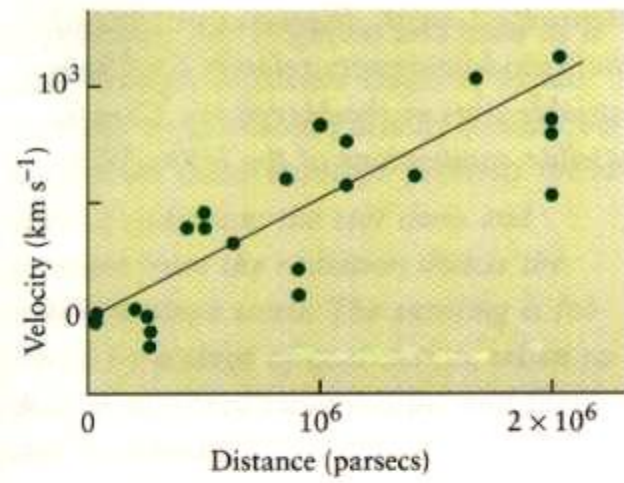


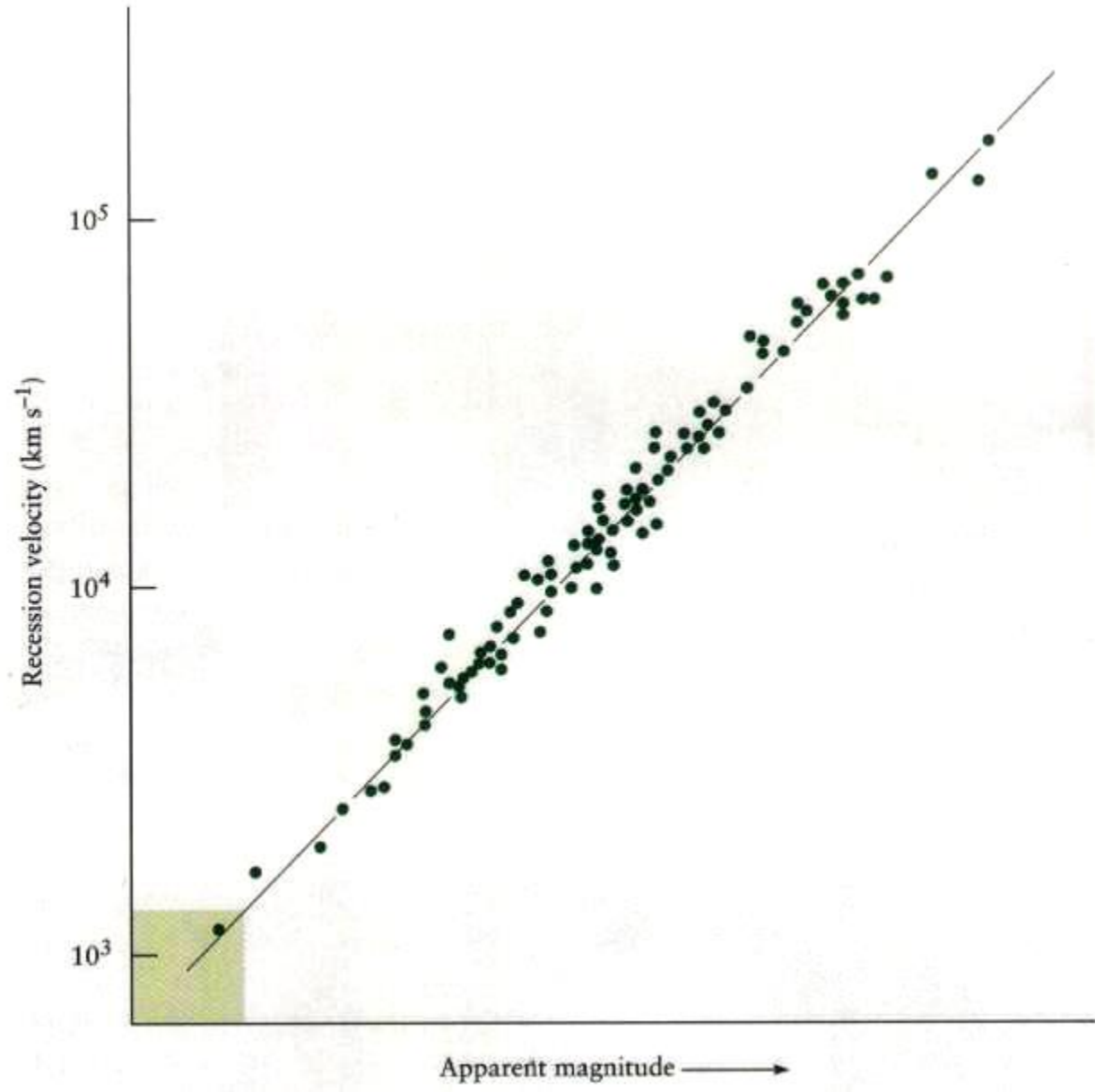


Introduction to Cosmology

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University of Warsaw*



Edwin Hubble made this plot of galaxy velocities versus distances in the 1930s (above left). He only sampled a nearby volume of space, as far as the Virgo cluster. This so-called Hubble diagram was greatly extended by Alan Sandage and his collaborators (right), who compared the recession velocities of the brightest galaxies in galaxy clusters with distances to the clusters as inferred from the apparent magnitudes of these galaxies. A galaxy cluster is so luminous that it can be recognized at a large distance from us. Both plots show that recession velocity increases proportionately to distance.



The background of the image is a vast field of galaxies, likely from a deep space survey. The galaxies are scattered across the frame, appearing in various colors including red, blue, and white. Some are bright and clear, while others are faint and distant. The overall effect is a rich, multi-colored cosmic landscape.

The Universe is a
dynamical system

Modeling the Universe

Basic assumption -
the laws of physics and
mathematics hold
everywhere in the Universe

Cosmological Copernican Principle

The Milky Way is just a normal galaxy and it does not occupy a preferred position in the Universe

A background image of a starry night sky. In the center, there is a prominent nebula with blue and red hues. The sky is filled with numerous bright stars of various colors, including white, yellow, and blue. The overall scene is dark, with the stars and nebula providing the primary light source.

Bold assumption:

on a sufficiently large scale

the Universe is

homogeneous and isotropic

Homogeneous and isotropic distribution of matter

$$\vec{r}(t) = R(t)\vec{r}_0$$

$R(t)$ – scale factor

$$\frac{d}{dt}\vec{r}(t) = \frac{d}{dt}R(t)\vec{r}_0$$

$$\vec{v}(t) = \frac{\dot{R}(t)}{R(t)}\vec{r}(t)$$

The Hubble constant – $H(t) = \frac{\dot{R}(t)}{R(t)}$

$$\vec{v}(t) = H(t)\vec{r}(t)$$

How to determine $R(t)$?

First attempt -
Newtonian dynamics

PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA.

Autore J. S. NEWTON, Trin. Coll. Cantab. Soc. Matheseos
Professore *Lucasiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPY S, Reg. Soc. PRÆSES.
Julii 5. 1686.

LONDINI,

Jussu Societatis Regiæ ac Typis *Josephi Streater*. Prostat apud
plures Bibliopolas. Anno MDCLXXXVII.

M. Warrick
OPTICKS:

OR, A
TREATISE
OF THE
REFLEXIONS, REFRACTIONS,
INFLEXIONS and COLOURS
OF

LIGHT.

By S. Isaac. ALSO *Newton.*

TWO TREATISES
OF THE
SPECIES and MAGNITUDE
OF

Curvilinear Figures.

LONDON,

Printed for SAM. SMITH, and BENJ. WALFORD,
Printers to the Royal Society, at the *Prince's Arms* in
St. Paul's Church-yard. MDCCIV.

Continuous medium (homogeneous and isotropic)

Newtonian mechanics

$$\vec{r}(t) = R(t) \cdot \vec{r}_0, \quad \vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{\dot{R}(t)}{R(t)} \cdot \vec{r}(t),$$

The continuity equation

$$\frac{\delta \varrho}{\delta t} + \operatorname{div}(\varrho \vec{v}) = 0,$$

$$\varrho = \varrho(t); \quad \dot{\varrho} + 3 \frac{\dot{R}(t)}{R(t)} \cdot \varrho = 0,$$

$$\Rightarrow \varrho(t) \cdot R^3 = \text{const},$$

The Euler equation

$$\frac{d\vec{v}}{dt} = -\frac{1}{\rho} \text{grad} p - \text{grad} \Phi, \quad \Phi - \text{gravitational potential},$$

