Introduction to Cosmology

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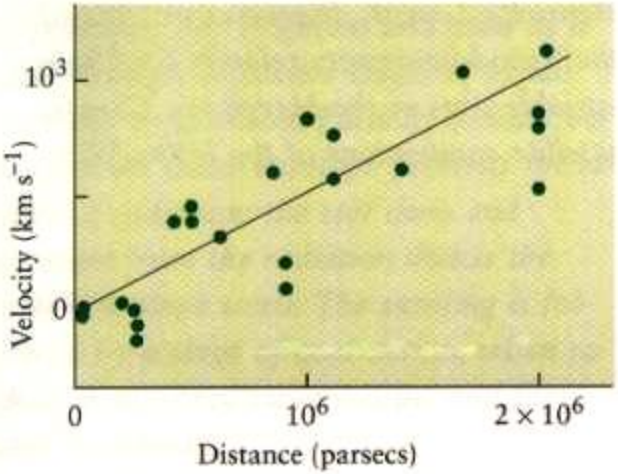
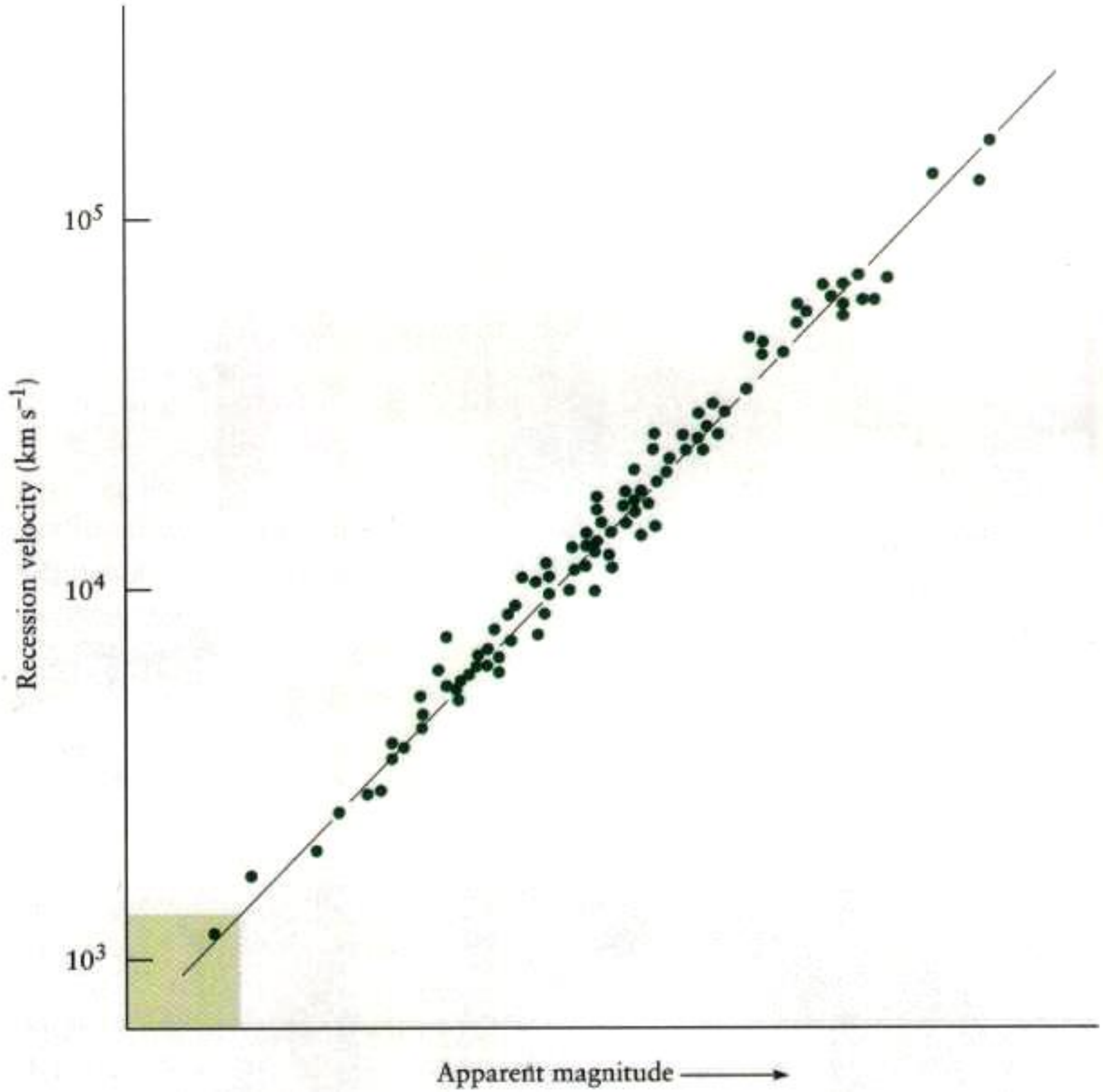
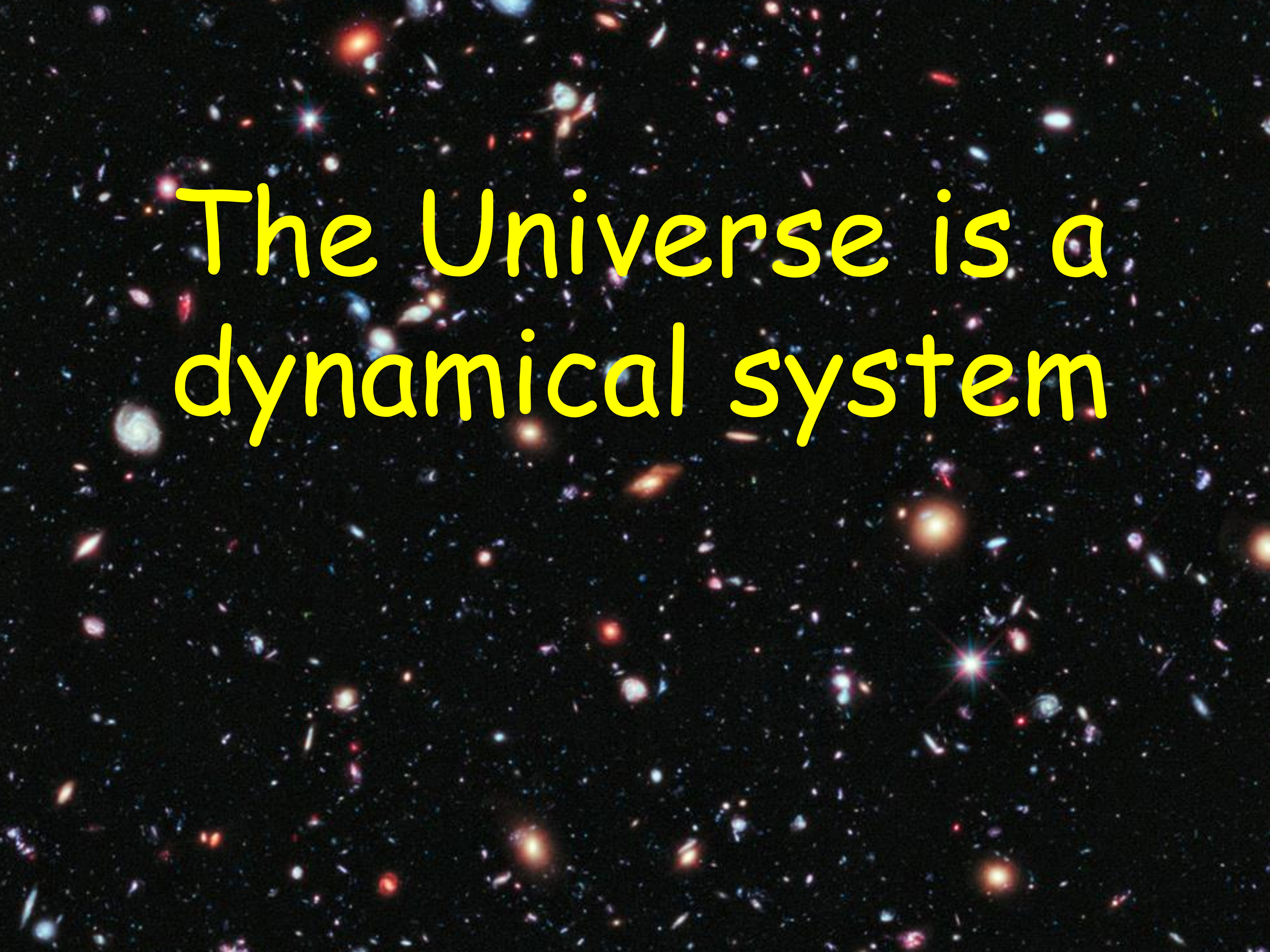




