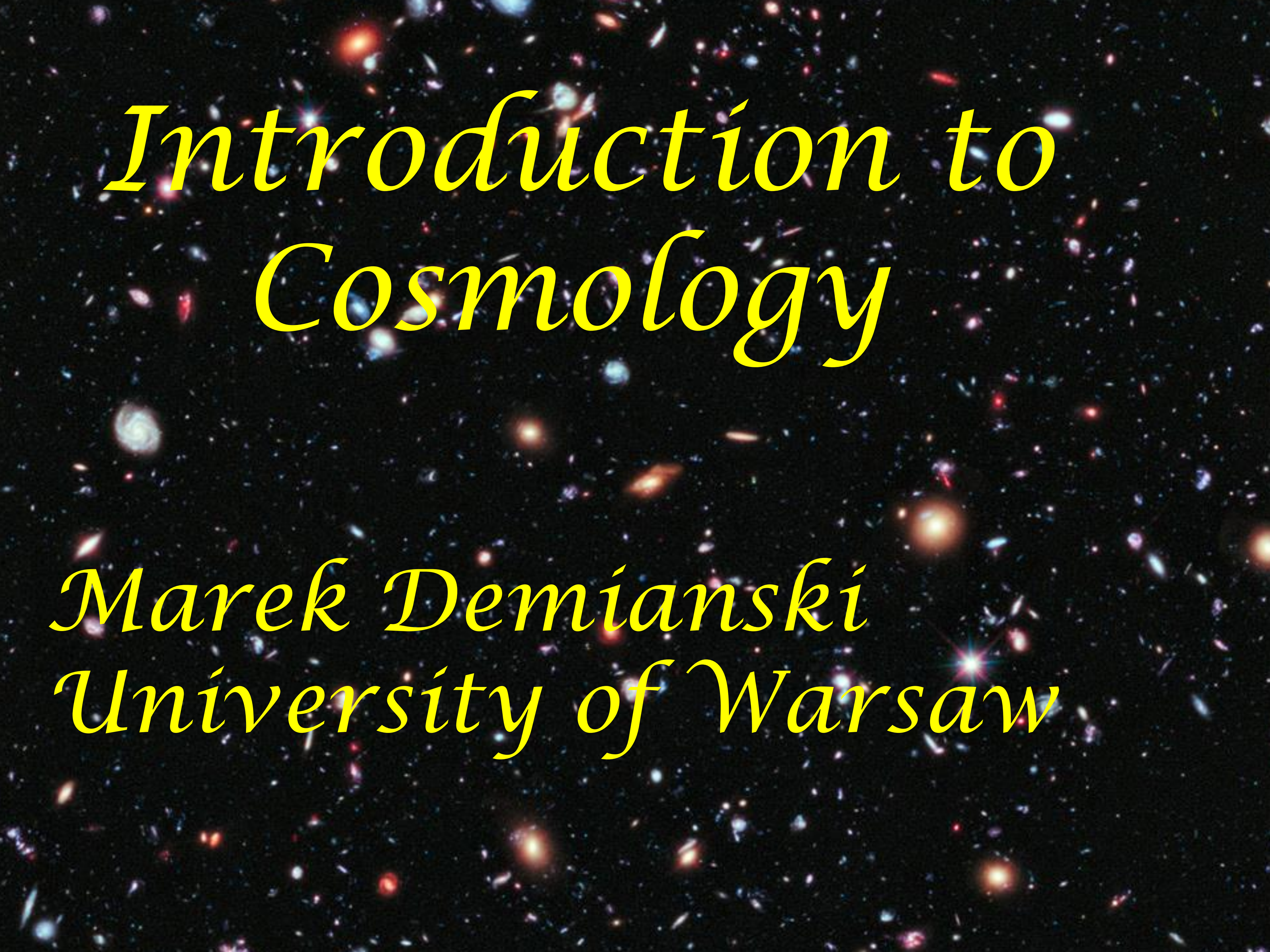
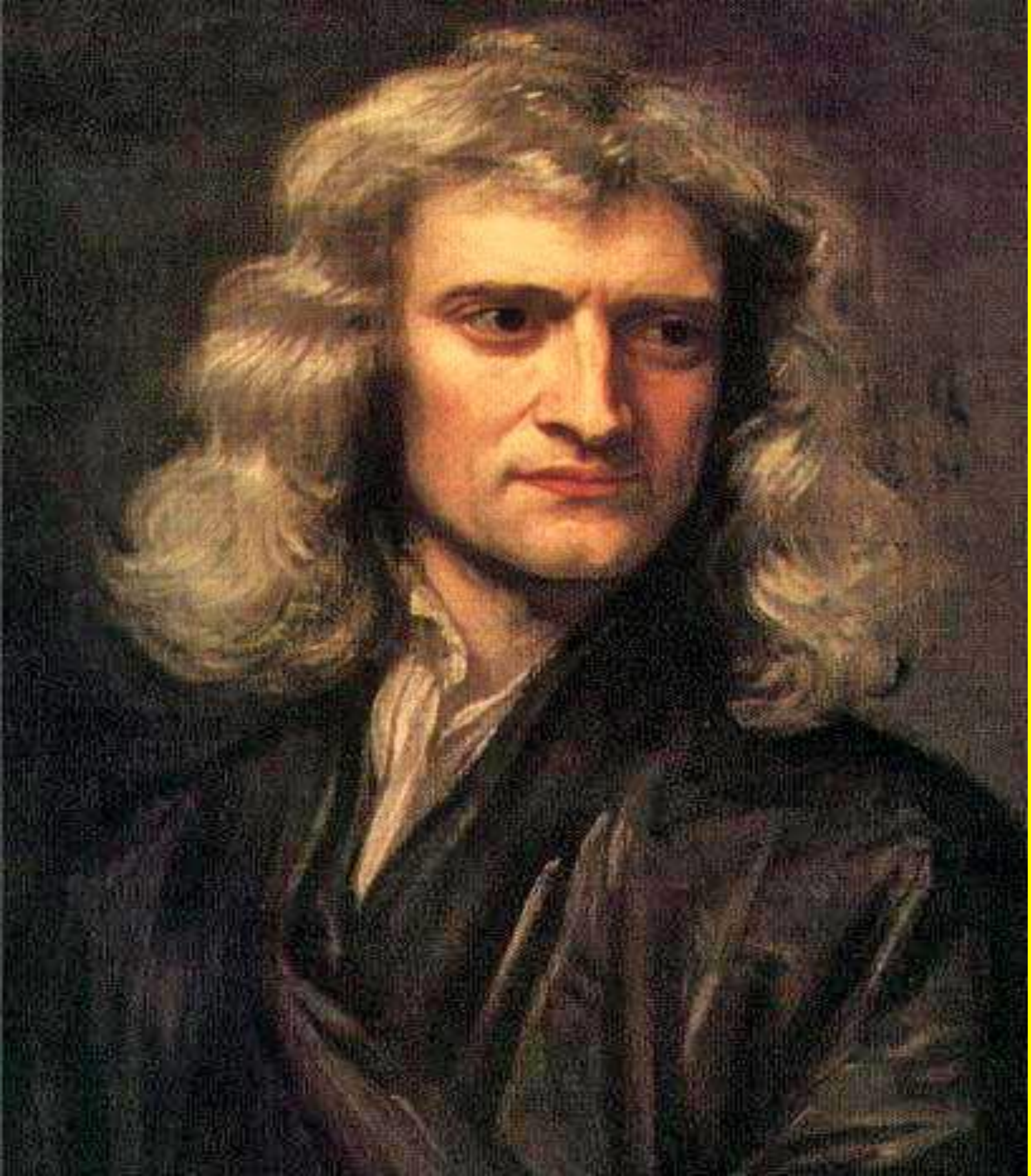
