Introduction to Cosmology

Marek Demianski
University of Warsaw

At the onset of nucleosynthesis ($T >> 1$ MeV, $t << 1$ sec) the balance between neutrons and protons is maintained by the week interactions (here $\nu = \nu_e$):

$$n \leftrightarrow p + e^- + \bar{\nu} \,,$$

$$\nu + n \leftrightarrow p + e^- \,,$$

$$e^+ + n \leftrightarrow p + \bar{\nu} \,.$$

When the rates for these reaction are rapid compared to the expansion rate H, chemical equilibrium obtains,

$$\mu_n + \mu_\nu = \mu_p + \mu_e \,,$$

what implies that in chemical equilibrium

$$\frac{n_n}{n_p} = \exp(-Q/T + (\mu_e - \mu_\nu)/T) \,,$$

where $Q = m_n - m_p = 1.293$Mev. Neglecting the chemical potential, the equilibrium value of the neutron-to-proton ratio is

$$\frac{n_n}{n_p} = \exp(-Q/T) \,,$$

Temperature	— Time
$10^{11} K$	— 0.01secs
$10^{10} K$	— 1.07secs
$10^9 K$	— 3 mins
10^8 K	— 5.3 hrs
$4 \times 10^3 K$	— 10^5 yrs

$$t \sim \frac{1}{T^2}$$

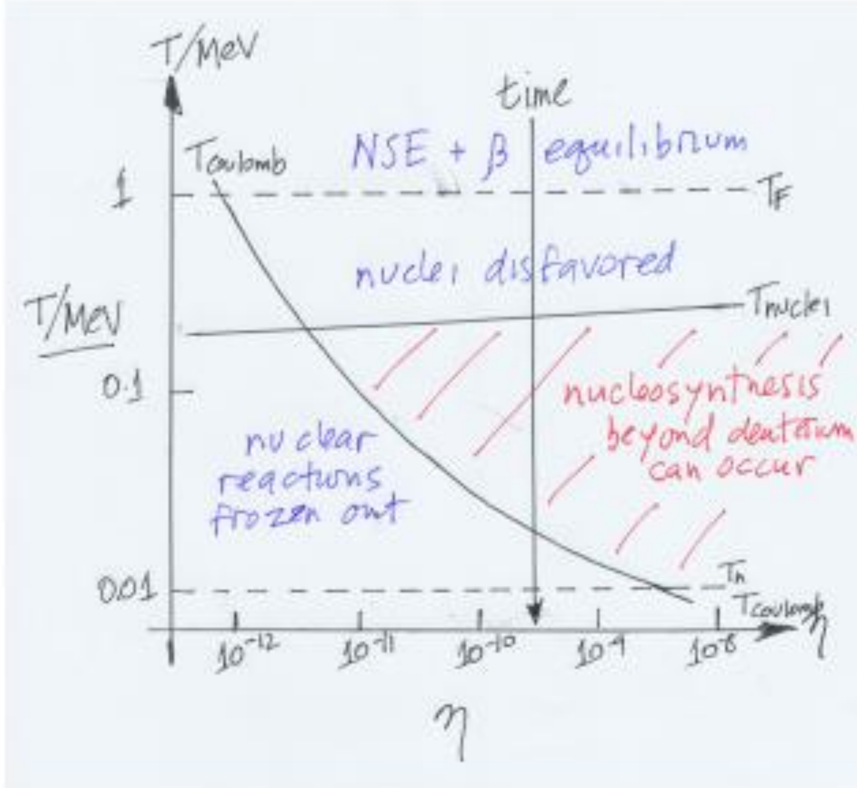


Figure 1: Key temperatures during BBN vs. the baryon-to-photon ratio η: T_F, the freeze in of the n/p ratio; $T_{\rm nuclei}$, the temperature below which nuclei are thermodynamically favored over free nucleons; $T_{\rm Coulomb}$, the temperature below which charged-particle nuclear reactions cease occurring; and T_n, the temperature at which the age of the Universe is the lifetime of a free neutron. Any significant nucleosynthesis beyond deuterium requires $T_{\rm nuclei} \geq T_{\rm Coulomb}$, or $\eta \geq 10^{-11}$. The vertical line marked "time" shows the timeline of successful BBN: freeze in of the n/p ratio at $T = T_F \longrightarrow$ period of waiting until nuclei are favored $T_F > T > T_{\rm nuclei} \longrightarrow$ nucleosynthesis $T_{\rm nuclei} > T > T_{\rm Coulomb} \longrightarrow$ frozen out nuclear reactions $T_{\rm Coulomb} > T \longrightarrow$ any free neutrons remaining decay $T_n > T$.

The proces of primordial nucleosynthesis terminates when temperature T drops below 10^8 K

Physical conditions at the end of primordial nucleosynthesis:

Temperature ~ 10^8 K

Density ~ $10^{-2}\ g/cm^3$!!!

Number density of photons $n_\gamma \sim 2.04 \cdot 10^{32}\ \frac{1}{cm^3}$

Number density of baryons $n_B \sim 6 \cdot 10^{21}\ \frac{1}{cm^3}$

George
Gamow

Det Kongelige Danske Videnskabernes Selskab
Matematisk-fysiske Meddelelser, bind **27**, nr. 10

Dan. Mat. Fys. Medd. **27**, no. 10 (1953)

EXPANDING UNIVERSE AND THE ORIGIN OF GALAXIES

BY

G. GAMOW

Similarly, noticing that, in the present era, the radiation-density must vary in inverse proportion to the fourth power of the time (because $\rho \approx T^4$, $T \sim l^{-1}$, and $l \sim t$), we find,

$$\rho_{rad.}\ (\text{present}) = \frac{3 \cdot 1 \times 10^{37}}{t^4}\ \text{gm per cm}^3. \qquad \ldots.(10)$$

For the present density of residual radiation we obtain 6×10^{-32}, corresponding to about 6 K. Thus we may conclude that the residual heat found at present in the Universe is comparable with the heat provided by nuclear transformations in stars.

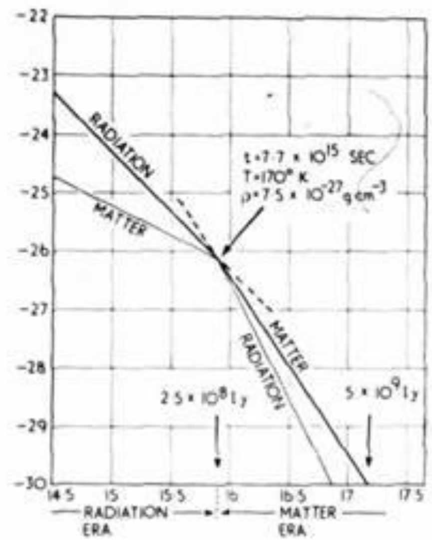
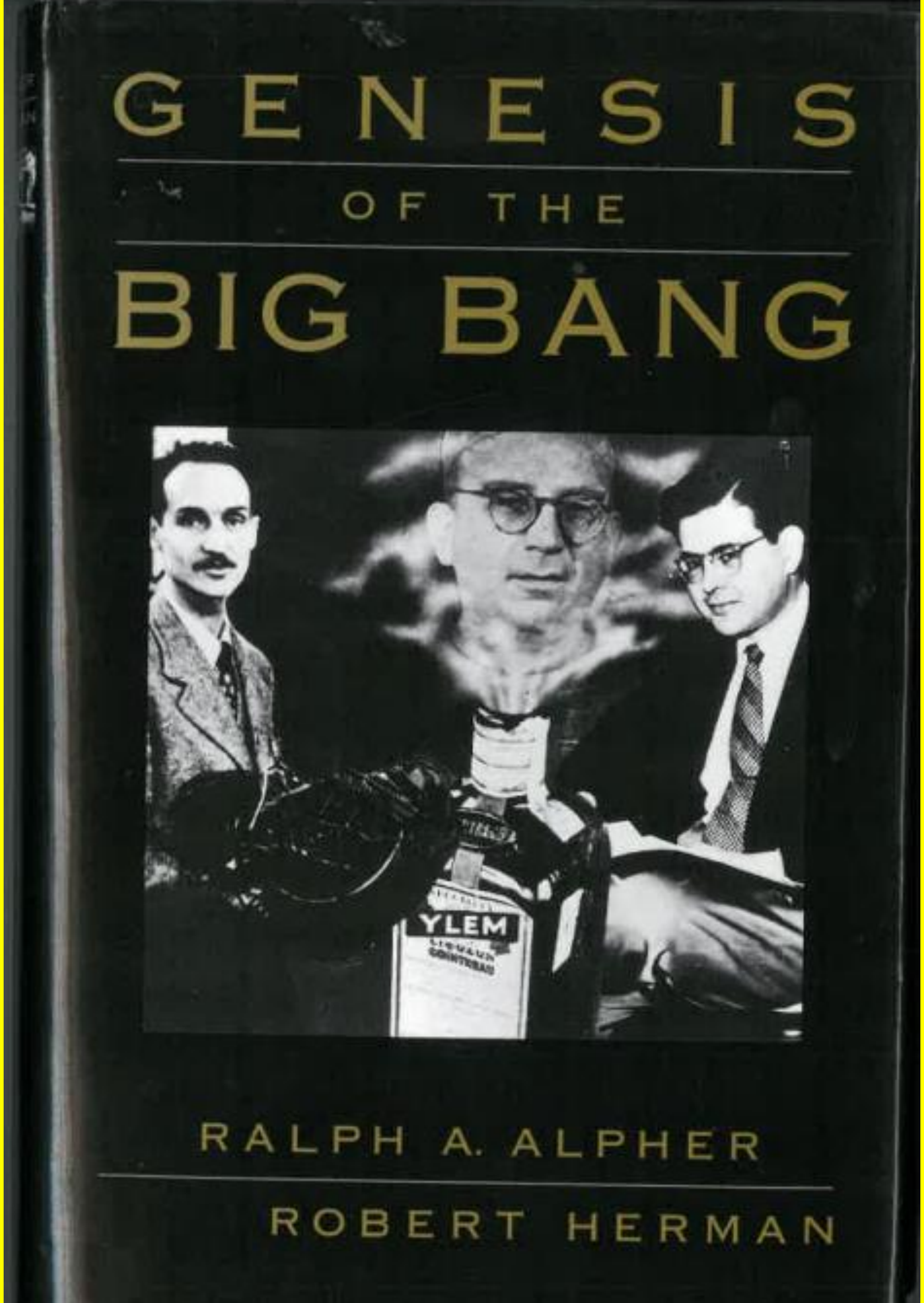


Fig. 1. The densities, in gm per cm³, of matter and radiation (ordinates) plotted against time in seconds (abscissae); logarithmic scale

4. Formation of Chemical Elements and Origin of Galaxies

The above considerations give us a general picture of changing physical conditions characteristic of the evolutionary history of our Universe. We will indicate here only quite briefly how this information can be used for the explanation of various characteristic properties of the Universe as we know it to-day. First of all, it may be suggested that, at least partially, the relative abundances of the atoms of various chemical elements were conditioned by thermonuclear reactions which took place at high speed during the very early stages of expansion while the temperature of the Universe was exceedingly high. And, in fact, the calculations in that direction, carried out by the present writer*, and later in some more detail by Fermi and Turkevich,† lead to a value of the H/He ratio which is in good agreement with observational data. However, there are still some difficulties to be overcome in understanding the abundances of heavier elements, and there is a possibility that the original distribution was partially modified by various processes during the later stages of the evolution.



Oxford
University
Press

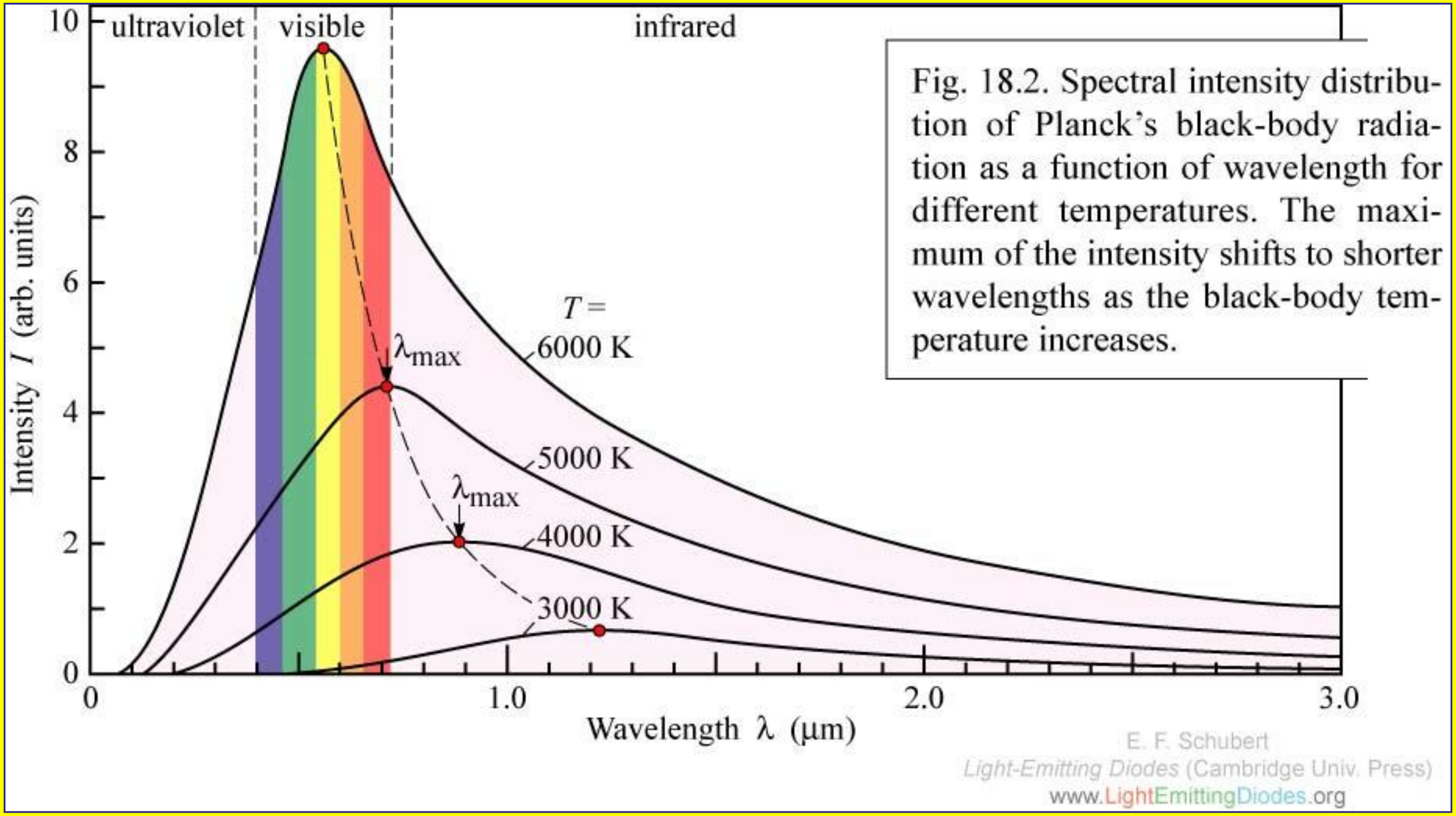
Max Planck black body quantum description