Edwin Hubble made this plot of galaxy velocities versus distances in the 1930s (above left). He only sampled a nearby volume of space, as far as the Virgo cluster. This so-called Hubble diagram was greatly extended by Alan Sandage and his collaborators (right), who compared the recession velocities of the brightest galaxies in galaxy clusters with distances to the clusters as inferred from the apparent magnitudes of these galaxies. A galaxy cluster is so luminous that it can be recognized at a large distance from us. Both plots show that recession velocity increases proportionately to distance.


The Universe is a dynamical system

Modeling the Universe

Basic assumption -
the laws of physics and mathematics hold everywhere in the Universe

Cosmological Copernican Principle

The Milky Way is just a normal galaxy and it does not occupy a preferred position in the Universe

Bold assumption:

on a sufficiently large scale

the Universe is

homogeneous and isotropic

Homogeneous and isotropic distribution of matter

$$\vec{r}(t) = R(t)\vec{r}_0$$

R(t) – scale factor

$$\frac{d}{dt}\vec{r}(t) = \frac{d}{dt}R(t)\vec{r}_0$$

$$\vec{v}(t) = \frac{\dot{R}(t)}{R(t)}\vec{r}(t)$$

The Hubble constant – $H(t) = \frac{\dot{R}(t)}{R(t)}$

$$\vec{v}(t) = H(t)\vec{r}(t)$$

How to determine R(t) ?

First attempt -
Newtonian dynamics

PHILOSOPHIÆ NATURALIS PRINCIPIA MATHEMATICA.

Autore *JS. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheseos Professore *Lucasiano*, & Societatis Regalis Sodali.

IMPRIMATUR.
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,

Jussu *Societatis Regiæ* ac Typis *Josephi Streater*. Prostat apud plures Bibliopolas. *Anno* MDCLXXXVII.

M Warwick

OPTICKS: OR, A TREATISE OF THE REFLEXIONS, REFRACTIONS, INFLEXIONS and COLOURS OF LIGHT.

by Sr. Isaac Newton.

ALSO Two TREATISES OF THE SPECIES and MAGNITUDE OF Curvilinear Figures.

LONDON,

Printed for SAM. SMITH, and BENJ. WALFORD, Printers to the Royal Society, at the *Prince's Arms* in St. *Paul's* Church-yard. MDCCIV.

Continuous medium (homogeneous and isotropic)

Newtonian mechanics

$$\vec{r}(t) = R(t) \cdot \vec{r}_0, \quad \vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \frac{\dot{R}(t)}{R(t)} \cdot \vec{r}(t),$$

The continuity equation

$$\frac{\delta \varrho}{\delta t} + \mathrm{div}(\varrho \vec{v}) = 0,$$

$$\varrho = \varrho(t); \quad \dot{\varrho} + 3\frac{\dot{R}(t)}{R(t)} \cdot \varrho = 0,$$

$$\Rightarrow \varrho(t) \cdot R^3 = \mathrm{const},$$

The Euler equation

$$\frac{d\vec{v}}{dt} = -\frac{1}{\varrho}\mathrm{grad}p - \mathrm{grad}\Phi, \quad \Phi - \text{gravitational potential},$$