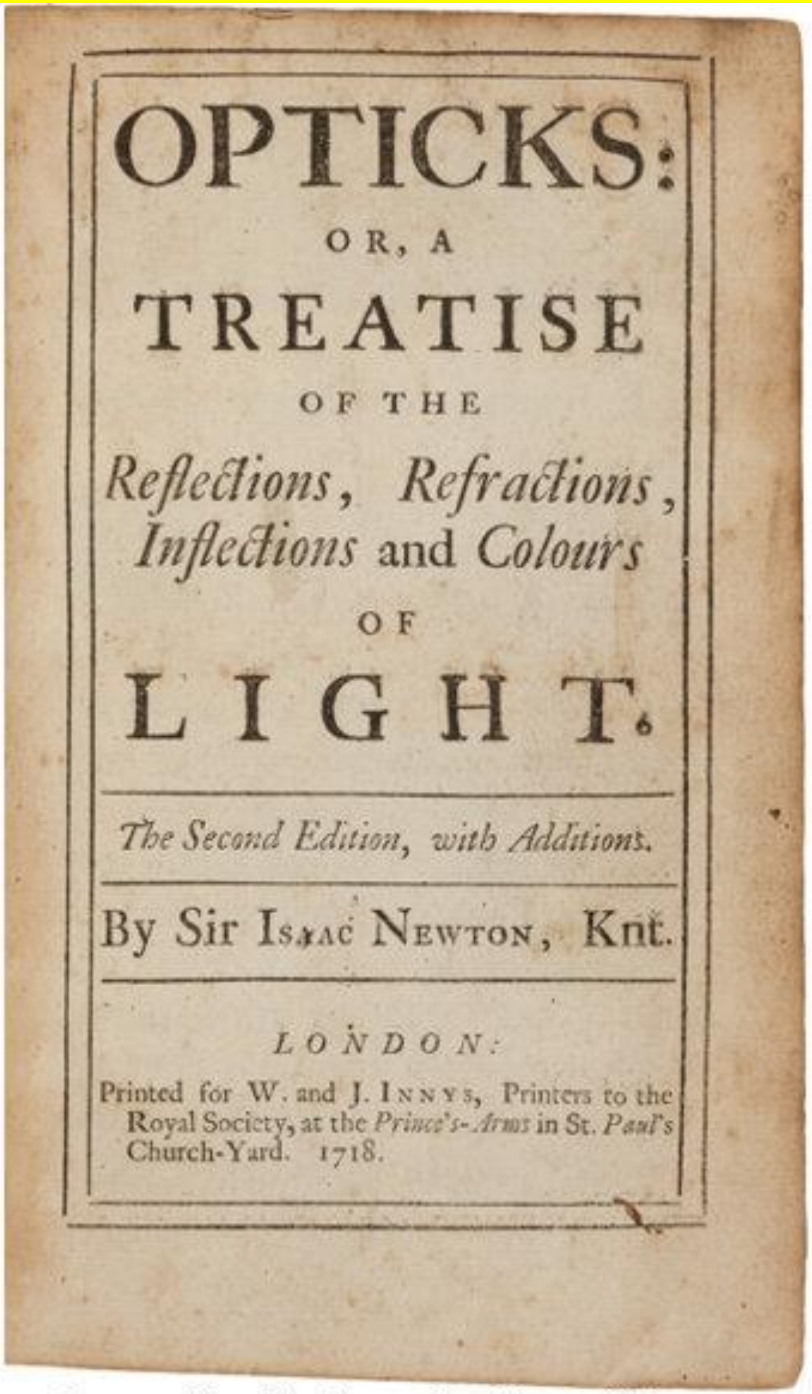


