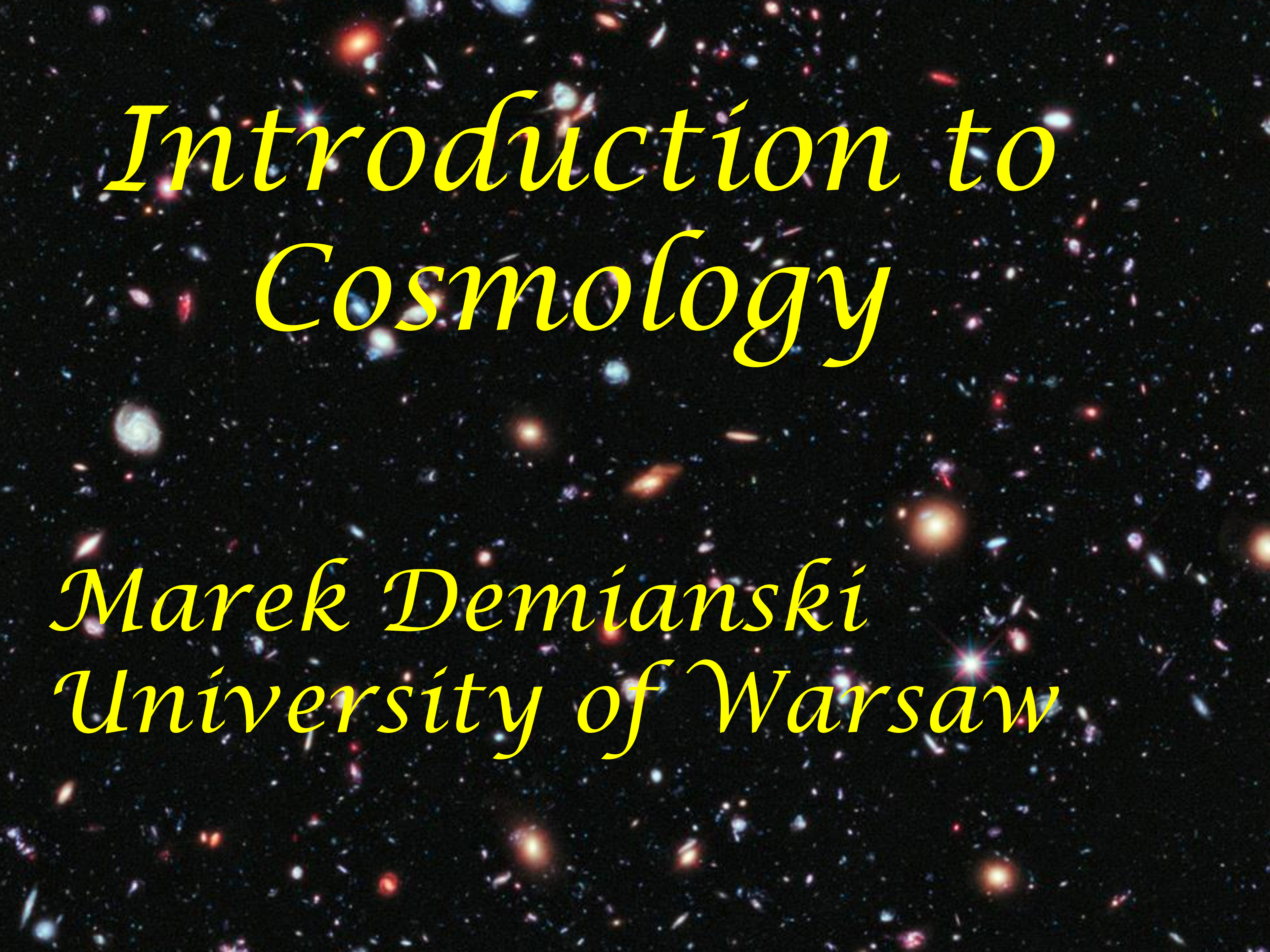
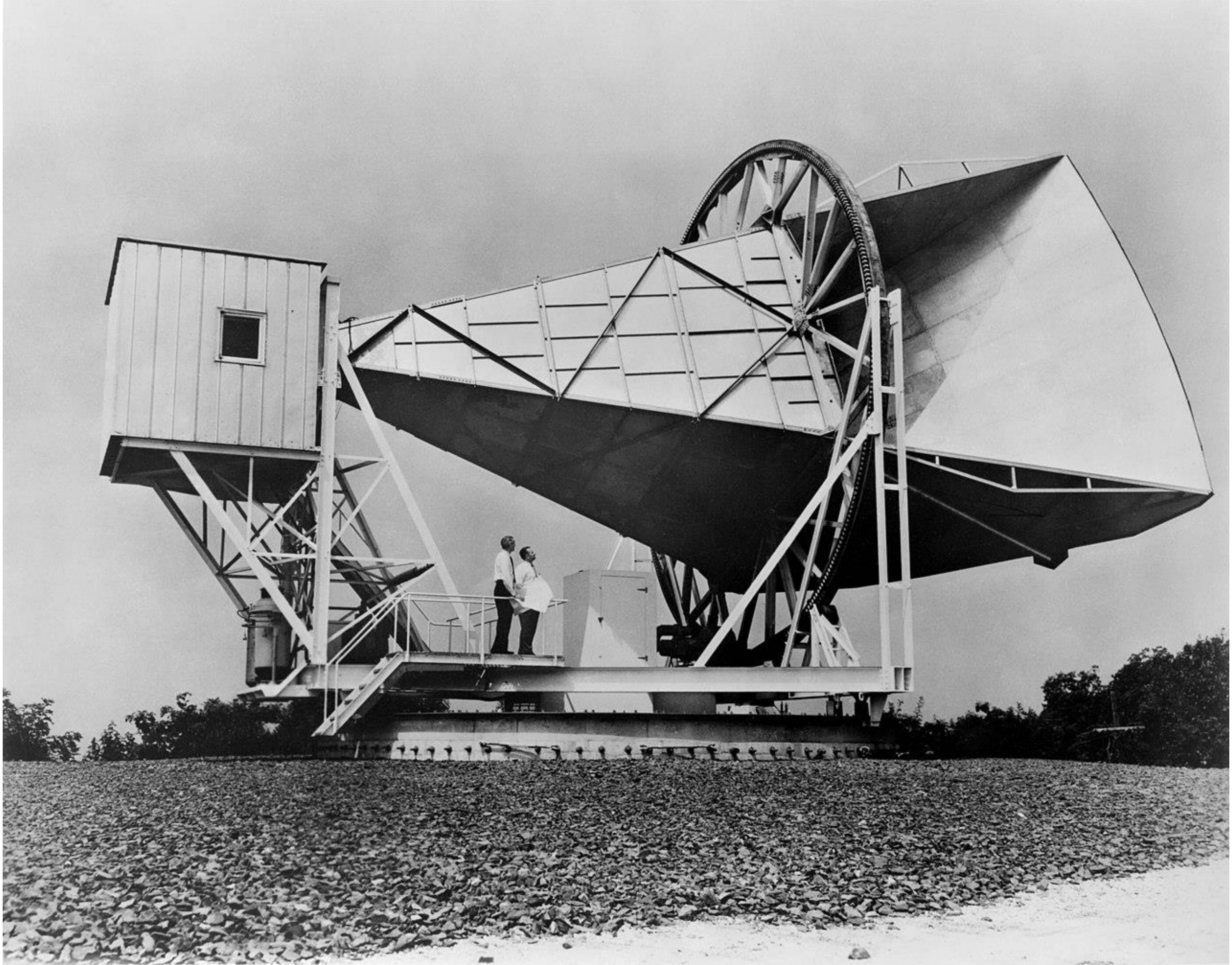
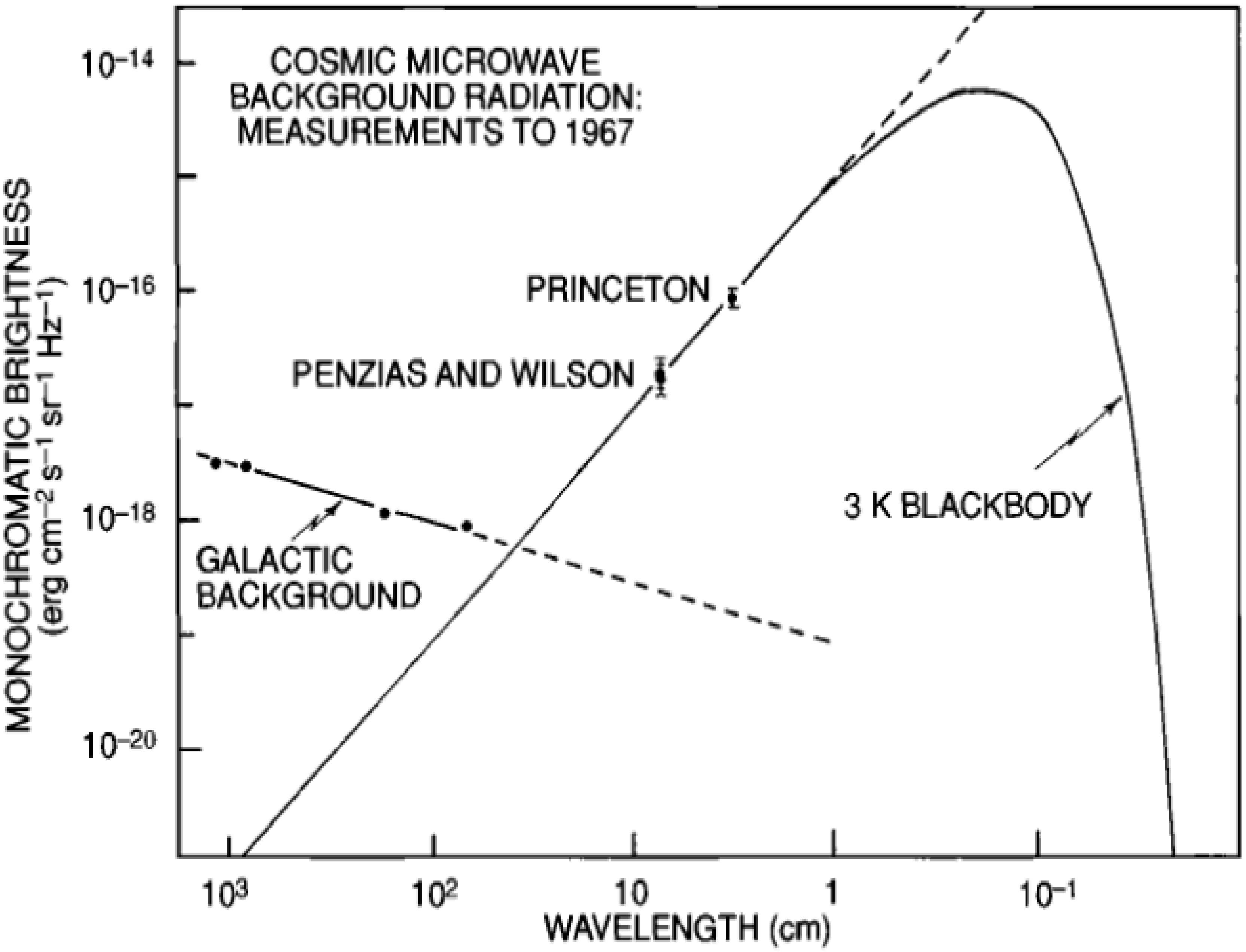
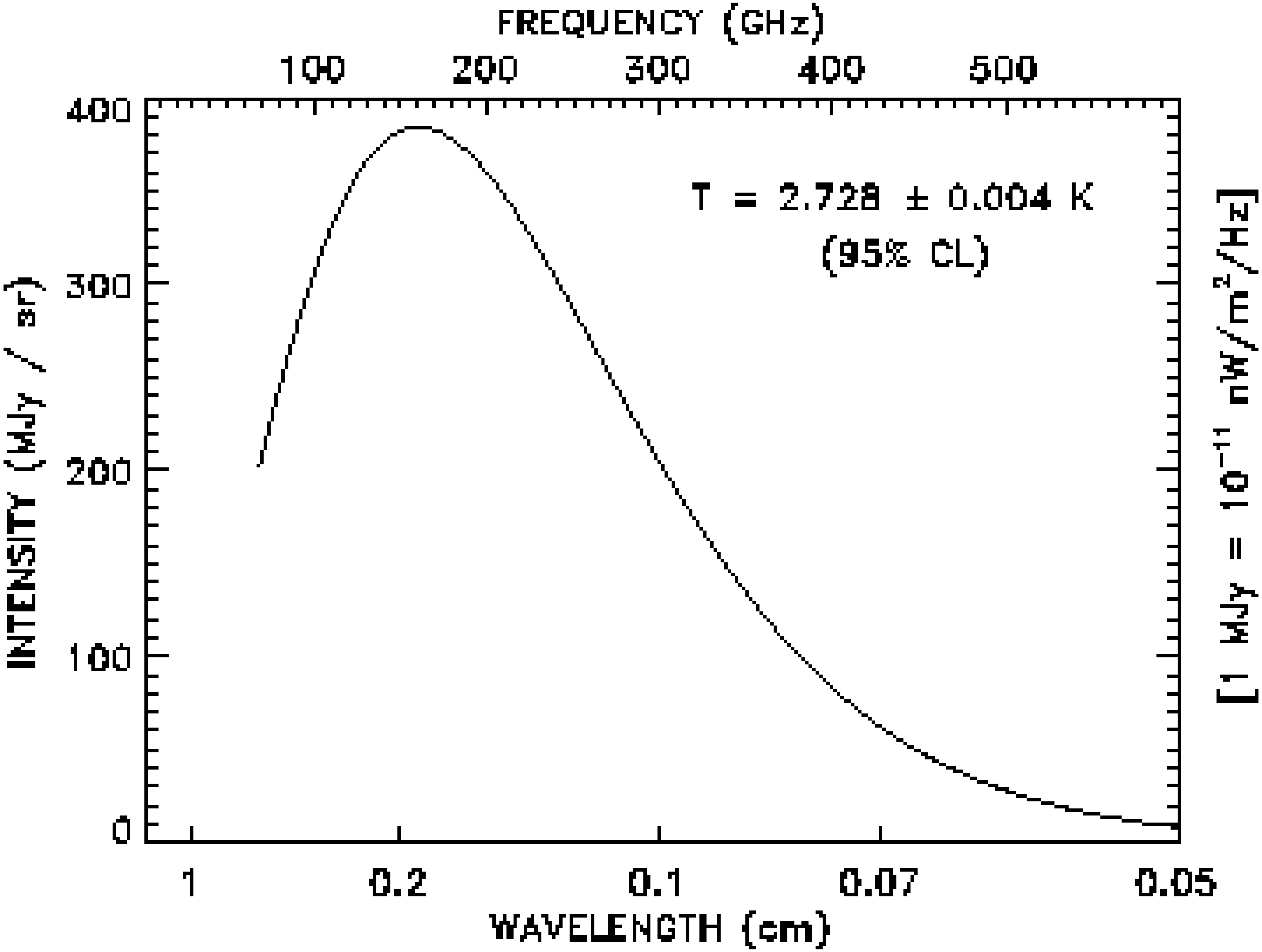


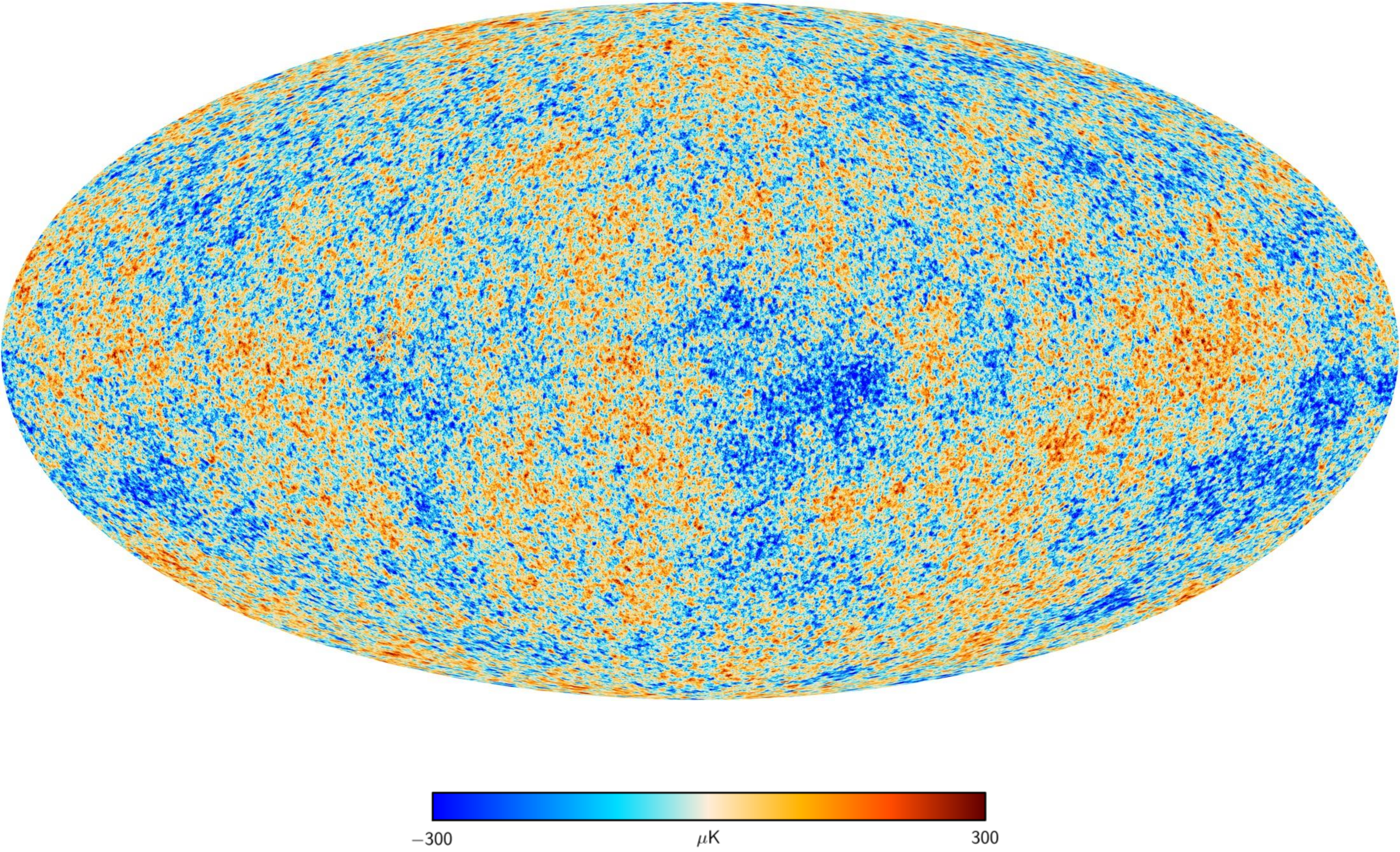
Introduction to Cosmology

Marek Demianski
University of Warsaw




COSMIC MICROWAVE
BACKGROUND RADIATION:
MEASUREMENTS TO 1967
MONOCHROMATIC BRIGHTNESS
(erg cm^{-2} s^{-1} sr^{-1} Hz^{-1})
10^{-14}
10^{-16}
10^{-18}
10^{-20}
PRINCETON
PENZIAS AND WILSON
GALACTIC
BACKGROUND
3 K BLACKBODY
10^{3}
10^{2}
10
1
10^{-1}
WAVELENGTH (cm)


FREQUENCY (GHz)
100
200
300
400
500
400
300
200
100
0
INTENSITY (MJy / sr)
T = 2.728 ± 0.004 K
(95% CL)
[1 MJy = 10^{-11} nW/m^2/Hz]
1
0.2
0.1
0.07
0.05
WAVELENGTH (cm)


−300
μK
300

Problems of the standard Big Bang cosmological model:

1. Why the universe is homogeneous and isotropic ?
2. Why the universe is almost flat?
3. Why the temperature of the CMB is so uniform all over the sky?
4. Why the universe is dominated by matter rather then by antimatter?