The Poisson equation

$$\Delta \Phi = 4\pi G \rho,$$

$$\rho = \rho(t); \quad p = p(t), \quad \Delta \Phi = 4\pi G \rho(t) \Rightarrow \Phi = 4\pi G \rho(t) \frac{r^2}{6},$$

$$\frac{d\vec{v}}{dt} = -\text{grad}\Phi, \Rightarrow \frac{d}{dt}\left(\frac{\dot{R}}{R}\right) \cdot \vec{r}(t) + \left(\frac{\dot{R}}{R}\right)^2 \cdot \vec{r}(t) = -\frac{4\pi G\rho}{3} \cdot \vec{r}(t),$$

$$\frac{d}{dt}\left(\frac{\dot{R}}{R}\right) + \left(\frac{\dot{R}}{R}\right)^2 = -\frac{4\pi G\rho}{3},$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G\rho}{3}, \Rightarrow \ddot{R} < 0,$$

gravitational interactions slow down the expansion

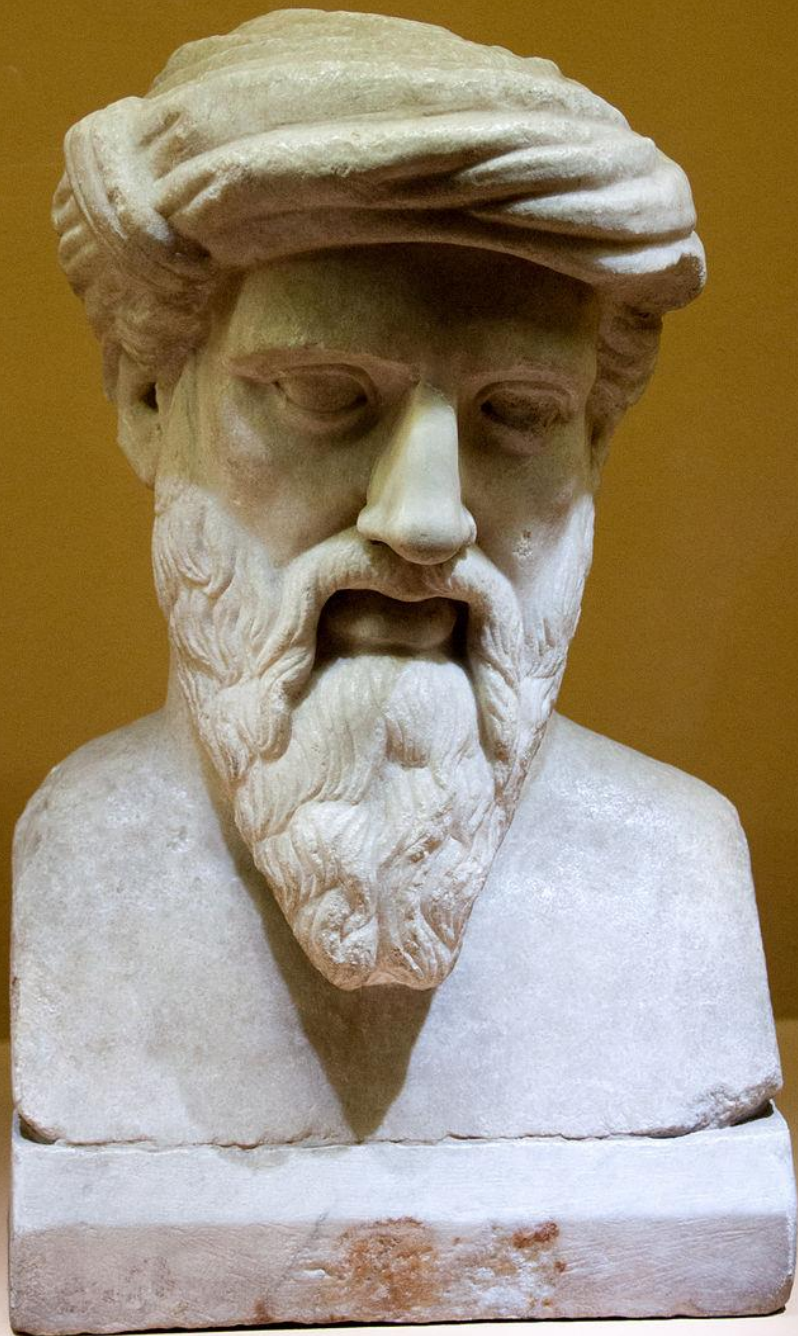
$$\left(\frac{\ddot{R}}{R} = -\frac{4\pi G \varrho}{3}\right) \cdot (\dot{R}R)$$

$$\ddot{R} \cdot \dot{R} = -\frac{4\pi GA}{3} \cdot \frac{\dot{R}}{R^2}, \quad A = \varrho(t_0) \cdot R^3(t_0),$$