Introduction to Cosmology

Marek Demianski
University of Warsaw



Isaac Newton
1642 - 1727



OPTICKS:

OR, A

TREATISE

OF THE

Reflections, *Refractions*,
Inflections and *Colours*

OF

LIGHT.

The Second Edition, with Additions.

By Sir Isaac Newton, Knt.

LONDON:

Printed for W. and J. Innys, Printers to the
Royal Society, at the *Prince's-Arms* in St. *Paul's*
Church-Yard. 1718.

Imaged by Heritage Auctions, HA.com

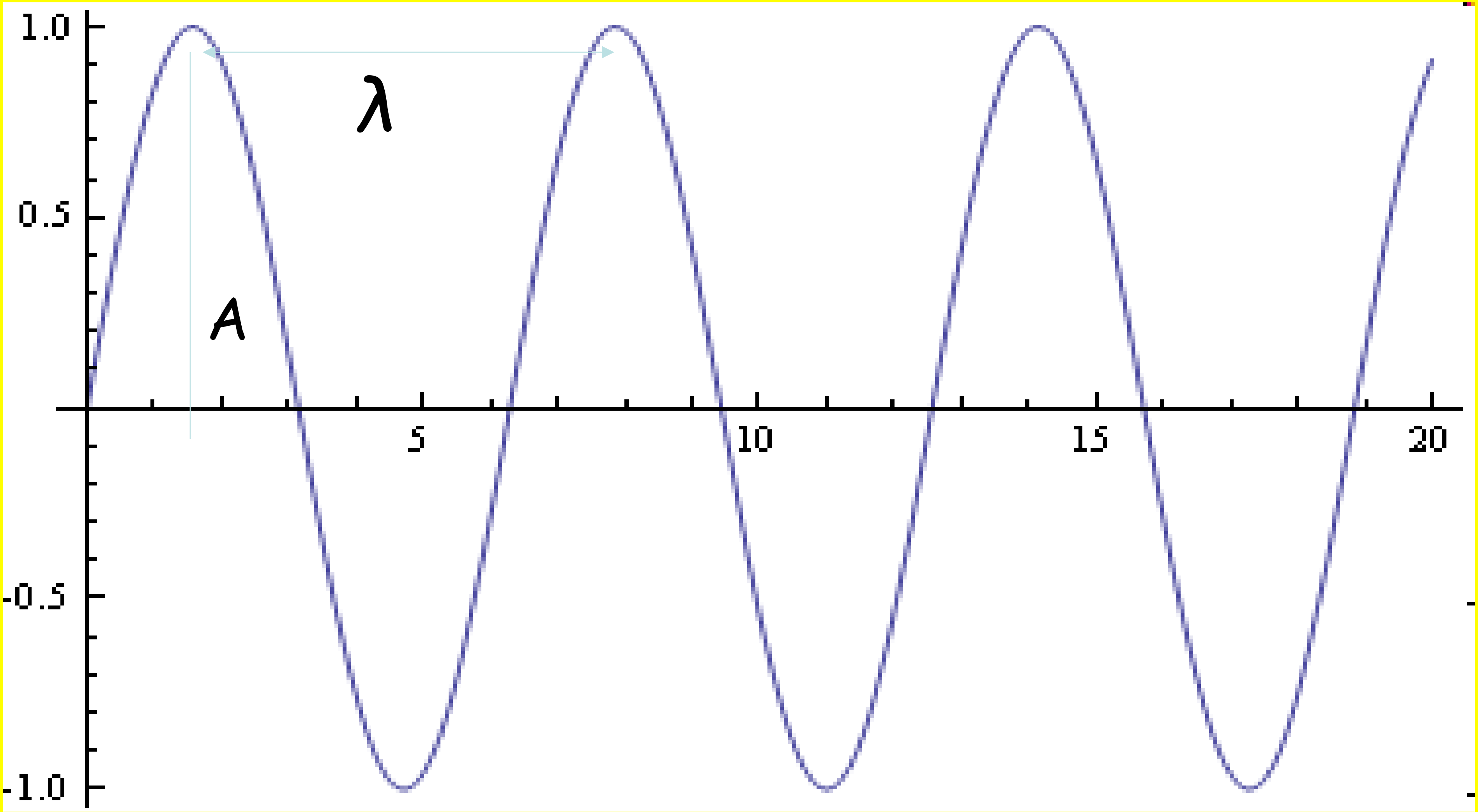
Light is a stream of particles

Christiaan Huygens
1629 - 1695

Light is a wave

water waves

Simple wave



A - amplitude   λ – wavelength

Characteristics of a wave

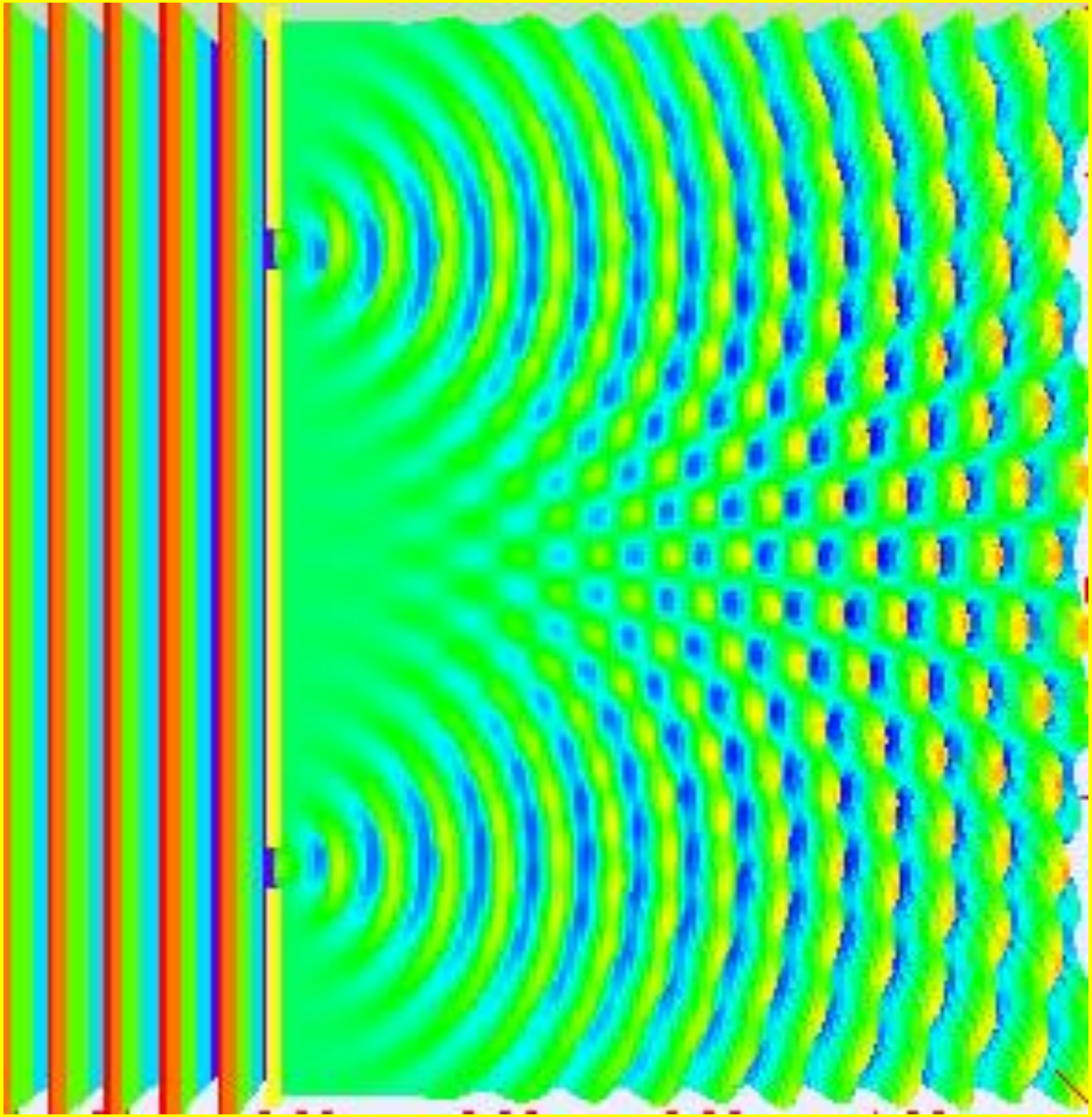
Wavelength - λ

Frequency - f

Period - $P = 1/f$

Velocity $c = \lambda/P = \lambda f$

Young’s double-slit experiment



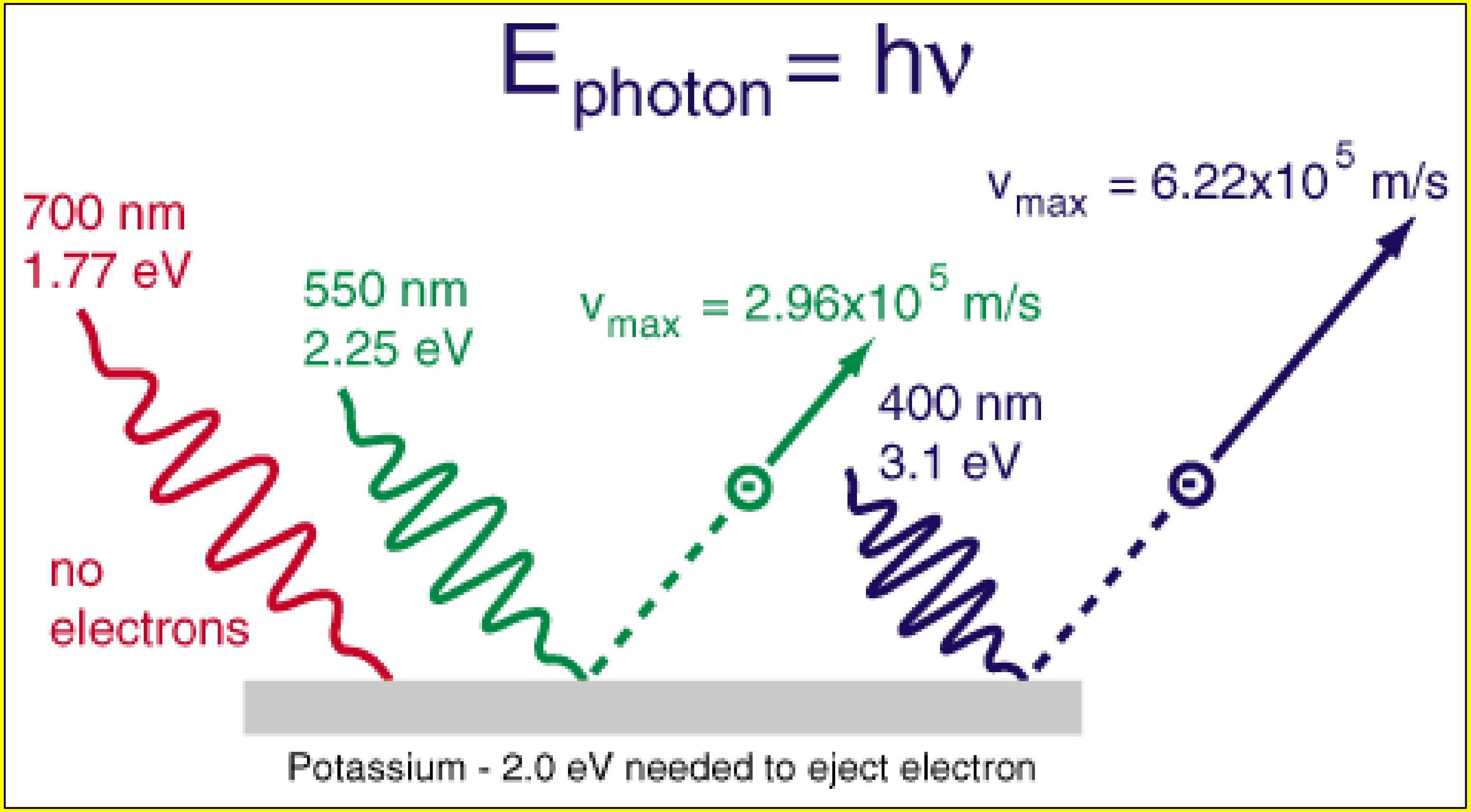
Young's double-slit experiment (1801)

- Light passes through two apertures. You can think of these two apertures as two light *sources*.

- Young showed that light from these sources produces an interference pattern (recall the animation with the interference of water waves).

- So, Young showed that light is a wave!

The Photoelectric Effect



No matter how many photons impinge on the surface, **no electrons** will be ejected unless $E_{photon} \geq E_{min}$

http://hyperphysics.phy-astr.gsu.edu/hbase/imgmod2/pelec.gif

Light is
a stream
of particles
(photons)

Nobel Prize
1921

So what is light ?

Sometimes light appears as a wave and sometimes as a stream of photons

What kind of wave is light?

In 1864, Maxwell completed a unified theory of electricity and magnetism.

Maxwell used his equations to show that, if
an electric charge oscillates back and forth,
it produces an *electromagnetic wave*.

