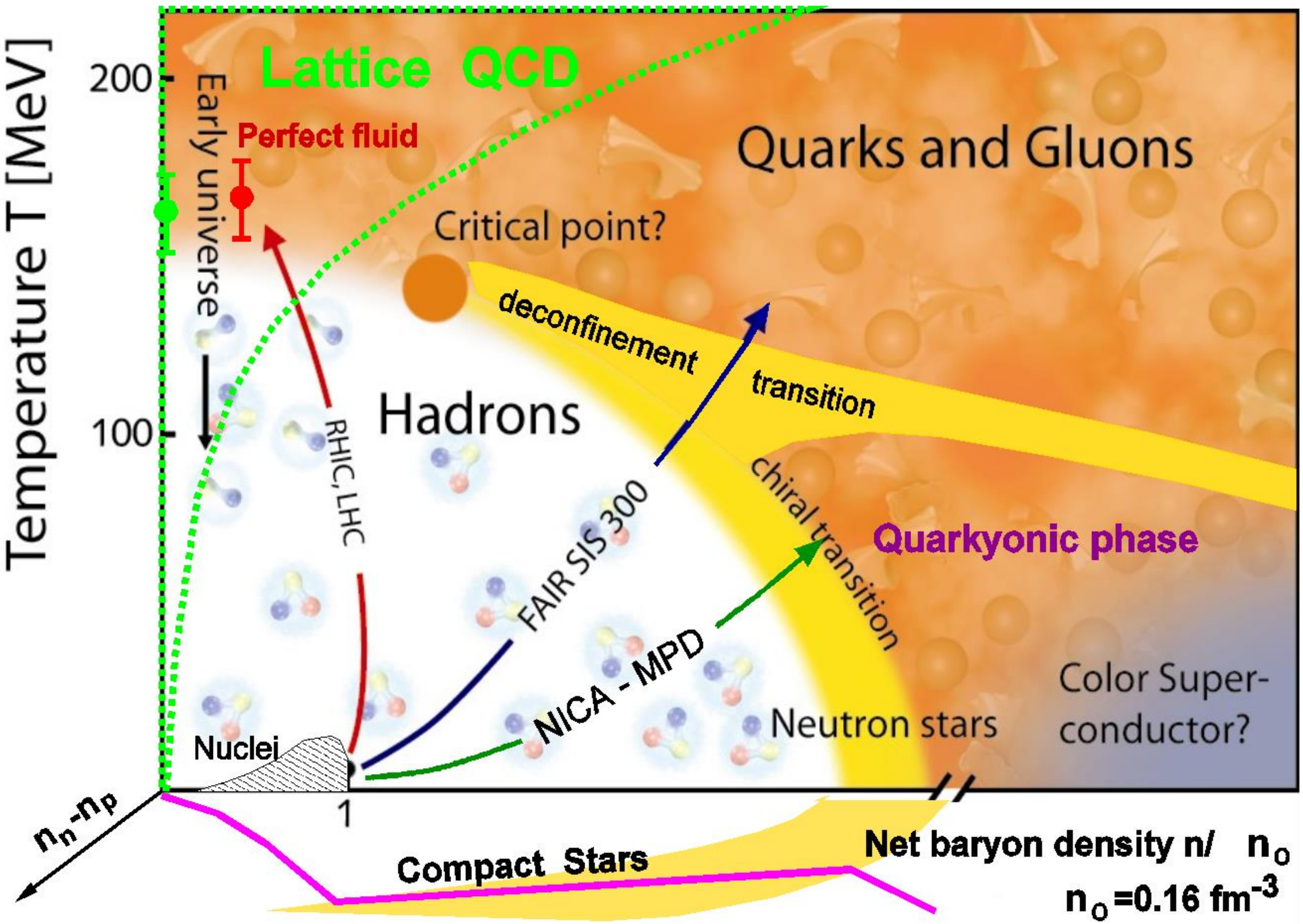
Outline

- A brief introduction to the physics of compact stars and their location in the QCD phase diagram.
- The compact star mass twins scenario.
- f-mode future detection with ET and implications for the EoS.
- Outlook and perspectives.

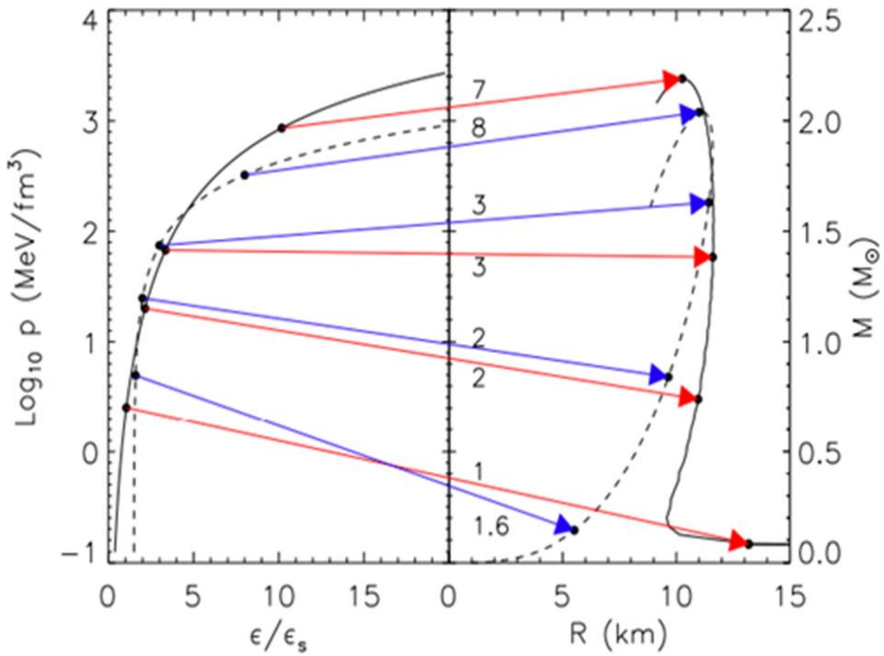
Superdense objects – what is inside?



Critical Endpoint in QCD



Compact Star Sequences
(M-R ⇔ EoS)



James Lattimer,
Annu. Rev. Nucl. Part. Sci. 62, 485 (2012), arXiv:1305.3510

- TOV Equations

$$\frac{dp}{dr} = -\frac{(\varepsilon + p/c^2)G(m + 4\pi r^3 p/c^2)}{r^2(1 - 2Gm/rc^2)}$$

- Equation of State (EoS)

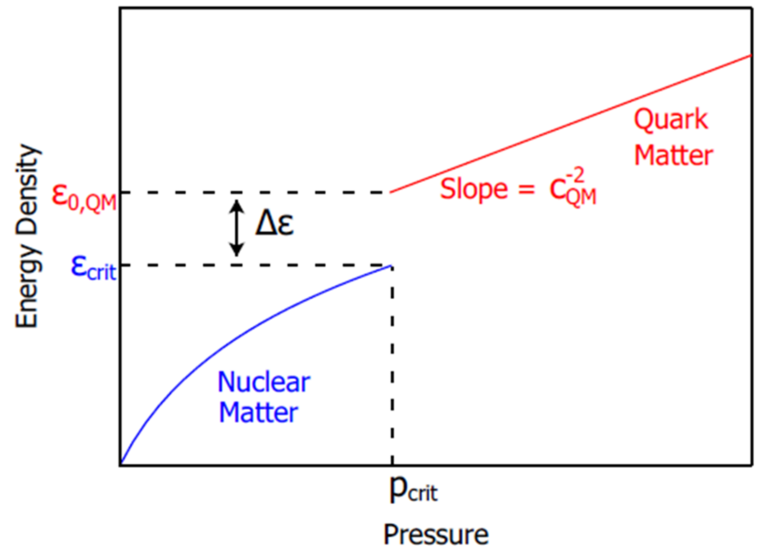
$$\frac{dm}{dr} = 4\pi r^2 \varepsilon \qquad p(\varepsilon)$$