$E_\gamma = h\,\nu$

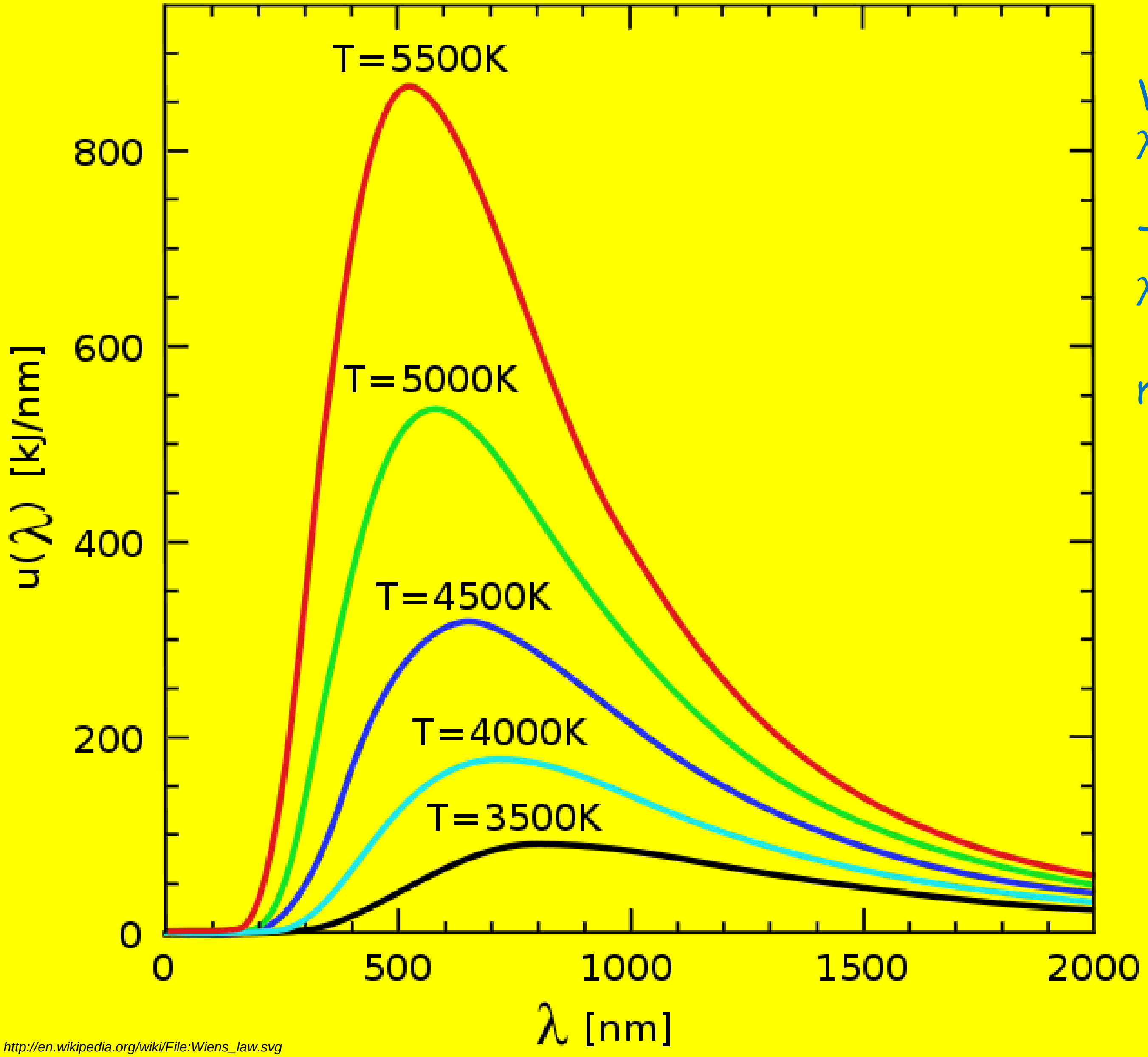
$$I_n = B_n = \frac{2hn^3}{c^2}\,\frac{1}{e^{hn/kT} - 1}$$

research-in-germany.de

h = Planck's cst k = Boltzmann's cst c = speed of light



Black Body Radiation



http://en.wikipedia.org/wiki/File:Wiens_law.svg

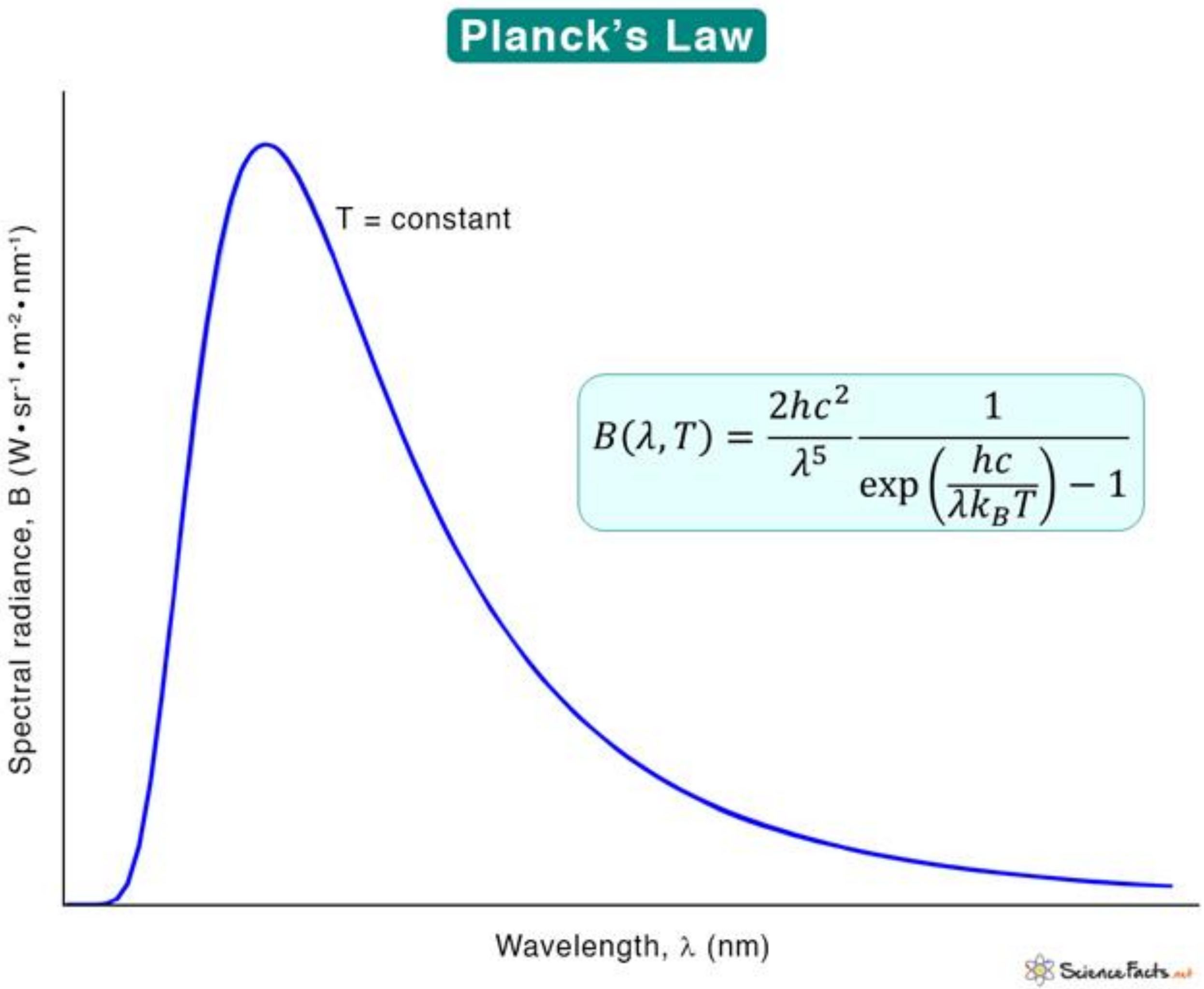
Wien's Law

λ_{max} ·T = $2.9 \cdot 10^6\ nm \cdot K$

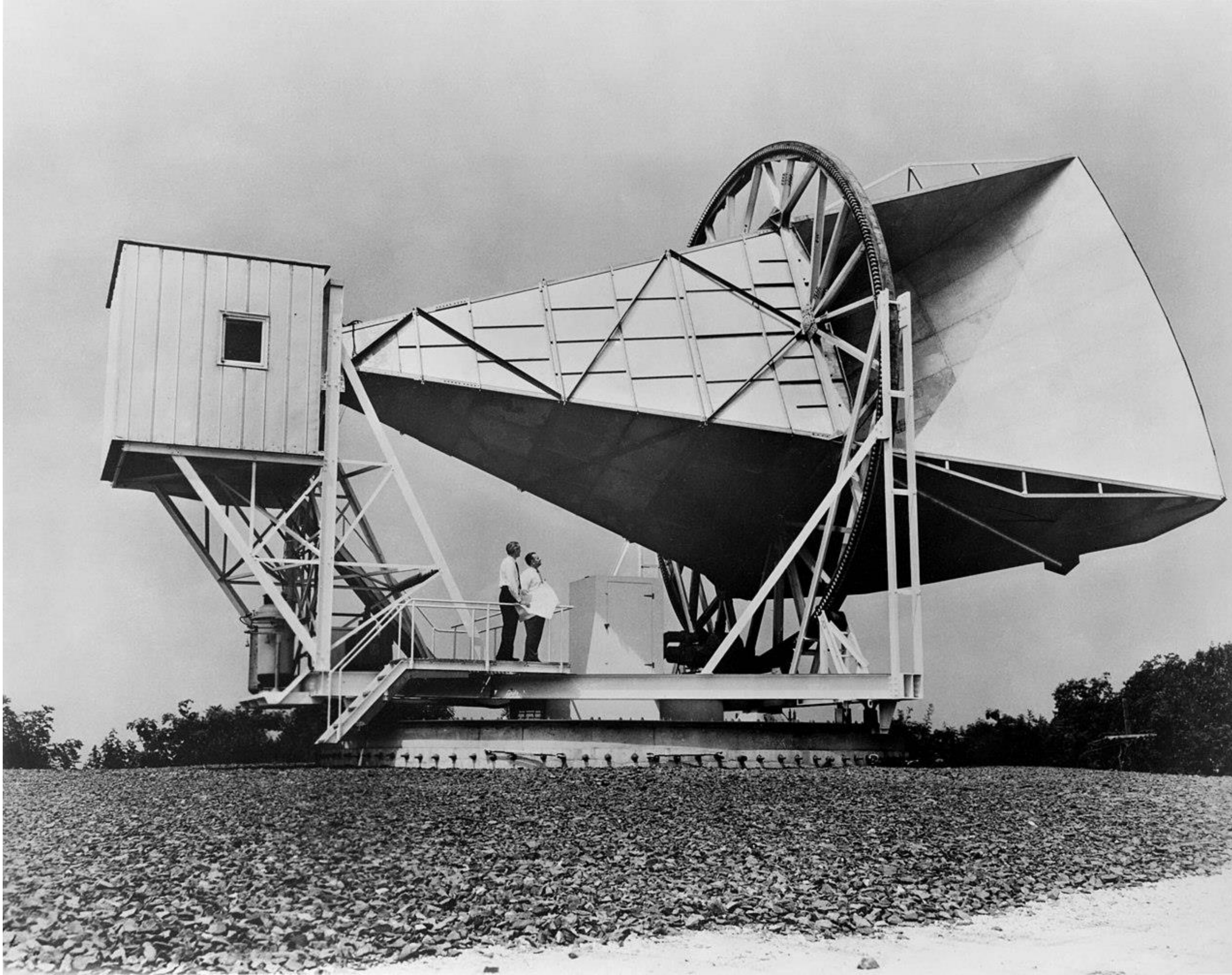
T = 6K

λ_{max} = 0.05 cm

microwaves



In an expanding universe the Planck distribution is preserved because T ~ 1/R, and λ ~ R !!



Homdale://www.nytimes.com/2023/09/04/science/astronomy-holmdel-antenna-microwaves.htm high pressure, such as the zero-mass scalar, capable of speeding the universe through the period of helium formation. To have a closed space, an energy density of 2×10^{-29} gm/cm^3 is needed. Without a zero-mass scalar, or some other "hard" interaction, the energy could not be in the form of ordinary matter and may be presumed to be gravitational radiation (Wheeler 1958).

One other possibility for closing the universe, with matter providing the energy content of the universe, is the assumption that the universe contains a net electron-type neutrino abundance (in excess of antineutrinos) greatly larger than the nucleon abundance. In this case, if the neutrino abundance were so great that these neutrinos are degenerate, the degeneracy would have forced a negligible equilibrium neutron abundance in the early, highly contracted universe, thus removing the possibility of nuclear reactions leading to helium formation. However, the required ratio of lepton to baryon number must be $> 10^9$.

We deeply appreciate the helpfulness of Drs. Penzias and Wilson of the Bell Telephone Laboratories, Crawford Hill, Holmdel, New Jersey, in discussing with us the result of their measurements and in showing us their receiving system. We are also grateful for several helpful suggestions of Professor J. A. Wheeler.

R. H. Dicke
P. J. E. Peebles
P. G. Roll
D. T. Wilkinson

May 7, 1965
Palmer Physical Laboratory
Princeton, New Jersey

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——— 1964, in *Relativity, Groups and Topology*, ed. C. DeWitt and B. DeWitt (New York: Gordon & Breach).
Zel'dovich, Ya. B. 1962, *Soviet Phys.—J.E.T.P.*, **14**, 1143.

A MEASUREMENT OF EXCESS ANTENNA TEMPERATURE AT 4080 Mc/s

Measurements of the effective zenith noise temperature of the 20-foot horn-reflector antenna (Crawford, Hogg, and Hunt 1961) at the Crawford Hill Laboratory, Holmdel, New Jersey, at 4080 Mc/s have yielded a value about 3.5° K higher than expected. This excess temperature is, within the limits of our observations, isotropic, unpolarized, and free from seasonal variations (July, 1964–April, 1965). A possible explanation for the observed excess noise temperature is the one given by Dicke, Peebles, Roll, and Wilkinson (1965) in a companion letter in this issue.

The total antenna temperature measured at the zenith is 6.7° K of which 2.3° K is due to atmospheric absorption. The calculated contribution due to ohmic losses in the antenna and back-lobe response is 0.9° K.

The radiometer used in this investigation has been described elsewhere (Penzias and Wilson 1965). It employs a traveling-wave maser, a low-loss (0.027-db) comparison switch, and a liquid helium–cooled reference termination (Penzias 1965). Measurements were made by switching manually between the antenna input and the reference termination. The antenna, reference termination, and radiometer were well matched so that a round-trip return loss of more than 55 db existed throughout the measurement; thus errors in the measurement of the effective temperature due to impedance mismatch can be neglected. The estimated error in the measured value of the total antenna temperature is 0.3° K and comes largely from uncertainty in the absolute calibration of the reference termination.

The contribution to the antenna temperature due to atmospheric absorption was obtained by recording the variation in antenna temperature with elevation angle and employing the secant law. The result, 2.3° ± 0.3° K, is in good agreement with published values (Hogg 1959; DeGrasse, Hogg, Ohm, and Scovil 1959; Ohm 1961).

The contribution to the antenna temperature from ohmic losses is computed to be 0.8° ± 0.4° K. In this calculation we have divided the antenna into three parts: (1) two non-uniform tapers approximately 1 m in total length which transform between the $2\frac{1}{8}$-inch round output waveguide and the 6-inch-square antenna throat opening; (2) a double-choke rotary joint located between these two tapers; (3) the antenna itself. Care was taken to clean and align joints between these parts so that they would not significantly increase the loss in the structure. Appropriate tests were made for leakage and loss in the rotary joint with negative results.

The possibility of losses in the antenna horn due to imperfections in its seams was eliminated by means of a taping test. Taping all the seams in the section near the throat and most of the others with aluminum tape caused no observable change in antenna temperature.

The backlobe response to ground radiation is taken to be less than 0.1° K for two reasons: (1) Measurements of the response of the antenna to a small transmitter located on the ground in its vicinity indicate that the average back-lobe level is more than 30 db below isotropic response. The horn-reflector antenna was pointed to the zenith for these measurements, and complete rotations in azimuth were made with the transmitter in each of ten locations using horizontal and vertical transmitted polarization from each position. (2) Measurements on smaller horn-reflector antennas at these laboratories, using pulsed measuring sets on flat antenna ranges, have consistently shown a back-lobe level of 30 db below isotropic response. Our larger antenna would be expected to have an even lower back-lobe level.

From a combination of the above, we compute the remaining unaccounted-for antenna temperature to be 3.5° ± 1.0° K at 4080 Mc/s. In connection with this result it should be noted that DeGrasse *et al.* (1959) and Ohm (1961) give total system temperatures at 5650 Mc/s and 2390 Mc/s, respectively. From these it is possible to infer upper limits to the background temperatures at these frequencies. These limits are, in both cases, of the same general magnitude as our value.

We are grateful to R. H. Dicke and his associates for fruitful discussions of their results prior to publication. We also wish to acknowledge with thanks the useful comments and advice of A. B. Crawford, D. C. Hogg, and E. A. Ohm in connection with the problems associated with this measurement.