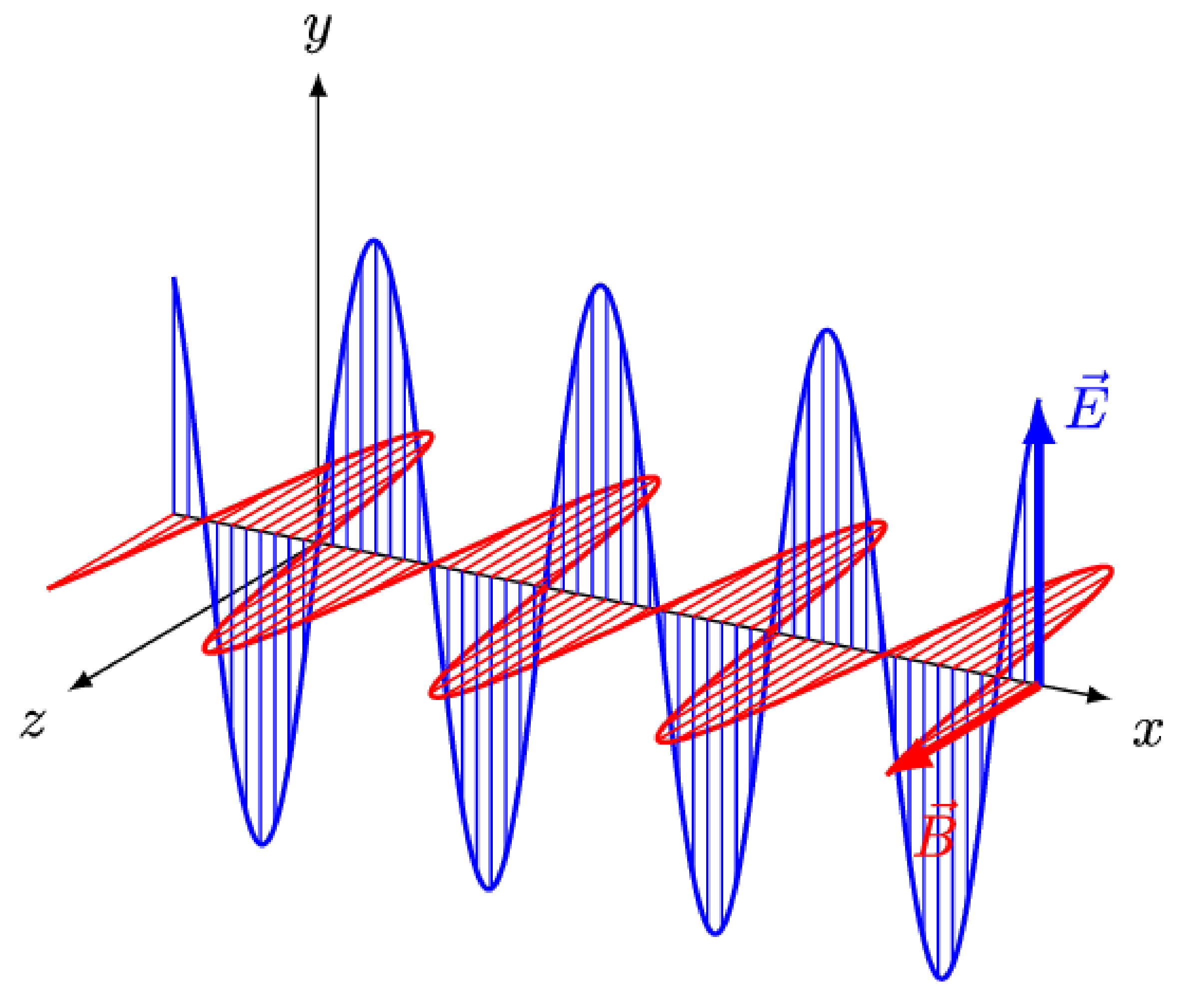
He was also able to predict that the speed of the wave is

$$c = 3 \cdot 10^8\ m/s$$

This speed agreed well with the measured value of the speed of light!

His conclusion: *light IS an electromagnetic wave*!

Light is an electromagnetic wave
It can propagate in vacuum!



Basic properties of light

- Light is an electromagnetic wave !
- Light can propagate in vacuum !
- The speed of light does not depend on the relative motion of the source of light and an observer.
- Light is a stream of particles (photons)!

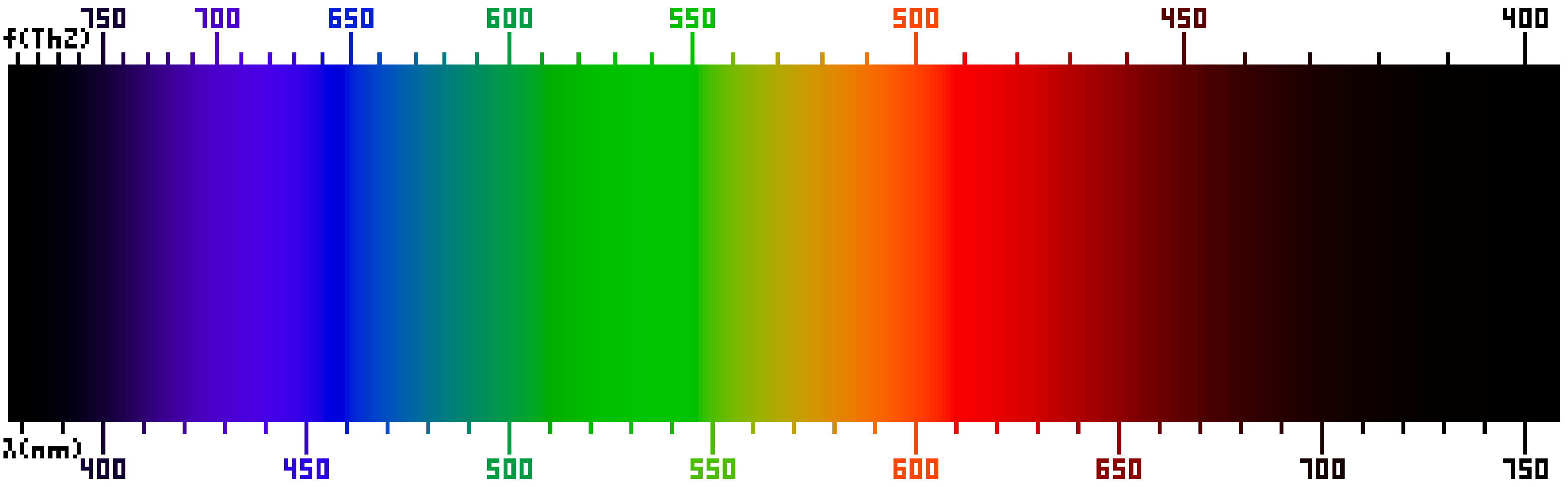
$$E_{photon}=h\cdot\nu$$

h – Planck constant

Color depends on temperature

http://en.wikipedia.org/wiki/File:Embers_01.JPG

Colors and the Spectrum



But what governs the ***distribution*** of light across the spectrum and the color we perceive?