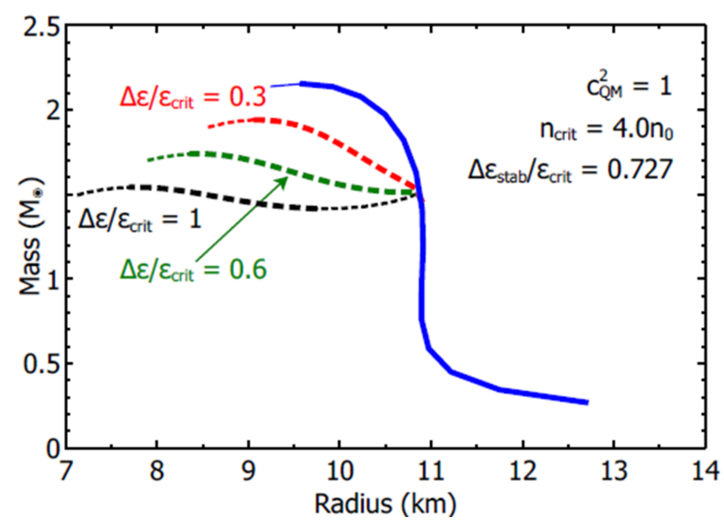
Compact Star Mass Twins and the AHP scheme

- First order PT can lead to a stable branch of hybrid stars with quark matter cores which, depending on the size of the "latent heat" (jump in energy density), can even be disconnected from the hadronic one by an unstable branch → "**third family of CS**".

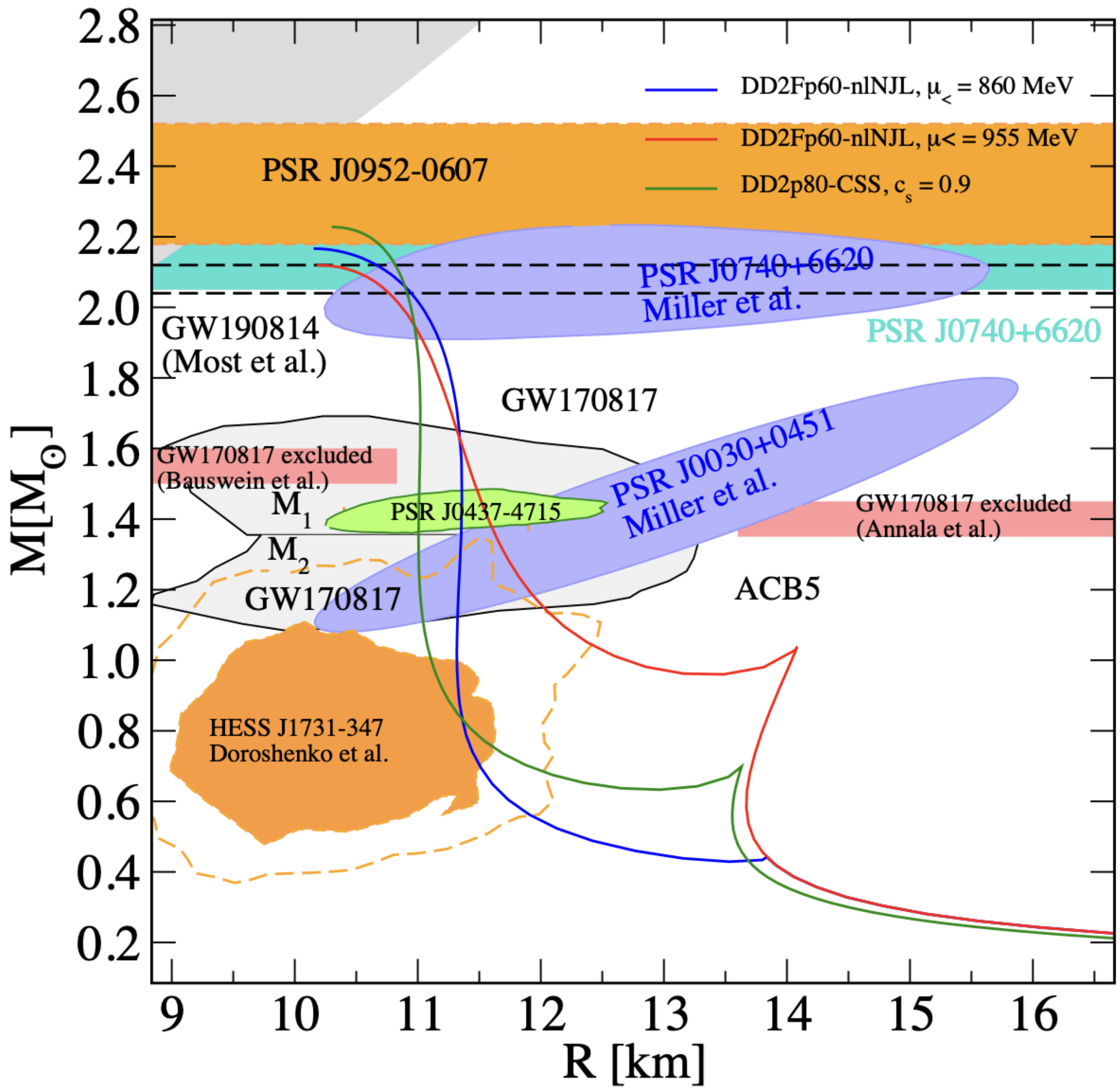
- Measuring two **disconnected populations** of compact stars in the M-R diagram would represent **the detection of a first order phase transition** in compact star matter and thus the indirect proof for the existence of a **critical endpoint (CEP) in the QCD phase diagram**!