Proposed solutions:

Very special initial conditions

Alternative theories of gravity

Very rapid expansion at very early epoch –

Inflationary model

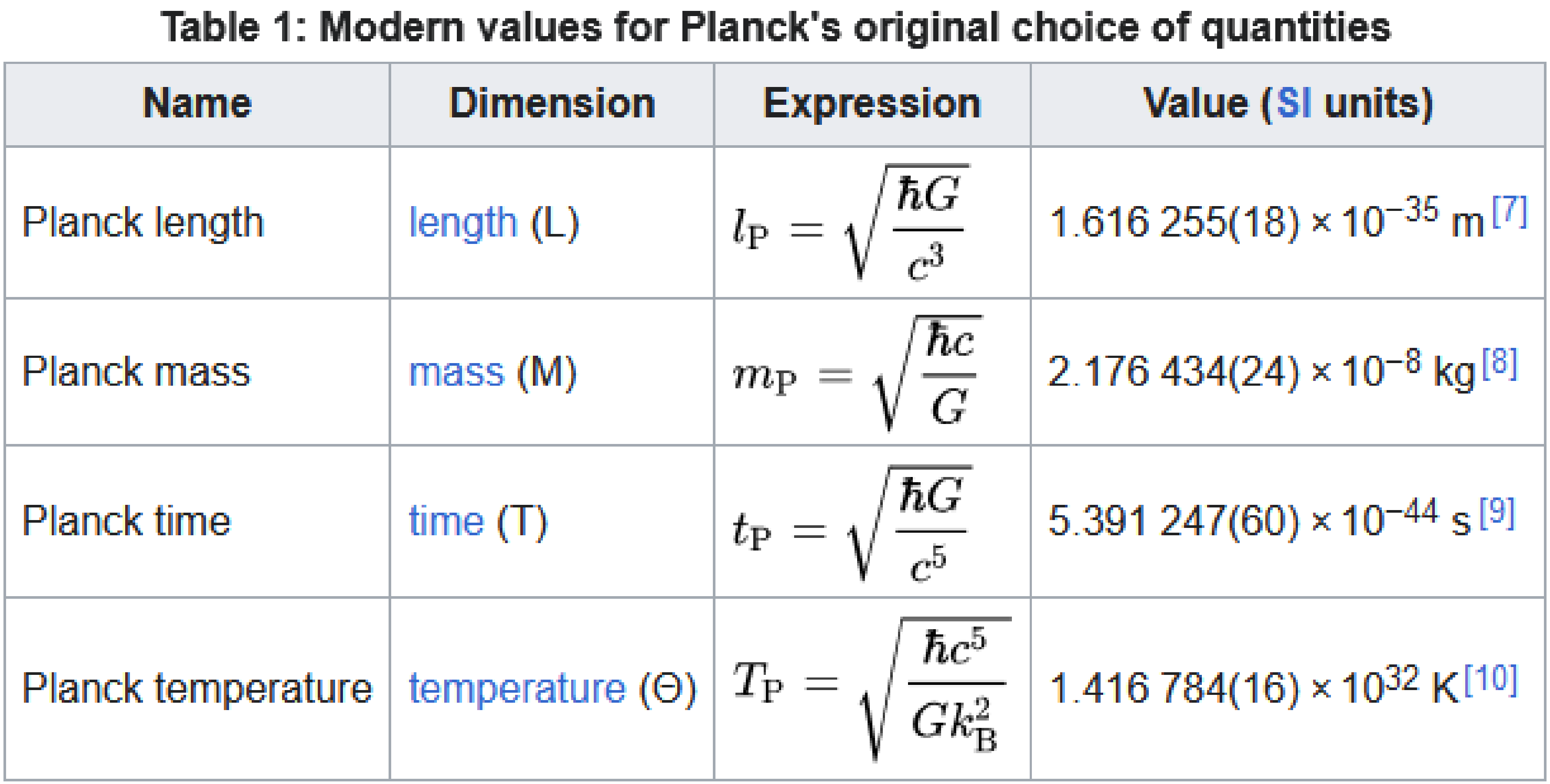
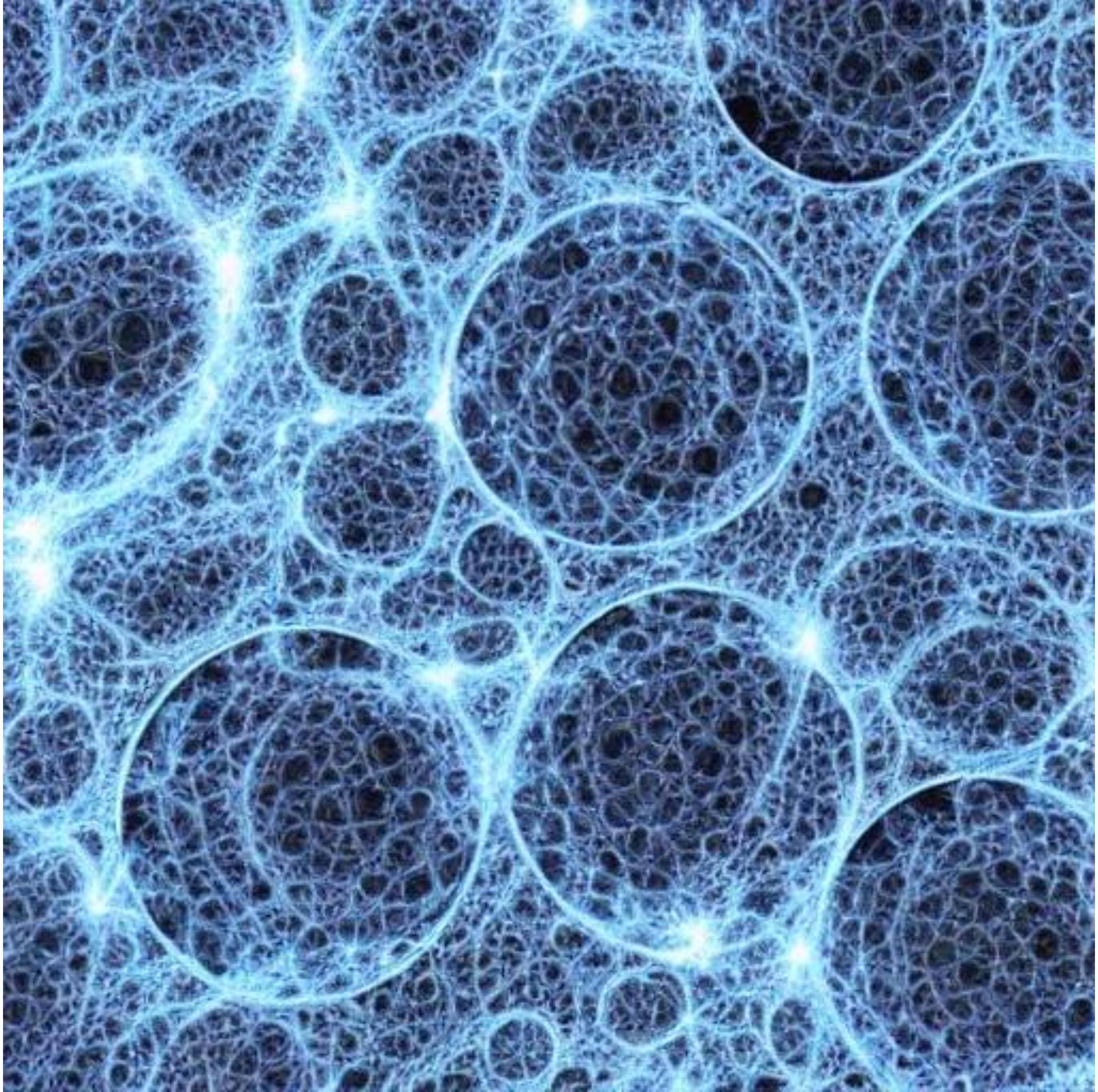


Table 1: Modern values for Planck's original choice of quantities

Name	Dimension	Expression	Value (SI units)
Planck length	length (L)	$l_\text{P} = \sqrt{\frac{\hbar G}{c^3}}$	1.616 255(18) × 10^{-35} m[7]
Planck mass	mass (M)	$m_\text{P} = \sqrt{\frac{\hbar c}{G}}$	2.176 434(24) × 10^{-8} kg[8]
Planck time	time (T)	$t_\text{P} = \sqrt{\frac{\hbar G}{c^5}}$	5.391 247(60) × 10^{-44} s[9]
Planck temperature	temperature (Θ)	$T_\text{P} = \sqrt{\frac{\hbar c^5}{G k_\text{B}^2}}$	1.416 784(16) × 10^{32} K[10]

Planck energy $E_P = m_P\, c^2 = 1.95 \cdot 10^9$ J = $1.21\ 10^{19}$ GeV



Force	**Approximate relative strength**	**Range**	**+/−**[1]	**Carrier particle**
Gravity	10^{-38}	∞	+ only	Graviton (conjectured)
Electromagnetic	10^{-2}	∞	+/−	Photon (observed)
Weak force	10^{-13}	$< 10^{-18}$ m </td>	+/−	W^+, W^-, Z^0 \ (observed[2])
Strong force	11	$< 10^{-15}$ m </td>	+/−	Gluons (conjectured[3])

Properties of the Interactions

The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction	Electromagnetic Interaction	Strong Interaction
		(Electroweak)		
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	W^+ W^- Z^0	γ	Gluons
Strength at 10^{-18} m	10^{-41}	0.8	1	25
3×10^{-17} m	10^{-41}	10^{-4}	1	60

THE STANDARD MODEL OF
FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model is a quantum theory that summarizes our current knowledge of the physics of fundamental particles and fundamental interactions (interactions are manifested by forces and by decay rates of unstable particles).

FERMIONS
matter constituents
spin = 1/2, 3/2, 5/2, ...

Leptons spin =1/2

Flavor	Mass GeV/c^2	Electric charge
ν_L lightest neutrino*	$(0-0.8)\times10^{-9}$	0
e electron	0.000511	−1
ν_M middle neutrino*	$(0.009-0.8)\times10^{-9}$	0
μ muon	0.1057	−1
ν_H heaviest neutrino*	$(0.05-0.8)\times10^{-9}$	0
τ tau	1.777	−1

Quarks spin =1/2

Flavor	Approx. Mass GeV/c^2	Electric charge
u up	0.0022	2/3
d down	0.0047	−1/3
c charm	1.27	2/3
s strange	0.0934	−1/3
t top	172.7	2/3
b bottom	4.18	−1/3

*See the neutrino paragraph below.

Spin is the intrinsic angular momentum of particles. Spin is given in units of $\hbar$, which is the quantum unit of angular momentum where $\hbar = h/2\pi = 6.58\times10^{-25}$ GeV s $=1.05\times10^{-34}$ J s.

Electric charges are given in units of the proton's charge. In SI units the electric charge of the proton is 1.60×10^{-19} coulombs.

The energy unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c^2 (remember $E = mc^2$) where 1 GeV $= 10^9$ eV $=1.60\times10^{-10}$ joule. The mass of the proton is 0.938 GeV/c^2 $= 1.67\times10^{-27}$ kg.

Neutrinos

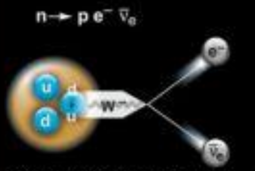
Neutrinos are produced in the sun, supernovae, reactors, accelerator collisions, and many other processes. Any produced neutrino can be described as one of three neutrino flavor states ν_e, ν_μ, or ν_τ, labelled by the type of charged lepton associated with its production. Each is a defined quantum mixture of the three definite-mass neutrinos ν_L, ν_M, and ν_H for which currently allowed mass ranges are shown in the table. Further exploration of the properties of neutrinos may yield powerful clues to puzzles about matter and antimatter and the evolution of stars and galaxy structures.

Matter and Antimatter

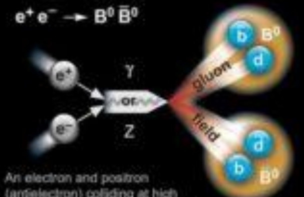
For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or – charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g., Z^0, γ, and $\eta_c = c\bar{c}$ but not $K^0 = d\bar{s}$) are their own antiparticles.

Particle Processes

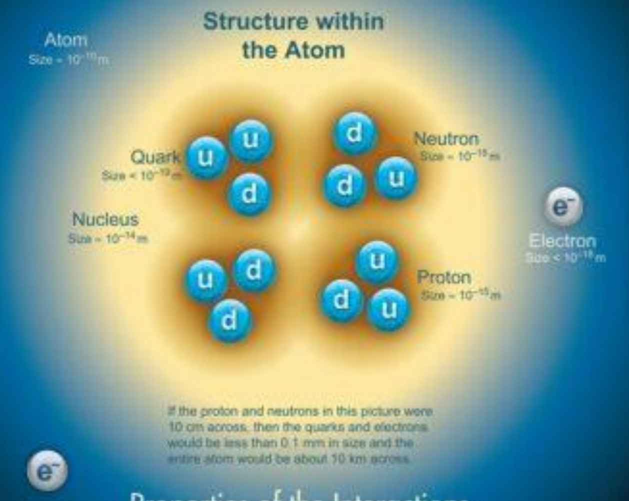
These diagrams are an artist's conception. Orange shaded areas represent the cloud of gluons.



A free neutron (udd) decays to a proton (uud), an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron β (beta) decay.



An electron and positron (antielectron) colliding at high energy can annihilate to produce $\bar{B}^0$ and B^0 mesons via a virtual Z boson or a virtual photon.



If the proton and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

BOSONS
force carriers
spin = 0, 1, 2, ...

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.38	−1
W^+ W bosons	80.38	+1
Z^0 Z boson	91.188	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Higgs Boson spin = 0

Name	Mass GeV/c^2	Electric charge
H Higgs	125.25	0

Higgs Boson

The Higgs boson is a critical component of the Standard Model. The associated Higgs field provides the mechanism by which fundamental particles get mass. Particles that interact more strongly with the Higgs field are more massive.

Color Charge

Only quarks and gluons carry "strong charge" (also called "color charge") and can have strong interactions. Each quark carries three types of color charge. These charges have nothing to do with the colors of visible light. Just as electrically-charged particles interact by exchanging photons, in strong interactions, color-charged particles interact by exchanging gluons.

Quarks Confined in Mesons and Baryons

Quarks and gluons cannot be isolated – they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs. The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge.

Two types of hadrons have been observed in nature **mesons** $q\bar{q}$ and **baryons** qqq. Among the many types of baryons observed are the proton (uud), antiproton ($\bar{u}\bar{u}\bar{d}$), and neutron (udd). Quark charges add in such a way as to make the proton have charge 1 and the neutron charge 0. Among the many types of mesons are the pion π^+ ($u\bar{d}$), kaon K^- ($s\bar{u}$), and B^0 ($d\bar{b}$).

Learn more at **ParticleAdventure.org**

Properties of the Interactions

The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
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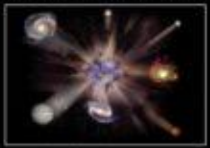
Unsolved Mysteries

Driven by new puzzles in our understanding of the physical world, particle physicists are following paths to new wonders and startling discoveries. Experiments may even find extra dimensions of space, microscopic black holes, and/or evidence of string theory.

Why is the Universe Accelerating?



The expansion of the universe appears to be accelerating. Is this due to Einstein's Cosmological Constant? If not, will experiments reveal a new force of nature or even extra (hidden) dimensions of space?

Why No Antimatter?

Matter and antimatter were created in the Big Bang. Why do we now see only matter except for the tiny amounts of antimatter that we make in the lab and observe in cosmic rays?

What is Dark Matter?

Invisible forms of matter make up much of the mass observed in galaxies and clusters of galaxies. Does this dark matter consist of new types of particles that interact very weakly with ordinary matter?

Are there Extra Dimensions?

An indication for extra dimensions may be the extreme weakness of gravity compared with the other three fundamental forces (gravity is so weak that a small magnet can pick up a paper clip overwhelming Earth's gravity).

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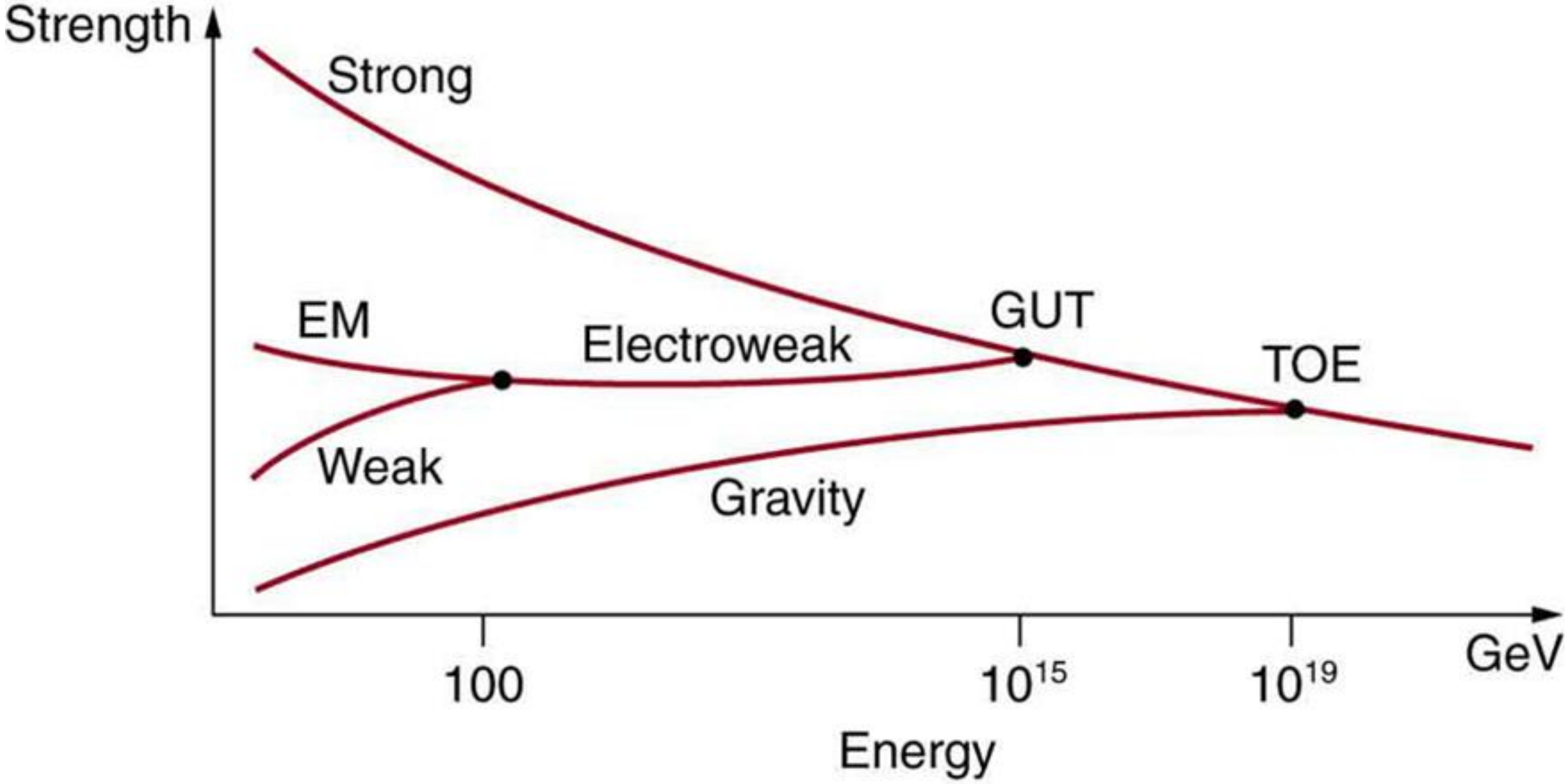
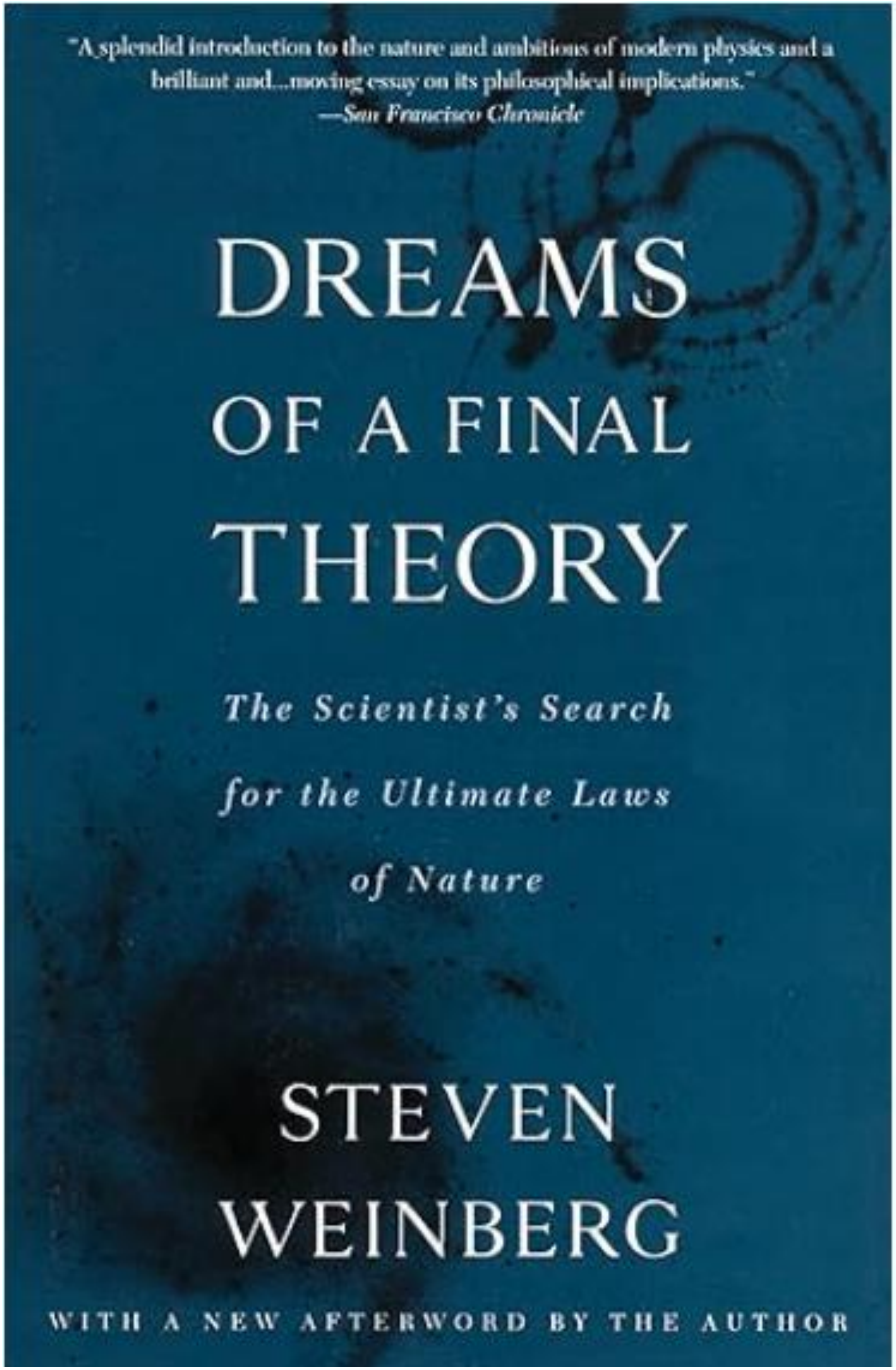