The Poisson equation

$$\Delta\Phi = 4\pi G\varrho\,,$$

$$\varrho = \varrho(t);\ \ p = p(t),\ \ \Delta\Phi = 4\pi G\varrho(t) \Rightarrow \Phi = 4\pi G\varrho(t)\frac{r^2}{6}\,,$$

$$\frac{d\vec{v}}{dt} = -\mathrm{grad}\Phi\,, \ \Rightarrow \ \frac{d}{dt}(\frac{\dot{R}}{R}) \cdot \vec{r}(t) + (\frac{\dot{R}}{R})^2 \cdot \vec{r}(t) = -\frac{4\pi G\varrho}{3} \cdot \vec{r}(t)\,,$$

$$\frac{d}{dt}(\frac{\dot{R}}{R}) + (\frac{\dot{R}}{R})^2 = -\frac{4\pi G\varrho}{3}\,,$$

$$\frac{\ddot{R}}{R} = -\frac{4\pi G\varrho}{3}\,, \ \Rightarrow \ \ddot{R} < 0\,,$$

gravitational interactions slow down the expansion

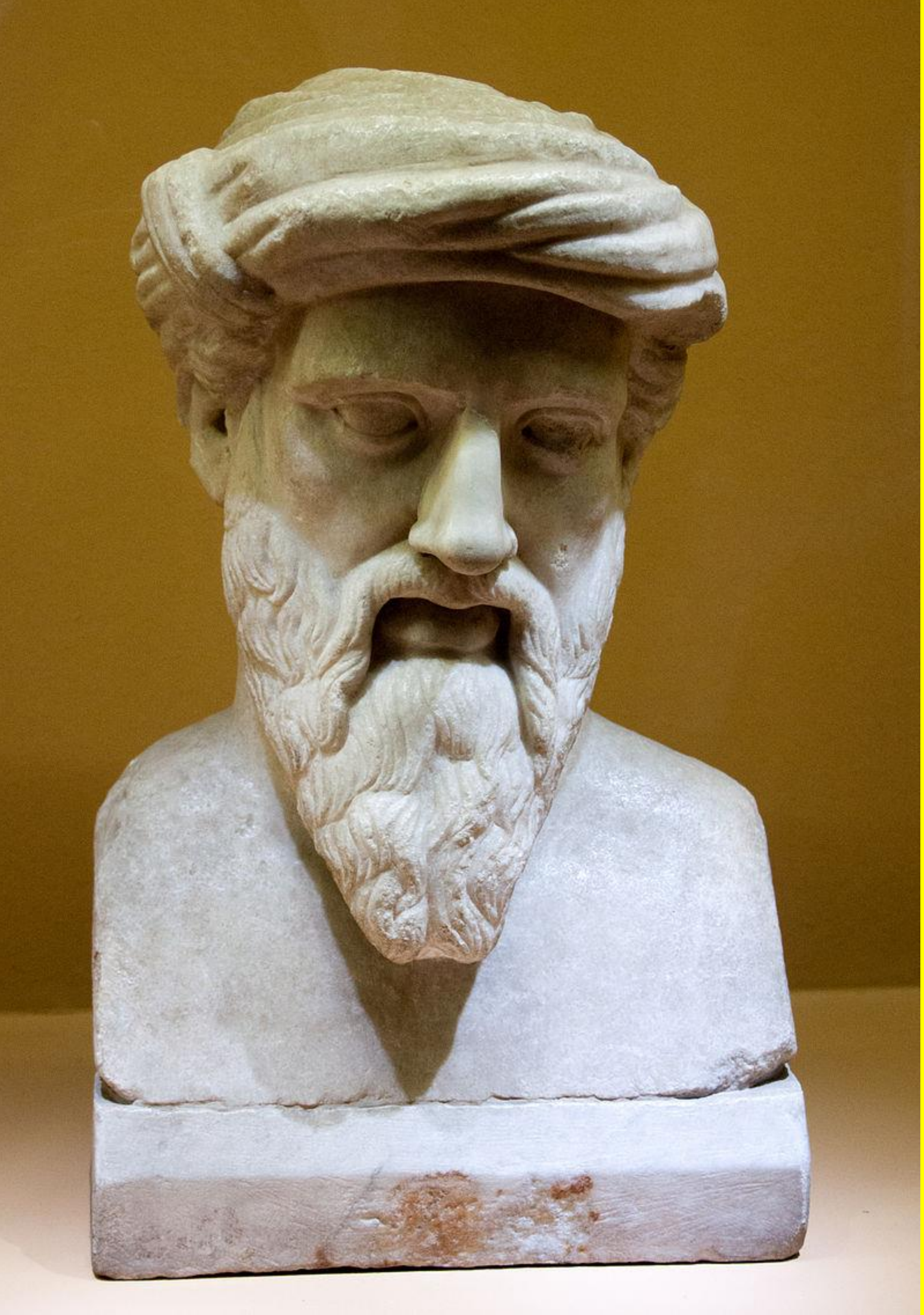
$$(\frac{\ddot{R}}{R} = -\frac{4\pi G\varrho}{3}) \cdot (\dot{R}R)$$