Max Planck to the Rescue!

research-in-germany.de

$$I_\nu = B_\nu = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT} - 1}$$

h = Planck's cst   k = Boltzmann's cst   c = speed of light

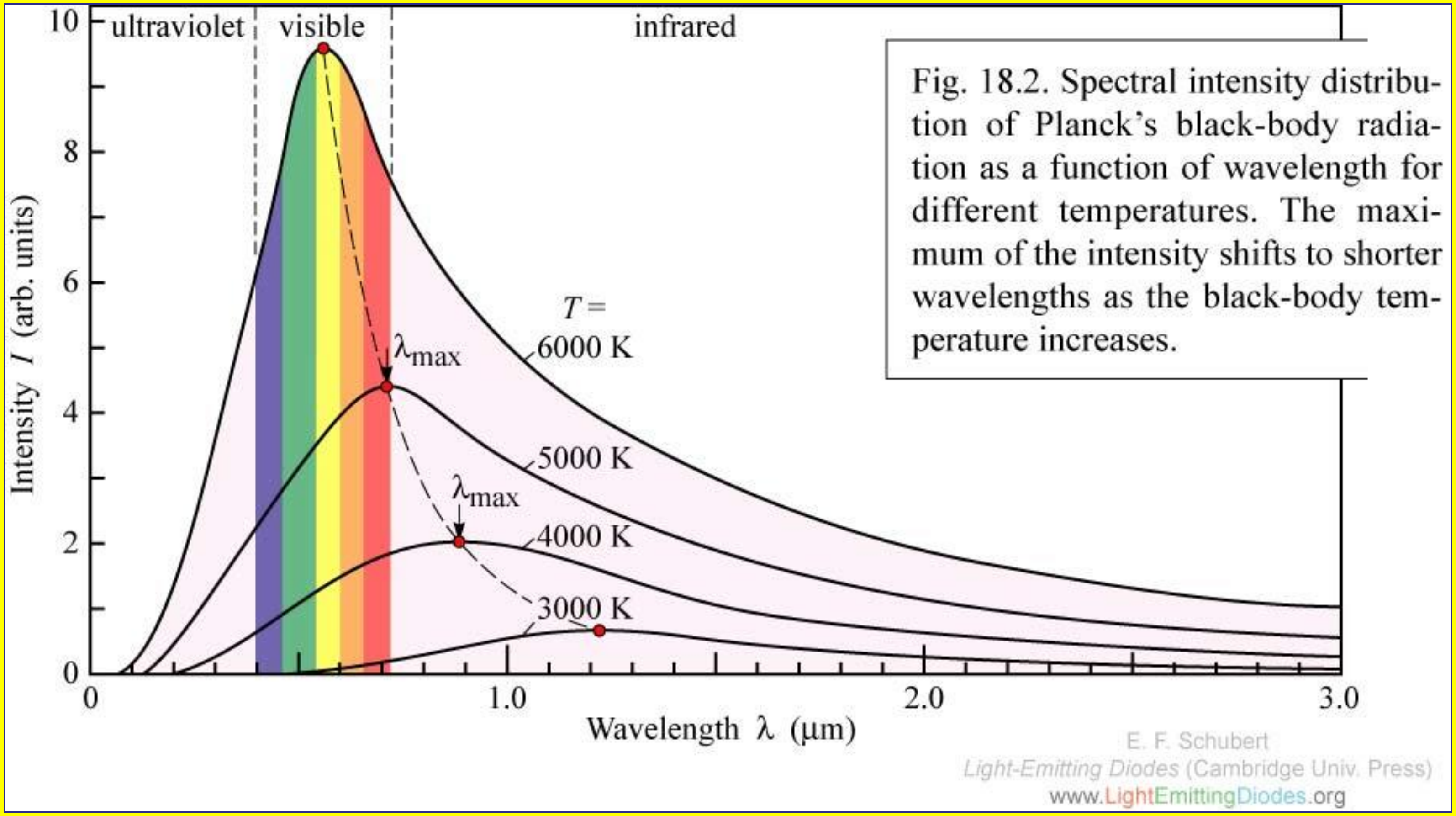


Fig. 18.2. Spectral intensity distribution of Planck's black-body radiation as a function of wavelength for different temperatures. The maximum of the intensity shifts to shorter wavelengths as the black-body temperature increases.

E. F. Schubert
Light-Emitting Diodes (Cambridge Univ. Press)
www.LightEmittingDiodes.org

Two Additional Useful Relations

The Stefan-Boltzmann law

Says: integral under Planck curve equals the total power emitted per unit area of a blackbody:

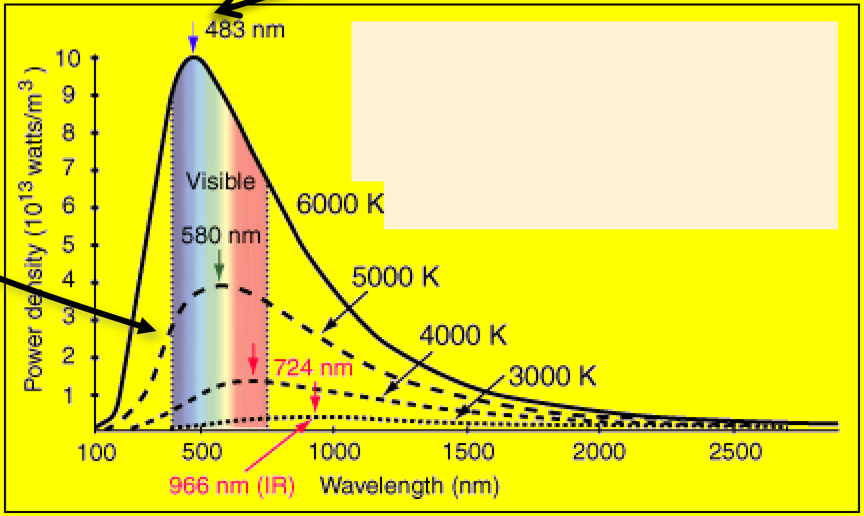
$$E_{total} = \int_0^\infty E_\nu d\nu = \sigma T^4$$