GUT – Grand Unified Theory
TOE – Theory of Everything



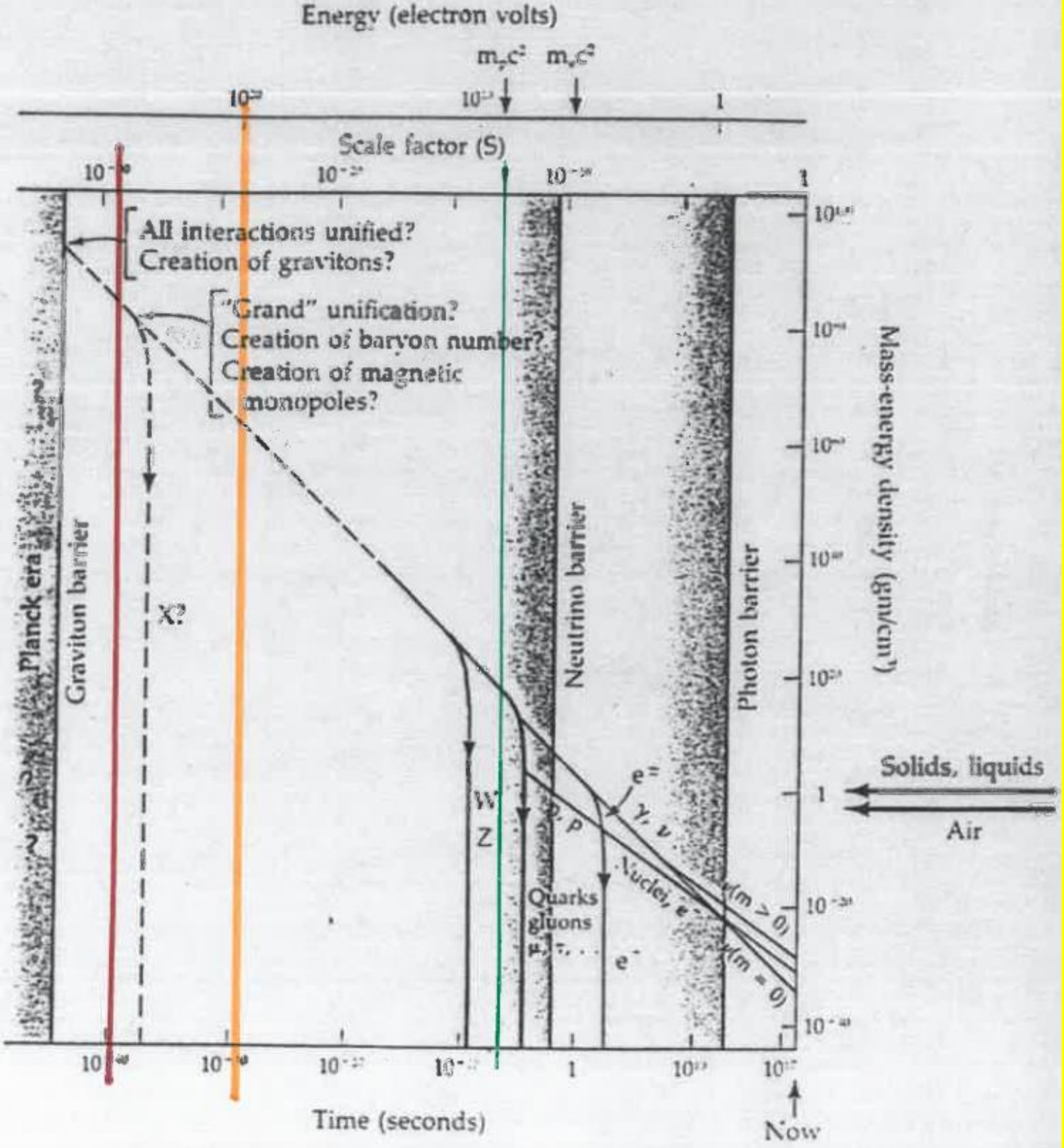
"A splendid introduction to the nature and ambitions of modern physics and a brilliant and...moving essay on its philosophical implications."
—*San Francisco Chronicle*

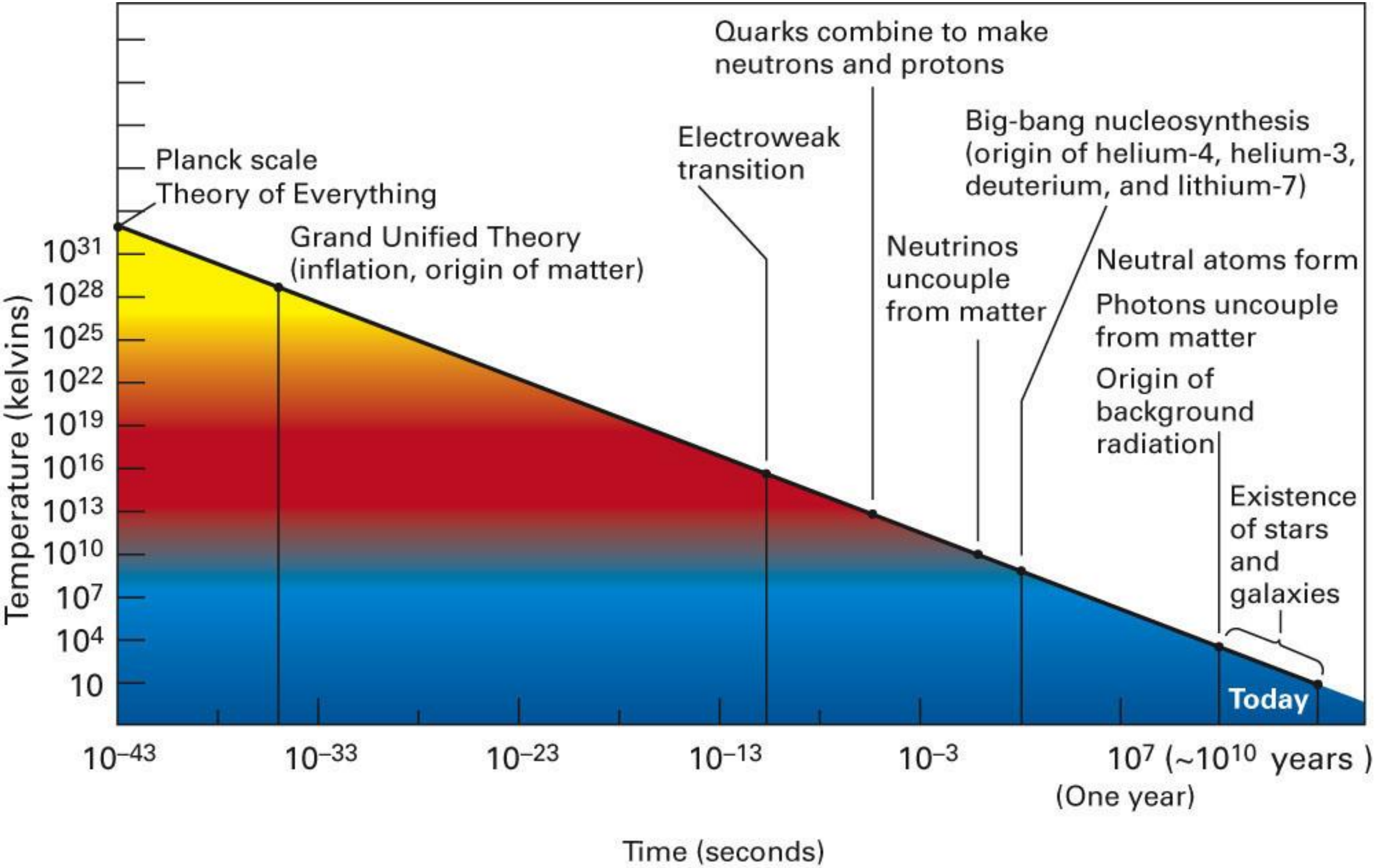
DREAMS OF A FINAL THEORY

The Scientist's Search for the Ultimate Laws of Nature

STEVEN WEINBERG

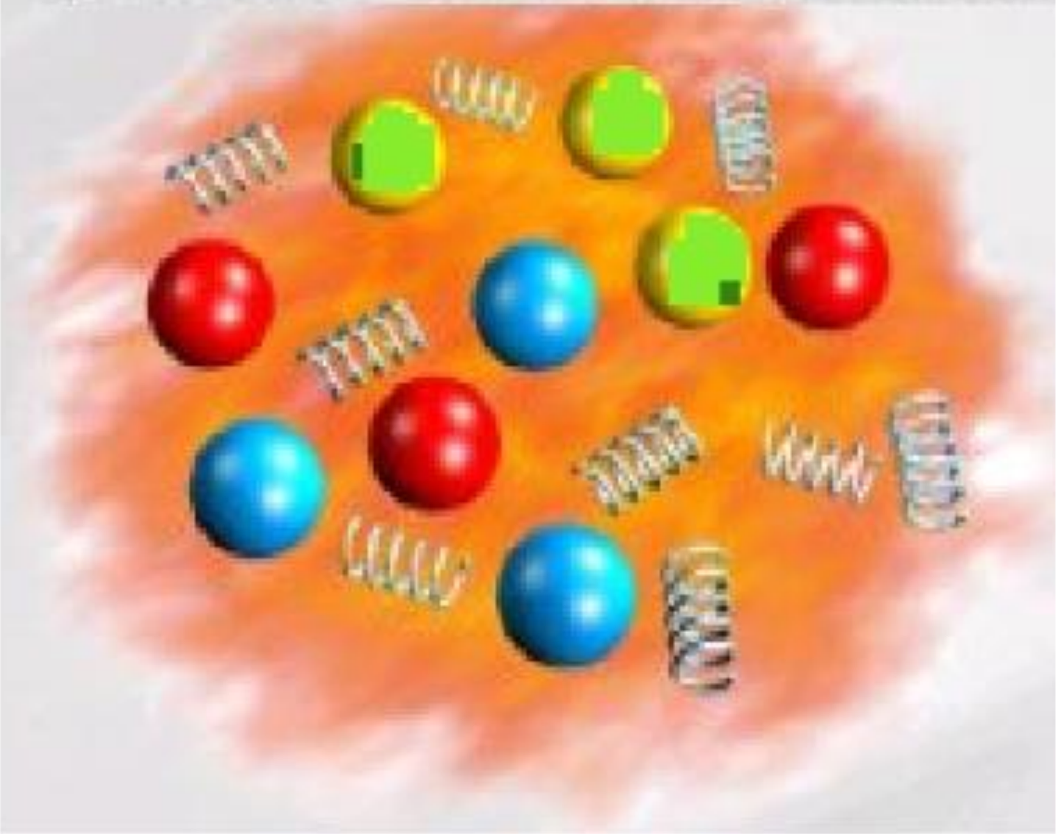
WITH A NEW AFTERWORD BY THE AUTHOR


Energy (electron volts)
m_pc^2
m_ec^2
1
Scale factor (S)
1
All interactions unified?
Creation of gravitons?
"Grand" unification?
Creation of baryon number?
Creation of magnetic monopoles?
X?
Planck era
Graviton barrier
Neutrino barrier
Photon barrier
W
Z
Quarks
gluons
$\mu, \tau, \ldots$
n, p
e^-
$e^\pm$
γ, ν
Nuclei, e^-
$\nu(m > 0)$
$\nu(m = 0)$
Mass-energy density (gm/cm³)
1
Solids, liquids
Air
Time (seconds)
1
Now



(Adapted from "Particle Accelerators Test Cosmological Theory" by David N. Schramm and Gary Steigman. © 1988 by Scientific American, Inc. All rights reserved. Drawn by Andrew Christie.)

Quark-Gluon-Plasma



Very early epoch of evolution of the Universe:

10^3 GeV – electroweak symmetry broken

100 GeV – baryogenesis – quark – antiquark annihilation

200 MeV – quark-gluon plasma recombination, formations of protons and neutrons

1 MeV – neutrinos decouple

0.5 MeV – electron positron annihilation

0.07 MeV – primordial nucleosynthesis

1 eV – energy-density of matter = energy-density of radiation

0.3 eV – recombination of plasma, formation on neutral atoms

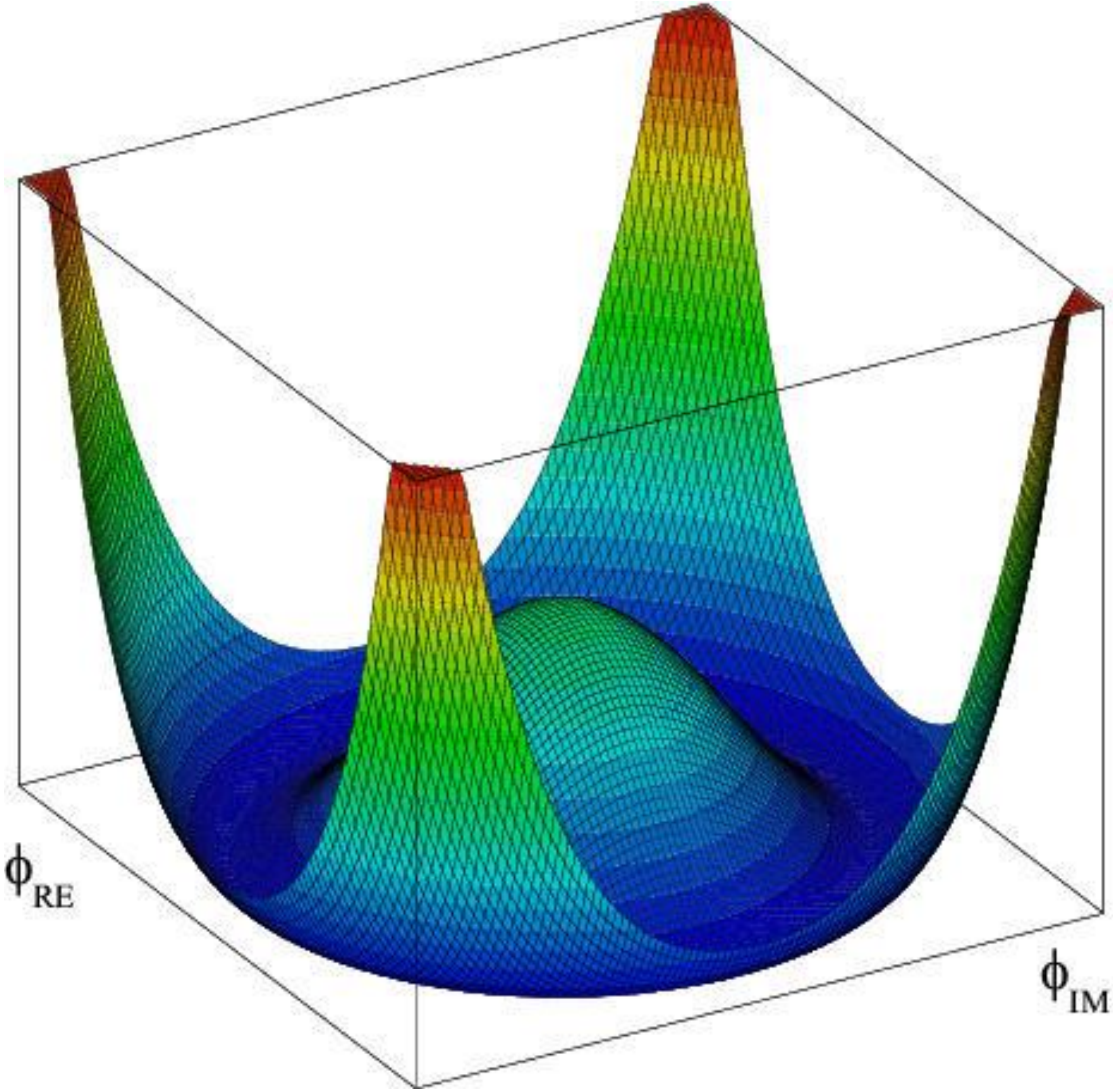
Scalar fields in cosmology

The Klein-Gordon equation on in FRWL background

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0\,.$$

$$\varrho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \quad p_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi)\,.$$

Equation of state $\quad p_\phi = w_\phi \varrho_\phi\,, \quad w_\phi = \dfrac{\frac{1}{2}\dot{\phi}^2 - V(\phi)}{\frac{1}{2}\dot{\phi}^2 + V(\phi)}\,.$



Potential of the Higgs field

$$3H^2 = \kappa^2\left(\rho + \frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

$$-\dot{a}^2 - 2a\ddot{a} = \kappa^2\left(pa^2 + a^2\left(\frac{1}{2}\dot{\phi}^2 - V(\phi)\right)\right)$$

$$-\frac{\dot{a}^2}{a^2} - 2\frac{\ddot{a}}{a} = \kappa^2\left(w\rho + \frac{1}{2}\dot{\phi}^2 - V(\phi)\right)$$

$$2\dot{H} + 3H^2 = -\kappa^2\left(w\rho + \frac{1}{2}\dot{\phi}^2 - V(\phi)\right) \ .$$