Note added in proof.—The highest frequency at which the background temperature of the sky had been measured previously was 404 Mc/s (Pauliny-Toth and Shakeshaft 1962), where a minimum temperature of 16° K was observed. Combining this value with our result, we find that the average spectrum of the background radiation over this frequency range can be no steeper than $\lambda^{0.7}$. This clearly eliminates the possibility that the radiation we observe is due to radio sources of types known to exist, since in this event, the spectrum would have to be very much steeper.

A. A. PENZIAS
R. W. WILSON

May 13, 1965
BELL TELEPHONE LABORATORIES, INC
CRAWFORD HILL, HOLMDEL, NEW JERSEY

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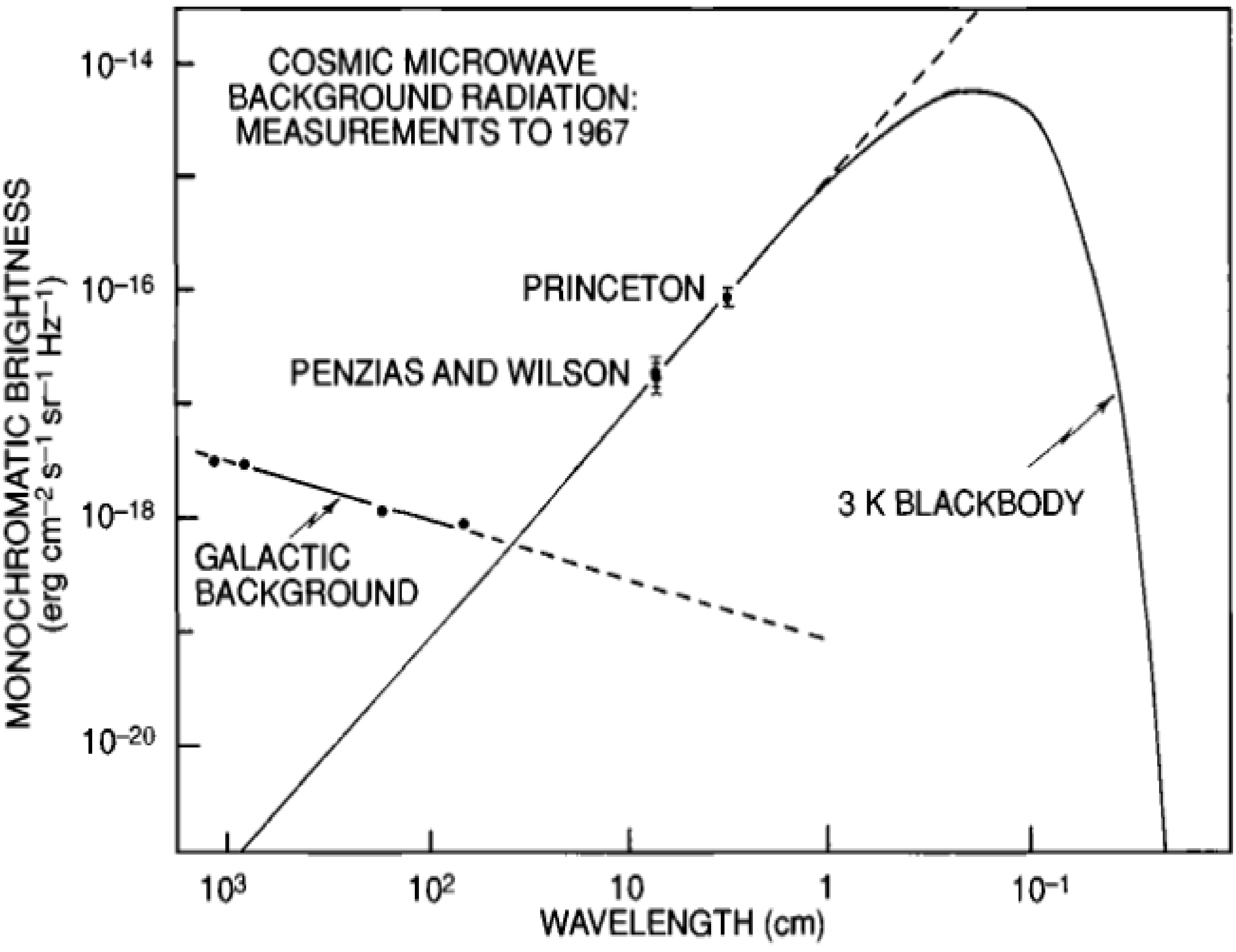
ERRATUM

In the paper "Stellar Evolution. I. The Approach to the Main Sequence" (*Ap. J.*, **141**, 993), the following corrections are to be made: page 993, line 1, replace "popuation" by "population"; page 997, line 18, delete the last word "energy"; page 999, line 2, replace "expanding" by "contracting"; page 1007, section heading VI—replace "8" by "9"; page 1007, line 1, replace "Figure 12" by "Figure 17"; page 1017, line 5, replace "equation (19)" by "equation (B9)"; page 1018, line 6, replace "W. Z. Fowler" by "W. A. Fowler."

ICKO IBEN, JR.

June 7, 1965
MASSACHUSETTS INSTITUTE
OF TECHNOLOGY


COSMIC MICROWAVE BACKGROUND RADIATION: MEASUREMENTS TO 1967
MONOCHROMATIC BRIGHTNESS (erg cm−2 s−1 sr−1 Hz−1)
10−14
10−16
10−18
10−20
PRINCETON
PENZIAS AND WILSON
GALACTIC BACKGROUND
3 K BLACKBODY
10³
10²
10
1
10−1
WAVELENGTH (cm)

The Nobel Prize in Physics 1978

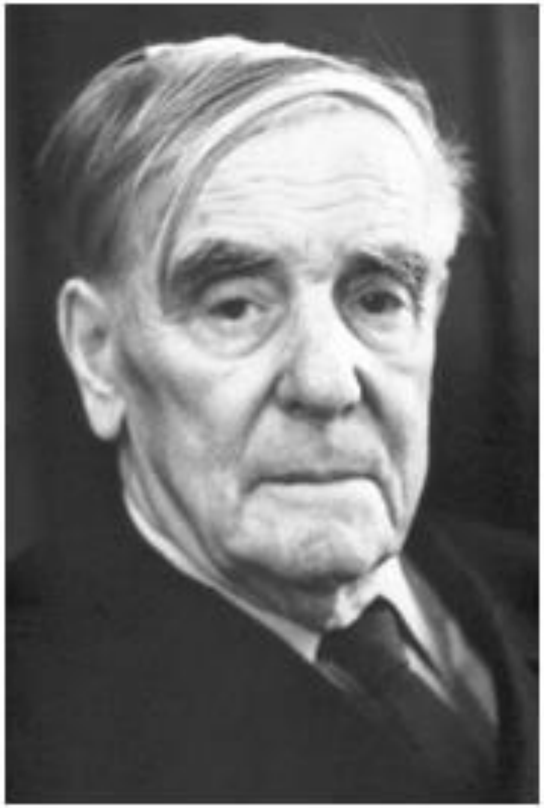


Photo from the Nobel Foundation archive.

Pyotr Leonidovich Kapitsa

Prize share: 1/2

Photo from the Nobel Foundation archive.

Arno Allan Penzias

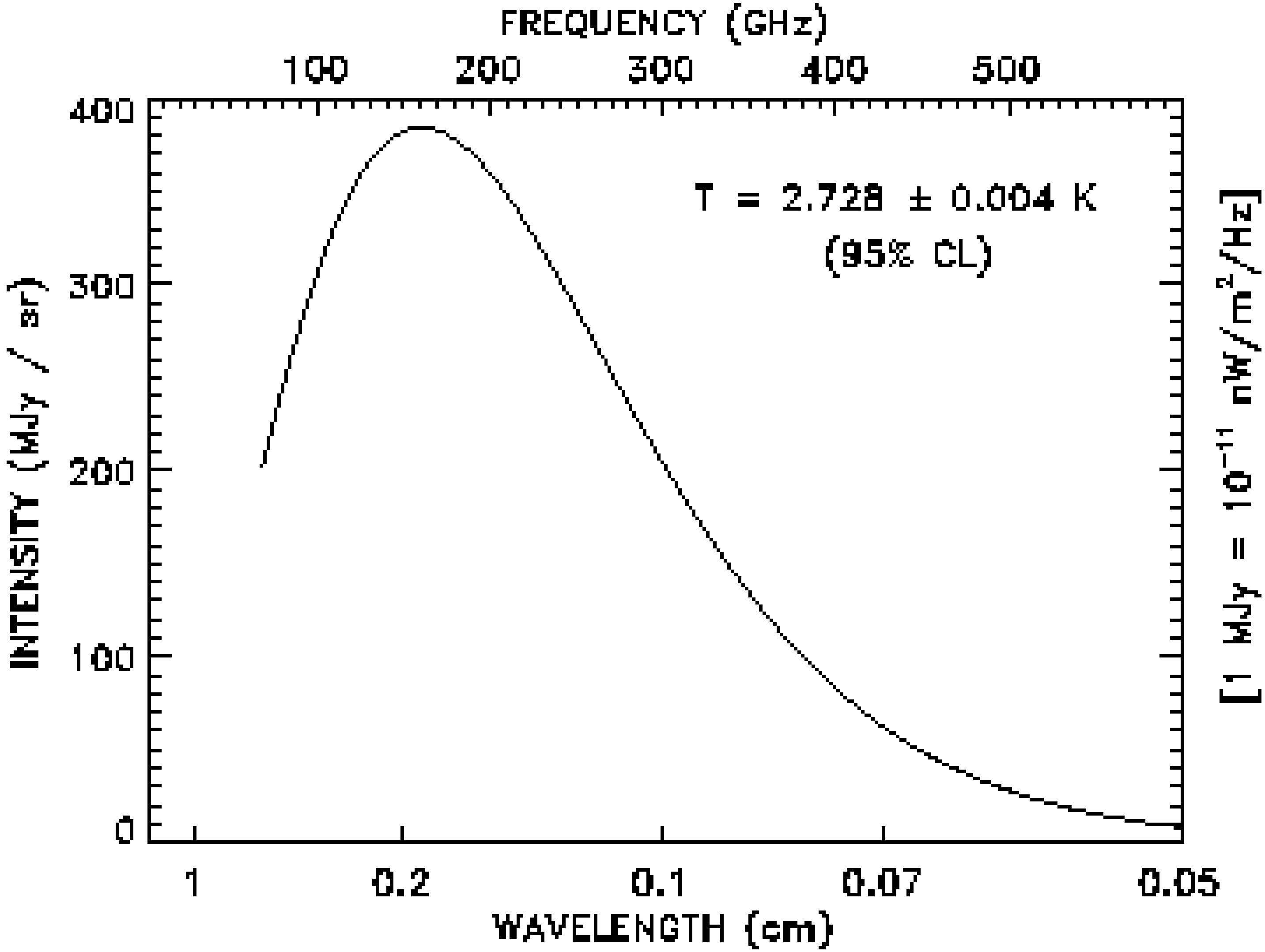
Prize share: 1/4

Photo from the Nobel Foundation archive.

Robert Woodrow Wilson

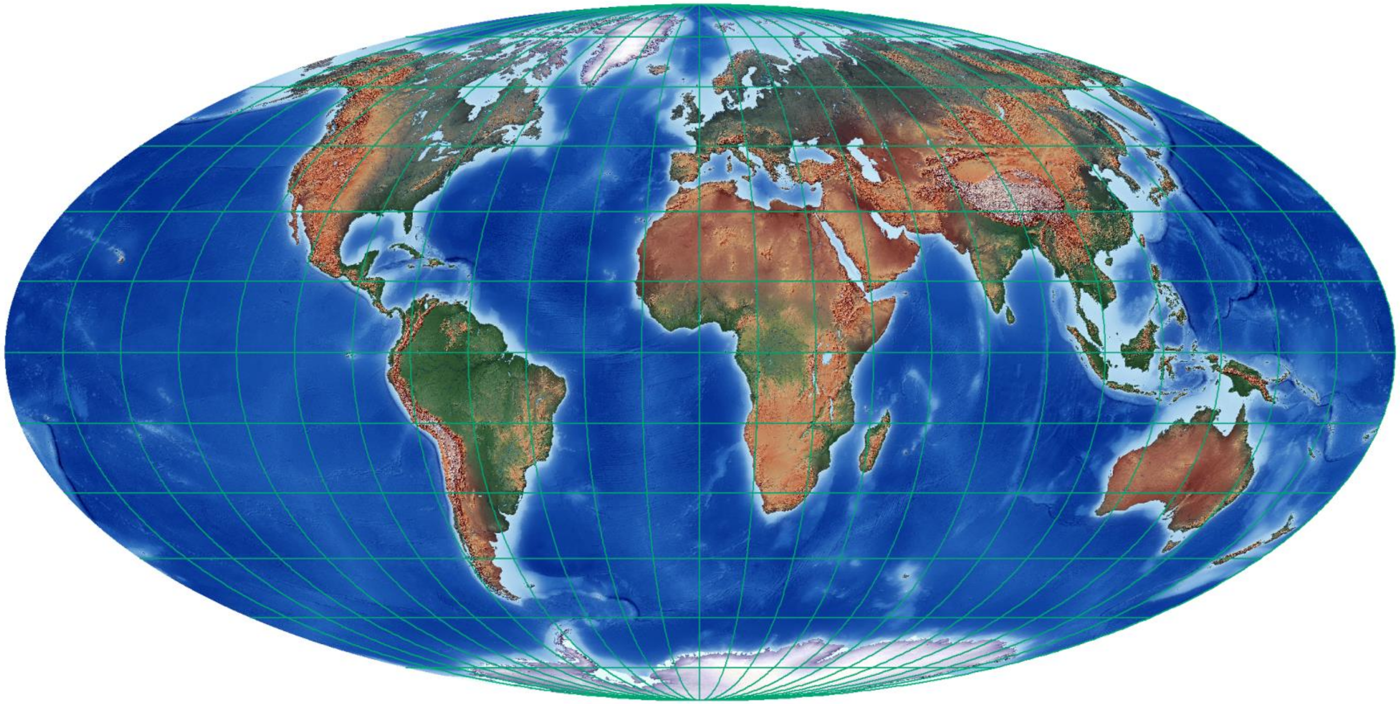
Prize share: 1/4

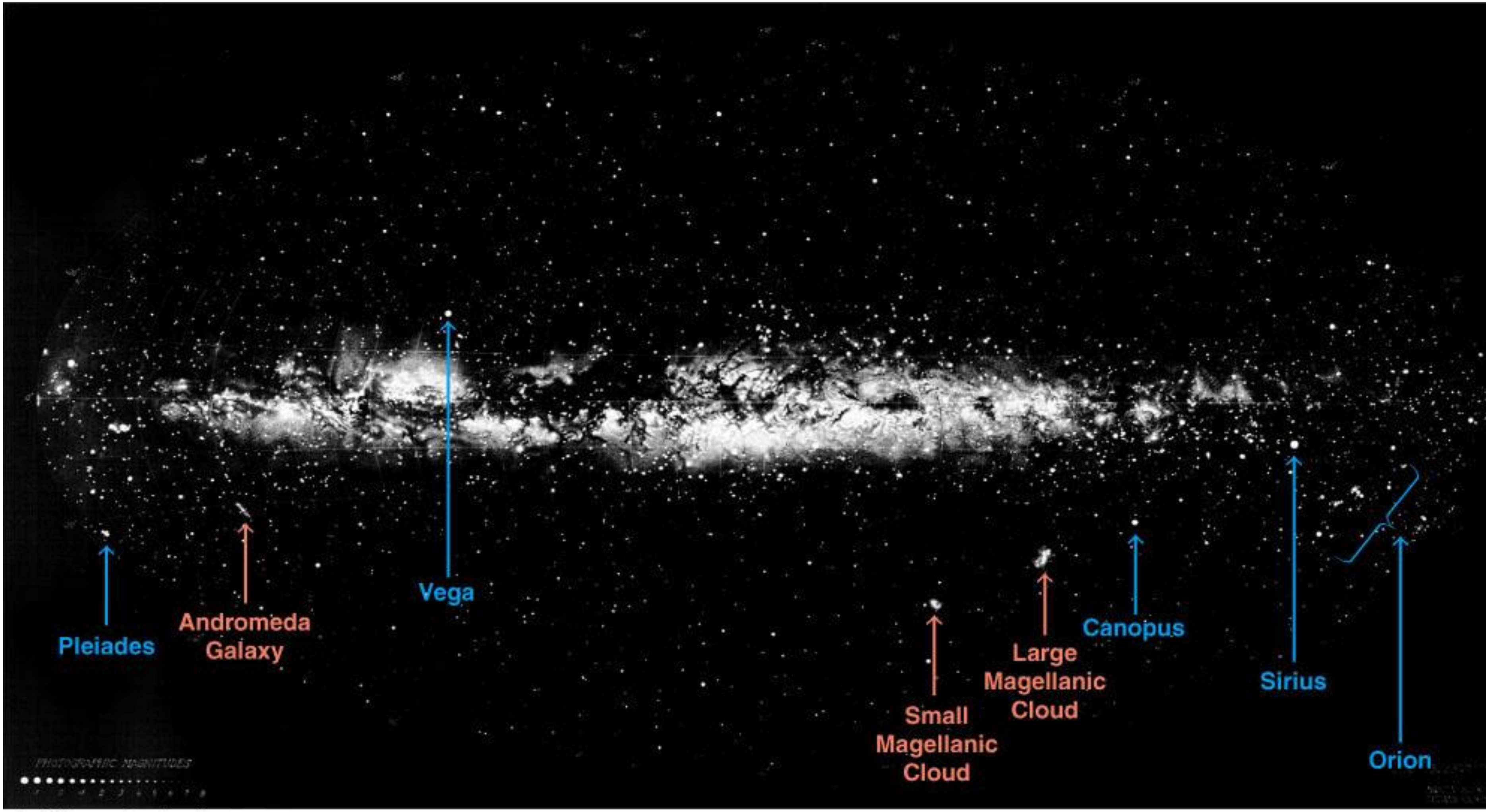

COBE
Cosmic Background Explorer


FREQUENCY (GHz)
100
200
300
400
500
400
300
200
100
0
INTENSITY (MJy / sr)
T = 2.728 ± 0.004 K
(95% CL)
[1 MJy = 10^-11 nW/m^2/Hz]
1
0.2
0.1
0.07
0.05
WAVELENGTH (cm)