σ = Stefan - Boltzmann cst

Wien's Law

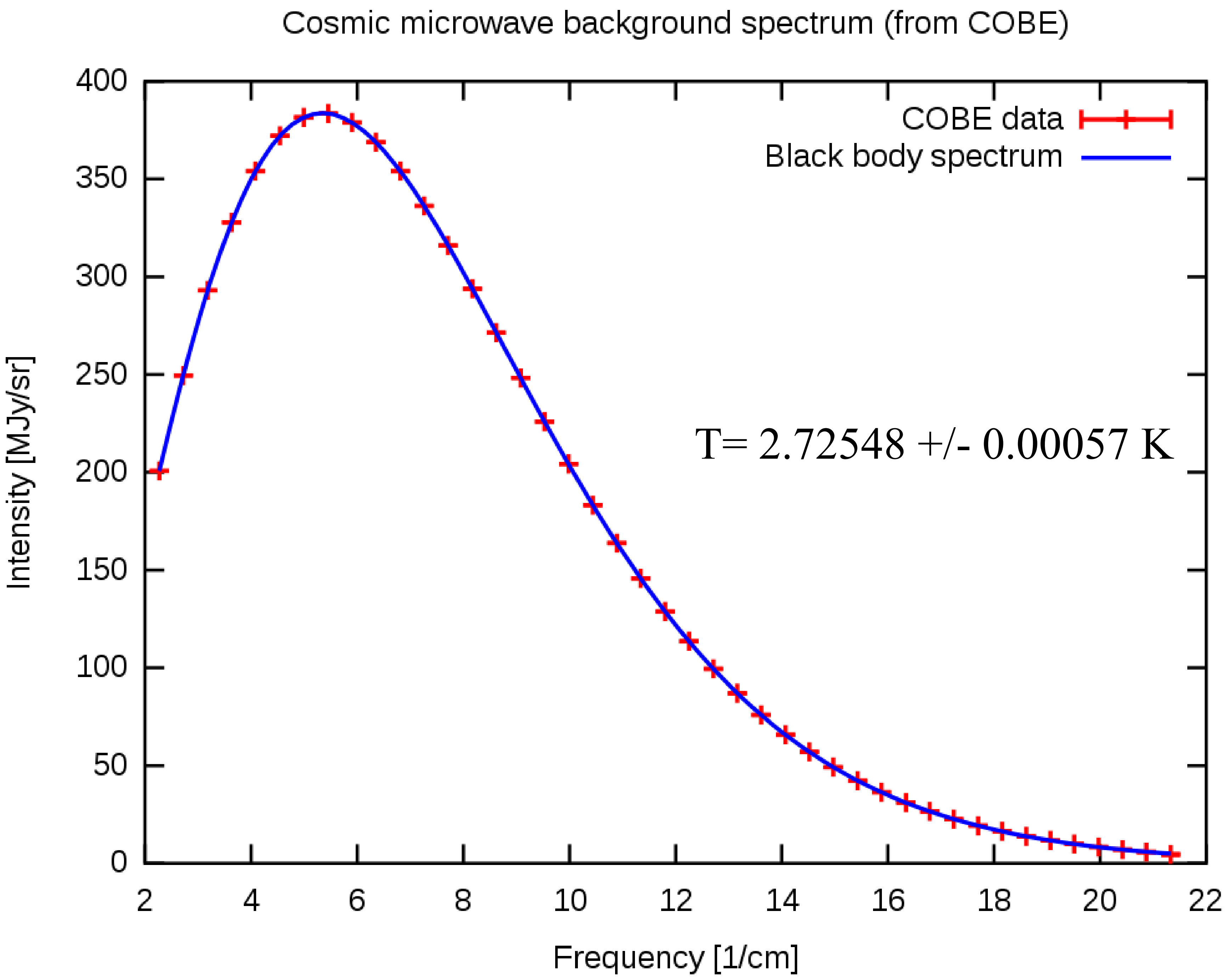
Says: wavelength of peak emission is where derivative of Planck curve is zero, and is given by:

$$\lambda_{max} \cdot T = 2.898 \cdot 10^6 \text{ nm} \cdot \text{K}$$



$E_\nu = I_\nu$

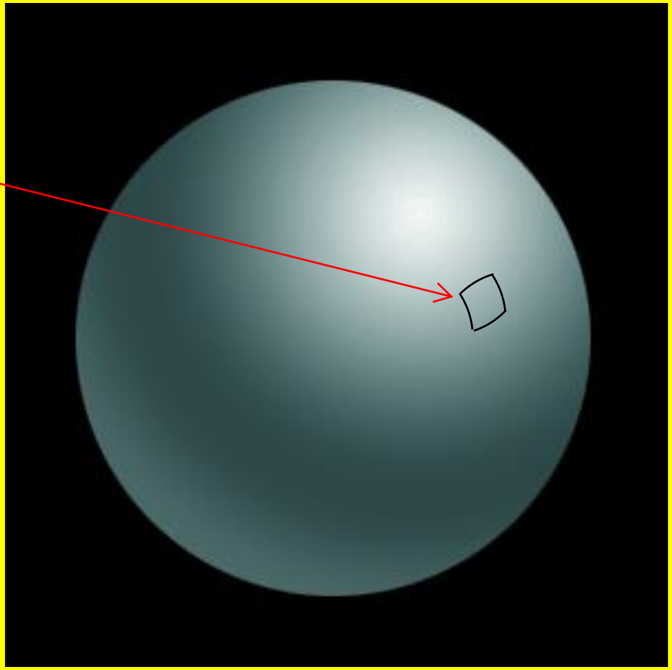
Both laws can be used to derive the surface temperature of a star!
How…?



Cosmic microwave background spectrum (from COBE)

T= 2.72548 +/- 0.00057 K

Stellar Luminosity = total power (E/sec)