$$\ddot{R} \cdot \dot{R} = -\frac{4\pi GA}{3} \cdot \frac{\dot{R}}{R^2}\,, \quad A = \varrho(t_0) \cdot R^3(t_0)\,,$$

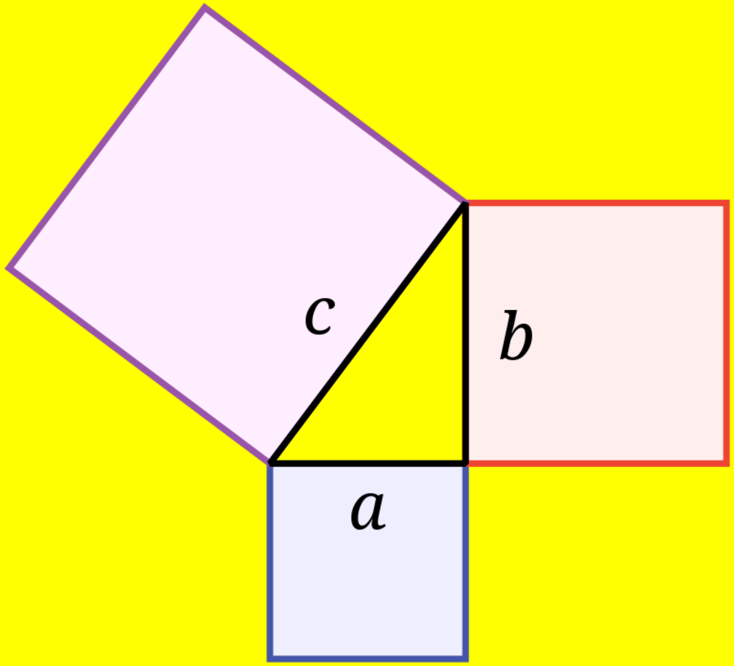
$$\frac{1}{2}\frac{d}{dt}(\dot{R})^2) = \frac{4\pi GA}{3}\frac{d}{dt}(\frac{1}{R})\,,$$

$$\dot{R}^2 = \frac{8\pi GA}{3}\frac{1}{R} + \text{const}, \quad \text{we will take const} = 0\,,$$

$$(\frac{\dot{R}}{R})^2 = \frac{8\pi G}{3} \cdot \varrho(t)\,, \quad H^2(t) = \frac{8\pi G}{3} \cdot \varrho(t)\,.$$