Alford, Han, Prakash,
Phys. Rev. D 88, 083013 (2013)
arxiv:1302.4732

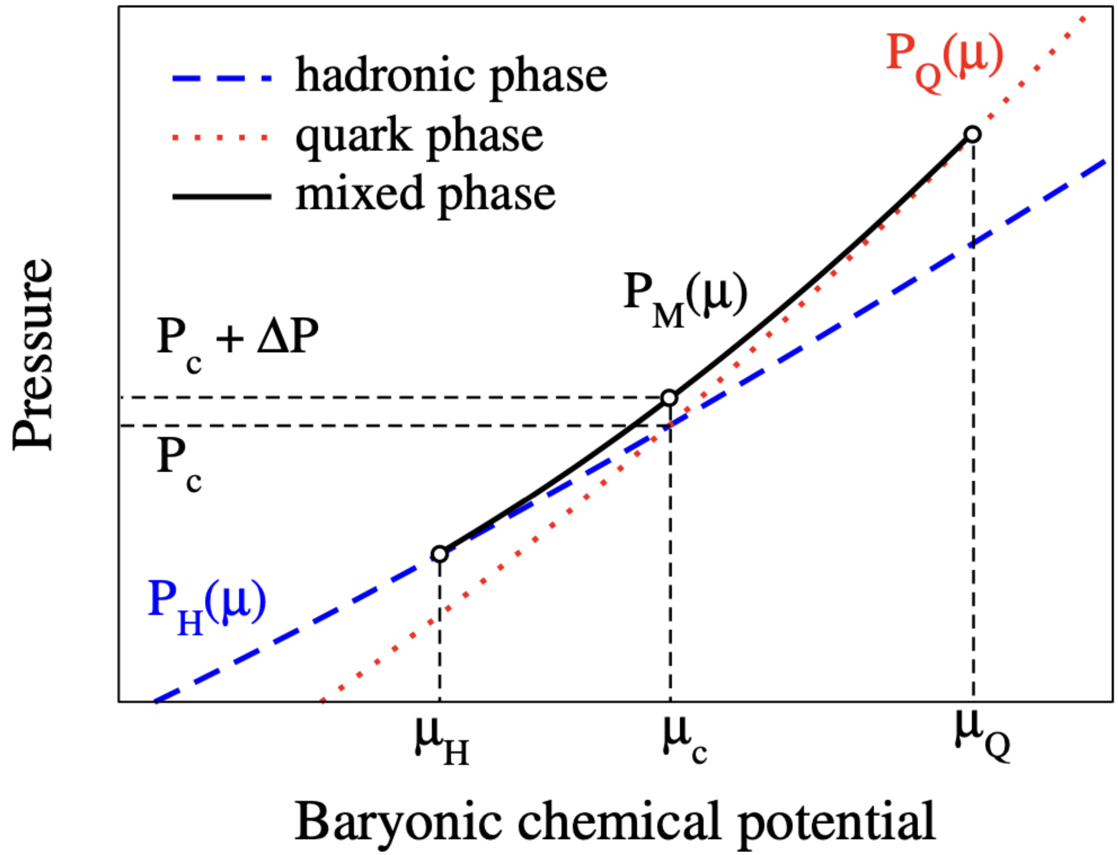




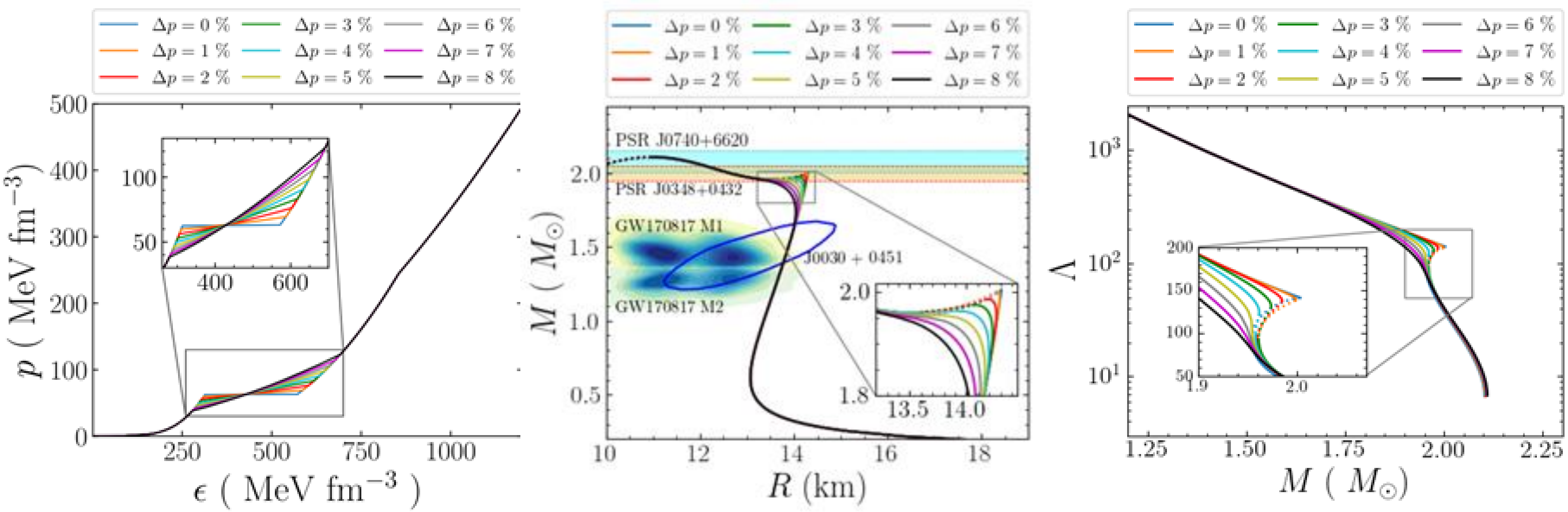
Compact Star Twins



Compact Star Twins (Pasta Phases)



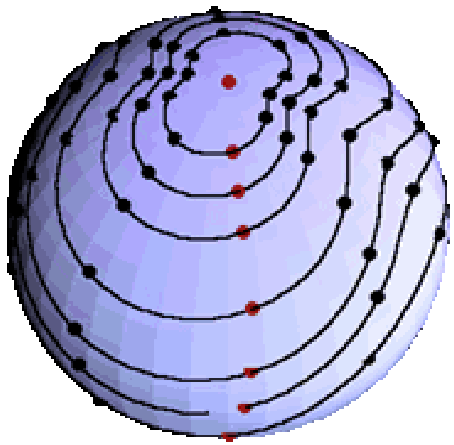
Stellar Properties



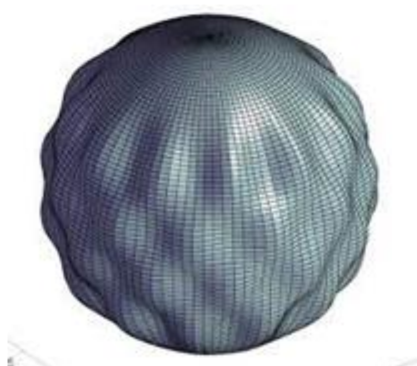
- The second and third family merge to form a single branch for Δp > 4% .
- Precise measurement of *M-R* required for detection of twin star.
- The jump Δp (if any) can be measured ~15 % (< 90% CI) with next-generation GW detectors ([P. Landry & K. Chakravarti , arXiv:2212.09733 ,2022]).