$\kappa^2 = \frac{8\pi G}{c^4}$ a – scale factor

Possible simple model of inflation

$\rho + \frac{1}{2}\dot{\phi}$ = 0, $V(\phi)$ = const

$3H^2 = \kappa^2 V(\phi)$ = const

$$a = \exp\left(\sqrt{\frac{\kappa^2 V(\phi)}{3}} \cdot t\right)$$

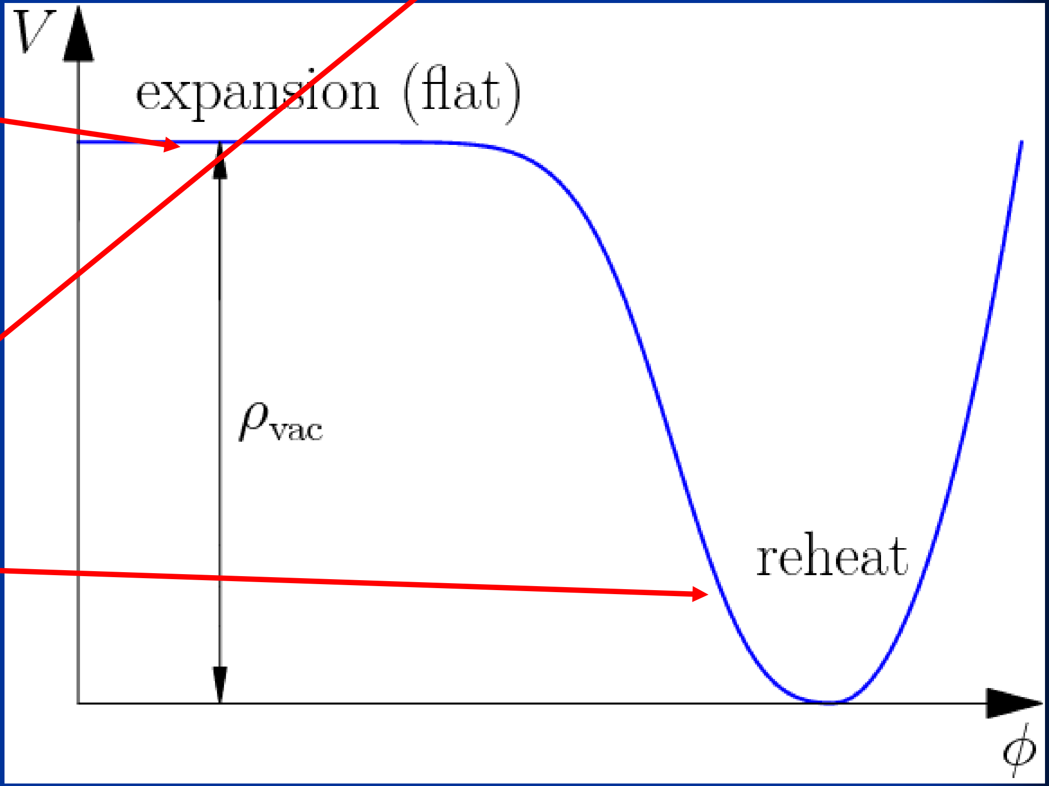
Rolling Models of Inflation

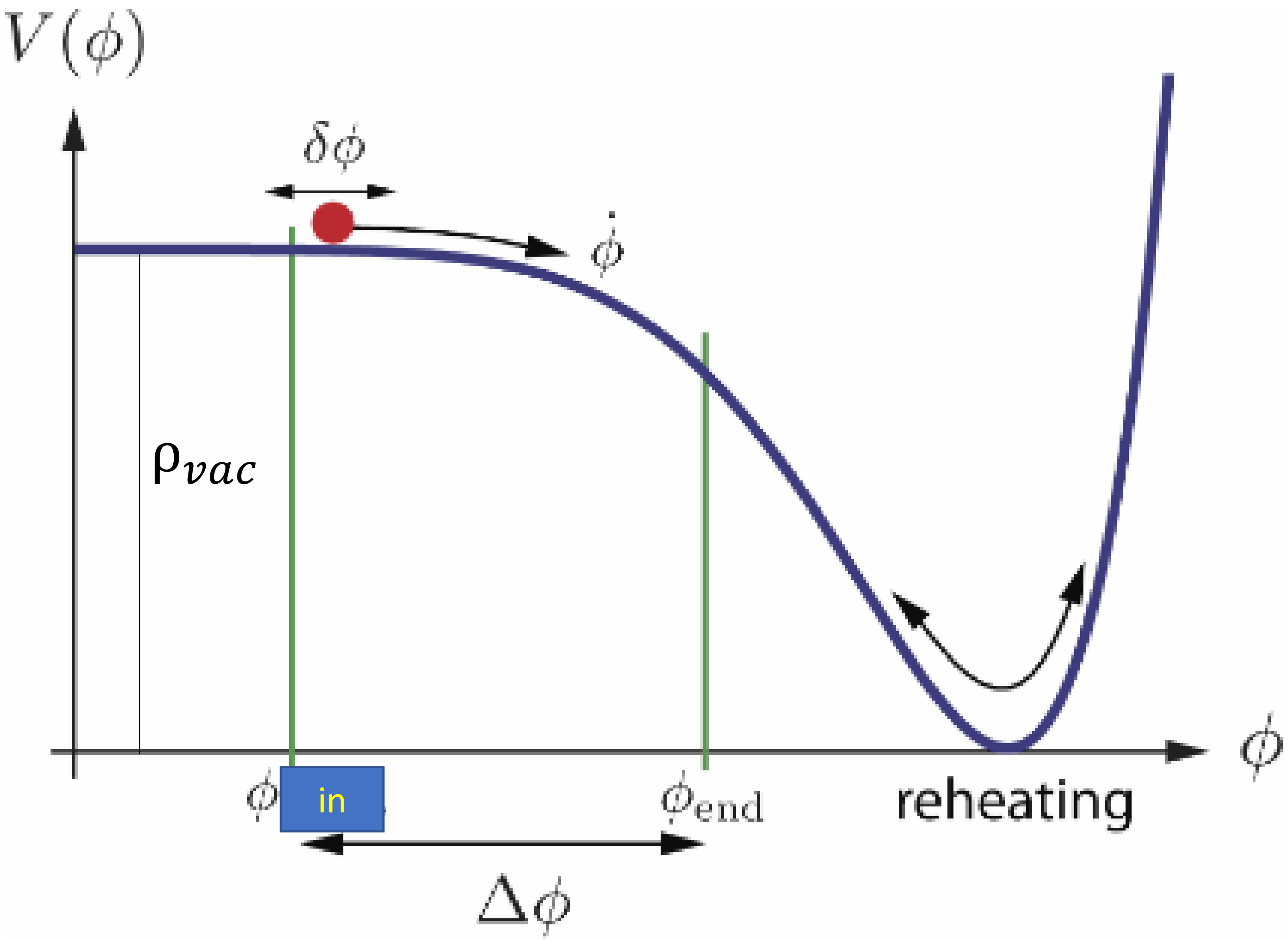
Linde (1982)
Albrecht & Steinhardt (1982)

- Equation of motion:

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0$$

- Flat region:
  - *V* almost constant
  - ρ_{vac} dominates energy density
- Decay of ϕ:
  - Particle production
  - Reheating





$V(\phi)$
$\delta\phi$
$\dot{\phi}$
ρ_{vac}
ϕ in
$\phi_{\rm end}$
reheating
ϕ
$\Delta\phi$

During the slow-roll inflation the scale factor is increasing exponentially:

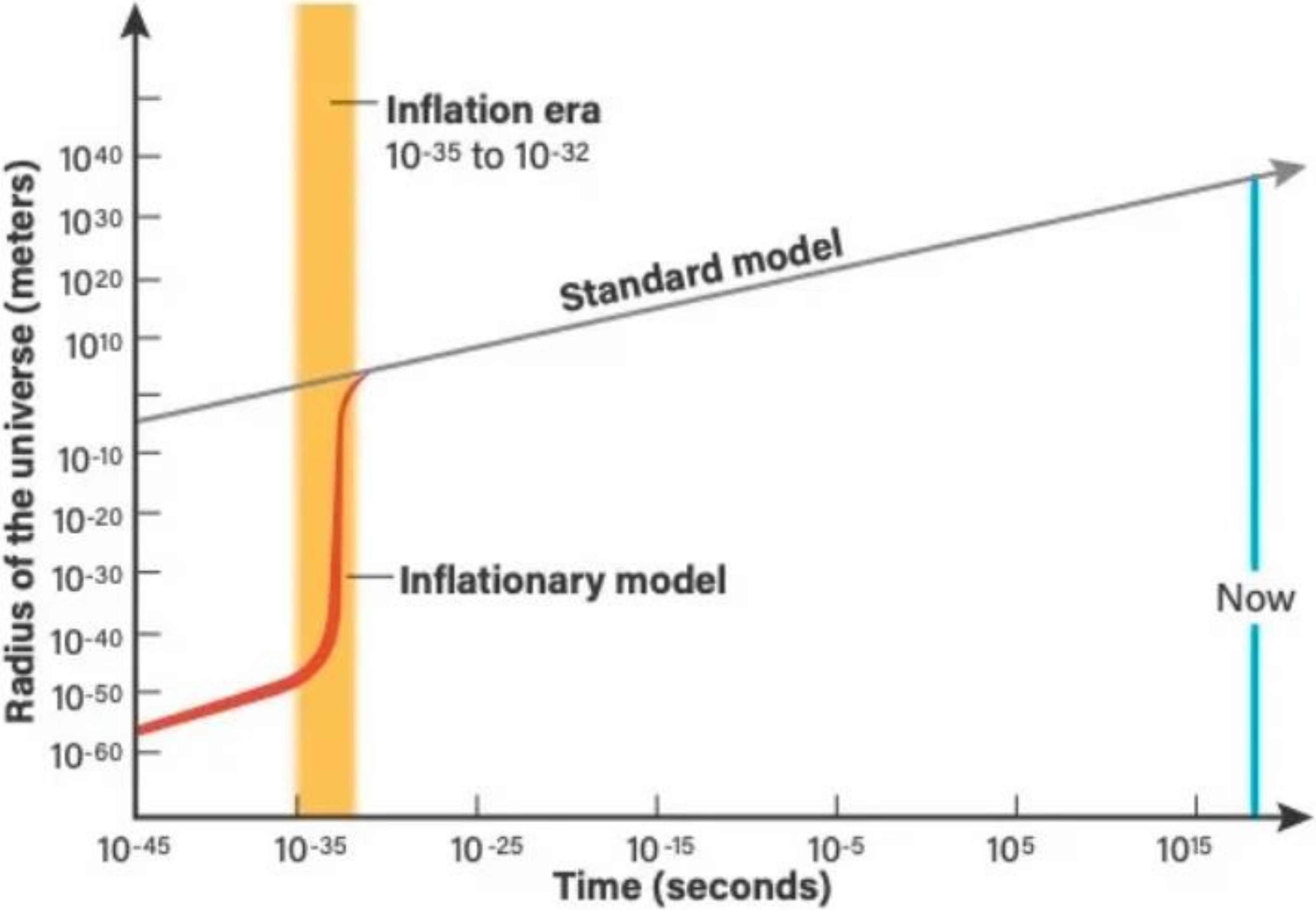
$$a = \exp\left(\sqrt{\frac{\kappa^2 V(\phi)}{3}} \cdot t\right)$$

$$\rho_m \sim a^{-3}\,; \quad T \sim a^{-1}$$

$$\frac{a_{final}}{a_{initial}} = \exp\left(\sqrt{\frac{\kappa^2 V(\phi)}{3}} \cdot (t_f - t_{in})\right) = \exp(60) \sim 10^{26}$$

So after the epoch of slow-roll inflation tempertaure of the Universe is almost zero and density of matter is almost zero.

During rapid oscillations around the true vacuum the universe becomes reheated and populated with particles. This epoch is usually identified with the Big Bang.



Inflation era
10-35 to 10-32
Standard model
Inflationary model
Now
Radius of the universe (meters)
1040
1030
1020
1010
10-10
10-20
10-30
10-40
10-50
10-60
10-45
10-35
10 -25
10 -15
10-5
105
1015
Time (seconds)

The scalar field driving inflation is quantum in nature.

A harmonic oscillator with frequency ω is governed by the equation:

$$\ddot{x} + \omega^2 x = 0\,.$$

Upon quantization, x becomes a quantum operator:

$$\hat{x} = \nu(\omega, t)\hat{a} + \nu^*(\omega, t)\hat{a}^\dagger\,, \tag{1}$$

$\nu(\omega, t) \sim e^{-i\omega t}$ - positive frequency solution of (1)

$\hat{a}$ - annihilation operator $\hat{a}|0>= 0$, $\dagger$ - denotes a Hermitian conjugate

$\hat{a}^\dagger$ - creation operator, adds one particle to a state upon which it acts

$$[\hat{a}, \hat{a}^\dagger] = \hat{a}\hat{a}^\dagger - \hat{a}^\dagger\hat{a} = 1\,,$$

$$[\hat{a}, \hat{a}] = [\hat{a}^\dagger, \hat{a}^\dagger] = 0$$

Assuming that the mass of the oscillator is equal to 1, we can introduce a momentum operator $\hat{p}$ as

$$\hat{p} = \frac{d\hat{x}}{dt}, \;\Rightarrow [\hat{x}, \hat{p}] = i\,.$$

We can now compute the quantum fluctuations of the operator $\hat{x}$ in the ground state $|0>$

$$< |\hat{x}|^2 >=< 0|\hat{x}\hat{x}^\dagger|0 >=< 0|(\nu^*\hat{a}^\dagger + \nu\hat{a})(\nu\hat{a} + \nu^*\hat{a}^\dagger|0 >=$$

$$|\nu(\omega,t)|^2 < 0|\hat{a}\hat{a}^\dagger|0 >= |\nu(\omega,t)|^2 < 0|[\hat{a},\hat{a}^\dagger] + \hat{a}^\dagger\hat{a}|0 >= |\nu(\omega,t)|^2\,,$$

$$\text{finally, we have} \;\; < |\hat{x}|^2 >= \frac{1}{2\omega}\,.$$

Quantum scalar field $\phi(t, \vec{x})$

$$\mathcal{L}(\phi) = \frac{1}{2}(\partial_t \phi)^2 - \frac{1}{2}(\partial_x \phi)^2 - \frac{1}{2} m \phi^2 - V(\phi)\,,$$

Momentum is defined as $\pi = \partial_t \phi\,.$

Equal time commutation relations are:

$$[\phi(x), \phi(y)] = [\pi(x), \pi(y)] = 0\,, \text{ and } [\phi(x), \pi(y)] = i\delta(x - y)\,.$$

It is convenient to introduce Fourier transformed fields

$$\phi_k = \int \phi(x) e^{-ikx} dx\,, \quad \pi_k = \int \pi(x) e^{-ikx} dx\,,$$

reality of the fields implies:

$$\phi_{-k} = \phi_k^\dagger\,, \quad \pi_{-k} = \pi_k^\dagger\,.$$

The classical Hamiltonian may be expressed in Fourier modes as:

$$H = \frac{1}{2} \sum_{k=-\infty}^{\infty} \left(\pi_k \pi_k^\dagger + {\omega_k}^2 \phi_k \phi_k^\dagger \right),$$

where $\omega_k = \sqrt{k^2 + m^2}$.

Creation and annihilation operators can be defined as:

$$a_k = \frac{1}{\sqrt{2\omega_k}} (\omega_k \phi_k + i\pi_k),\ \ a_k^\dagger = \frac{1}{\sqrt{2\omega_k}} (\omega_k \phi_k^\dagger - i\pi_k^\dagger).$$

They satisfy the standard commutation relations:

$$[a_k, a_k^\dagger] = 1.$$

Similarity with the case of harmonic oscillator suggests that

$$< 0|\phi_k \phi_k^\dagger|0 > \neq 0$$

To find out how the quantum fluctuations of the field Φ influence the evolution of the very earl universe it is necessary to consider small perturbations of the field $\Phi(t,x) \rightarrow \Phi(t) + \delta\Phi(t,x)$ and the geometry of the universe. This involves the whole non-linear set of the Einstein equations. It is outside the scope of this course and so we will discuss only the final results of such analysis. The final equation that governs the perturbations $\delta\Phi$ is

$$\delta\phi^{''} + 2aH\delta\Phi^{'} + k^2\delta\Phi = 0\,,$$

where $'$ denotes derivative with respect to the conformal time defined as $\tau = \int \frac{dt}{a(t)}$.

In analogy to the case of harmonic oscillator one can find the power spectrum of fluctuations in $\delta\Phi$ as

$$P_{\delta\Phi} = \frac{H^2}{2k^3}\,.$$

Brief summary:

Inflation predicts that quantum-mechanical perturbations in the very early universe are first produced when the relevant scales are causally connected. Then these scales are whisked outside the horizon by inflation, only to re-enter much later to serve as initial conditions for the growth of structure in the universe. The perturbations are best described in terms of their Fourier modes. The mean of a given Fourier mode, for example for the gravitational potential, is zero:

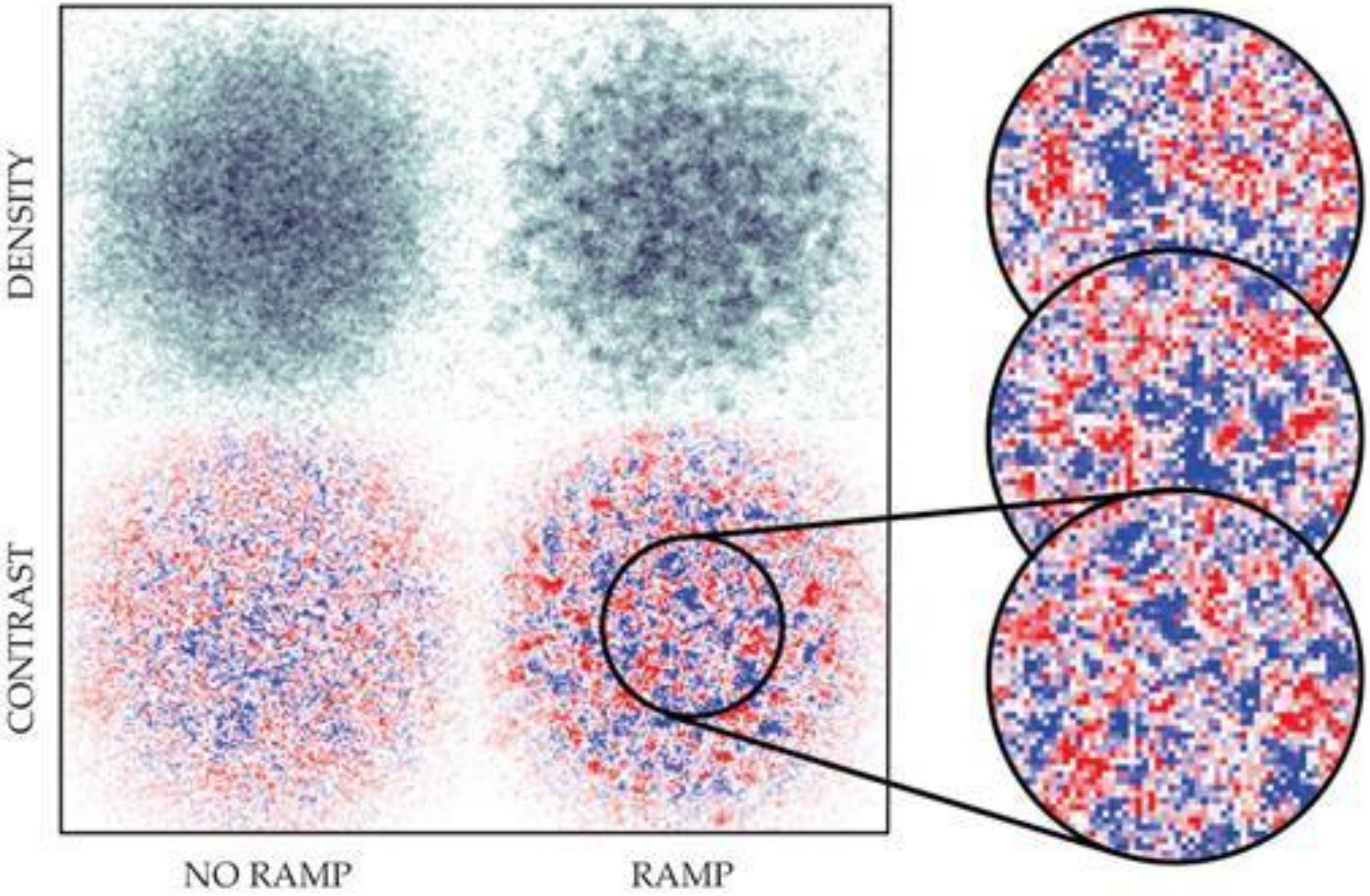
$$< \Phi(\mathbf{k}) >= 0 \, .$$

Further, any given Fourier mode is uncorrelated with a different one. However, a given mode has nonzero variance, so

$$< \Phi(\mathbf{k})\Phi^*(\mathbf{k}^{'}) >= P_\Phi(\mathbf{k})(2\pi)^3\delta_D^{(3)}(\mathbf{k}-\mathbf{k}^{'}) \, ,$$

the Dirac delta function is enforcing the independence of the different modes. This is the characteristic feature of a Gaussian random process.

A spectrum in which $k^3 P_{\Phi(k)}$ is constant (i.e., does not depend on k) is called a scale-invariant or scale-free spectrum.



By ramping a magnetic field in the vicinity of a so-called Feshbach resonance, researchers trick a Bose–Einstein condensate into behaving like the expanding early universe—even though the condensate's physical size remains the same. Under the ramped field, the condensate picks up density fluctuations; in the early universe, such quantum field fluctuations became particle–antiparticle pairs. The cutouts at right show how additional data can be gathered by repeating the experiment. (Adapted from ref. 1.)

Cosmic inflation is a quantum process.

Creation of primordial density perturbations of random nature.

Smoothing out early irregularities.
The Universe after inflation becomes flat and homogeneous and isotropic

Predictions of the inflationary model:

- The universe is flat
- Density perturbations have flat spectrum
- Primordial gravitational waves

Self gravitating gas

The continuity equations

$$\frac{\partial \varrho}{\partial t} + \nabla(\varrho \vec{v}) = 0\,, \tag{1}$$

The Euler equation

$$\frac{D}{dt}\vec{v} = \frac{\partial \vec{v}}{\partial t} + (\vec{v} \cdot \nabla)\vec{v} = -\frac{1}{\varrho}\nabla p - \nabla \varphi\,, \tag{2}$$

The Poisson equation

$$\triangle \varphi = 4\pi G \varrho\,. \tag{3}$$

Newtonian cosmological model - initial state: homogeneous and isotropic distribution of matter

$$\vec{r}(t) = R(t)\vec{r}_0, \;\; \vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{\dot{R}(t)}{R(t)}\vec{r}(t) = H(t)\vec{r}(t)\,, \;\; \varphi(t) = \frac{4\pi G}{3}\varrho(t)\cdot r^2\,.$$

$$(1) \rightarrow \dot{\varrho} + 3\frac{\dot{R}(t)}{R(t)}\varrho = 0\,, \;\; \rightarrow \varrho(t)R^3(t) = \mathrm{const}\,.$$

$$(2) \rightarrow \ddot{R}(t)\vec{r}_0 = -\frac{2\pi G}{3}\varrho(t)\vec{\nabla}r^2\,, \;\; \rightarrow \frac{\ddot{R}}{R} = -\frac{4\pi G}{3}\varrho(t)\,.$$

Let us consider small perturbations:

Types of perturbations:
scalar - example - density perturbations ,
vector - example - velocity perturbations ,
tensorial - example - perturbations of geometry .

$$\varrho \to \varrho(t) + \delta\varrho(t,\vec{x})\,,\;\; \vec{r} \to \vec{r} + \delta\vec{r}(t,\vec{x})\,,\;\; \vec{v} \to \vec{v} + \delta\vec{v}(t,\vec{x})\,, \varphi \to \varphi(t) + \delta\varphi(t,\vec{x})\,.$$

Substituting these perturbed quantities into (1), (2) and (3) and keeping only the linear terms in these equations and assuming that the perturbed quantities can be represented as $\frac{\delta\varrho}{\varrho_0(t)} \sim \mu(t)e^{i\vec{k}\cdot\vec{x}}$ after some algebraic manipulations we get the following equation:

$$\ddot{\mu} + 2H\dot{\mu} - (4\pi G\varrho_0(t) - c_s^2k^2)\mu = 0\,, \qquad (4)$$

where $H = \frac{\dot{a}}{a}$ is the Hubble constant and $c_s = \sqrt{\frac{\partial p}{\partial \varrho}}$ is the velocity of sound, and $k = \frac{2\pi}{L}$ where L describes the scale of the perturbation .

In the general case the equation (4) does not have exact solutions. There are two special cases:

a) $k \to 0$ - large scale perturbations, and

b) $c_s^2 k^2 >> 4\pi G \varrho_0(t)$ - small scale perturbations.