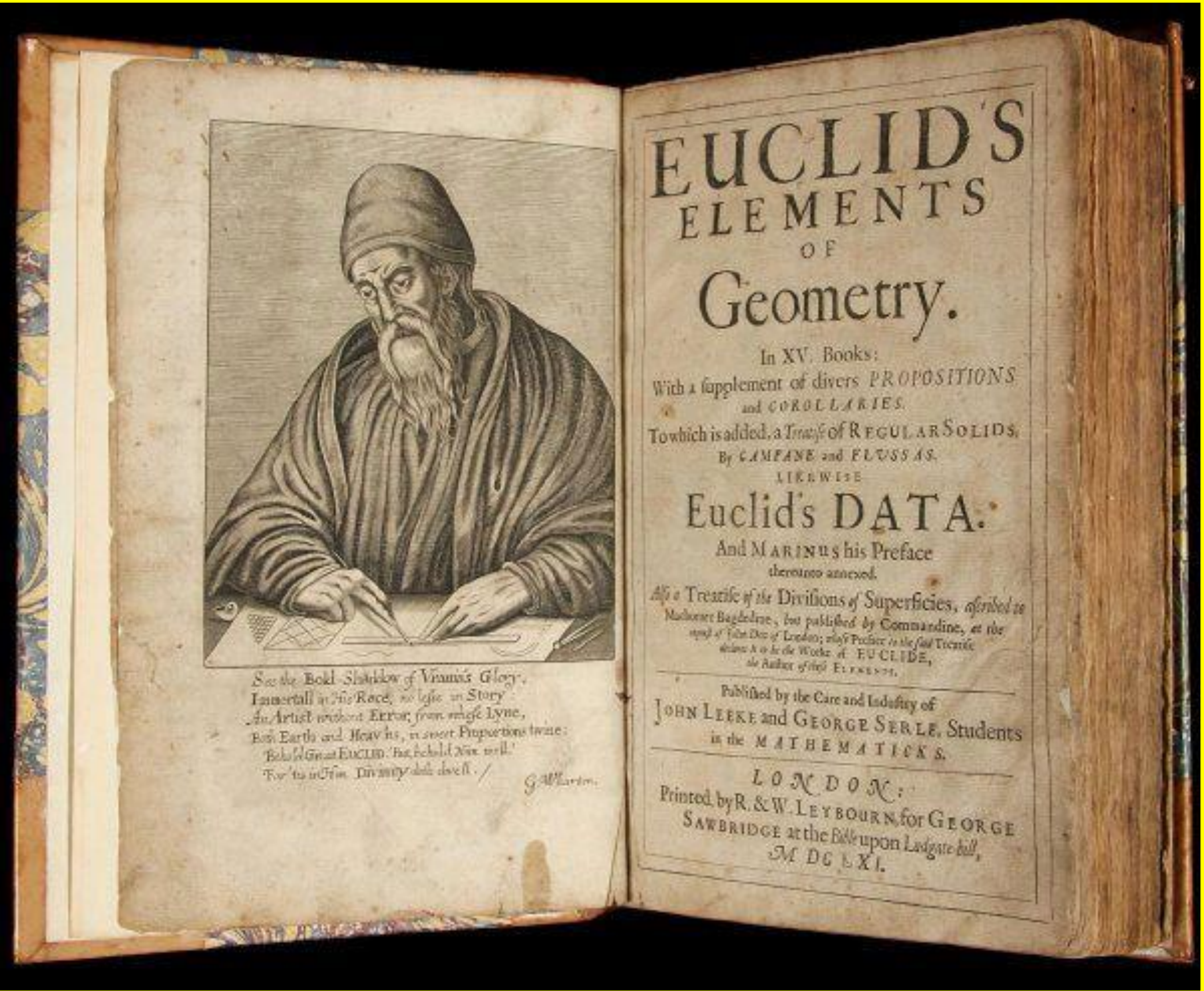


Pythagoras
570 – 495 BC



$$a^2 + b^2 = c^2$$

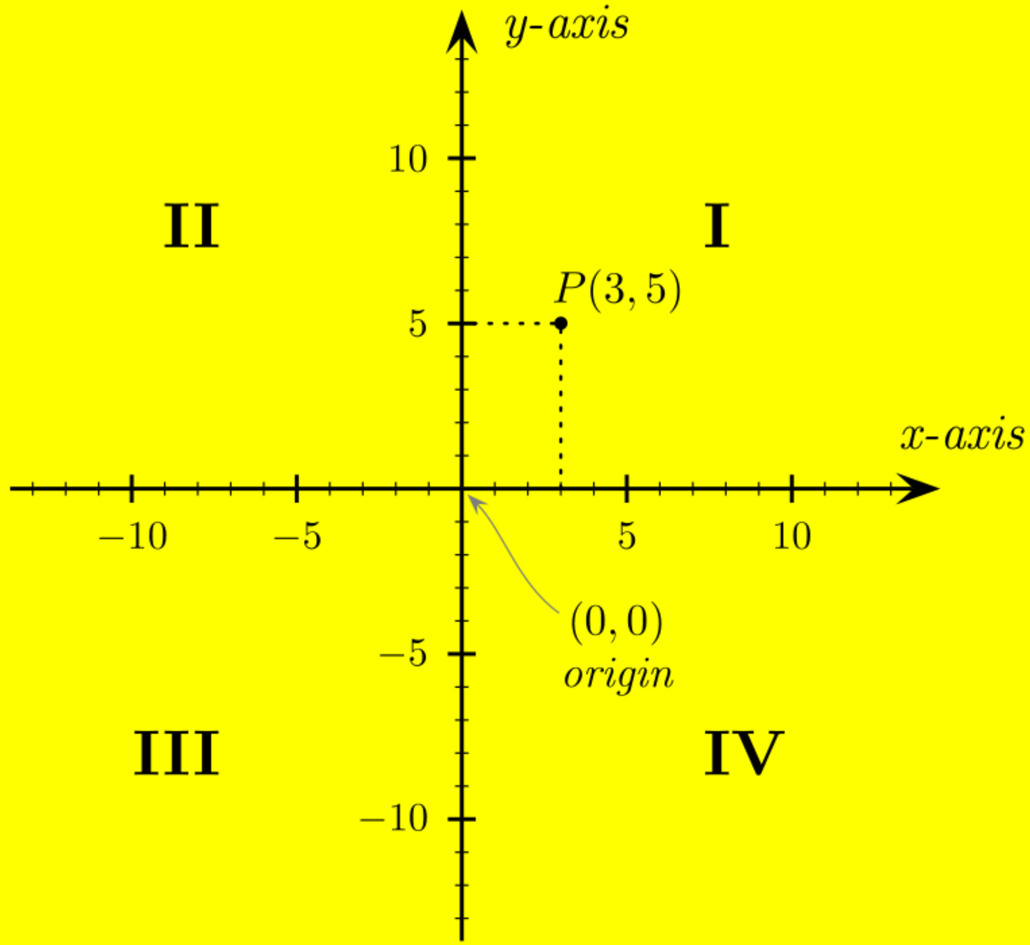
Geometry of space



Euclid
~ 304 -282 BC

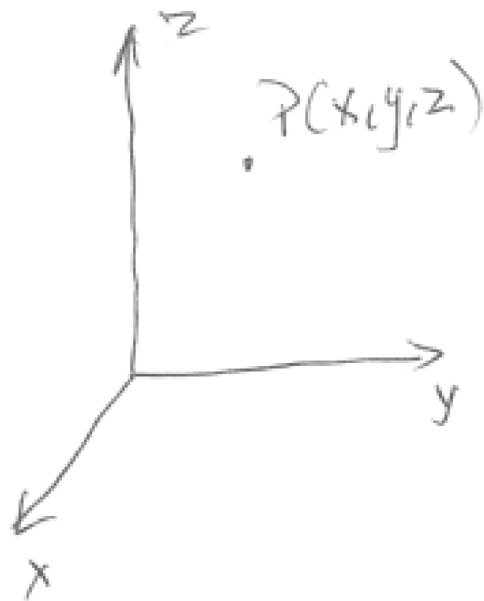
René Descartes 1596 - 1650

Cartesian coordinates



Elements of analytic geometry

Cartesian coordinates:

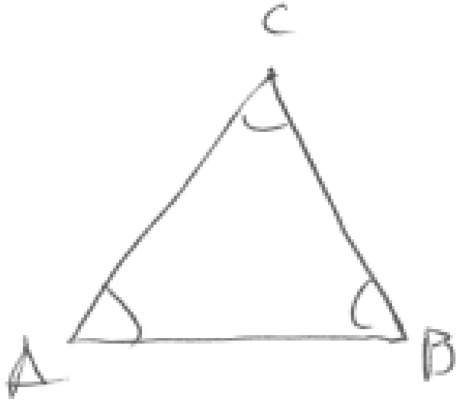


distance: $P(x_1, y_1, z_1)$; $Q(x_2, y_2, z_2)$

$$d_{PQ}^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2$$

$$d_{PQ} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$$d_{PQ} < d_{PR} + d_{RQ}$$