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee – arXiv:2309.08775
Monthly Notices of the Royal Astronomical Society, Vol 531, Issue 4, Pag 4640–4655 (2024)

Gravitational Waves from NS



Credit: C. Hanna and B. Owen

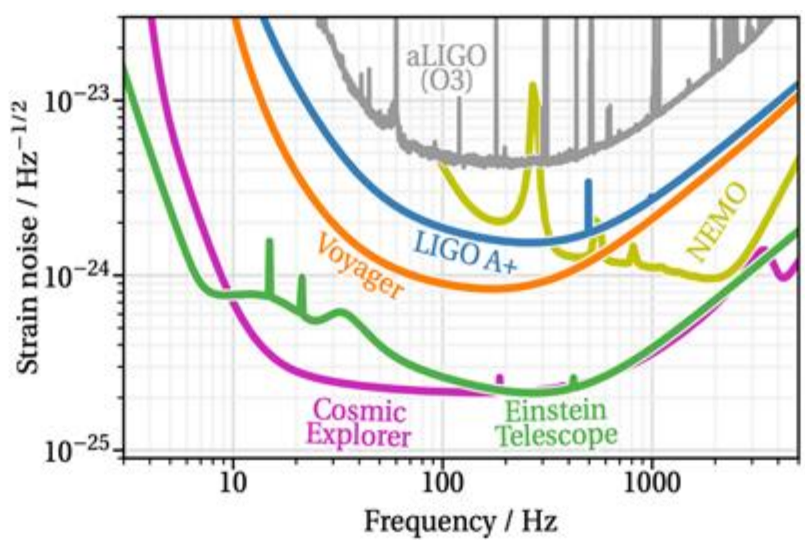


Credit: CERN/Indico

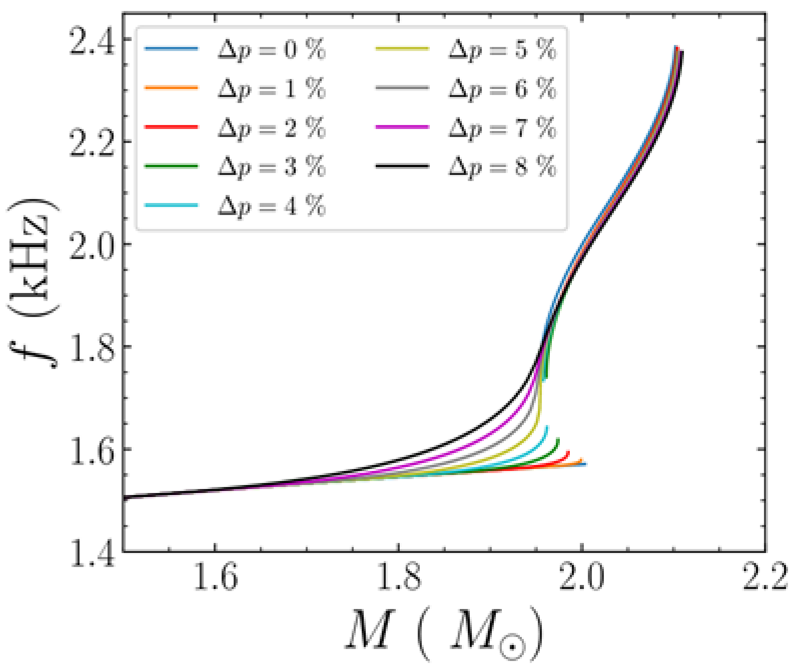
PC: cosmicexplorer.org/sensitivity

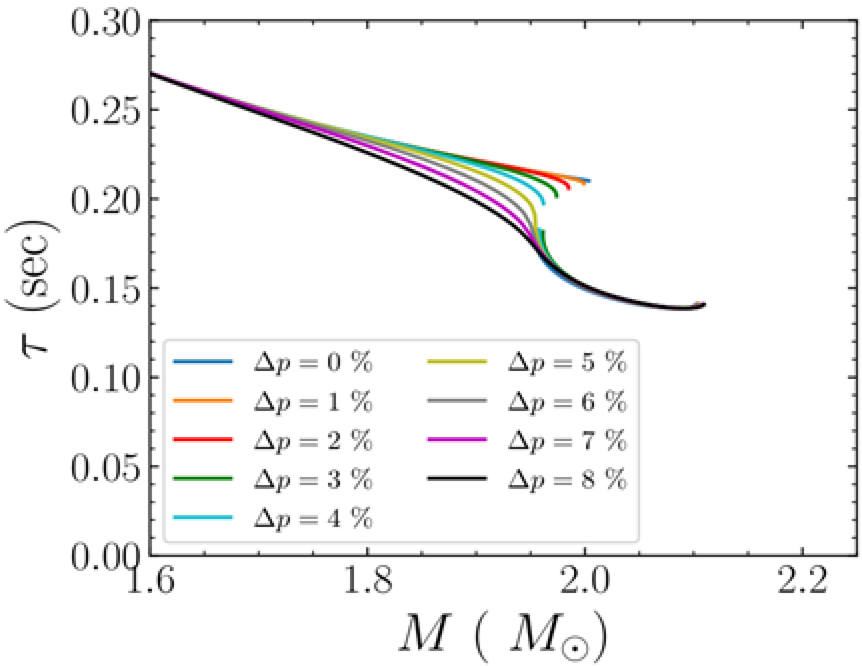
- Non-radial QNMs raised from time varying quadrupole deformations are source of GWs.

  - fundamental (f) mode,
    - no node, probe for mean density,(1 kHz < f < 3kHz)
  - pressure (p) mode,
    - Sound speed, (5 kHz < f < 10kHz)
  - gravity (g) mode,
    - (50 Hz < f < 500 Hz)
- R-mode, for rotating stars only.
  - Viscosity, (0.5 kHz < f < 2kHz)
- Space-time (w) mode.
  - 5 kHz< f



f-mode characteristics





- f-mode characteristics are obtained within ***General relativistic*** formalism.
- Sudden increase (decrease) in the frequency (damping time) observed with appearance of twin star.
- Detections of f-mode GWs from compact stars with known mass may reveal the presence of twin stars.
- Simultaneous measurement of *M-f* (from binary system) can be used to comment on twin stars.

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee – arXiv:2309.08775
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Asteroseismology and Universal Relations

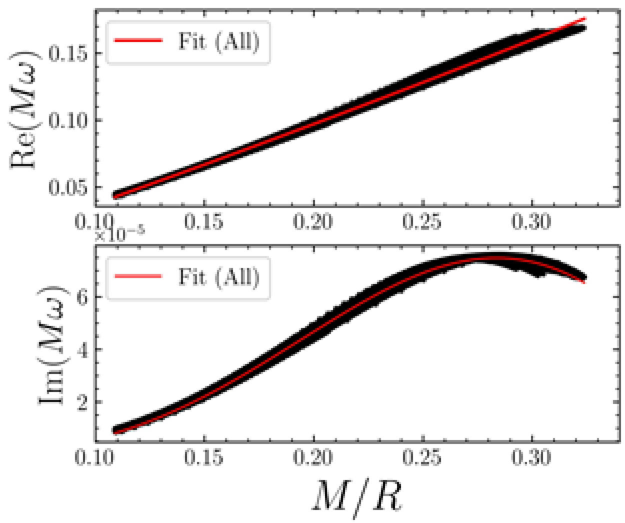
❑ URs among f-mode characteristics (f, τ_f or $\omega=2\pi f+1/\tau_f$) and NS observables.

$$f(\text{kHz}) = a_r + b_r\sqrt{\frac{M}{R^3}}$$

Empirical relations (EOS dependent)

$$\text{Re}(M\omega) = a_0 + a_1\left(\frac{M}{R}\right) + a_2\left(\frac{M}{R}\right)^2$$

$$\text{Im}(M\omega) = b_0\left(\frac{M}{R}\right)^4 + b_1\left(\frac{M}{R}\right)^5 + b_2\left(\frac{M}{R}\right)^6$$



	Re(Mω)		Im(Mω)
a_0	$-0.027\pm9\times10^{-5}$	b_0	$(9.81\pm0.004)\times10^{-2}$
a_1	0.610 ± 0.0015	b_1	$(-4.444\pm0.003)\times10^{-1}$
a_2	0.049 ± 0.002	b_2	$(4.91\pm0.0045)\times10^{-1}$

- Scaled Universal relations are more useful.
- The URs can be used for EoS inference.
- URs involving tidal deformability have also been examined.