Case b) describes damped harmonic oscillations.

Let us discuss the case a) in a flat universe dominated by matter. The background is then described by:

$$a(t) = a_0 t^{2/3}\,, H(t) = \frac{2}{3}\frac{1}{t}\,,\ \ \varrho(t) = \frac{1}{6\pi G t^2}\,.$$

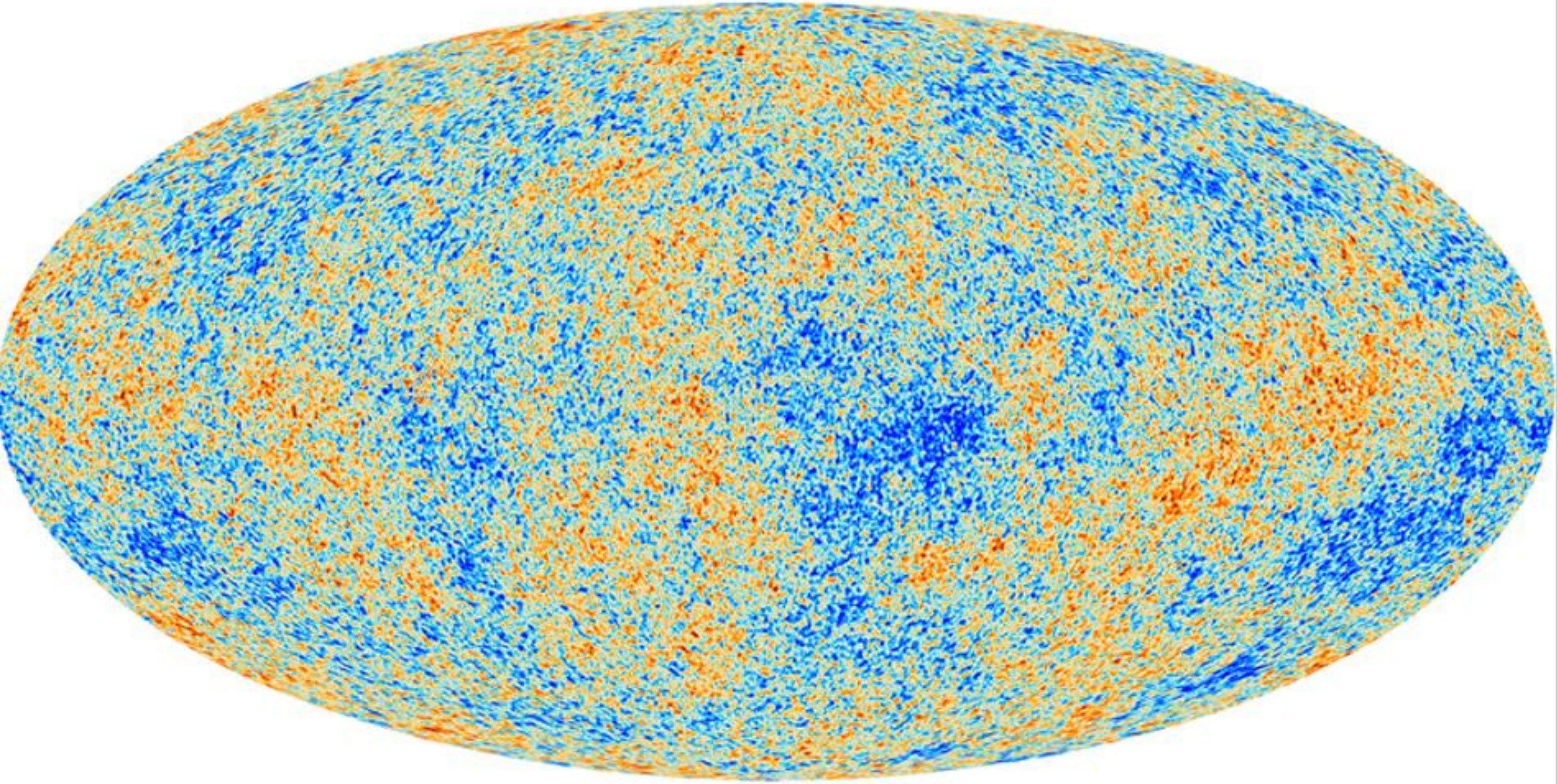
The equation (4) reduces to:

$$\ddot{\mu} + \frac{4}{3t}\dot{\mu} - \frac{2}{3t^2}\mu = 0\,,$$

It is easy to check that the generals solution is

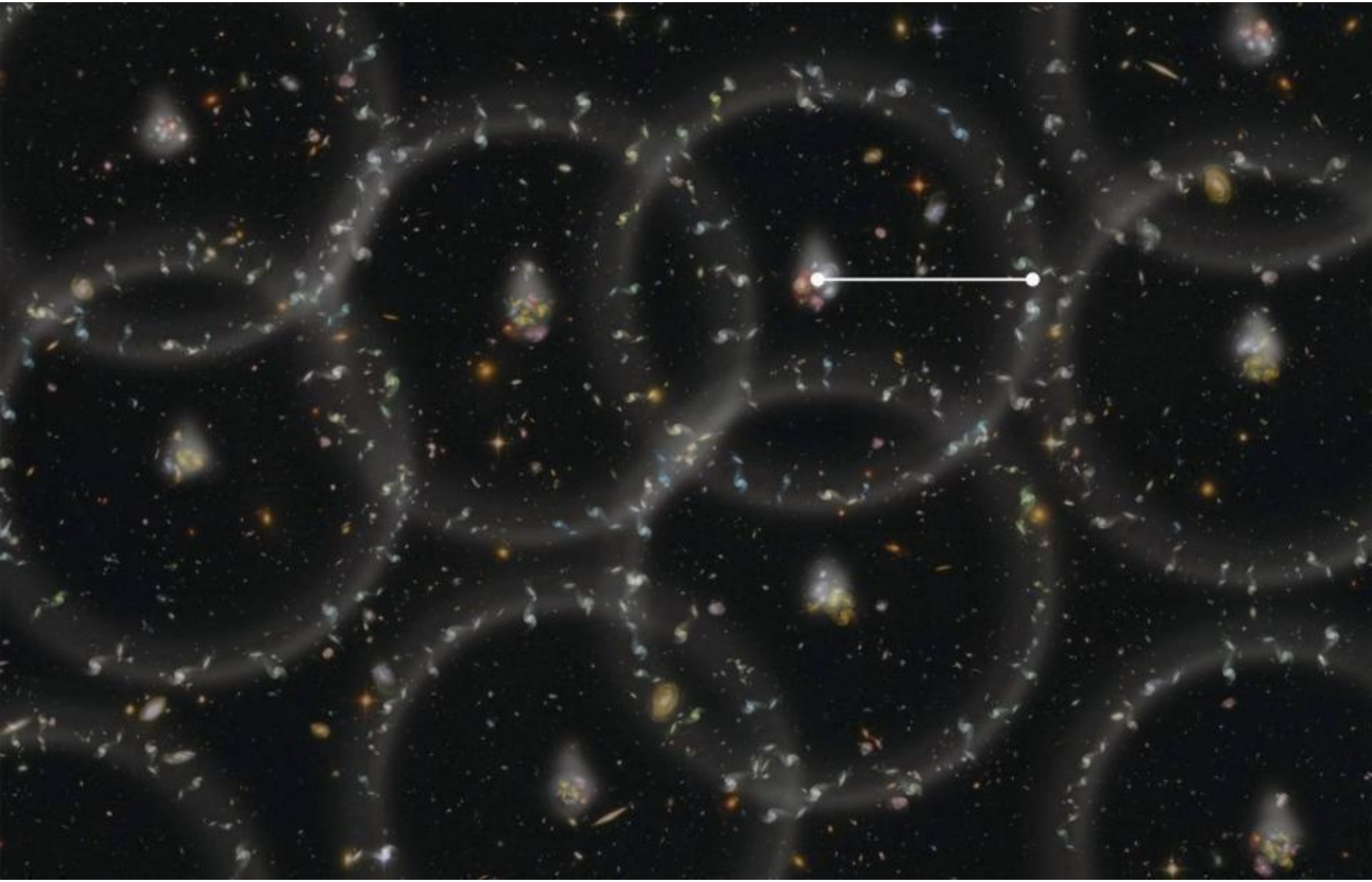
$$\mu(t) = At^{2/3} + Bt^{-1}, \text{where A and B are constants.}$$

Initial conditions

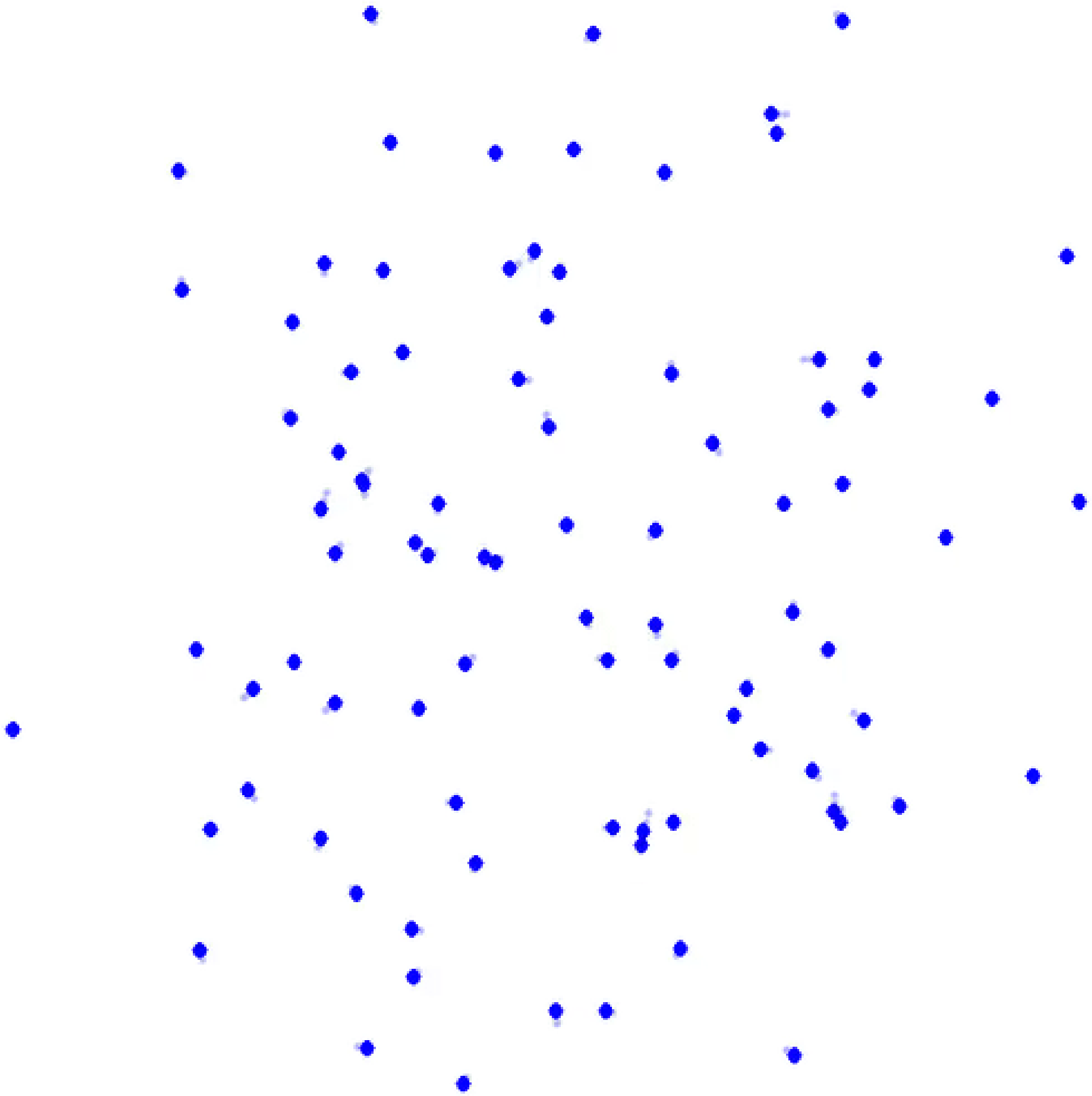


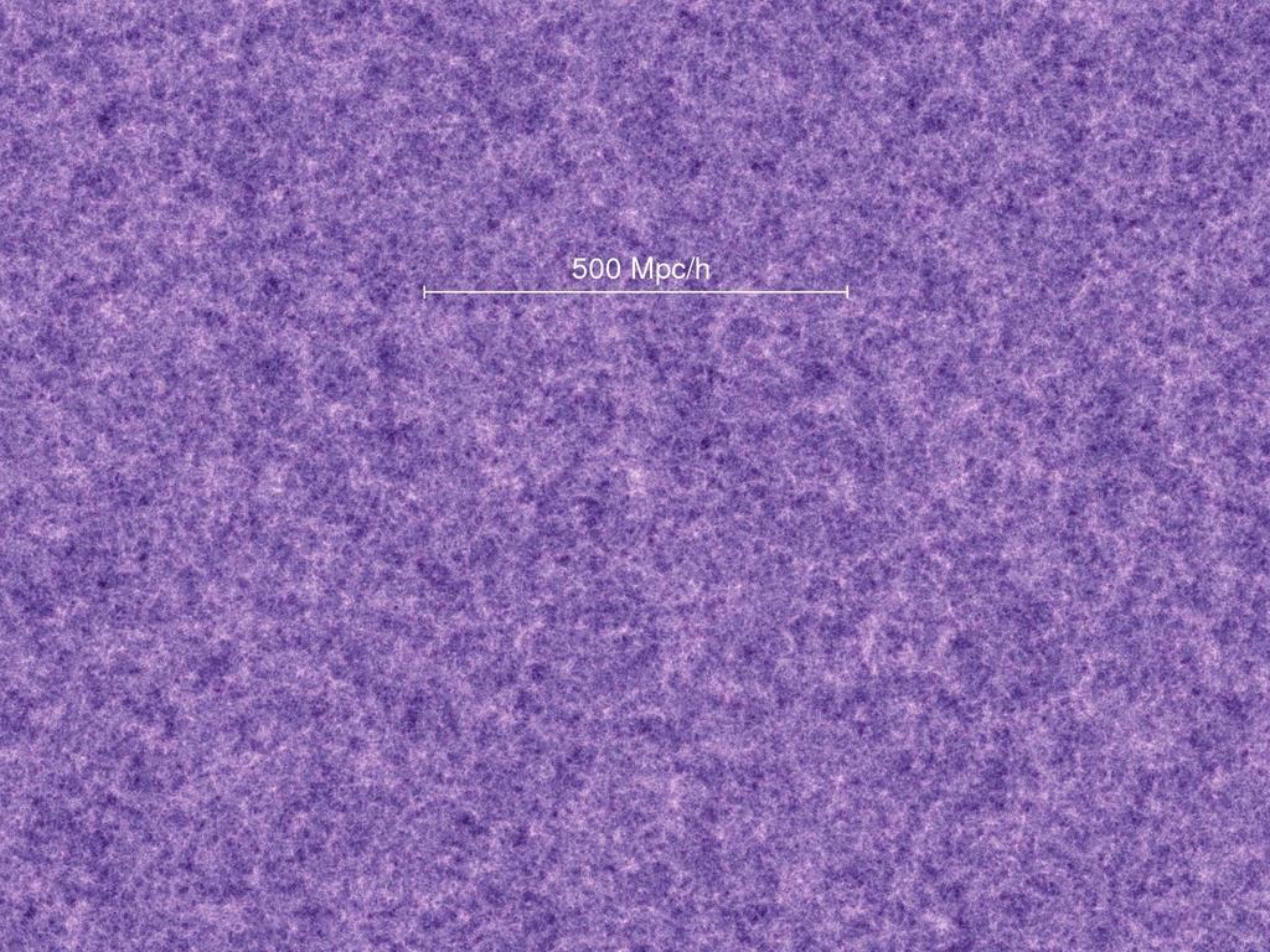
Random quantum generated density perturbations filtered by interaction with radiation. Small scale perturbations dumped. The density perturbations start to grow after recombination (matter-radiation decoupling).

https://svs.gsfc.nasa.gov/vis/a010000/a013700/a013768/13768_BAO_Narr_1080_Best.mp4