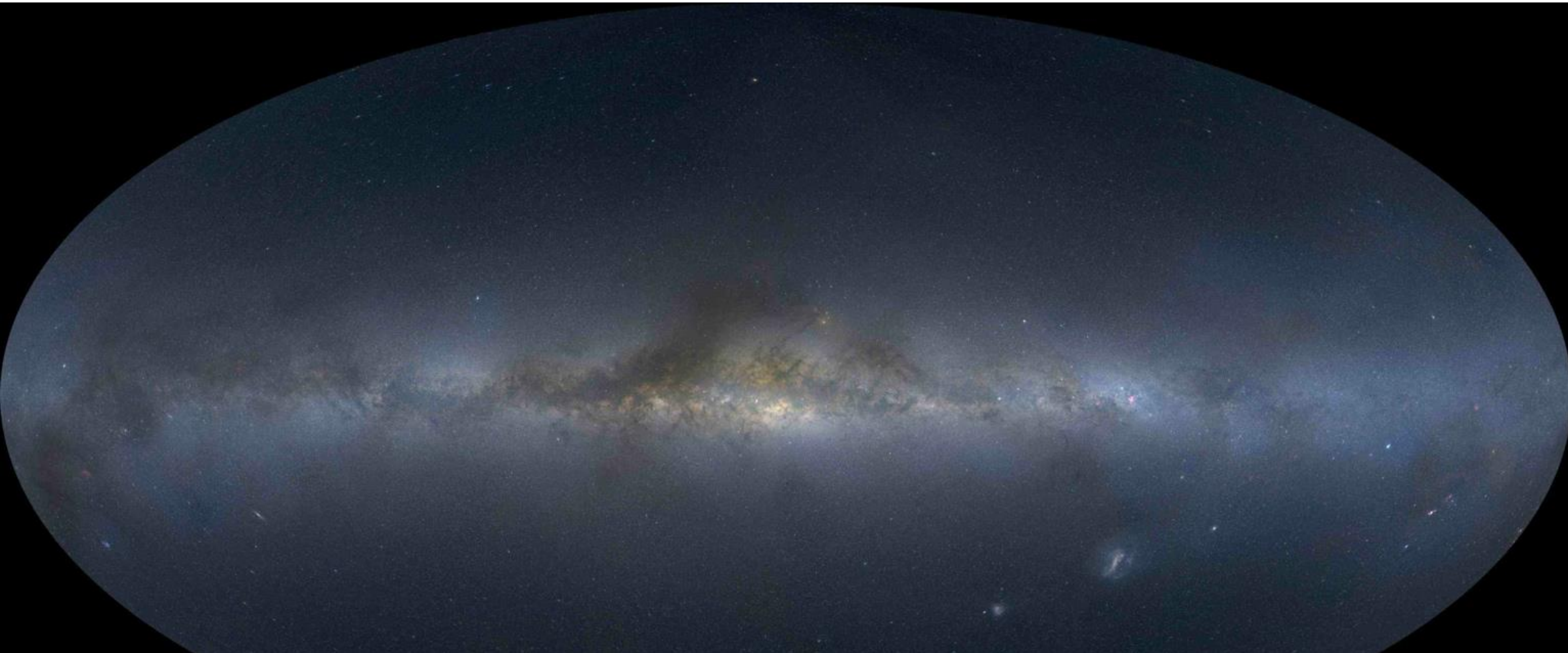
Mollweide projection

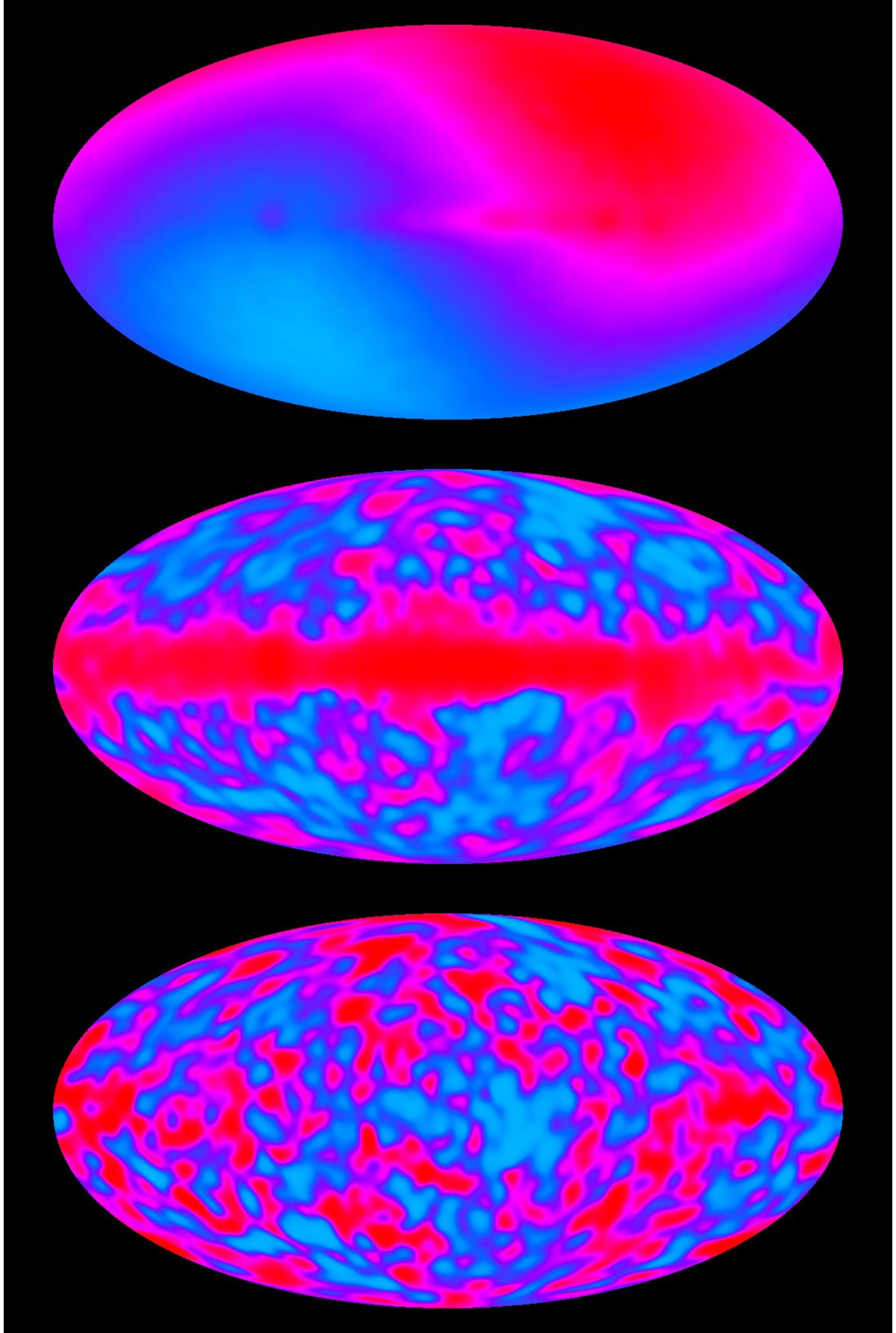
The map projection is the Mollweide projection. It's an equal area projection that is suitable for when you want to show an undistorted view of a sample of features. It's well suited for maps like the Planck results because the areas of features are not distorted. However, there is several angular distortion, especially along the prime meridian and polar regions.





Lund Observatory, Sweden

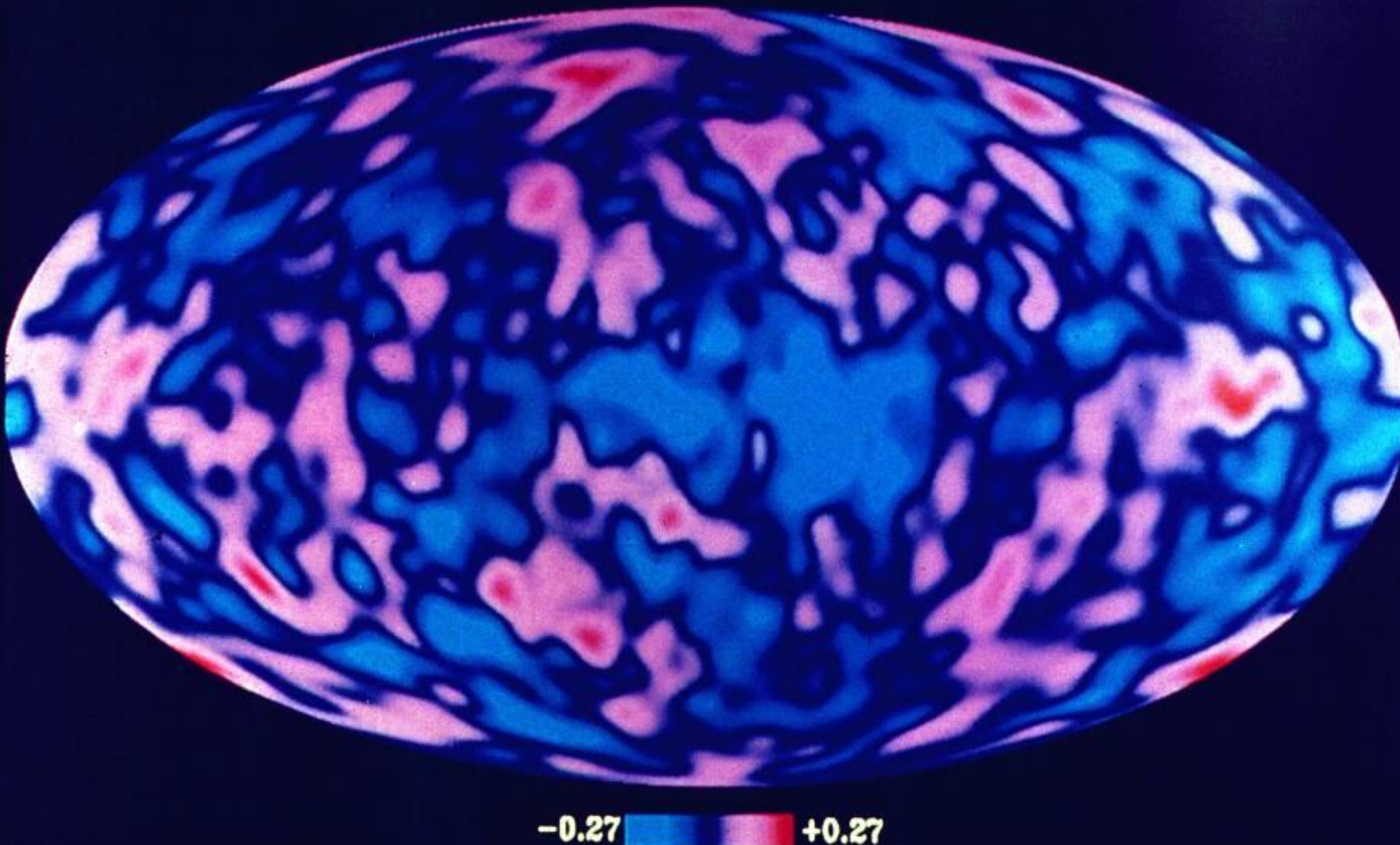




Dipol perturbation due to motion of the Solar System relative to CMB

Intrinsic perturbations

COBE DMR



Nobelprize.org
The Official Web Site of the Nobel Prize

BACK

The Nobel Prize in Physics 2006

The Royal Swedish Academy of Sciences has decided to award the Nobel Prize in Physics for 2006 jointly to John C. Mather and George F. Smoot "for their discovery of the blackbody form and anisotropy of the cosmic microwave background radiation".

John C. Mather
Senior Astrophysicist at NASA's Goddard Space Flight Center, USA.
Born 1946.

Mather co-ordinated the entire COBE project. He was also in charge of the FIRAS instrument. The results showing the blackbody spectrum of the microwave background radiation were published in 1990.

Photo: NASA

George F. Smoot
Professor of Physics at the University of California, Berkeley, USA.
Born 1945.

Smoot was in charge of the DMR instrument that measured small temperature variations of the microwave background radiation in different directions in the sky. The results, which show where matter began to accumulate in the Universe, were published in 1992.

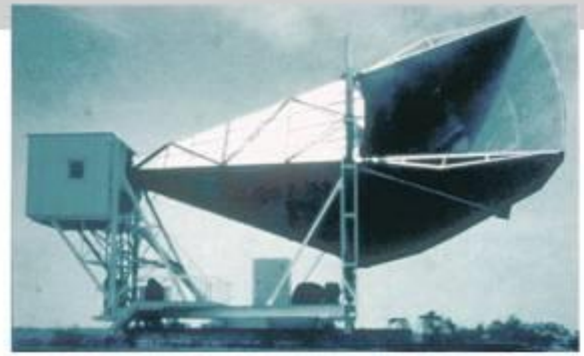
The first light in the Universe

In 1989 NASA launched the COBE – Cosmic Background Explorer – satellite. Its aim was to study the cosmic microwave background radiation in more detail than earlier measurements made from the earth or from balloons.