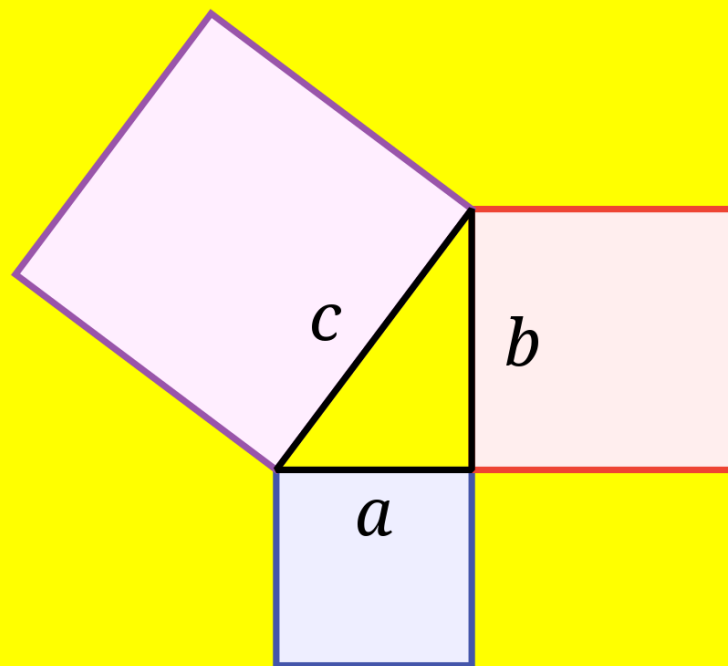
$$\frac{1}{2} \frac{d}{dt} (\dot{R})^2 = \frac{4\pi GA}{3} \frac{d}{dt} \left(\frac{1}{R}\right),$$

$$\dot{R}^2 = \frac{8\pi GA}{3} \frac{1}{R} + \text{const}, \quad \text{we will take const} = 0,$$

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3} \cdot \varrho(t), \quad H^2(t) = \frac{8\pi G}{3} \cdot \varrho(t).$$



Pythagoras
570 - 495 BC



$$a^2 + b^2 = c^2$$

Geometry of space

Euclid
~ 304 -282 BC



See the Bold Shakes of Virgil's Glory,
Immortal in the Race, no less in Story,
An Artful without Error, from which Lyne,
Both Earth and Heav'n, in secret Prophecies trace:
Behold Great EUCLID, has beheld thee well,
For 'tis in vain, to say thyself a well!

G. Waller.

EUCLID'S ELEMENTS OF Geometry.

In XV. Books:

With a Supplement of divers PROPOSITIONS
and COROLLARIES.

To which is added, a Treatise of REGULAR SOLIDS,
By CAMPEANE and FLUSSAS.

LIKewise

Euclid's DATA.

And MARINUS his Preface
thereto annexed.

Also a Treatise of the Divisions of Superficies, ascribed to
Machomet Baghelene, but published by Commandine, at the
request of John Dow of London; whose Preface to the said Treatise
declares it to be the Works of EUCLIDE,
the Author of these ELEMENTS.