N body simulations

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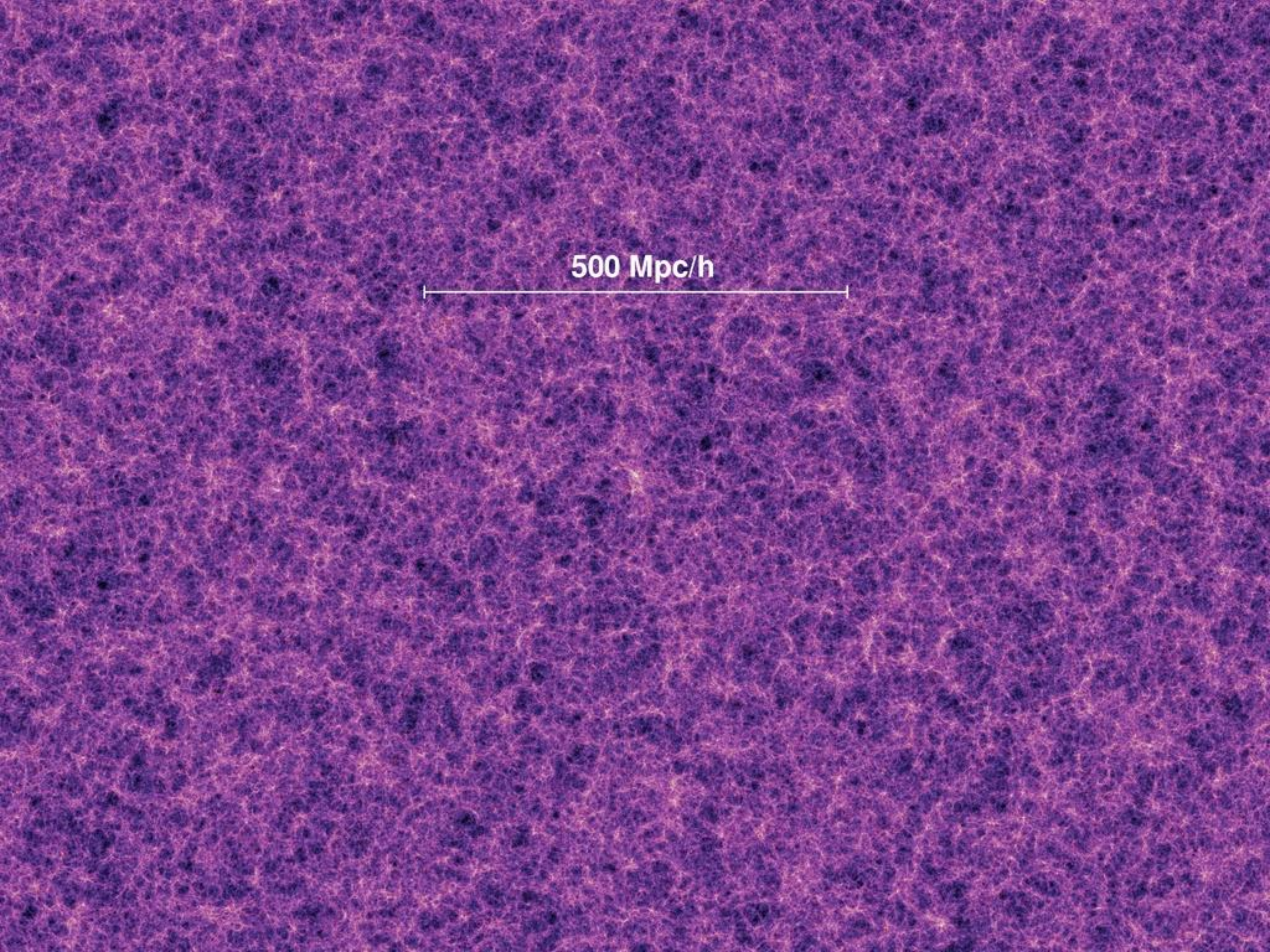
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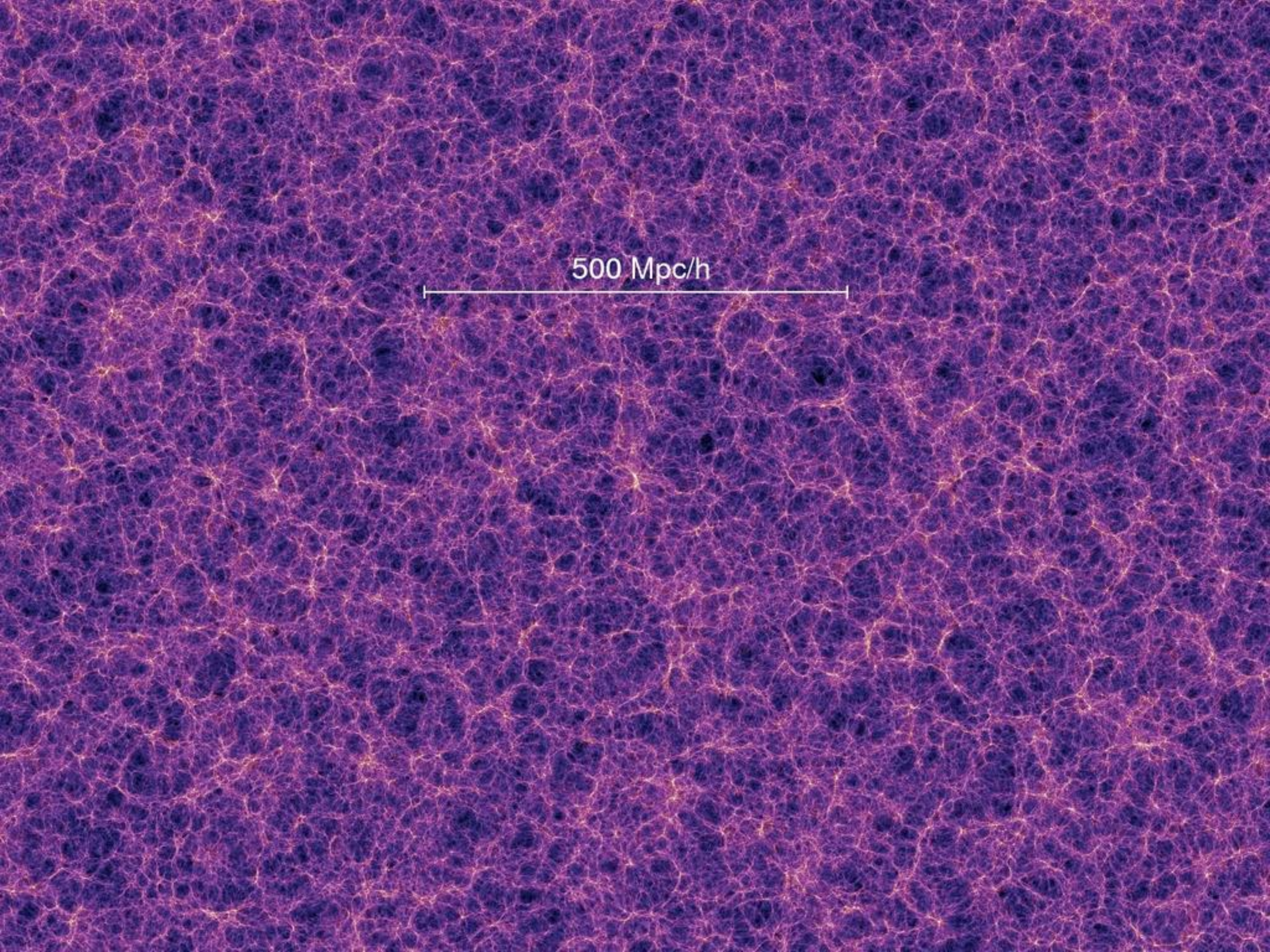
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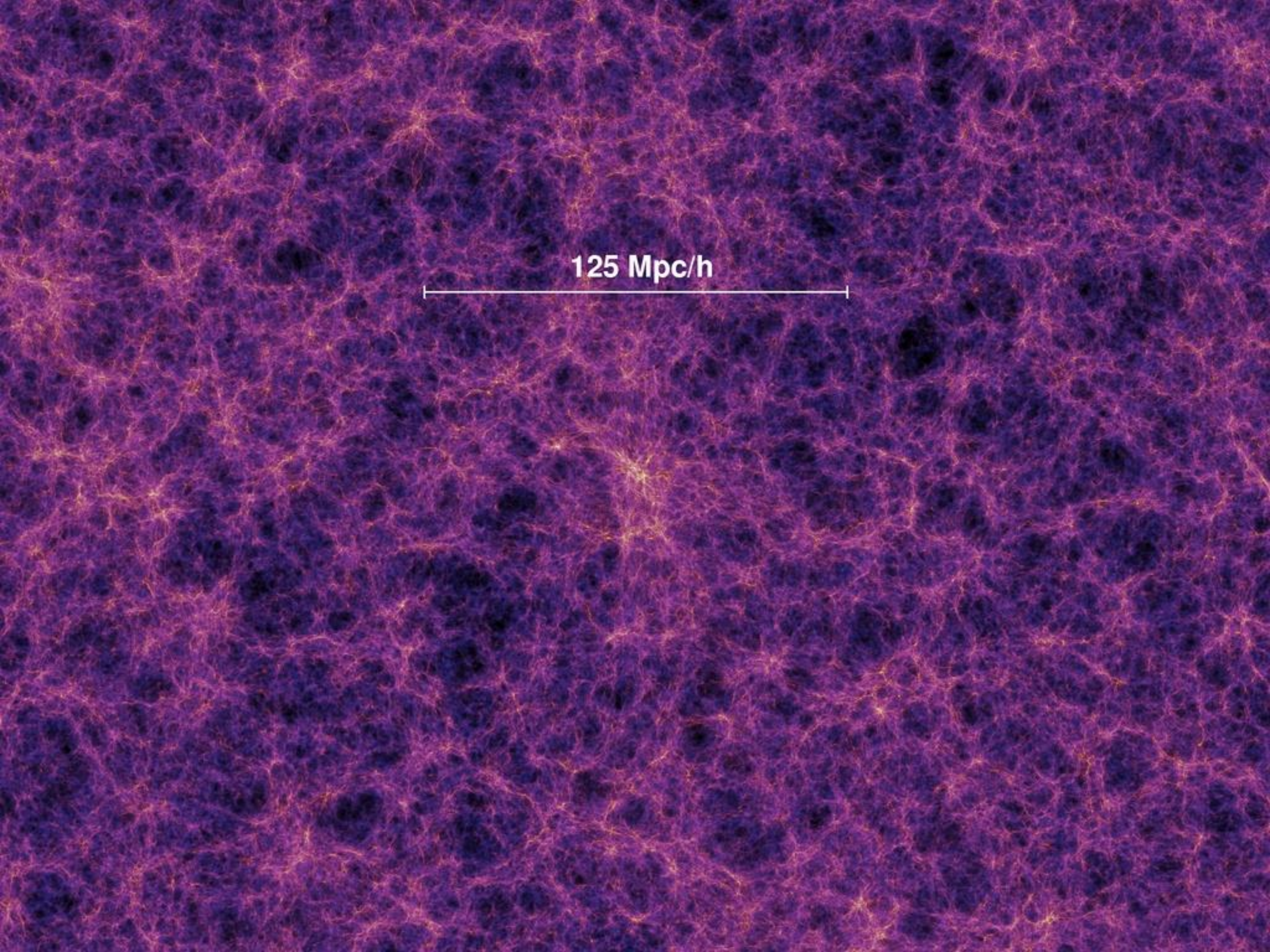
125 Mpc/h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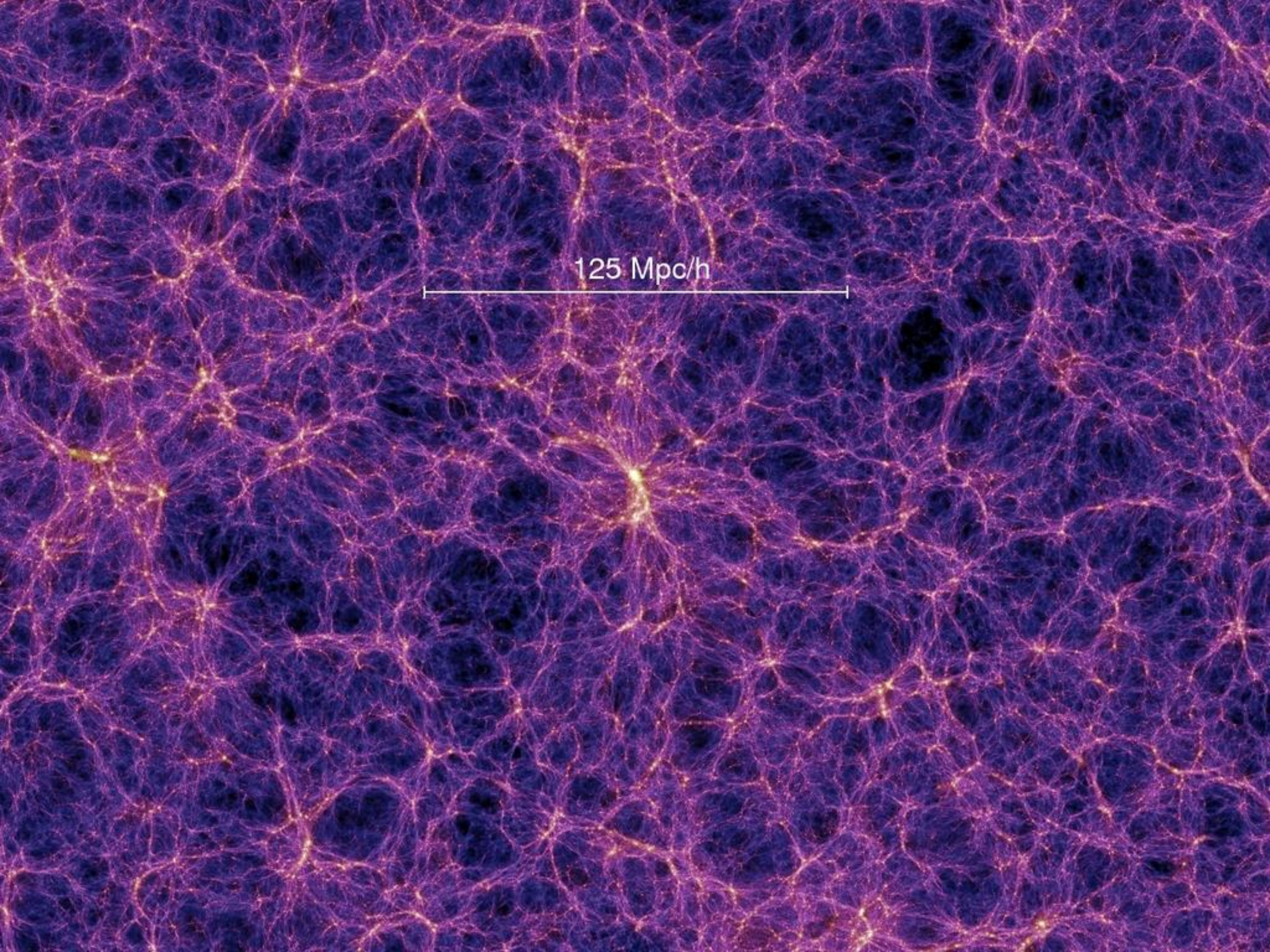
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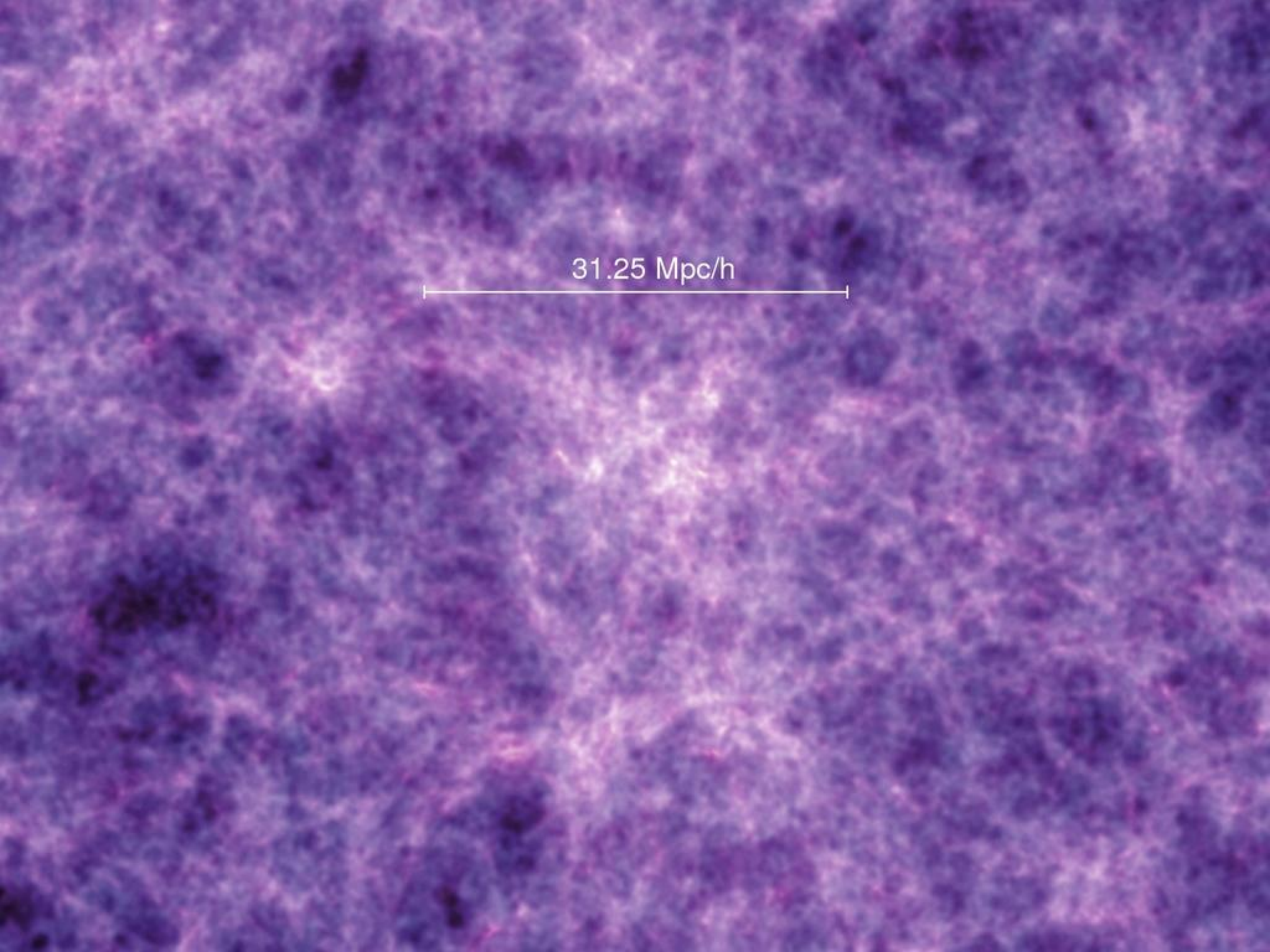
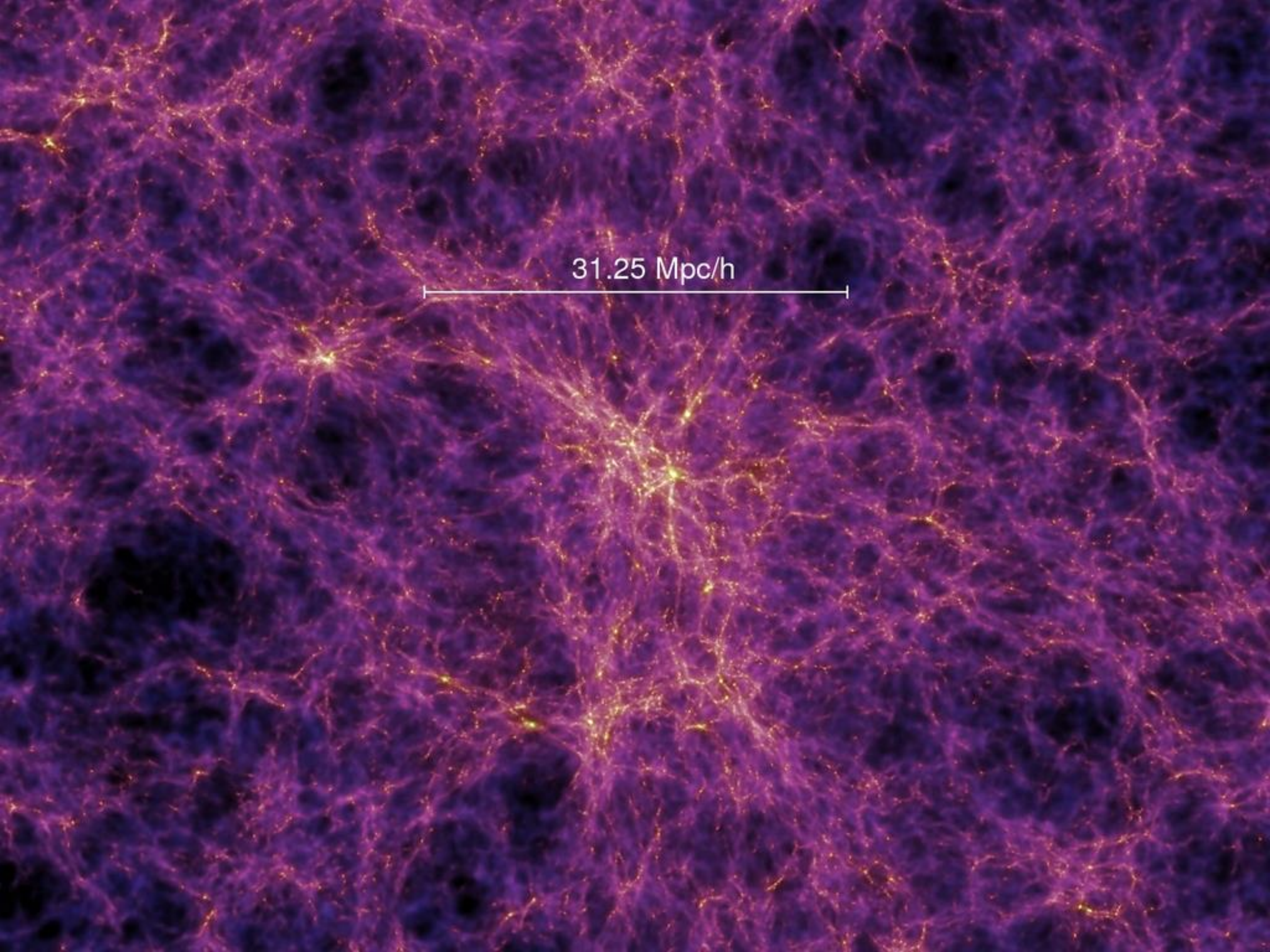