USAF
BOEING
DELTA
MAP

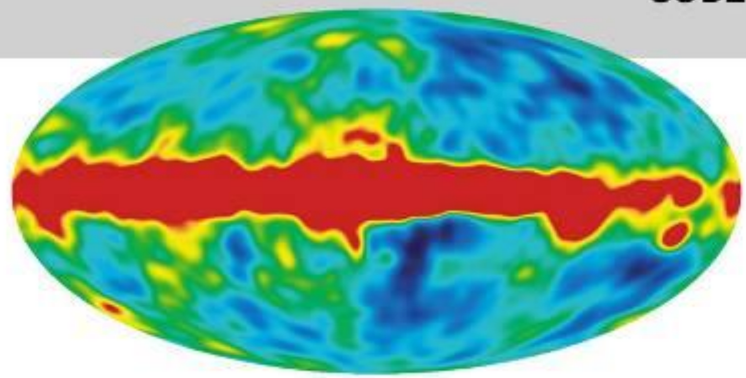
1965

Penzias and Wilson



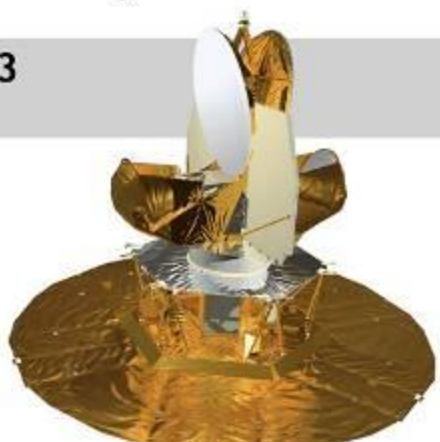
1992

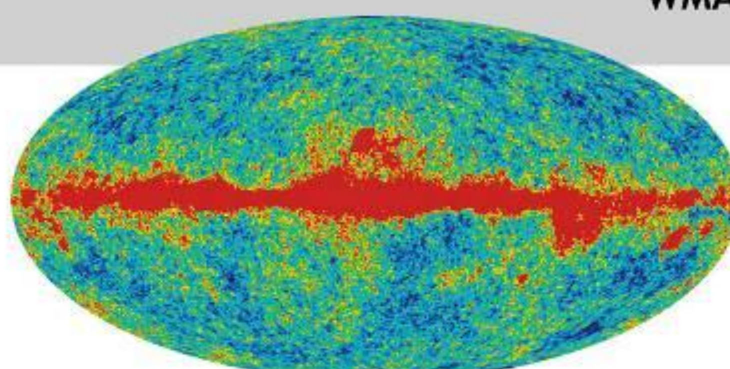
COBE

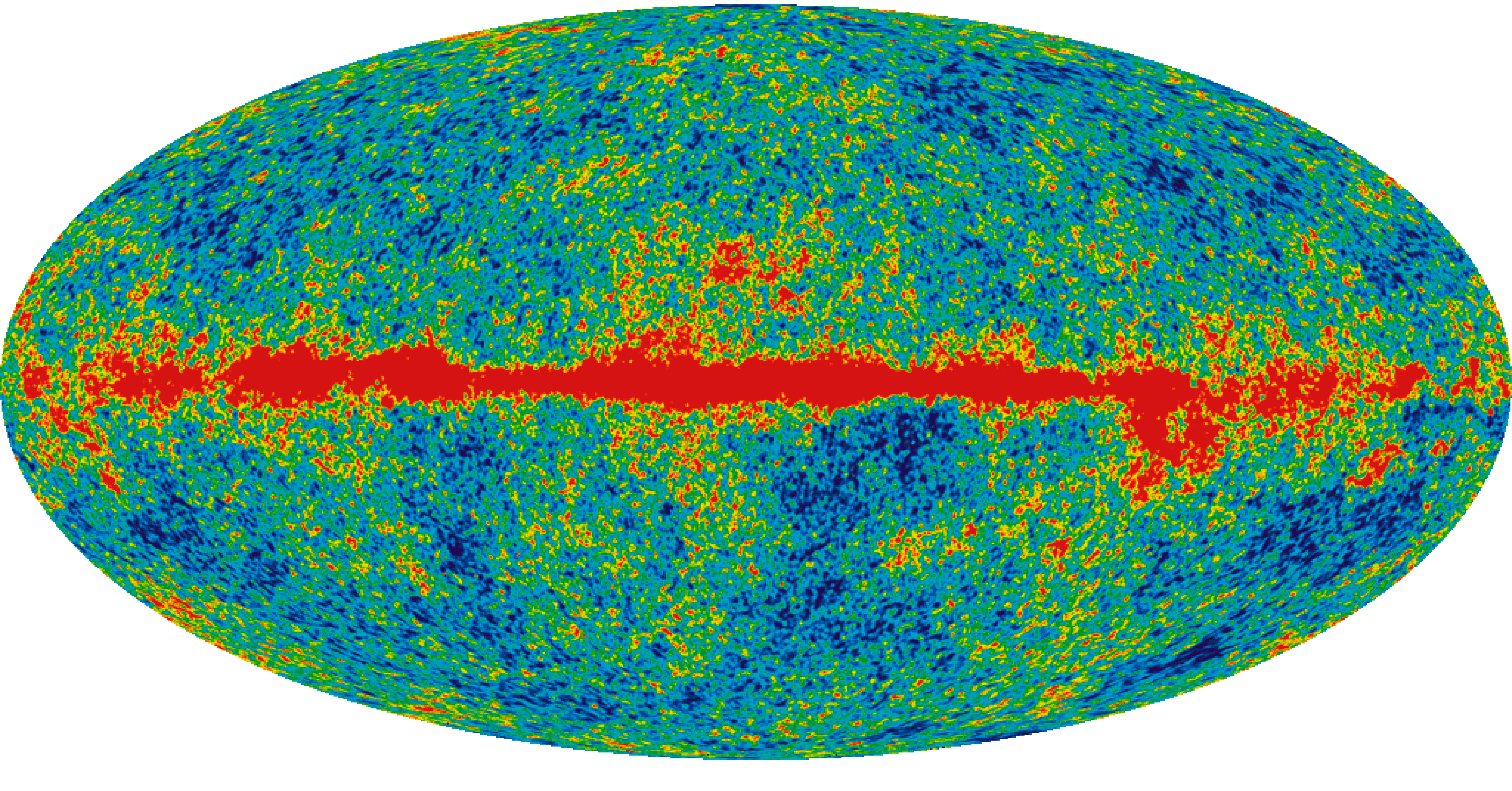


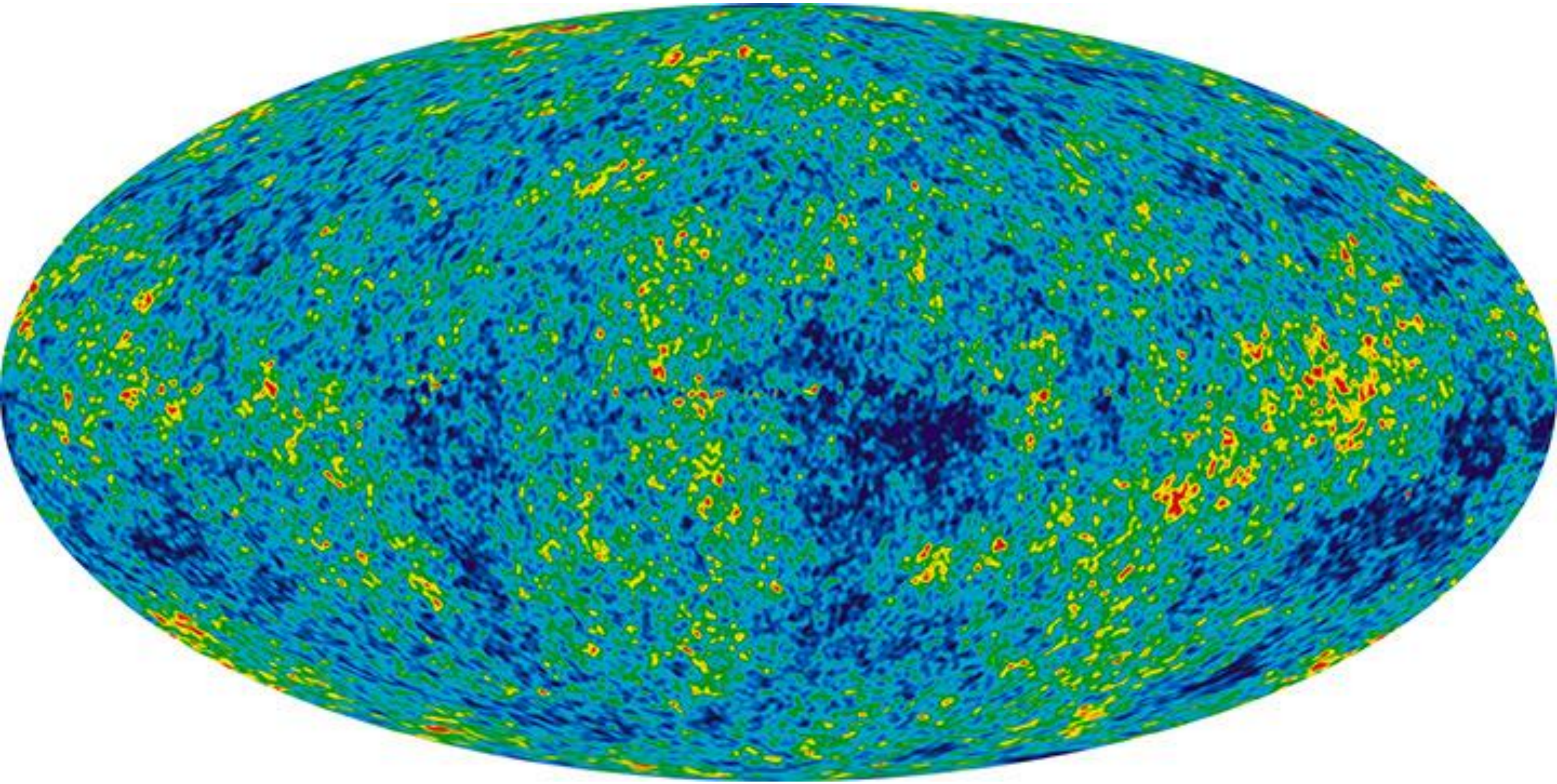
2003

WMAP

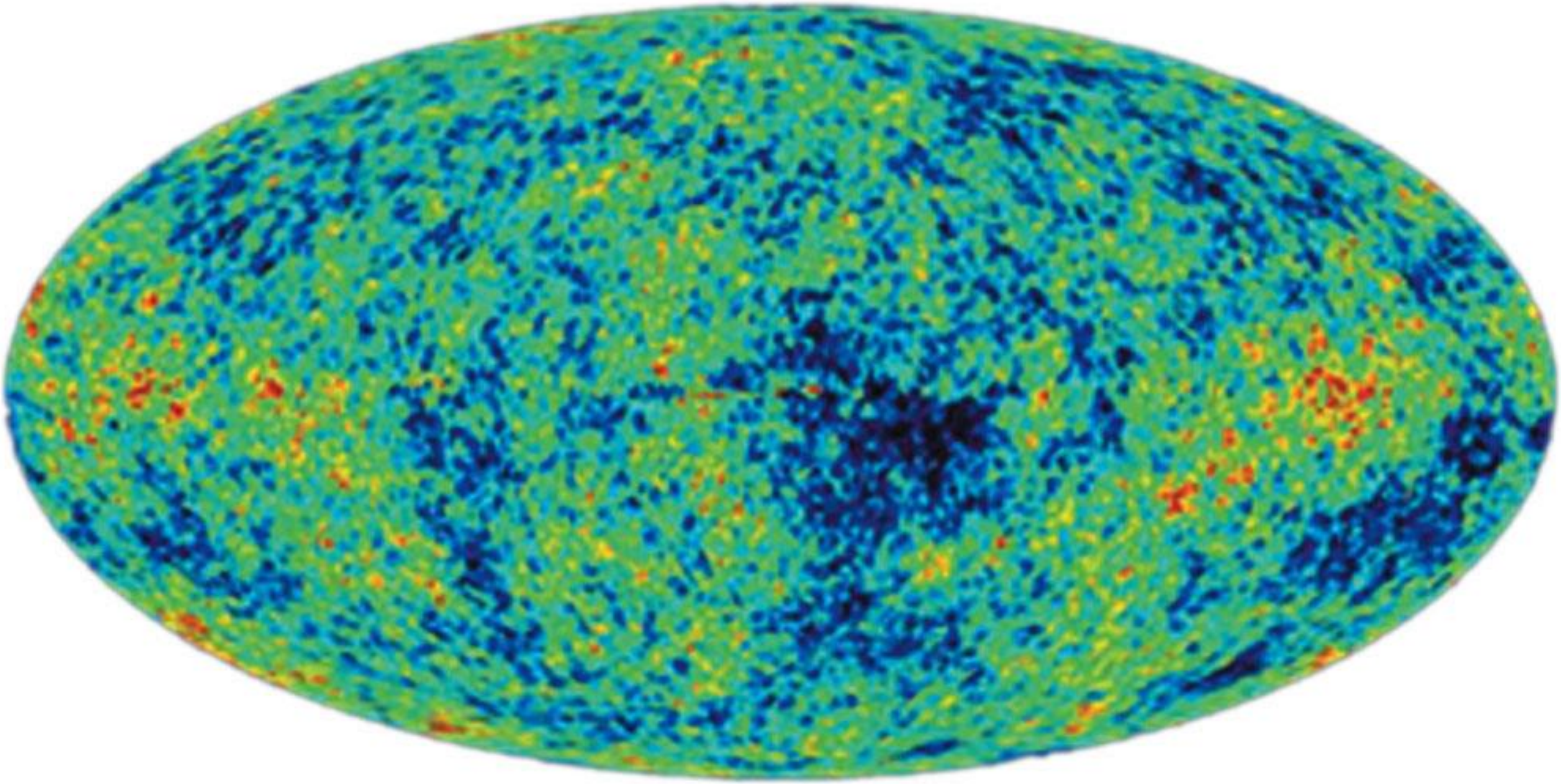








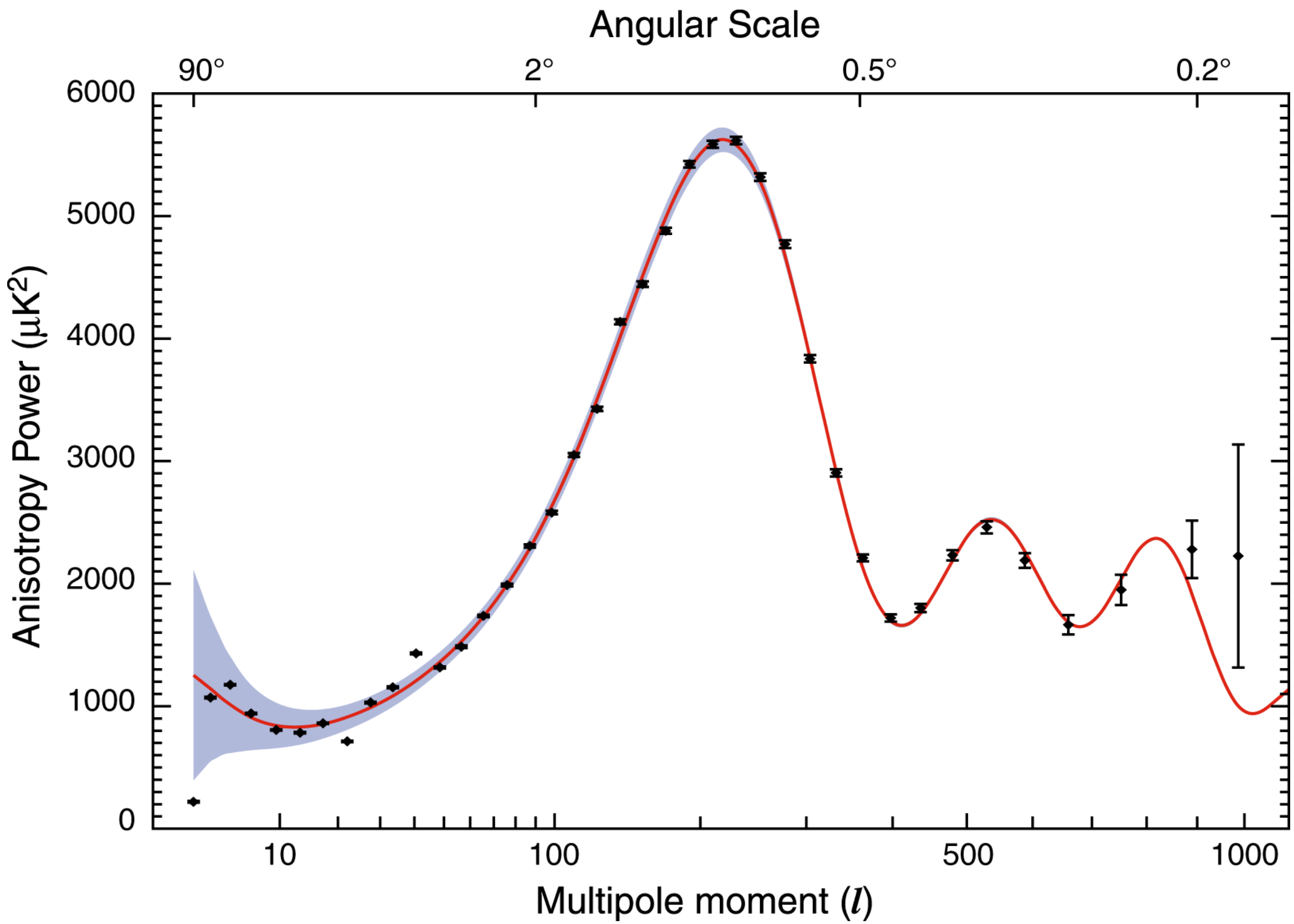
Wilkinson Microwave Anisotropy Probe (WMAP)