David Alvarez-Castillo, Bikram Keshari Pradhan, Debarati Chatterjee – arXiv:2309.08775

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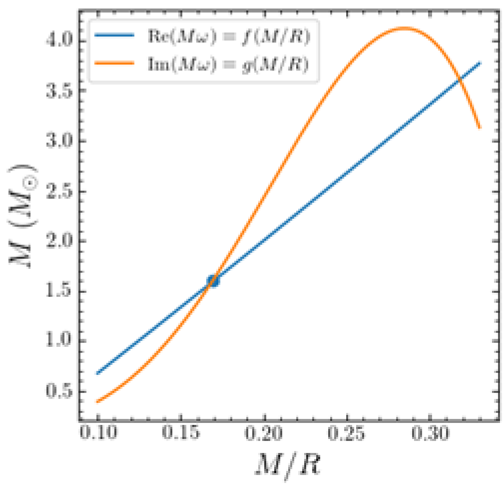
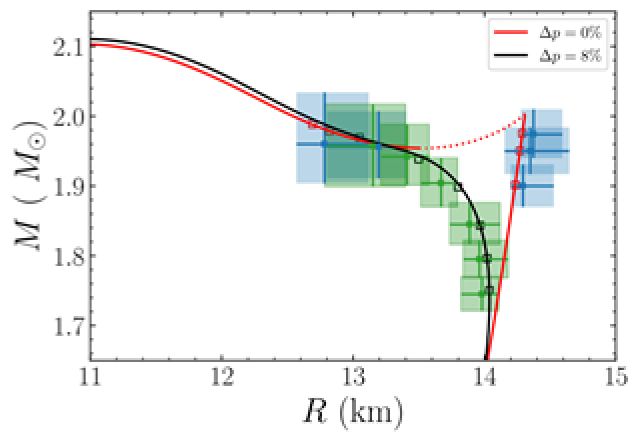
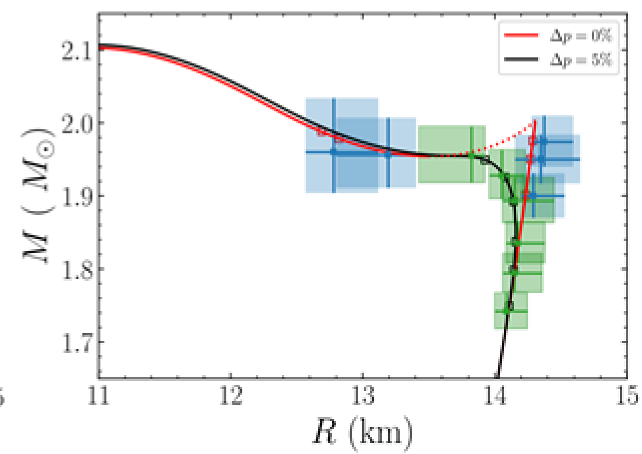
Compact star observables from f-mode observations: the role of UR Uncertainty

- ❖ Determination under the assumption that *f*, *τ* are measured precisely.
- ❖ Errors on UR results uncertainties on *M*-*R*.







- ★ The presence of the twins maybe confirmed with exact measurement of *f, and τ* .
- ★ The unstable branch of Δp = 0% can be distinguished from the connecting stable branch of Δp =8%. Differentiating among Δp =0% and Δp =5% is more challenging.
- ★ We perform a parameter estimation using Bilby to get the posteriors of (f,τ). From the recovered (f,τ), we reconstruct the (M,R) using Universal relations.

Inclusion of Observational Uncertainties

- F-mode being excited during pulsar glitches. All the energy radiated through GW.
- The burst waveform is modelled as an exponentially damped oscillation.

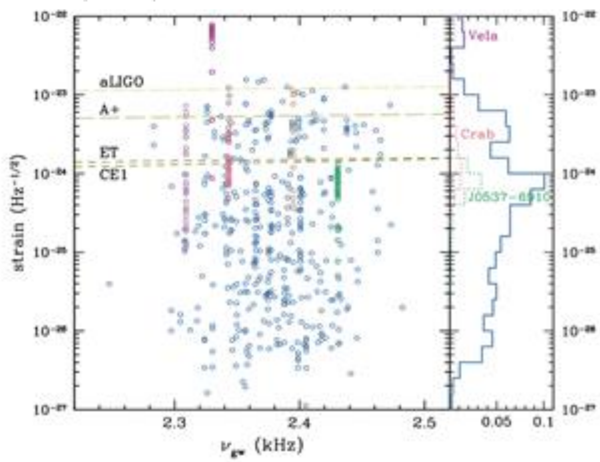
$$h(t) = h_0 \exp\left(-t/\tau_f\right) \sin\left(2\pi\nu_f t\right), \;\; t > 0$$

(B.J. Owen,2010, Ho et al. 2020)

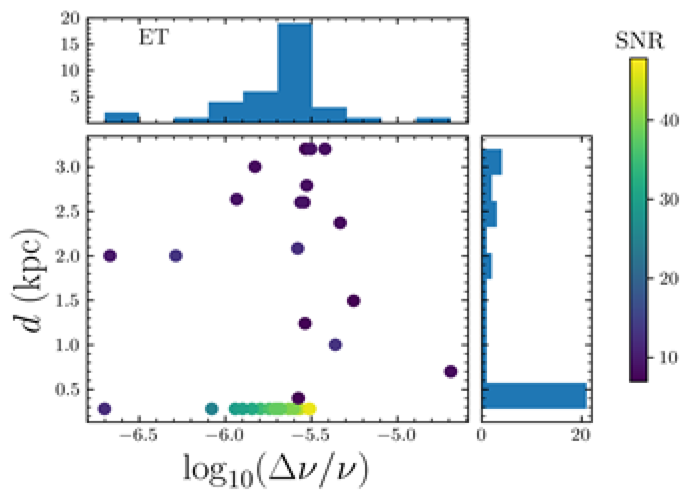
$$h_0 = 4.85\times10^{-17}\sqrt{\frac{E_{\rm gw}}{M_\odot c^2}}\sqrt{\frac{0.1\text{sec}}{\tau_f}\frac{1\text{kpc}}{d}}\left(\frac{1\text{kHz}}{\nu_f}\right)$$

f-modes GW

$$E_{\rm gw} = E_{\rm glitch} = 4\pi^2 I \nu^2 \left(\frac{\Delta\nu}{\nu}\right)$$



Ho et al ,PRD 101, 103009 (2020)