31.25 Mpc/h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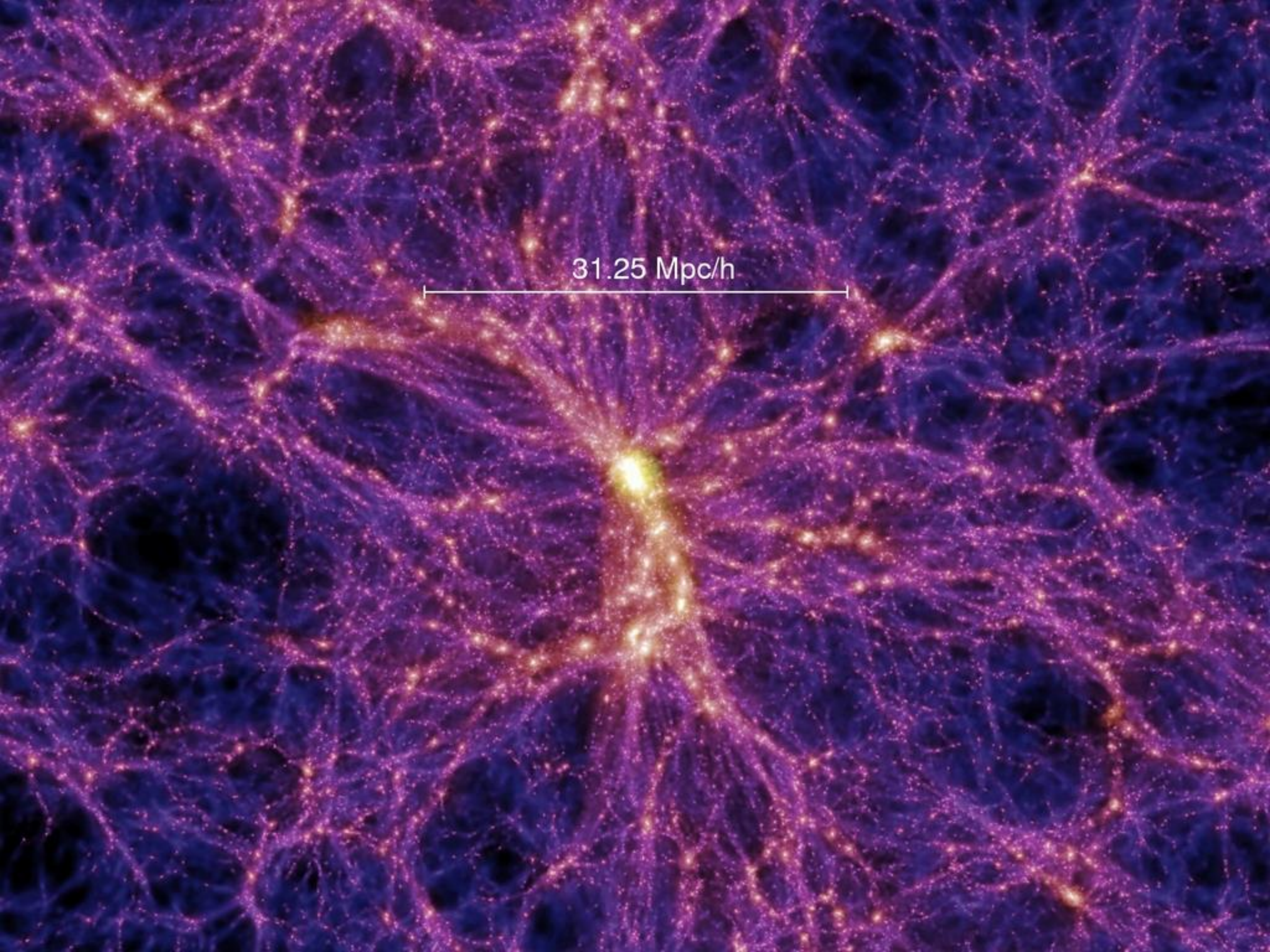
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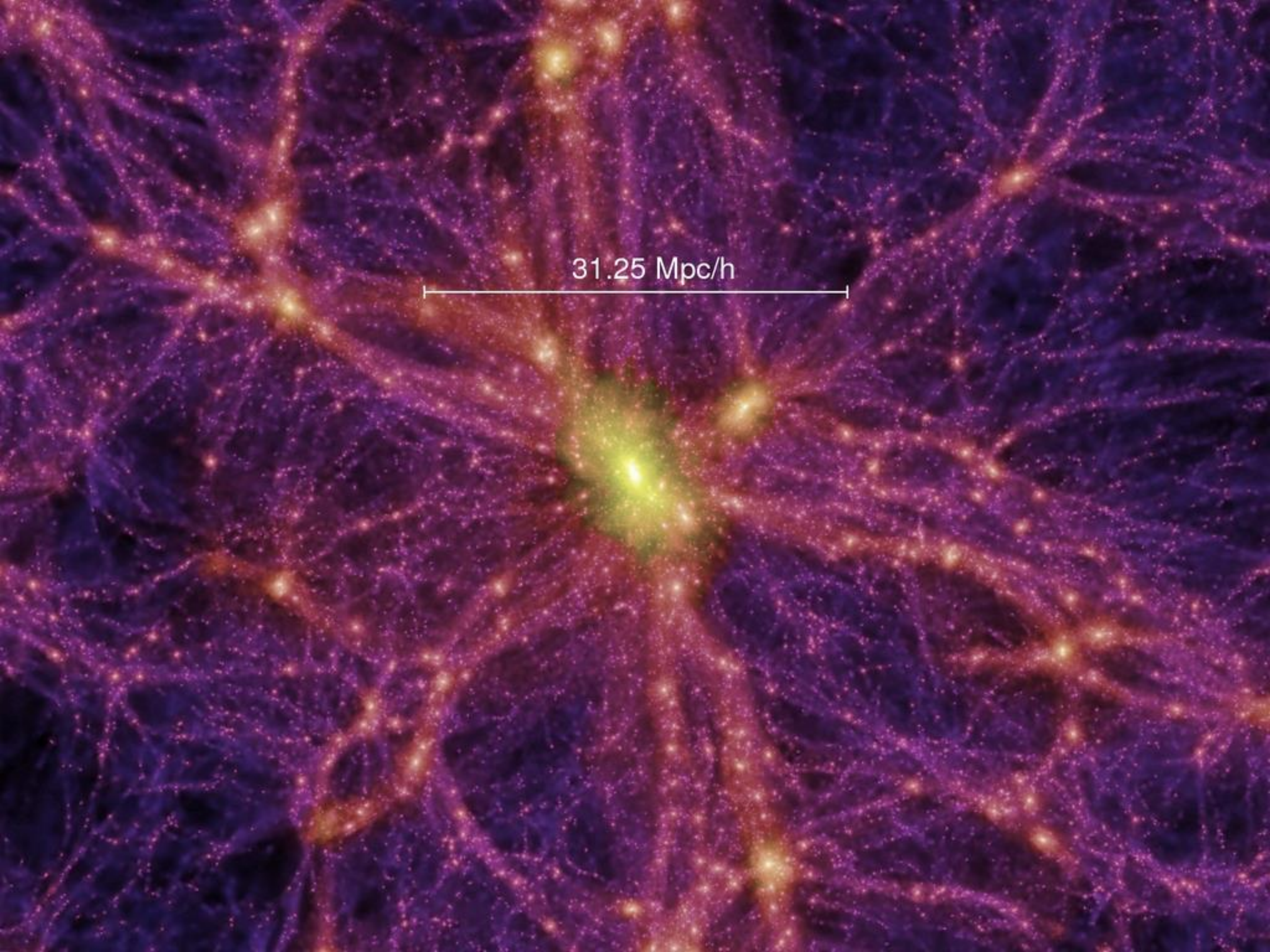
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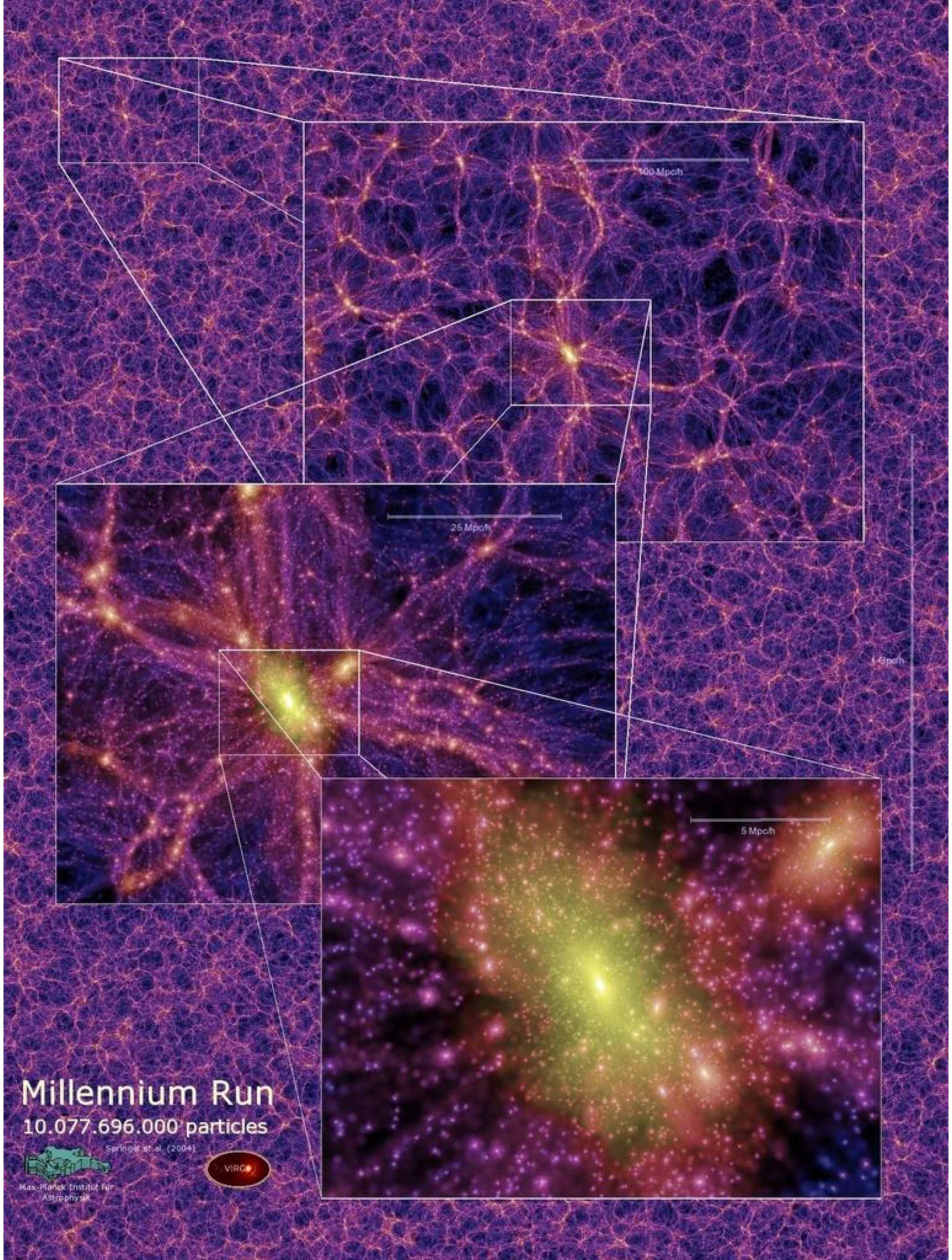
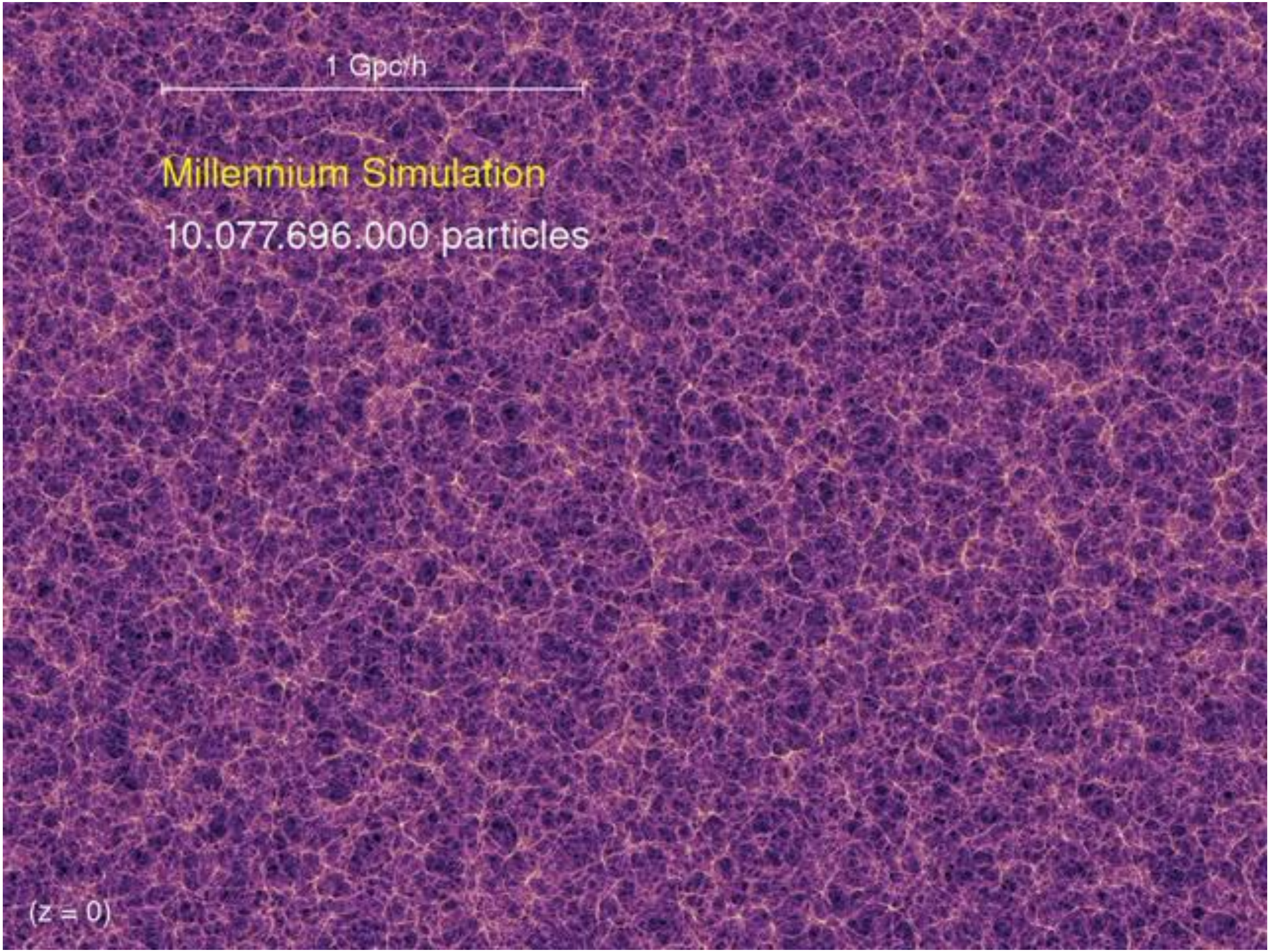
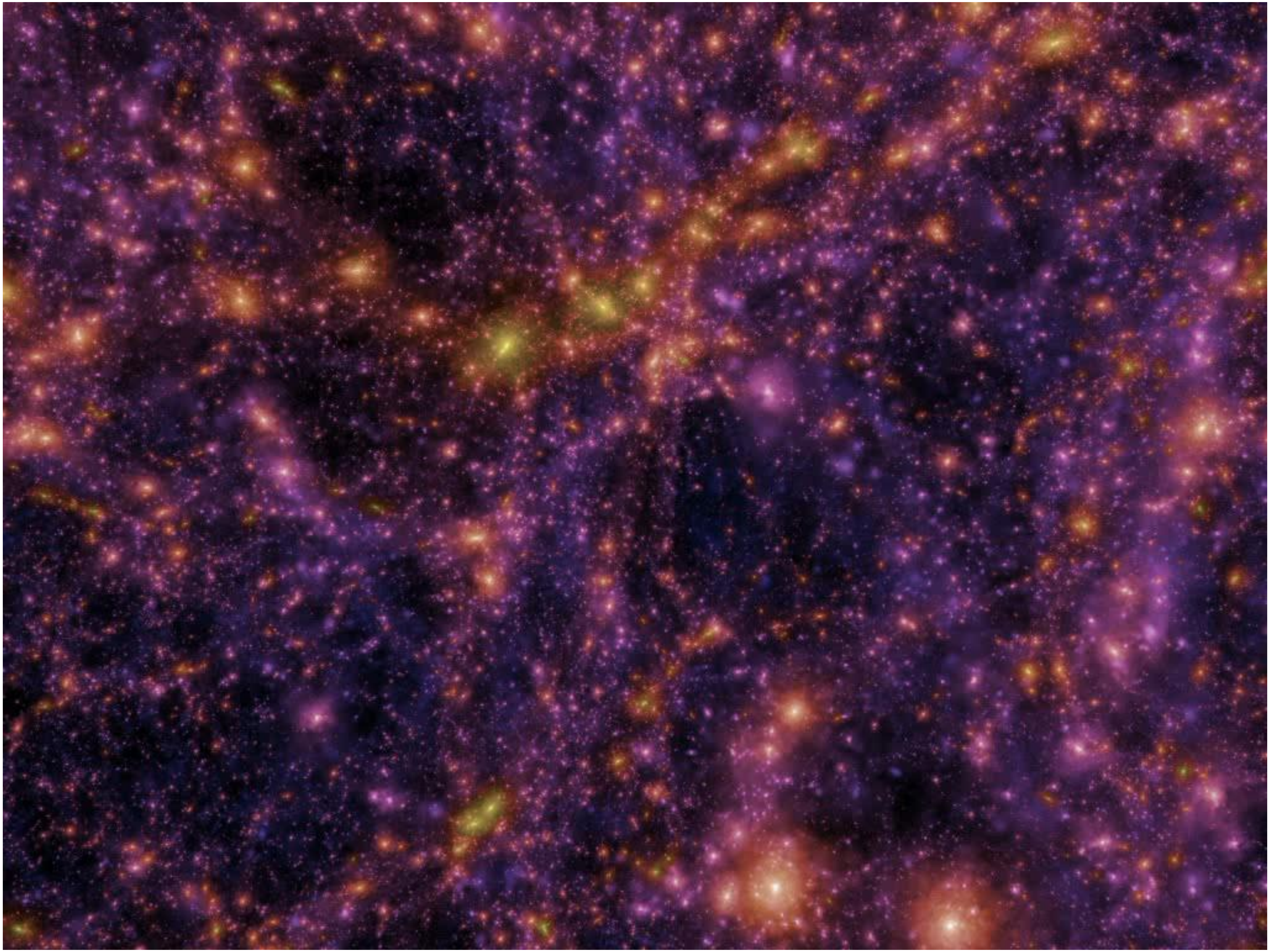
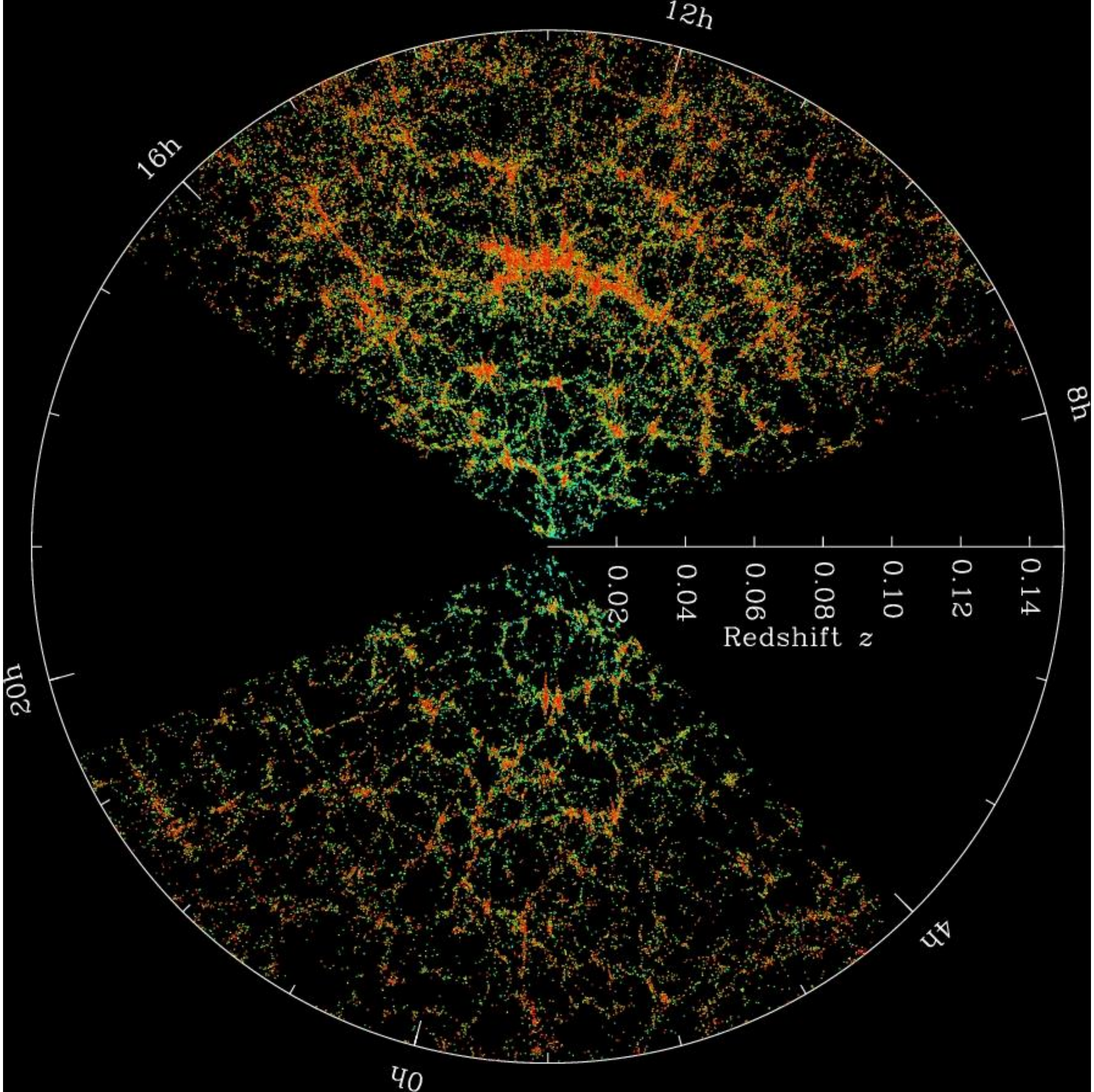

100 Mpc/h
25 Mpc/h
1 Gpc/h
5 Mpc/h
Millennium Run
10.077.696.000 particles
Springel et al. (2004)
Max-Planck Institut für Astrophysik
VIRGO