NASA's Wilkinson Microwave Anisotropy Probe (WMAP) Science Team

COBE to WMAP

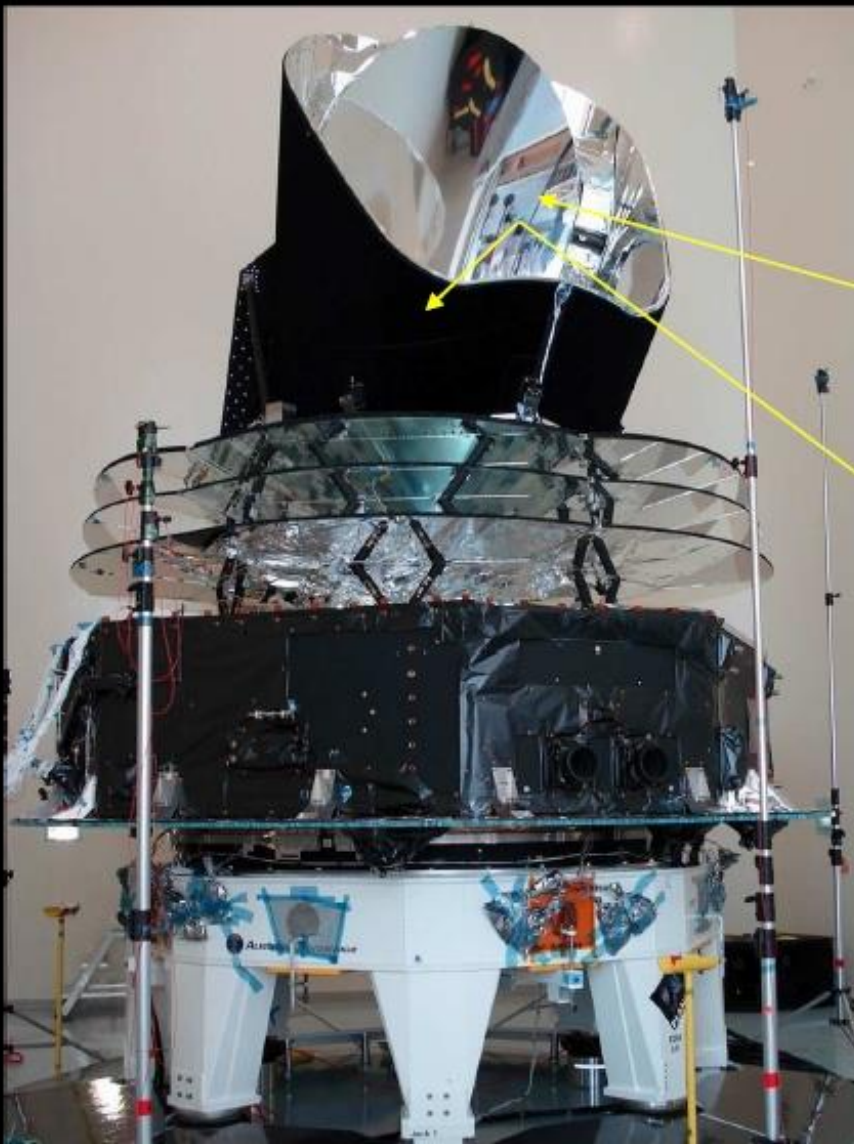
Power spectrum



Max Tegmark's concordance site

PLANCK

Looking back to the dawn of time



Planck Telescope
1.5x1.9m off-axis
Gregorian
T = 50 K

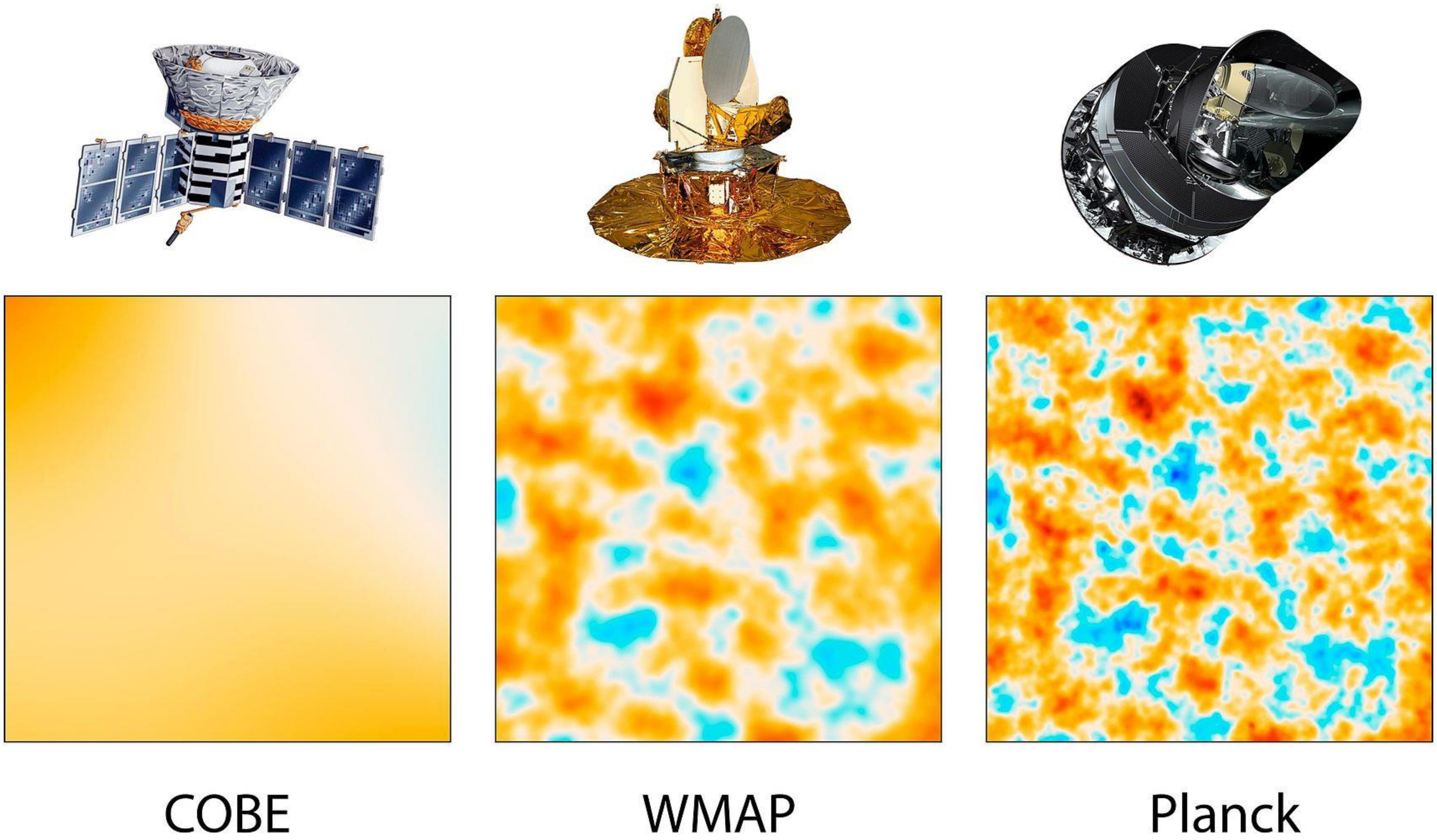
LFI Radiometers
30-70 GHz, T = 20 K

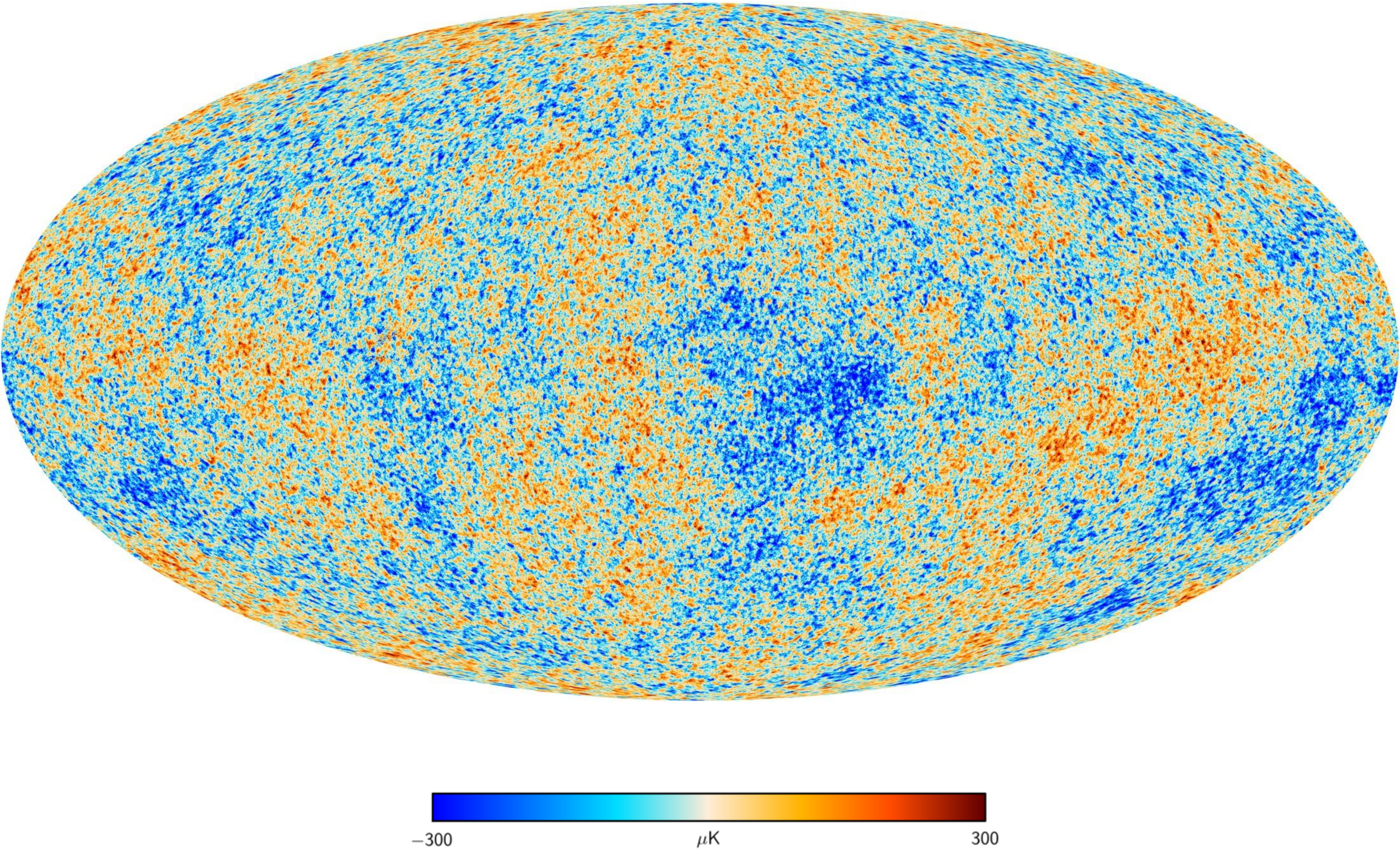
HFI Bolometers
100-857 GHz, T = 0.1 K

agenzia spaziale italiana

cnes
CENTRE NATIONAL D'ÉTUDES SPATIALES


esa


COBE
WMAP
Planck


−300
μK
300

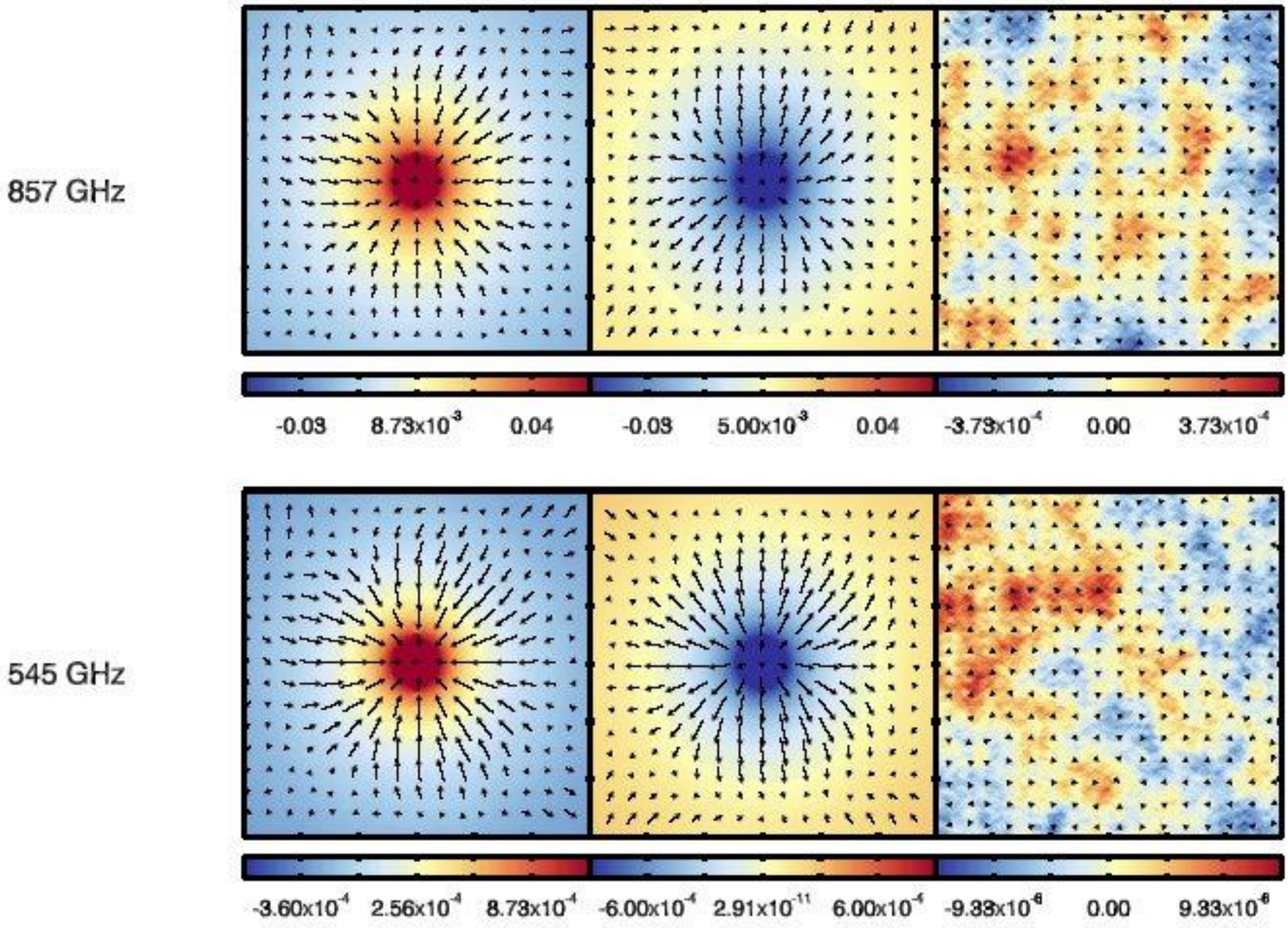
Very important conclusions:

Early universe was almost homogeneous and isotropic

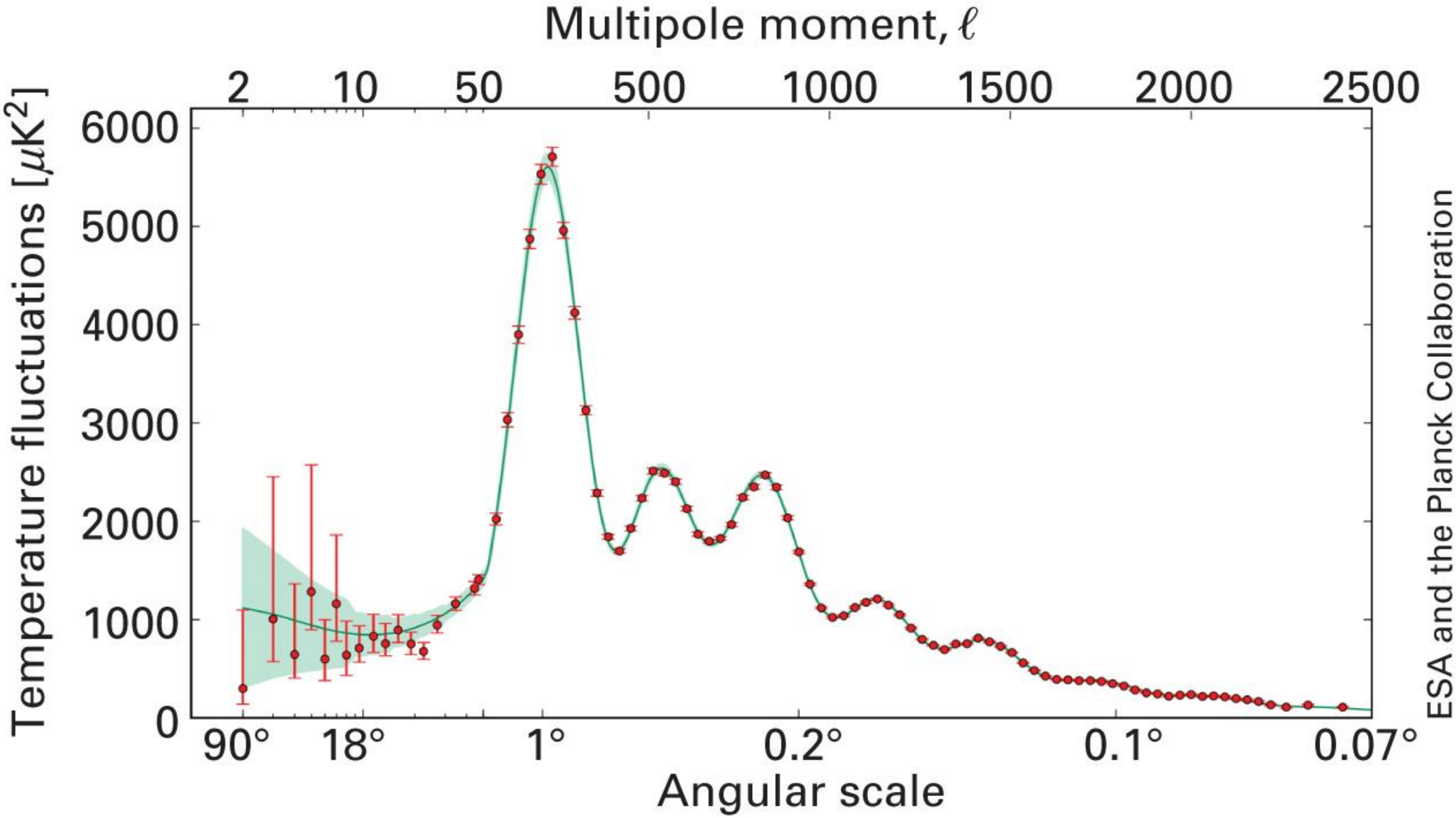
$$\frac{\delta T}{T} \sim 10^{-5} \quad => \quad \frac{\delta\rho}{\rho} \sim 10^{-5}$$

The small temperature fluctuations are random (Gaussian) in nature !

How the primordial fluctuations were formed ?



857 GHz
-0.03
8.73x10^-3
0.04
-0.03
5.00x10^-3
0.04
-3.73x10^-4
0.00
3.73x10^-4
545 GHz
-3.60x10^-4
2.56x10^-4
8.73x10^-4
-6.00x10^-4
2.91x10^-11
6.00x10^-4
-9.33x10^-6
0.00
9.33x10^-6



Power Spectrum from Planck

Angular power spectrum

$$T(\theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta\varphi)\,,$$

$$C_l = \frac{1}{2l+1} \sum_{m} |a_{lm}|^2\,.$$

https://background.uchicago.edu/~whu/index.html
WayneHu

Tegmark https://space.mit.edu/home/tegmark/cmb/pipeline.html

Old Universe – *New* Numbers

$\Omega_{tot} = 1.02^{+0.02}_{-0.02}$

$w < -0.78$ (95% CL)

$\Omega_{\Lambda} = 0.73^{+0.04}_{-0.04}$

$\Omega_b h^2 = 0.0224^{+0.0009}_{-0.0009}$

$\Omega_b = 0.044^{+0.004}_{-0.004}$

$n_b = 2.5 \times 10^{-7}{}^{+0.1\times10^{-7}}_{-0.1\times10^{-7}}$ cm^{-3}

$\Omega_m h^2 = 0.135^{+0.008}_{-0.009}$

$\Omega_m = 0.27^{+0.04}_{-0.04}$

$\Omega_\nu h^2 < 0.0076$ (95%CL)

$m_\nu < 0.23$ eV (95% CL)

$T_{cmb} = 2.725^{+0.002}_{-0.002}$ K

$n_\gamma = 410.4^{+0.9}_{-0.9}$ cm^{-3}

$\eta = 6.1 \times 10^{-10}{}^{+0.3\times10^{-10}}_{-0.2\times10^{-10}}$

$\Omega_b \Omega_m^{-1} = 0.17^{+0.01}_{-0.01}$

$\sigma_8 = 0.84^{+0.04}_{-0.04}$ Mpc

$\sigma_8 \Omega_m^{0.5} = 0.44^{+0.04}_{-0.05}$

$A = 0.833^{+0.086}_{-0.083}$

$n_s = 0.93^{+0.03}_{-0.03}$

$dn_s/d \ln k = -0.031^{+0.016}_{-0.018}$

$r < 0.71$ (95% CL)

$z_{dec} = 1089^{+1}_{-1}$

$\Delta z_{dec} = 195^{+2}_{-2}$

$h = 0.71^{+0.04}_{-0.03}$

$t_0 = 13.7^{+0.2}_{-0.2}$ Gyr

$t_{dec} = 379^{+8}_{-7}$ kyr

$t_r = 180^{+220}_{-80}$ Myr (95% CL)

$\Delta t_{dec} = 118^{+3}_{-2}$ kyr

$z_{eq} = 3233^{+194}_{-210}$

$\tau = 0.17^{+0.04}_{-0.04}$

$z_r = 20^{+10}_{-9}$ (95% CL)

$\theta_A = 0.^\circ598^{+0.002}_{-0.002}$

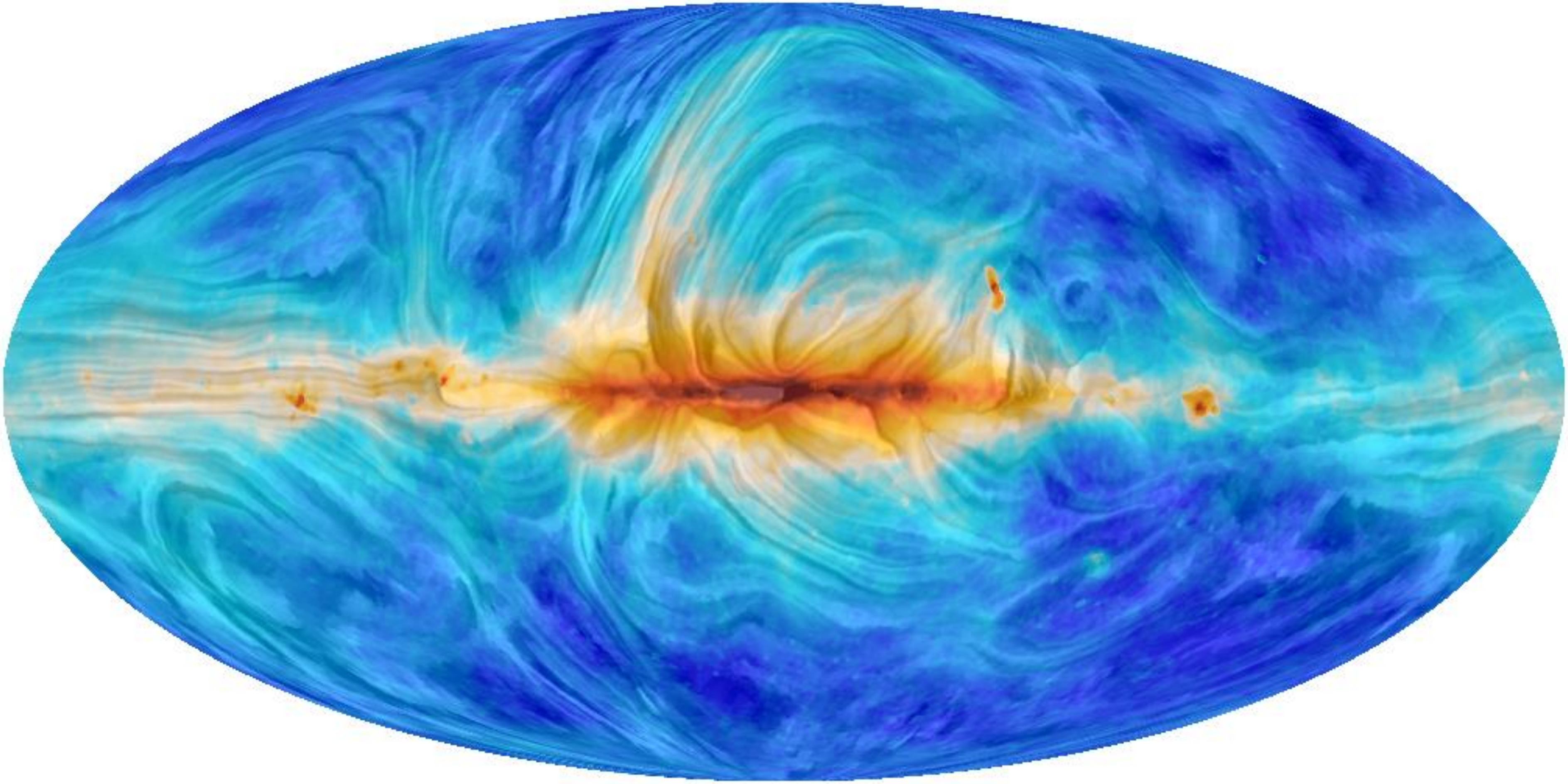
$d_A = 14.0^{+0.2}_{-0.3}$ Gpc

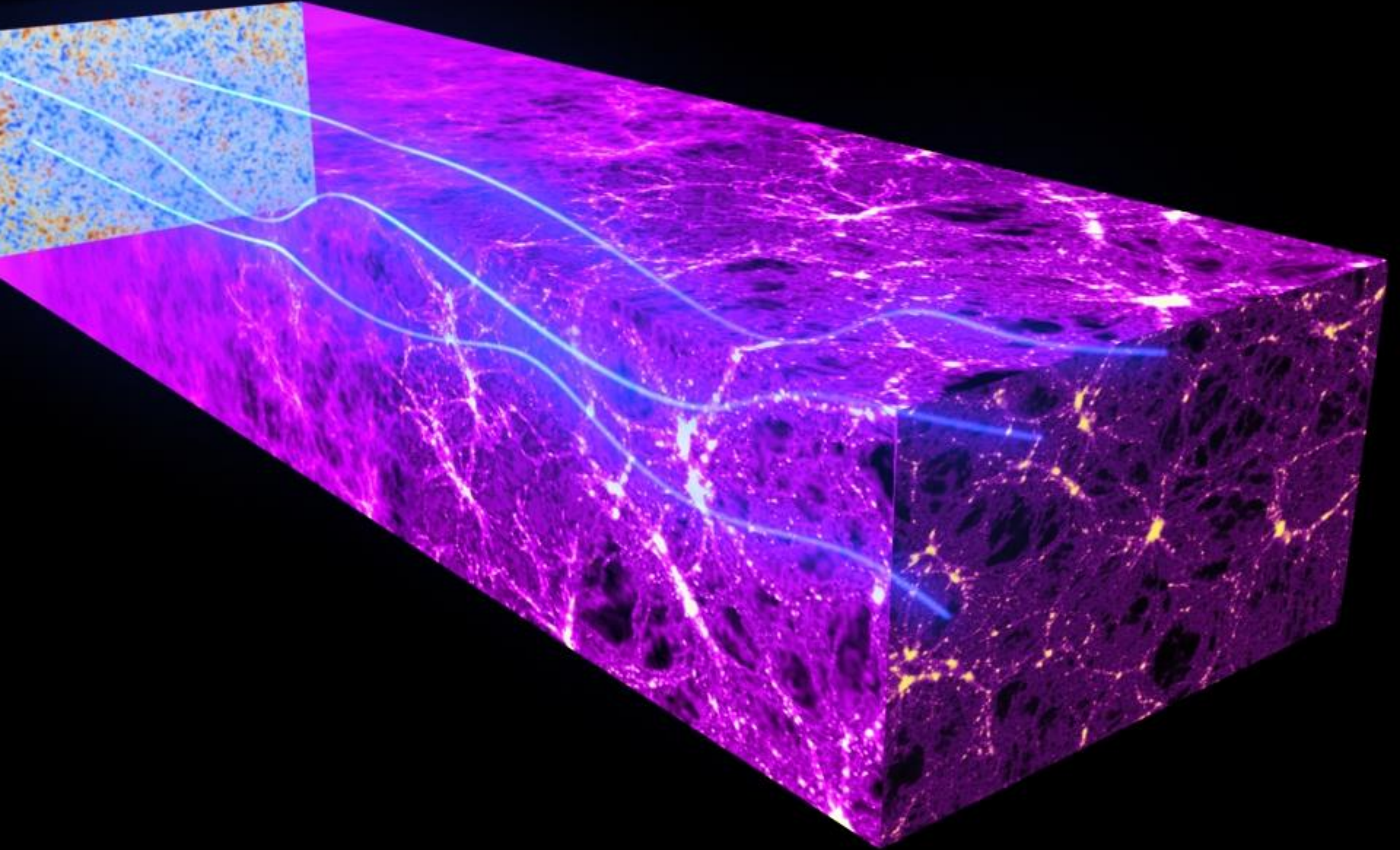
$l_A = 301^{+1}_{-1}$

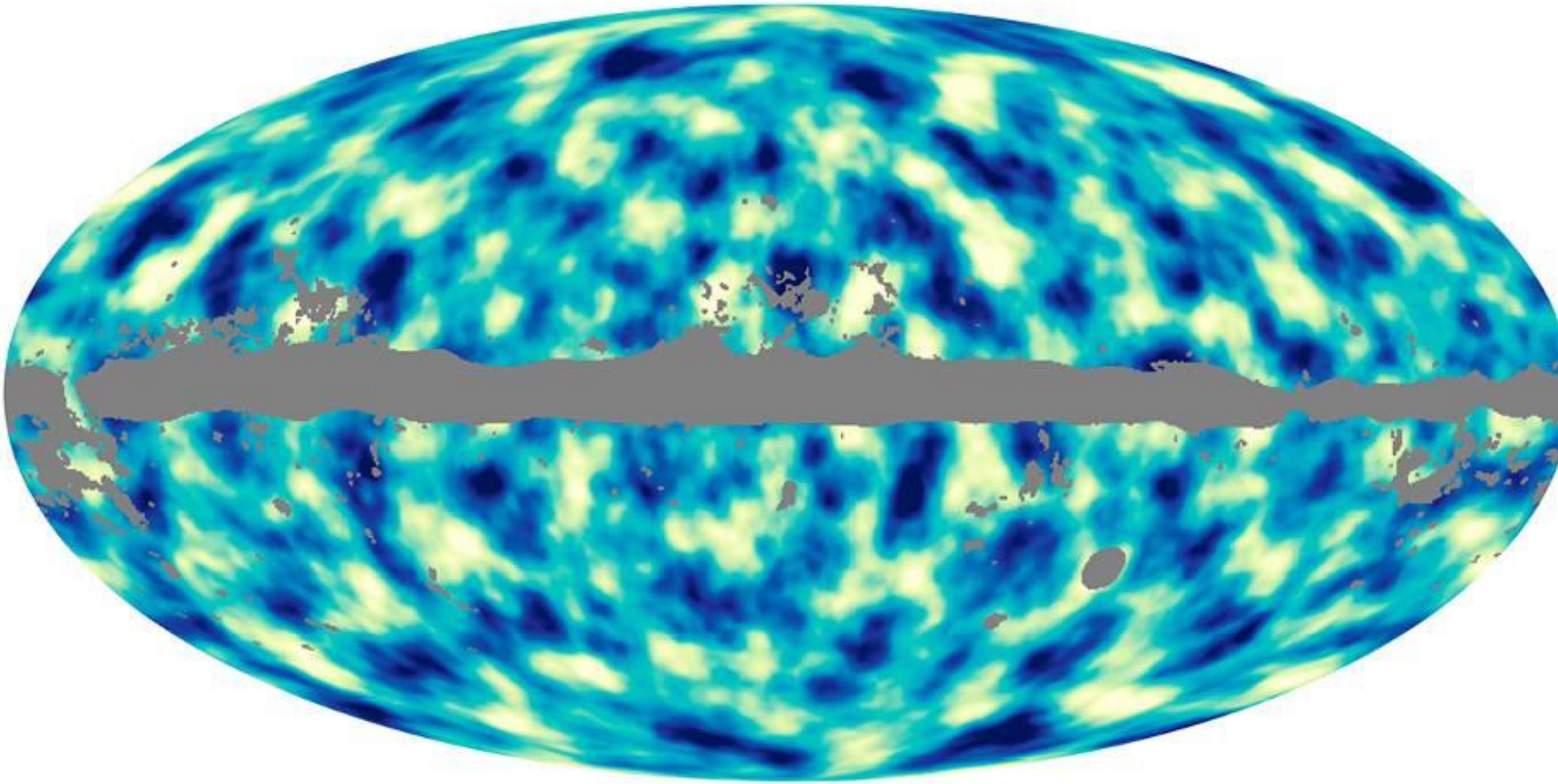
$r_s = 147^{+2}_{-2}$ Mpc

Table 1. Base-ΛCDM cosmological parameters from *Planck* TT,TE,EE+lowE+lensing.