$$\measuredangle A + \measuredangle B + \measuredangle C = 180^\circ$$

$P(x,y,z)$ $Q(x+dx, y+dy, z+dz)$

$$d_{PQ}^2 = dx^2 + dy^2 + dz^2 = dl^2$$ → line element

Line in 3-dim space

$L: x(t), y(t), z(t) \Rightarrow \vec{r}(t)$ $\quad \frac{d\vec{r}}{dt} = \vec{v}(t)$

$$dl^2 = [\dot{x}(t)^2 + \dot{y}(t)^2 + \dot{z}(t)^2] \cdot dt^2 \qquad \frac{d\vec{r}}{dt} = \frac{d\vec{r}}{dl} \cdot \frac{dl}{dt}$$

$\frac{d\vec r}{dl}\cdot\frac{d\vec r}{dl} = 1$ $\frac{d\vec r}{dl} = \vec t$ $\vec t\cdot\vec t = 1$ tangent vector

$\frac{d\vec t}{dl}\cdot\vec t = 0 \Rightarrow \frac{d\vec t}{dl}\perp\vec t$; $\frac{d\vec t}{dl} = \frac{1}{\rho}\cdot\vec n$ $\frac{1}{\rho}$ – curvature radius

$\vec n\cdot\vec n = 1$; $\vec n\cdot\vec t = 0$ $\frac{d\vec n}{dl} = \frac{\vec t}{\rho} - \frac{\vec b}{\tau}$ $\vec b\cdot\vec b = 1$

$\vec b\cdot\vec n = \vec b\cdot\vec t = 0$

$\vec n$ – normal vector

τ – torsion (radius) $\frac{d\vec b}{dl} = -\frac{\vec n}{\tau}$

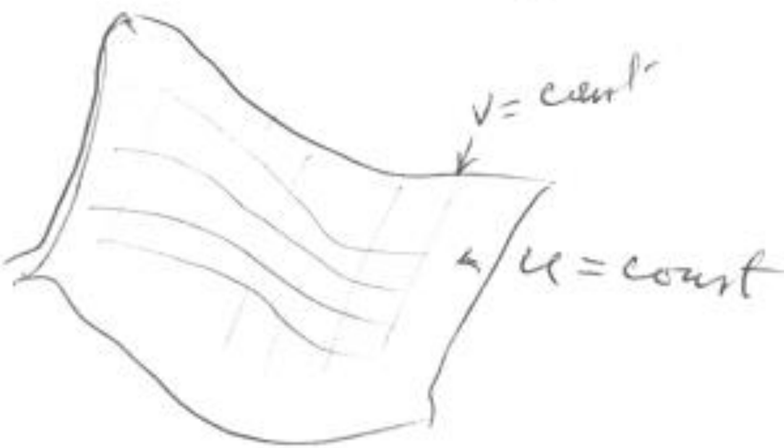
$\vec b$ – binormal vector

$(\vec t, \vec n, \vec b)$ (ρ, τ)

Surfaces: $x = x(u,v)$; $y = y(u,v)$; $z = z(u,v)$

$$dl^2 = dx^2 + dy^2 + dz^2$$

$$dl^2 = E(u,v)\,du^2 + 2F(u,v)\,du\,dv + G(u,v)\,dv^2$$



$$\vec{t}_1 \sim \left(\frac{\partial x}{\partial u}, \frac{\partial y}{\partial u}, \frac{\partial z}{\partial u}\right)$$

$$\vec{t}_2 \sim \left(\frac{\partial x}{\partial v}, \frac{\partial y}{\partial v}, \frac{\partial z}{\partial v}\right)$$

$\vec{n} \sim \vec{t}_1 \times \vec{t}_2$ (vector product).

Different coordinates:

$$x = r \sin\theta \cos\varphi \qquad dl^2 = dx^2 + dy^2 + dz^2$$

$$y = r \sin\theta \sin\varphi \qquad dl^2 = dr^2 + r^2 (d\theta^2 + \sin^2\theta \, d\varphi^2)$$

$$z = r \cos\theta$$

Curved spaces

$$dl^2 = g_{ab}(x^c) dx^a dx^b$$ (Einstein summation notation)

$$X_a X^a = \sum_{a=1}^{4} X_a X^a$$