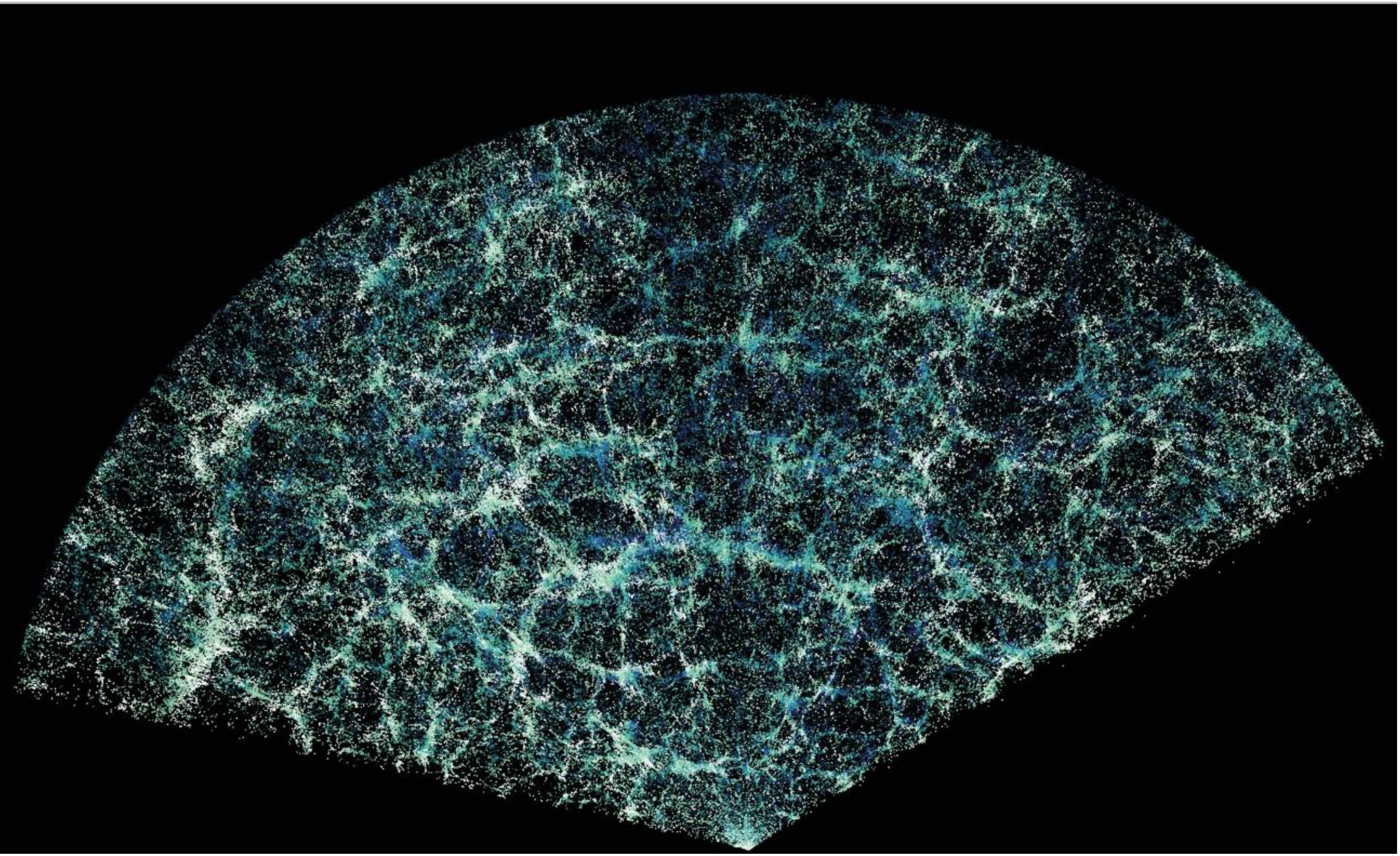

1 Gpc/h
Millennium Simulation
10.077.696.000 particles
(z = 0)



https://wwwmpa.millenium simulation.mpg.de/galform/virgo/millennium/

Mass resolution ~ $9{\cdot}10^8\ M_\odot$


12h
16h
8h
20h
4h
0h
0.02
0.04
0.06
0.08
0.10
0.12
0.14
Redshift z

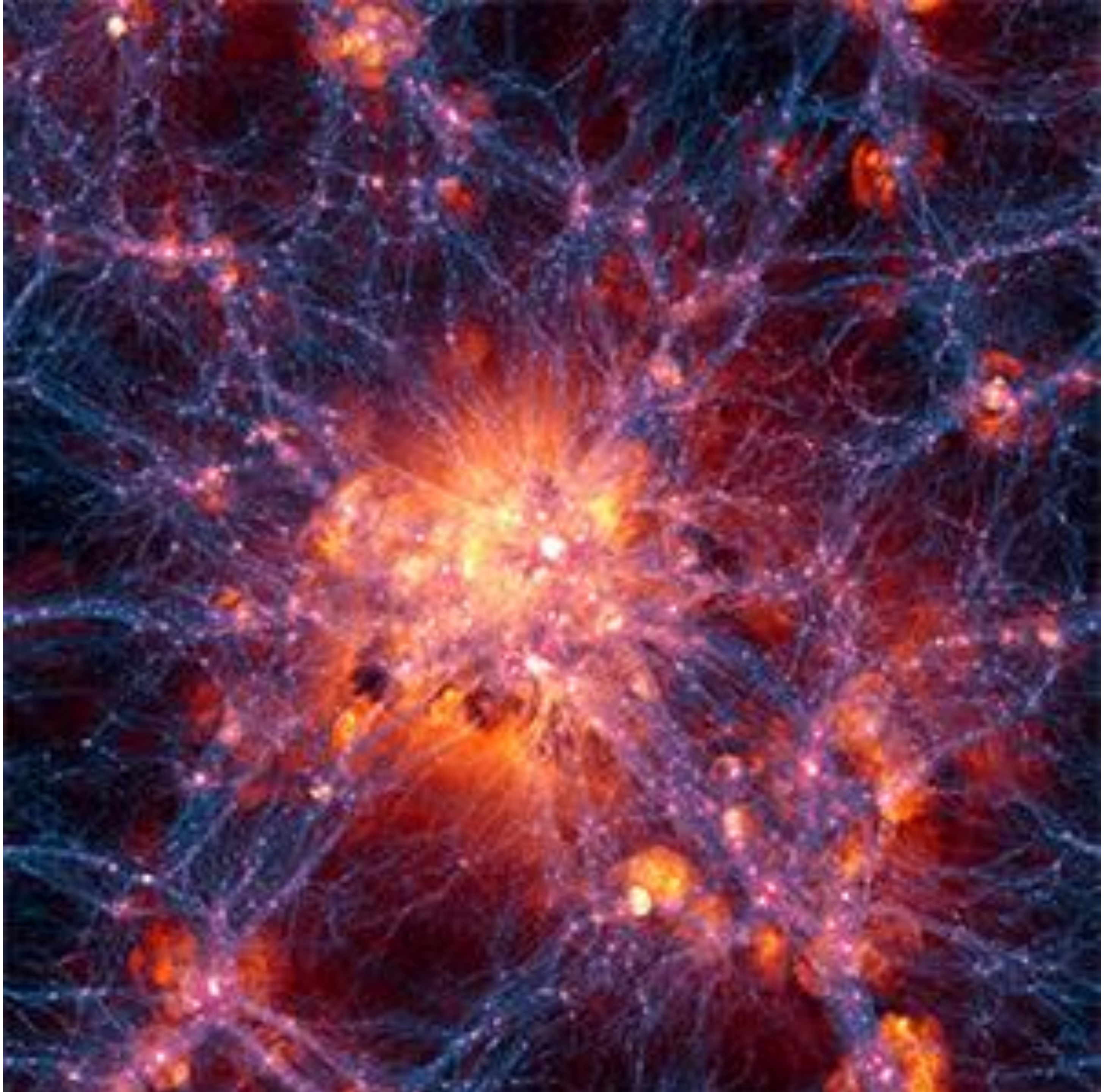


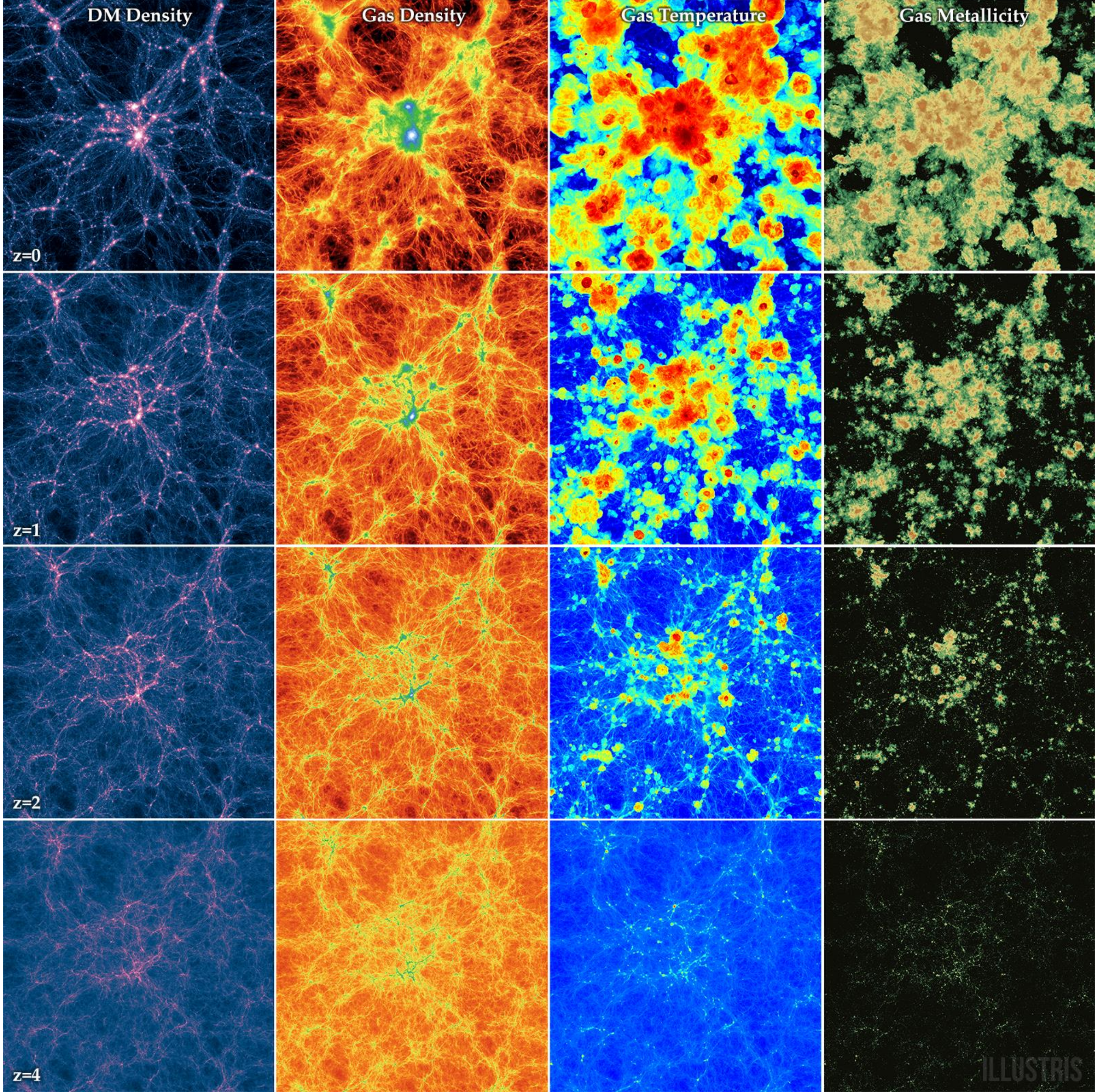
DESI May 2024

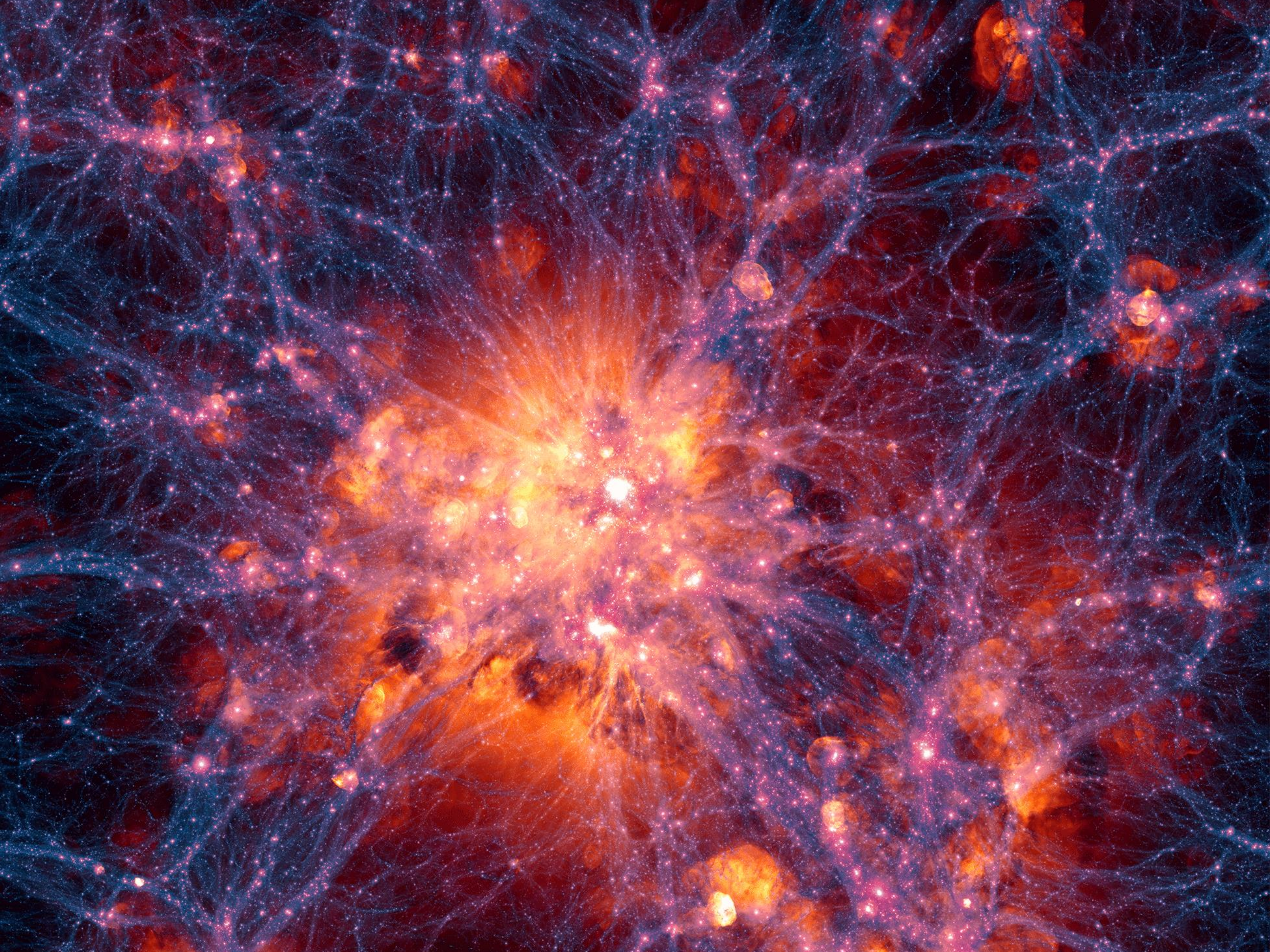
illustris https://www.illustris-project.org/media/

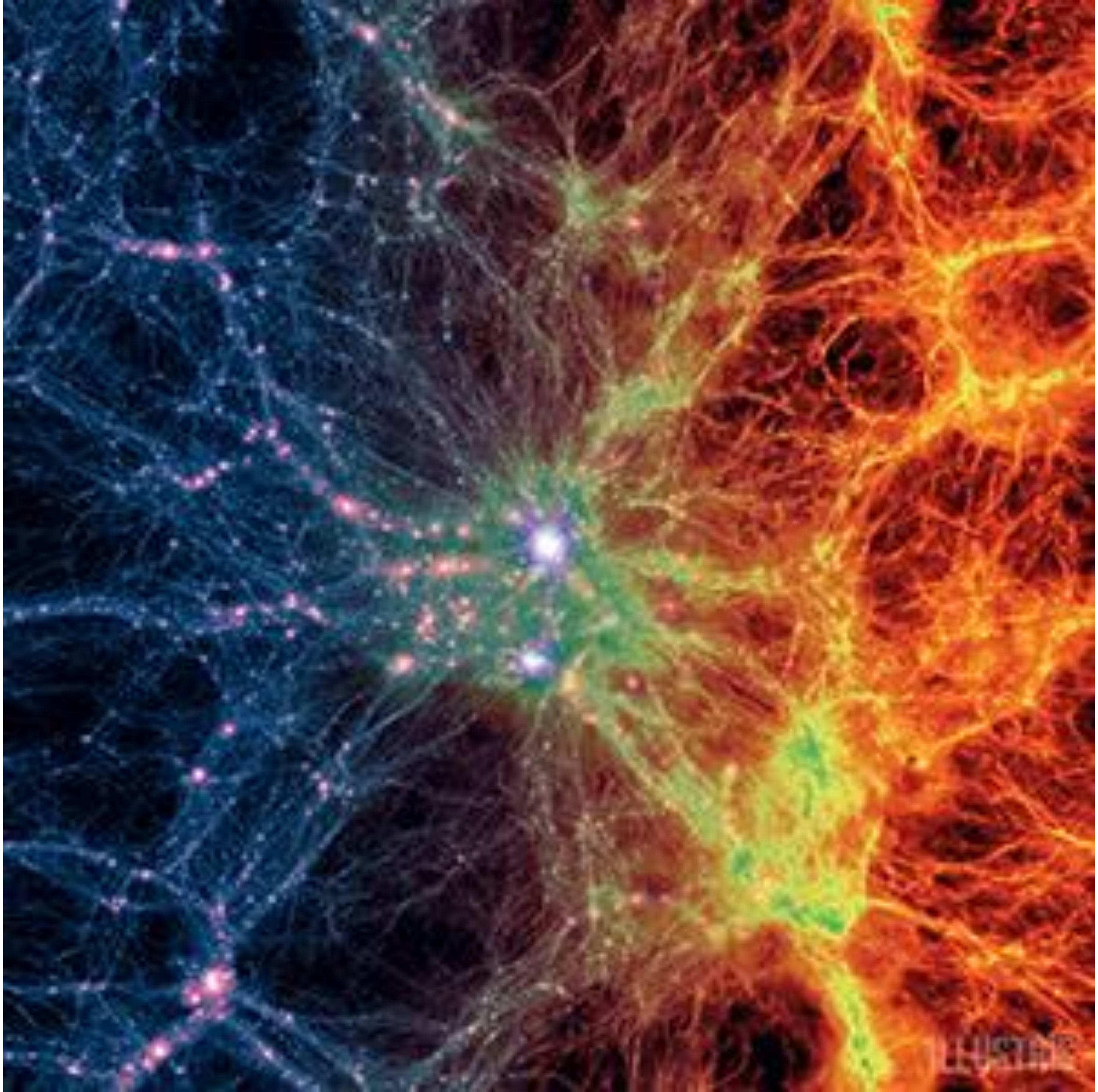
https://www.illustris-project.org/movies/illustris_movie_rot_sub_frame.mp4

illustris-simulation




DM Density
Gas Density
Gas Temperature
Gas Metallicity
z=0
z=1
z=2
z=4
ILLUSTRIS




ILLUSTRIS

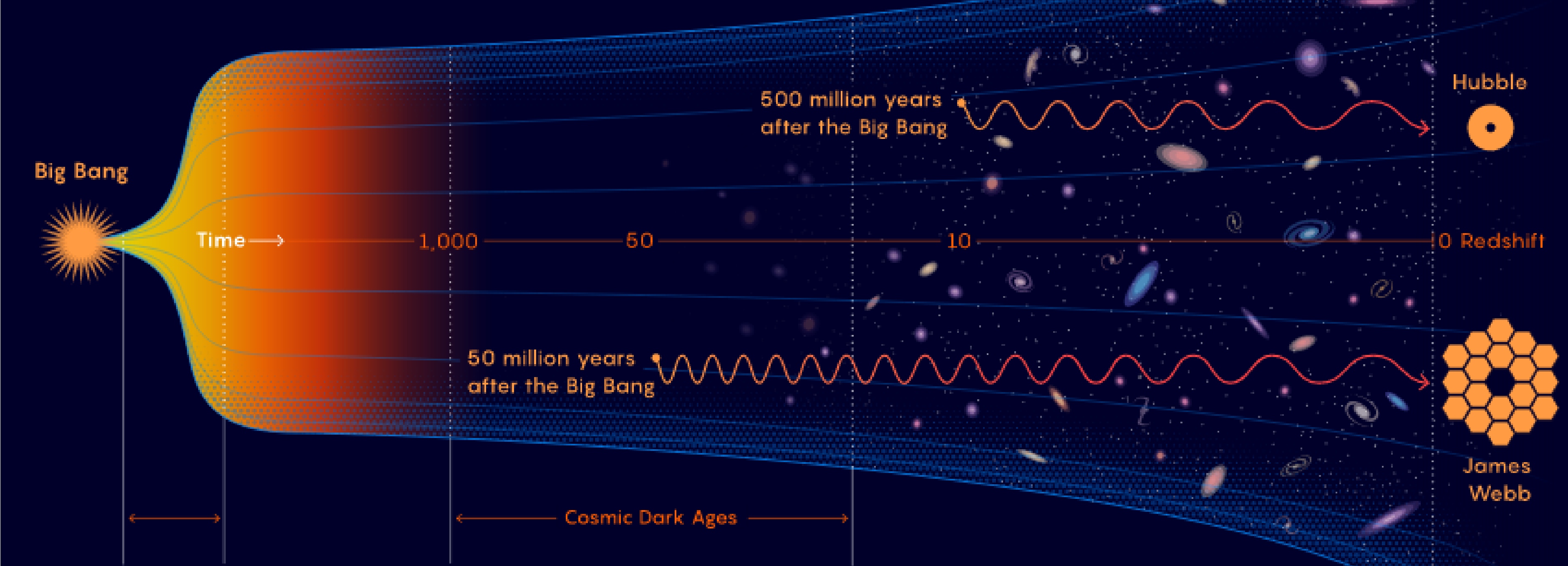
Flamingo simulation

[flamingo simulation](https://flamingo.strw.leidenuniv.nl/map_slider.html)

https://flamingo.strw.leidenuniv.nl/map_slider.html

The Growth of Cosmic Structure

Over billions of years, the universe went from smooth to structured. Powerful space telescopes have gradually uncovered much of the story of how this happened. The James Webb Space Telescope aims to reveal the crucial period when stars and galaxies first formed.



10^{-36} to 10^{-33} seconds

Inflation

During this period of exponential expansion, quantum fluctuations created density variations.

380,000 years

Cosmic Dark Ages Begin

After the first atoms formed, the universe was dark and structureless for eons.

100 million years?

Galaxies Form

At some point, dark matter corralled atoms into forming stars and proto-galaxies.

13.8 billion years

Today

Galaxies have become dimmer and more structured, appearing as spirals and ellipses.