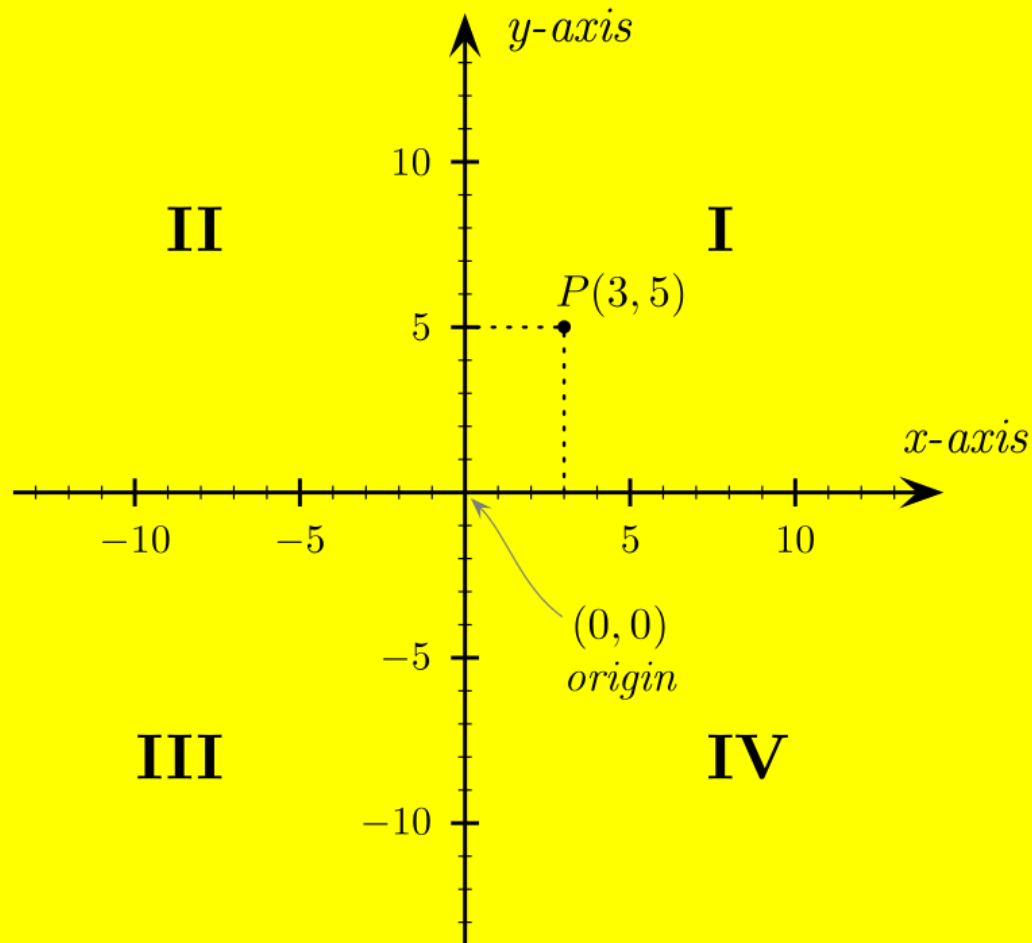
Published by the Care and Industry of
JOHN LEEKE and GEORGE SERLE, Students
in the MATHEMATICKS.

LONDON:
Printed, by R. & W. LEYBOURN, for GEORGE
SAWBIDGE at the Bell upon Ludgate-hill,
M D C L X I.



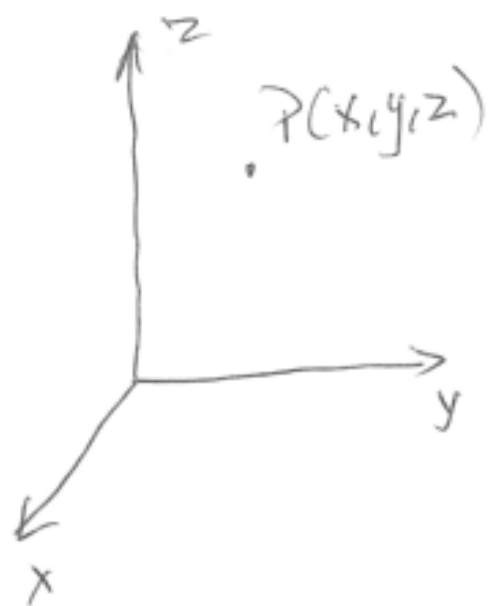
René
Descartes
1596 - 1650

Cartesian coordinates



Elements of analytic geometry

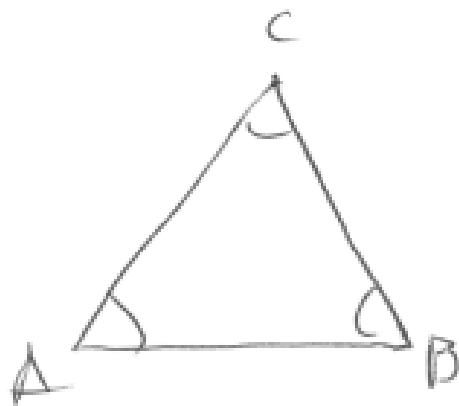
Cartesian coordinates:



distance: $P(x_1, y_1, z_1)$; $Q(x_2, y_2, z_2)$

$$d_{PQ}^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

$$d_{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$



P, Q, R

$$\angle P < \angle R + \angle Q$$

$$\angle A + \angle B + \angle C = 180^\circ$$

$$P(x, y, z) \quad Q(x+dx, y+dy, z+dz)$$

$$dPA^2 = dx^2 + dy^2 + dz^2 = dl^2 \quad \text{line element}$$

Line in 3-dim space

$$L: x(t), y(t), z(t) \Rightarrow \vec{r}(t) \quad \frac{d\vec{r}}{dt} = \vec{v}(t)$$

$$dl^2 = [\dot{x}(t)^2 + \dot{y}(t)^2 + \dot{z}(t)^2] \cdot dt^2 \quad \frac{d\vec{r}}{dl} = \frac{d\vec{r}}{dt} \cdot \frac{dt}{dl}$$

$$\frac{d\vec{r}}{ds} \cdot \frac{d\vec{r}}{ds} = 1 \quad \frac{d\vec{r}}{ds} = \vec{t} \quad \vec{t} \cdot \vec{t} = 1 \quad \text{tangent vector}$$

$$\frac{d\vec{t}}{ds} \cdot \vec{t} = 0 \Rightarrow \frac{d\vec{t}}{ds} \perp \vec{t} ; \quad \frac{d\vec{t}}{ds} = \frac{1}{\rho} \cdot \vec{n} \quad \frac{1}{\rho} - \text{curvature}$$

radius

$$\vec{n} \cdot \vec{n} = 1 ; \quad \vec{n} \cdot \vec{t} = 0$$

\vec{n} - normal vector

$$\frac{d\vec{n}}{ds} = \frac{\vec{t}}{\rho} - \frac{\vec{b}}{\tau} \quad \vec{b} \cdot \vec{b} = 1$$

$$\vec{b} \cdot \vec{n} = \vec{b} \cdot \vec{t} = 0$$

$$\vec{\tau} - \text{torsion (radius)} \quad \frac{d\vec{b}}{ds} = -\frac{\vec{n}}{\tau}$$

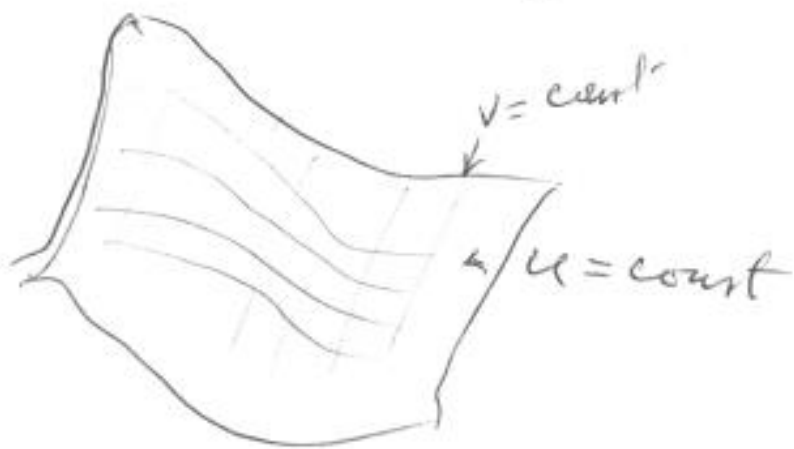
\vec{b} - binormal vector

$$(\vec{t}, \vec{n}, \vec{b}) \quad (\rho, \tau)$$

Surfaces: $x = x(u, v)$; $y = y(u, v)$; $z = z(u, v)$

$$dl^2 = dx^2 + dy^2 + dz^2$$

$$dl^2 = E(u, v) du^2 + 2F(u, v) du dv + G(u, v) dv^2$$



$$\vec{t}_1 \sim \begin{pmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} & \frac{\partial z}{\partial u} \end{pmatrix}$$

$$\vec{t}_2 \sim \begin{pmatrix} \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} & \frac{\partial z}{\partial v} \end{pmatrix}$$

$$\vec{n} \sim \vec{t}_1 \times \vec{t}_2 \quad (\text{vector product}).$$

Differential coordinates:

$$x = r \sin \theta \cos \varphi$$

$$y = r \sin \theta \sin \varphi$$

$$z = r \cos \theta$$

$$dl^2 = dx^2 + dy^2 + dz^2$$

$$dl^2 = dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\varphi^2)$$

Curved spaces

$$dl^2 = g_{ab}(x^c) dx^a dx^b$$

$$X_a X^a = \sum_{a=1}^4 X_a X^a$$

(Einstein summation notation)