Parameter	Plik best fit	Plik [1]	CamSpec [2]	$([2]-[1])/\sigma_1$	Combined
$\Omega_b h^2$	0.022383	0.02237 ± 0.00015	0.02229 ± 0.00015	−0.5	0.02233 ± 0.00015
$\Omega_c h^2$	0.12011	0.1200 ± 0.0012	0.1197 ± 0.0012	−0.3	0.1198 ± 0.0012
$100\theta_{MC}$	1.040909	1.04092 ± 0.00031	1.04087 ± 0.00031	−0.2	1.04089 ± 0.00031
τ	0.0543	0.0544 ± 0.0073	$0.0536^{+0.0069}_{-0.0077}$	−0.1	0.0540 ± 0.0074
$\ln(10^{10} A_s)$	3.0448	3.044 ± 0.014	3.041 ± 0.015	−0.3	3.043 ± 0.014
n_s	0.96605	0.9649 ± 0.0042	0.9656 ± 0.0042	+0.2	0.9652 ± 0.0042
$\Omega_m h^2$	0.14314	0.1430 ± 0.0011	0.1426 ± 0.0011	−0.3	0.1428 ± 0.0011
H_0 [km s^{-1} Mpc^{-1}]	67.32	67.36 ± 0.54	67.39 ± 0.54	+0.1	67.37 ± 0.54
Ω_m	0.3158	0.3153 ± 0.0073	0.3142 ± 0.0074	−0.2	0.3147 ± 0.0074
Age [Gyr]	13.7971	13.797 ± 0.023	13.805 ± 0.023	+0.4	13.801 ± 0.024
σ_8	0.8120	0.8111 ± 0.0060	0.8091 ± 0.0060	−0.3	0.8101 ± 0.0061
$S_8 \equiv \sigma_8(\Omega_m/0.3)^{0.5}$ . . .	0.8331	0.832 ± 0.013	0.828 ± 0.013	−0.3	0.830 ± 0.013
z_{re}	7.68	7.67 ± 0.73	7.61 ± 0.75	−0.1	7.64 ± 0.74
$100\theta_*$	1.041085	1.04110 ± 0.00031	1.04106 ± 0.00031	−0.1	1.04108 ± 0.00031
r_{drag} [Mpc]	147.049	147.09 ± 0.26	147.26 ± 0.28	+0.6	147.18 ± 0.29







The Saha equation

Consider a hydrogen atom with a single energy level (ground state), and consider the ionization reaction

$$H^+ + e \leftrightarrow H^0 + \chi_H \,,$$

where $\chi_H = 13.6$ eV is the ground state binding energy. Integration of the distribution function over the momenta yields the number densities

$$n_e = \frac{2(2\pi m_e kT)^{3/2}}{h^3} \exp\frac{\mu^-}{kT} \,,$$

$$n^+ = \frac{(2\pi m_p kT)^{3/2}}{h^3} \exp\frac{\mu^+}{kT} \,,$$

$$n_0 = \frac{2(2\pi(m_p + m_e)kT)^{3/2}}{h^3} \exp\frac{\mu^0}{kT} \exp\frac{\chi_H}{kT} \,.$$

μ – denotes the chemical potential

Forming the product $n^+ n_e / n_0$, we find

$$\frac{n^+ n_e}{n_0} = \frac{(2m_e kT)^{3/2}}{h^3} \exp \frac{(\mu^- + \mu^+ - \mu_0)}{kT} \exp \frac{-\chi_H}{kT} \, .$$

In equilibrium $(\mu^- + \mu^+ - \mu_0) = 0$, so we finally find that

$$\frac{n^+ n_e}{n_0} = \frac{(2m_e kT)^{3/2}}{h^3} \exp \frac{-\chi_H}{kT} \, .$$

Charge balance implies $n^+ = n$, and conservation of nucleons means that $n^0 + n^+ = n$, the total number density of protons (which equals the total number density of electrons). With these extra conditions, the degree of ionization $y = n^+/n = n^e/n$ is, numerically,

$$\frac{y^2}{1-y} = \frac{4 \cdot 10^{-9}}{\varrho} T^{3/2} \exp \frac{-1.6 \cdot 10^5}{T} \, ,$$

where T and ϱ are in the cgs units.

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In[•]:= Plot[3×10^{11} x^{3/2} Exp[{-1.6×10^{5}/x}], {x, 3500 , 2600}, AxesLabel → {T, y}]
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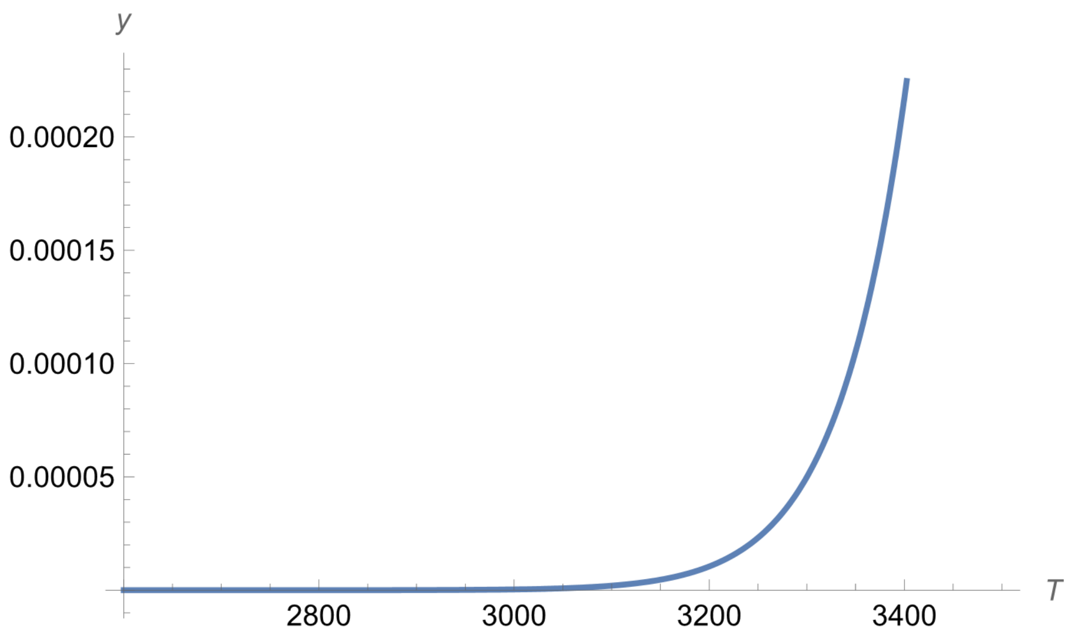
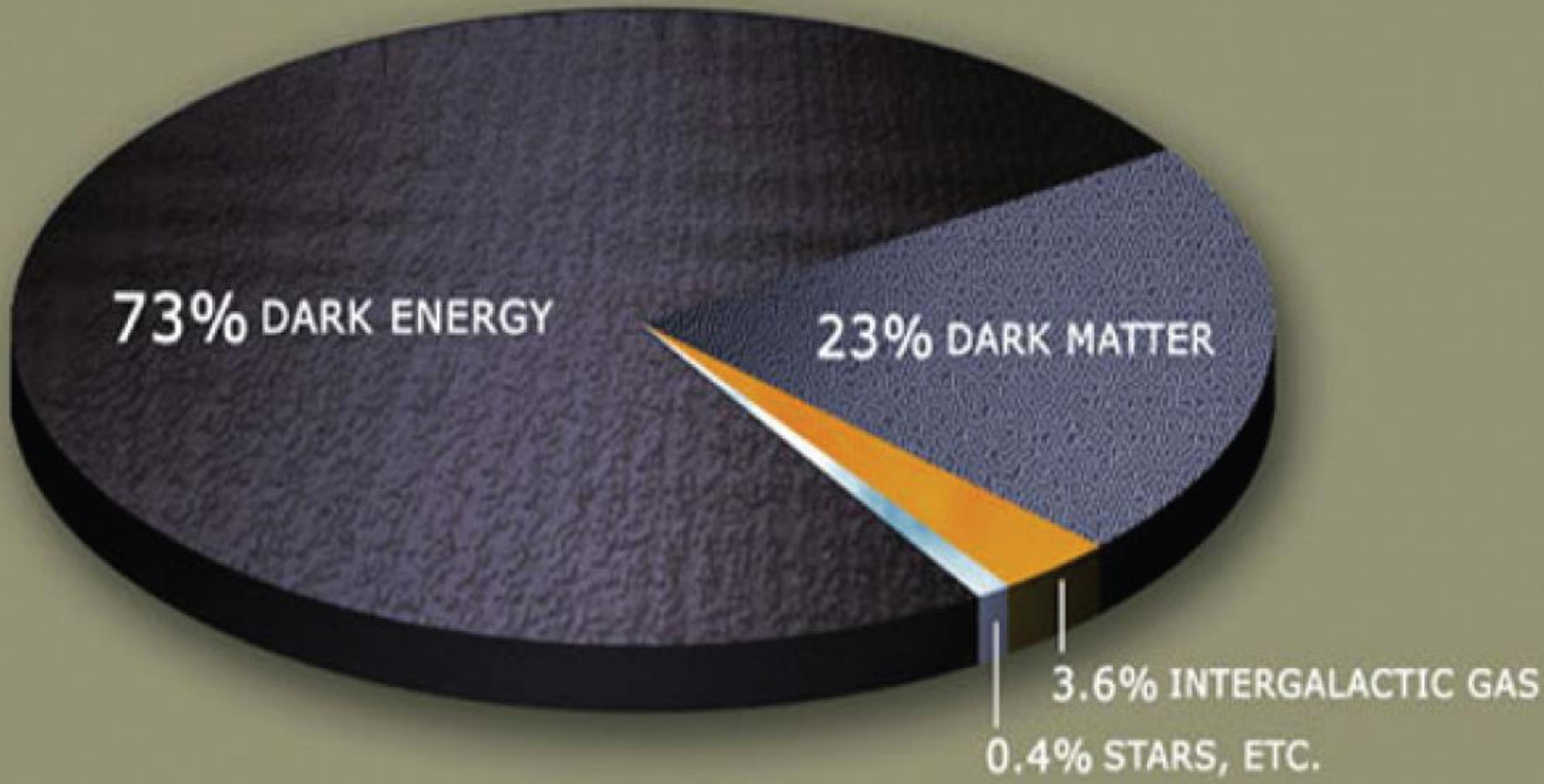
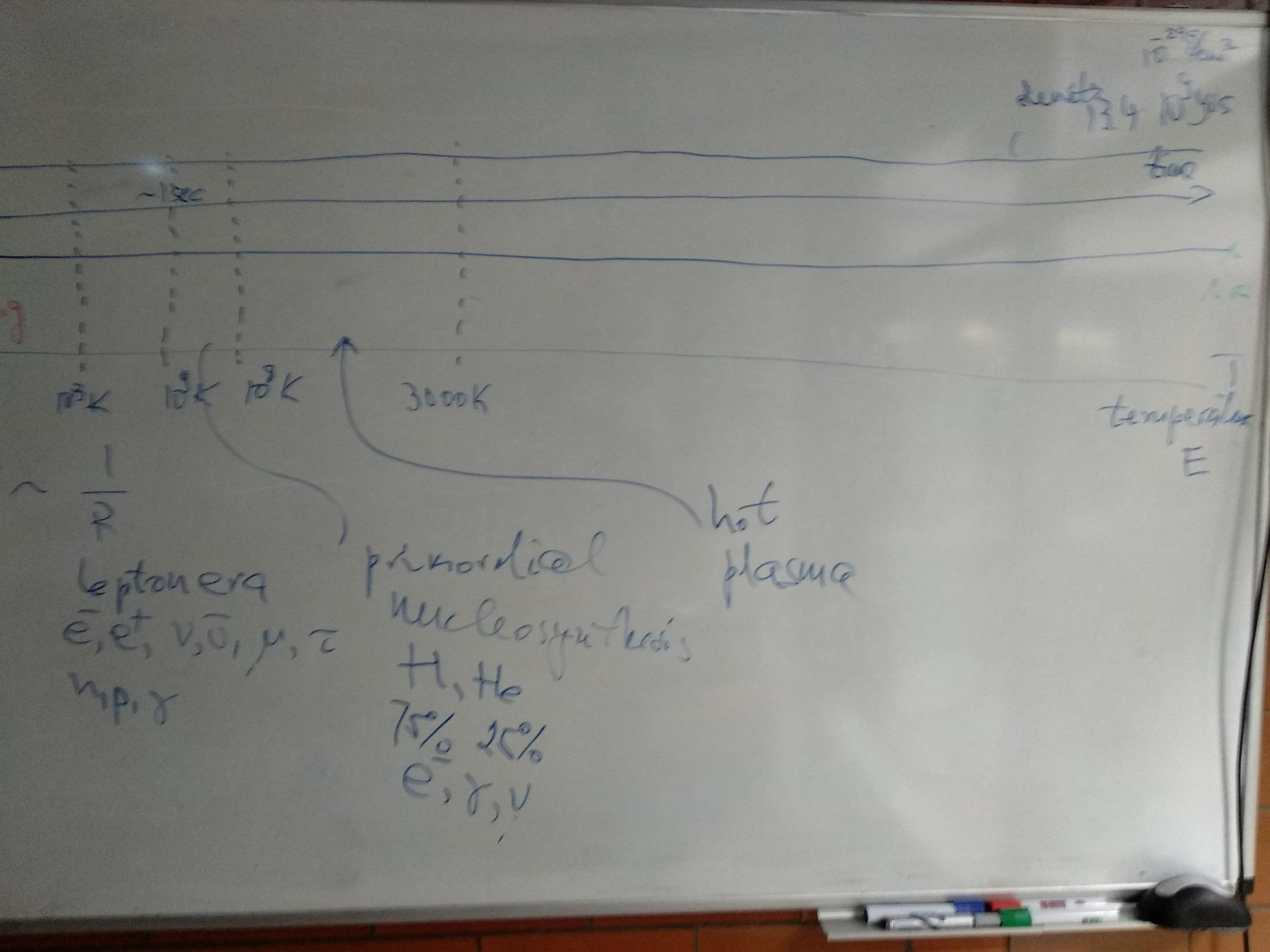
Out[•]=




73% DARK ENERGY
23% DARK MATTER
3.6% INTERGALACTIC GAS
0.4% STARS, ETC.



~1sec
time
10^13 K
10^9 K
10^8 K
3000K
temperature
E
~ 1/R
lepton era
e-, e+, ν, ν̄, μ, τ
n, p, γ
primordial nucleosynthesis
H, He
75% 25%
e-, γ, ν
hot plasma