B. K. Pradhan, D. Pathak, and D. Chatterjee, *ApJ* **956** 38, (2023)

- B. Abbott et al.,LVC, *ApJ* **874** 163, 2019.
- R. Abbott et al.,LVK, PhRvD, 104, 122004, 2021.
- R. Abbott, et al., LVK, arXiv:2210.10931, 2022.
- R. Abbott, et al., LVK, arXiv:2203.12038, 2022.
- D. Lopez et al., PhRvD, 106, 103037, 2022

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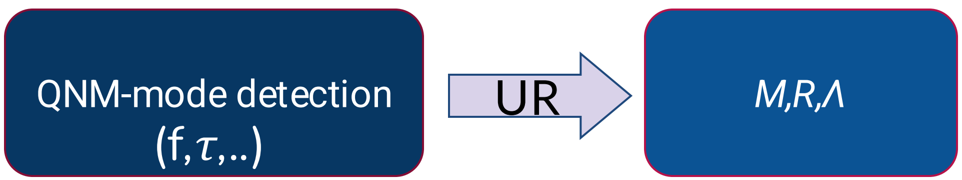
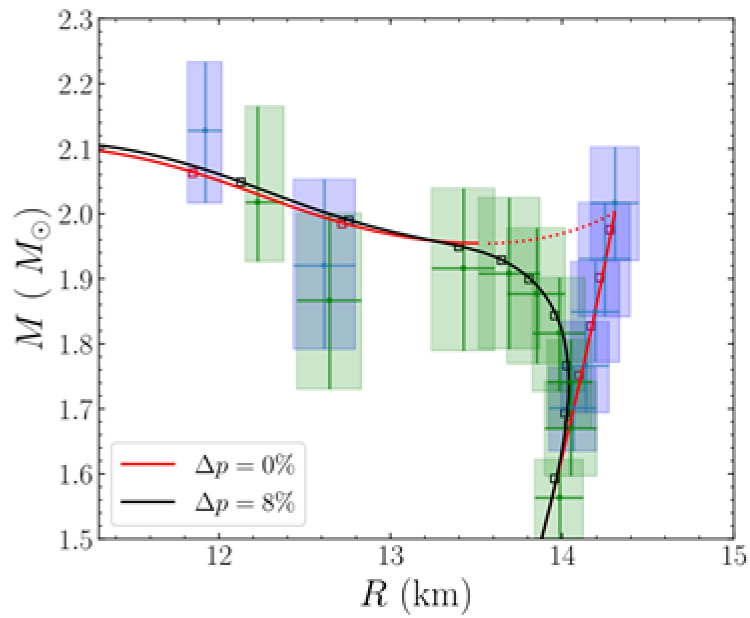
Inclusion of Observational Uncertainties

- Glitching pulsars data taken from the [Jodrell Bank Glitch catalogue].
- Spin frequency, distance (d) and sky position to each pulsar are assign from [ATNF Pulsar Catalogue].
- Consider few random mass configurations with an assumed EOS model .
- Then f-mode frequency, damping time, moment of inertia to pulsars from the assume EoS model.





- The measurement of *R* from f-mode observation may confirm the presence of twins.
- More challenging for low mass twins. However, we have more observations at low masses.
- Differentiating the nature of Δp is more challenging.

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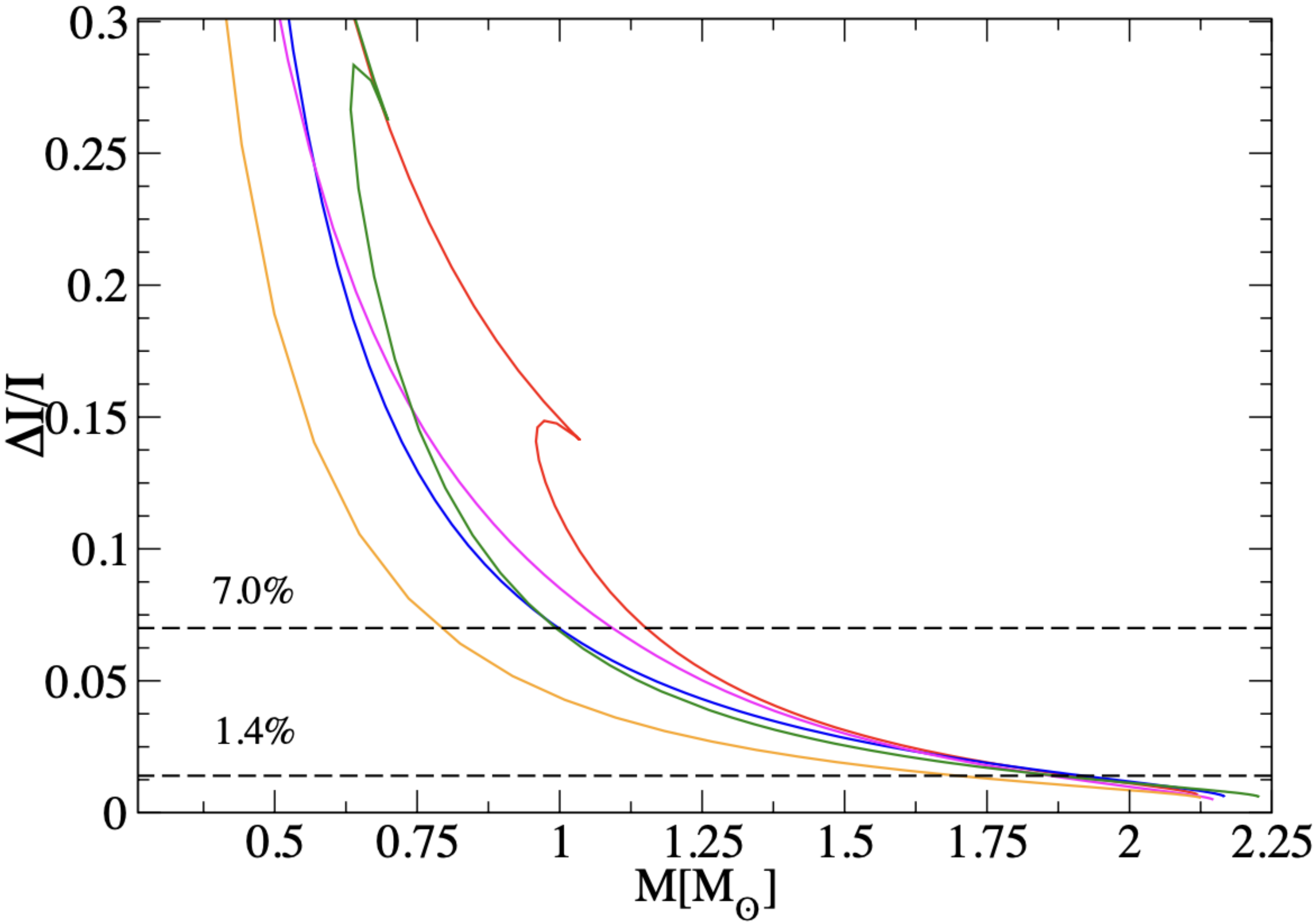
The Crust of Twin Compact Stars

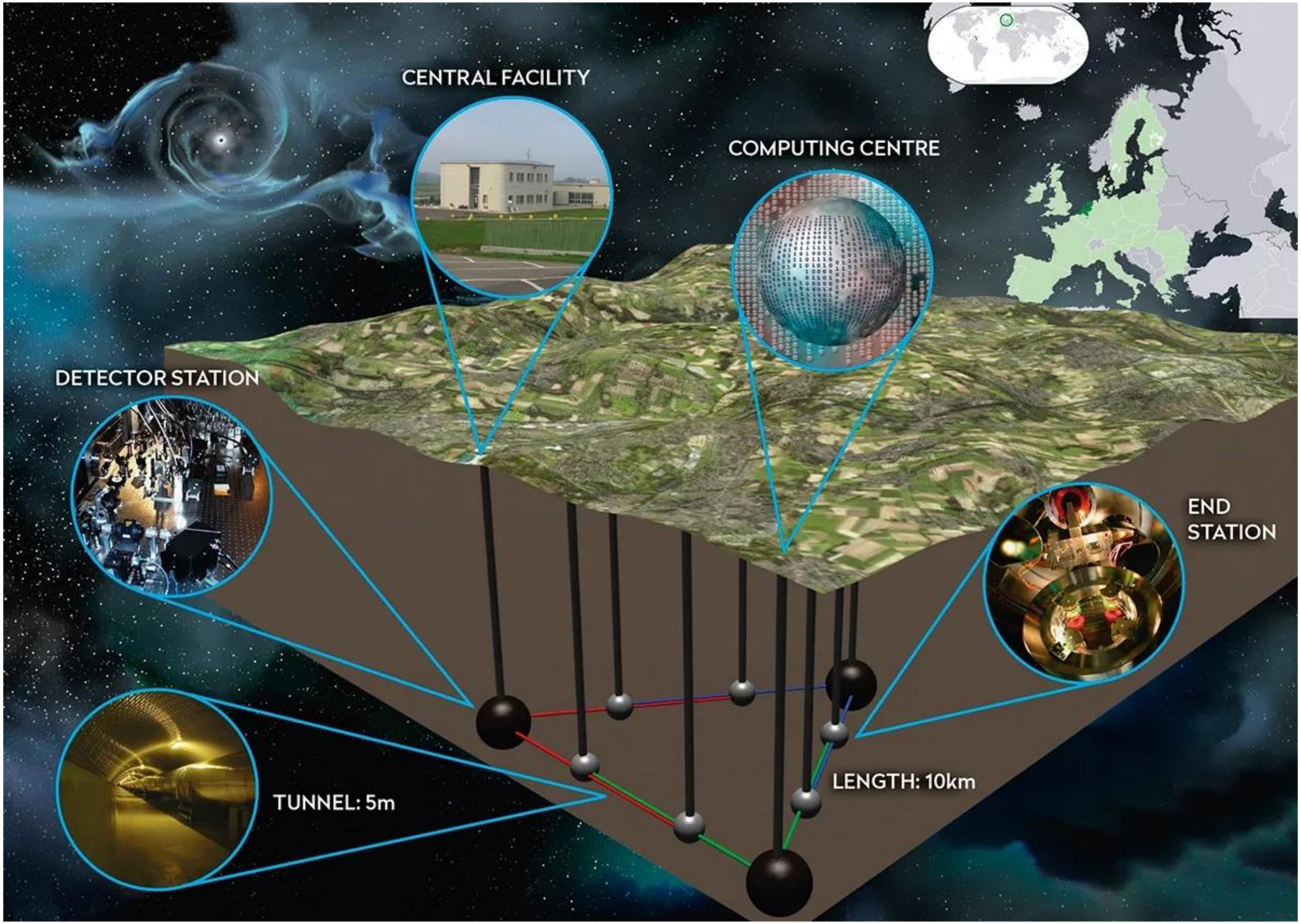


Outlook

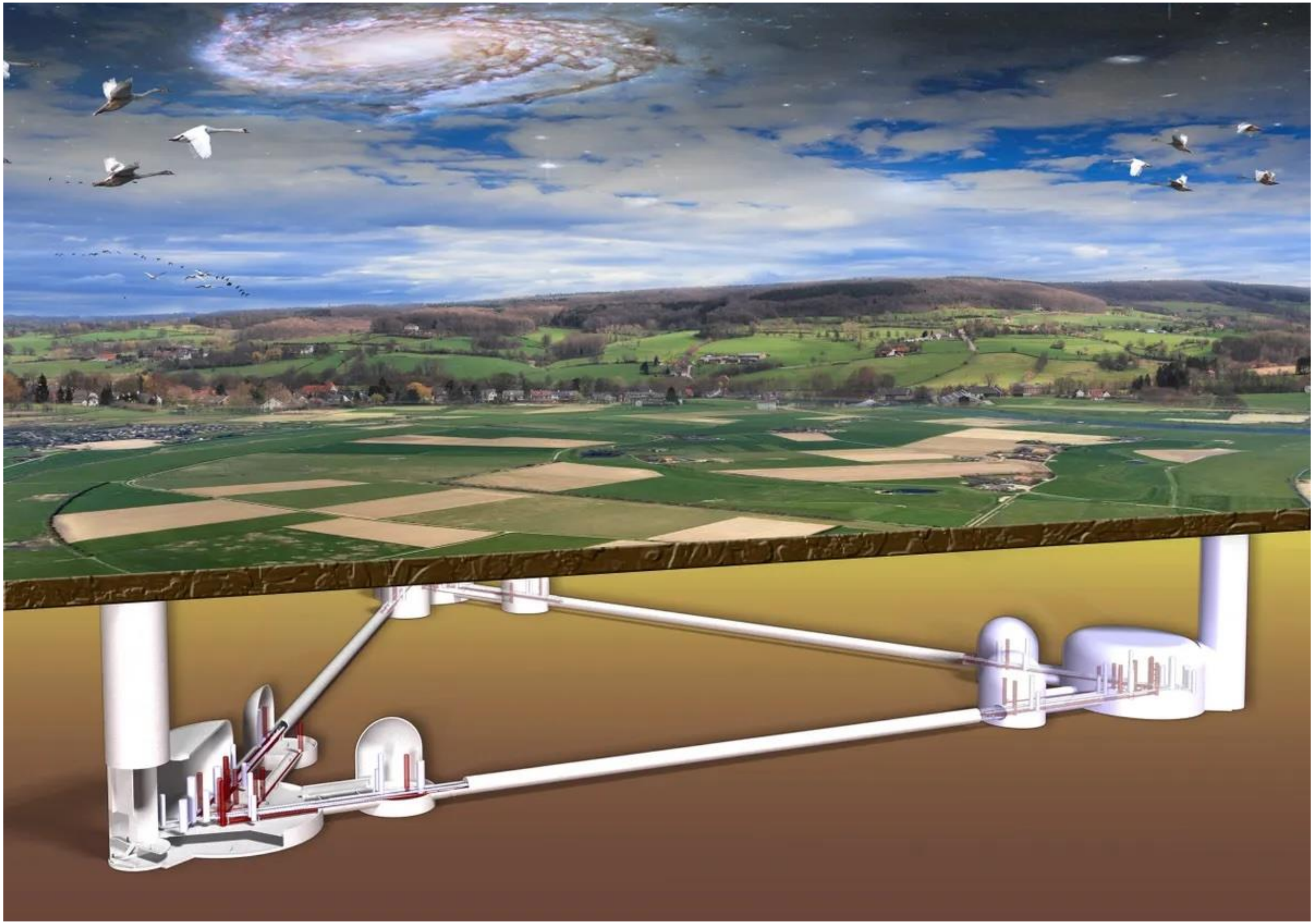
- Multi-messenger astronomy and collider experiments will continue probing the properties of dense matter.
- f-mode oscillation of hybrid stars and twin stars involving the “pasta phase” has been investigated. Nevertheless, complementary compact star measurements are necessary to confirm the nature of the quark-hadron phase transition.
- Bayesian Analysis and Machine Learning methods including compact star constraints and from laboratory experiment are useful for estimation of unknown physical parameters.
- Compact star crusts studies and pulsar observations are work in progress.
- Other physical effects can be studied with f-mode detections, like possible dark matter content in compact stars.

Gracias

Einstein Telescope



Einstein Telescope




NICER
Neutron Star Interior Composition Explorer
LIGO
KAGRA
Virgo
ET
EINSTEIN
TELESCOPE
ATHENA
STROBE-X
SK
A
SKA AFRICA
SQUARE KILOMETRE ARRAY
ngvla
MeerKAT
eXTP
enhanced X-ray Timing and Polarimetry Mission

Neutron Star Equation of State

The energy per nucleon in neutron star core matter is given by:

$$\begin{aligned} E_{\rm tot}(n,\{x_i\}) &= E_{\rm b}(n,x_p) + E_{\rm lep}(n,x_e,x_\mu)\ , \\ E_{\rm b}(n,x_p) &= E_0(n) + S(n,x_p) \\ E_{\rm lep}(n,x_e,x_\mu) &= E_e(n,x_e) + E_\mu(n,x_\mu)\ , \end{aligned}$$

where $n = n_p + n_n$ is the total baryon density and $x_i = n_i/n$, $i = p, e, \mu$ are the fractions of protons, electrons and muons, respectively. The baryonic part is very well described by the parabolic approximation w.r.t. the asymmetry

$$\alpha \quad = \quad \frac{n_n - n_p}{n_n + n_p} = 1 - 2x_p,$$

resulting in $S(n,x_p) = (1-2x_p)^2 E_s(n)$. The leptonic contribution is a sum of the Fermi gas expressions for the contributing leptons $l = e, \mu$

$$E_l(n,x_l) = \quad \tfrac{1}{n}\tfrac{p_{F,l}^4}{4\pi^2}\left[\sqrt{1+z_l^2}\left(1+\tfrac{z_l^2}{2}\right) - \tfrac{z_l^4}{2}{\rm Arsinh}\left(\tfrac{1}{z_l}\right)\right]\ ,$$

where $z_l = m_l/p_{F,l}$. For massless leptons ($z_l \to 0$), this expression goes over to

$$E_l(n,x_l)\big|_{m_l=0} = \frac{1}{n}\frac{p_{F,l}^4}{4\pi^2} = \frac{3}{4}\left(3\pi^2 n\right)^{1/3} x_l^{4/3}\ .$$

Charge neutrality and β-equillibrium

Under neutron star conditions charge neutrality holds,

$$x_p = x_e + x_\mu \ .$$

The $\beta-$ equilibrium with respect to the weak interaction processes $n \to p + e^- + \bar{\nu}_e$ and $p + e^- \to n + \nu_e$ (and similar for muons), for cold neutron stars (temperature T below the neutrino opacity criterion $T < T_\nu \sim 1$ MeV) implies

$$\mu_n - \mu_p = \mu_e = \mu_\mu \ .$$

The chemical potentials are defined as

$$\mu_i = \frac{\partial \varepsilon_i}{\partial n_i} = \frac{\partial}{\partial x_i} E_i(n, \{x_j\}) \ , \quad i, j = n, p, e, \mu \ ,$$

where $\varepsilon_i = n \ E_i(n, \{x_j\})$ is the partial energy density of species i in the system. From the above equations:

$$\mu_e = 4(1 - 2x) E_s(n) \ .$$

Since electrons in neutron star interiors are ultrarelativistic,

$$\mu_e = \sqrt{p_{F,e}^2 + m_e^2} \approx p_{F,e}, \text{ and } p_{F,e} = (3\pi^2 n_e)^{1/3} = (3\pi^2 n)^{1/3} (x - x_\mu)^{1/3} \ ,$$

$$\frac{x - x_\mu}{(1 - 2x)^3} = \frac{64 E_s^3(n)}{3\pi^2 n} \ , \qquad (x - x_\mu)^{2/3} - x_\mu^{2/3} = \frac{m_\mu^2}{(3\pi^2 n)^{2/3}} \ .$$

The total pressure is then given as $P(n) = n^2 \left(\frac{\partial E_{\text{tot}}}{\partial n}\right)$.

NJL model with multiquark interactions

$$\mathcal{L} = \bar{q}(i\partial\!\!\!/ - m)q + \mu_q \bar{q}\gamma^0 q + \mathcal{L}_4 + \mathcal{L}_8 \ , \ \mathcal{L}_4 = \frac{g_{20}}{\Lambda^2}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2] - \frac{g_{02}}{\Lambda^2}(\bar{q}\gamma_\mu q)^2 \ ,$$

$$\mathcal{L}_8 \ = \ \frac{g_{40}}{\Lambda^8}[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]^2 - \frac{g_{04}}{\Lambda^8}(\bar{q}\gamma_\mu q)^4 - \frac{g_{22}}{\Lambda^8}(\bar{q}\gamma_\mu q)^2[(\bar{q}q)^2 + (\bar{q}i\gamma_5\tau q)^2]$$

Meanfield approximation: $\mathcal{L}_{\mathrm{MF}} = \bar{q}(i\partial\!\!\!/ - M)q + \tilde{\mu}_q \bar{q}\gamma^0 q - U \ ,$

$$M = m + 2\frac{g_{20}}{\Lambda^2}\langle\bar{q}q\rangle + 4\frac{g_{40}}{\Lambda^8}\langle\bar{q}q\rangle^3 - 2\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle\langle q^\dagger q\rangle^2 \ ,$$

$$\tilde{\mu}_q = \mu_q - 2\frac{g_{02}}{\Lambda^2}\langle q^\dagger q\rangle - 4\frac{g_{04}}{\Lambda^8}\langle q^\dagger q\rangle^3 - 2\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle^2\langle q^\dagger q\rangle \ ,$$

$$U \ = \ \frac{g_{20}}{\Lambda^2}\langle\bar{q}q\rangle^2 + 3\frac{g_{40}}{\Lambda^8}\langle\bar{q}q\rangle^4 - 3\frac{g_{22}}{\Lambda^8}\langle\bar{q}q\rangle^2\langle q^\dagger q\rangle^2 - \frac{g_{02}}{\Lambda^2}\langle q^\dagger q\rangle^2 - 3\frac{g_{04}}{\Lambda^8}\langle q^\dagger q\rangle^4 \ .$$

Thermodynamic Potential:

$$\Omega \ = \ U - 2N_f N_c \int \frac{d^3p}{(2\pi)^3}\left\{E + T\log[1 + e^{-\beta(E-\tilde{\mu}_q)}] + T\log[1 + e^{-\beta(E+\tilde{\mu}_q)}]\right\} + \Omega_0$$

S. Benic - Eur.Phys.J. A50 (2014) 111, arXiv:1401.5380