Stefan-Boltzmann Law describes E/sec per unit area $=\sigma T^4$

***Luminosity* = total power from entire surface area ($4\pi R_*^2$):**

$$\boxed{L = 4\pi R_*^2 \sigma T^4}$$

$\sigma = 5.67\mathrm{x}10^{-8}\ \mathrm{W\ m^{-2}\ K^{-4}}$

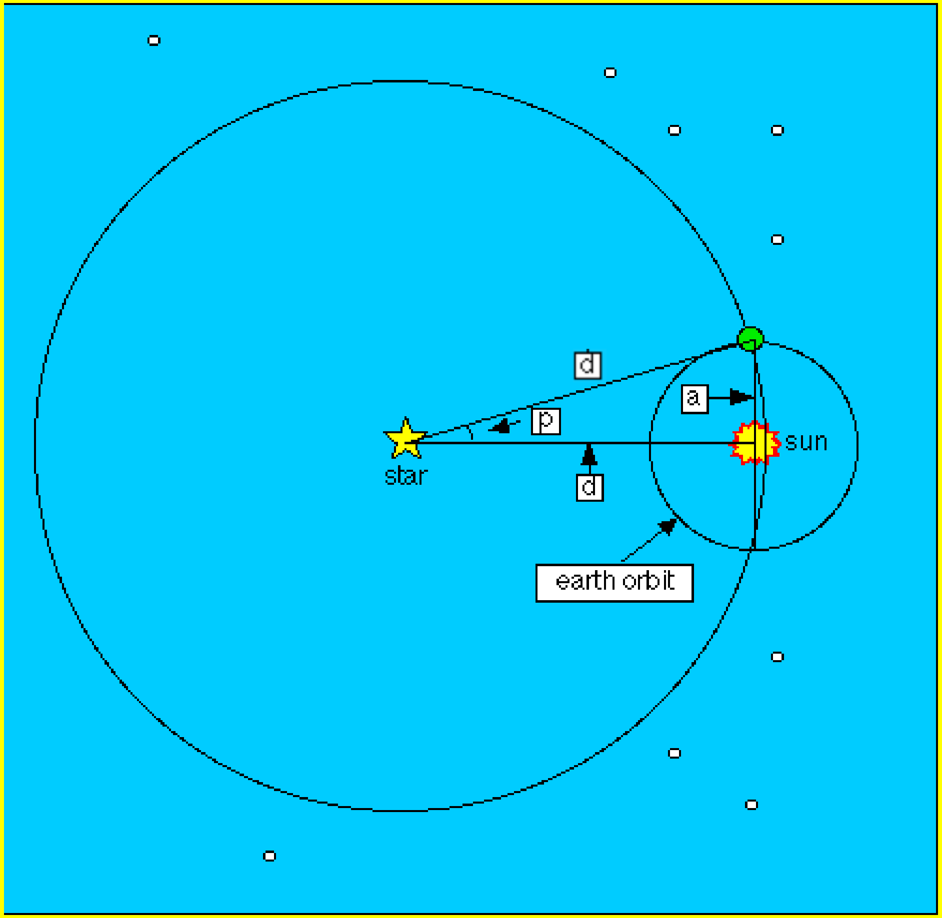
Example: $R_\odot = 7 \cdot 10^8$ m
$T_\odot = 5800$ K

$L_\odot = 4\pi \cdot (7\mathrm{x}10^8)^2 \cdot 5.67 \cdot 10^{-8} \cdot (5800)^4$

$= 3.95 \cdot 10^{26}$ W(= J/s)
= *1 Solar Luminosity* ($L_\odot$)

Distances to Stars:
The Principle of Parallax

a=1AU = 1.49 10^11 m **d**=distance to star in AU **p**=parallax angle*



$$\sin p \approx \tan p \approx \boxed{p_{rad} = 1/d_{AU}}$$

But parallaxes are tiny !

Let’s express them in arcsec (”):

$$p_{rad} = p''/206265 = 1/d_{AU}$$

Let’s define a new unit: *parsec* (pc)

1 pc = 206265 AU

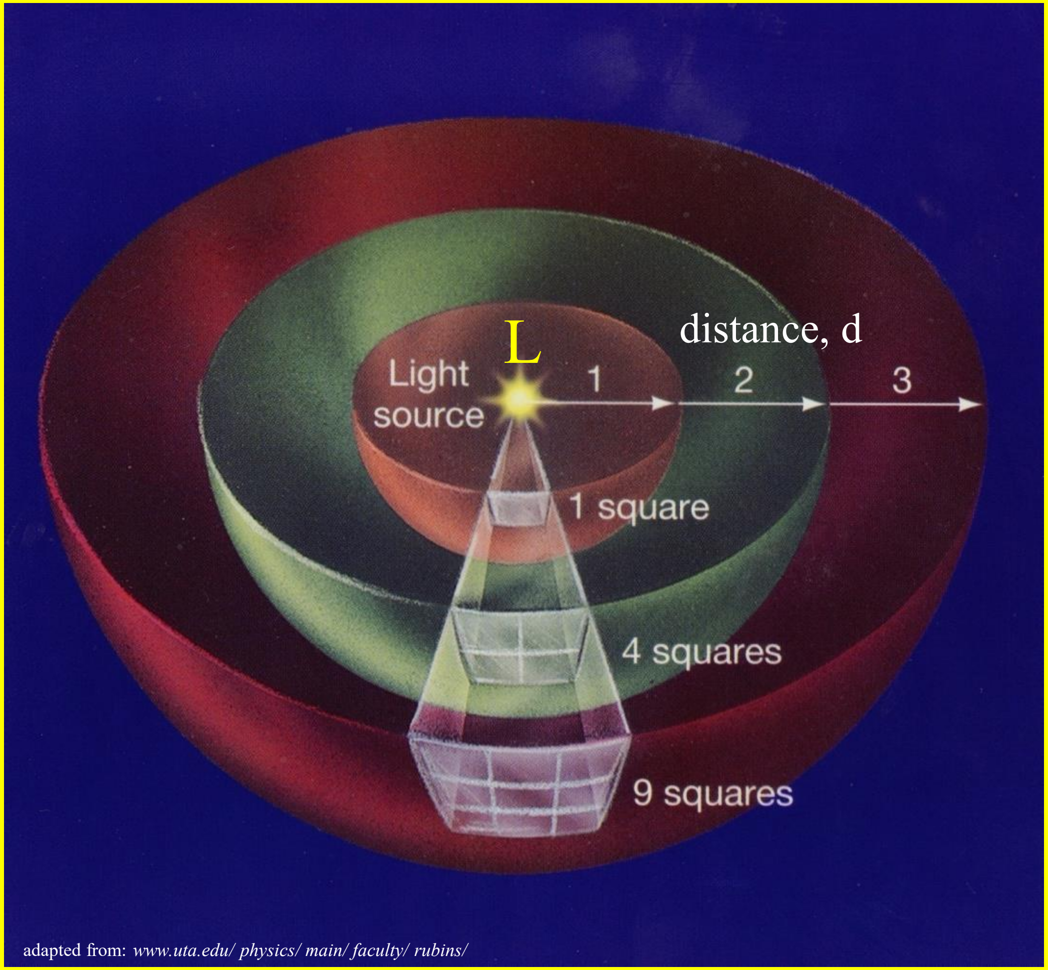
So: $$\frac{p''}{206265} = \frac{1}{d_{AU}} = \frac{1}{206265 d_{pc}}$$

➔ $\boxed{p'' = \frac{1}{d_{pc}}}$ and $\boxed{d_{pc} = \frac{1}{p''}}$

**In practice, we make measurements 6 months apart, so total angular shift is 2p.*

Inverse square law: *Flux* declines as $1/d^2$

Flux= energy received per unit area per unit time e.g. $W\,m^{-2}$



adapted from: *www.uta.edu/ physics/ main/ faculty/ rubins/*

At larger **d**, the same total amount of light is spread more thinly; less falls on each area:

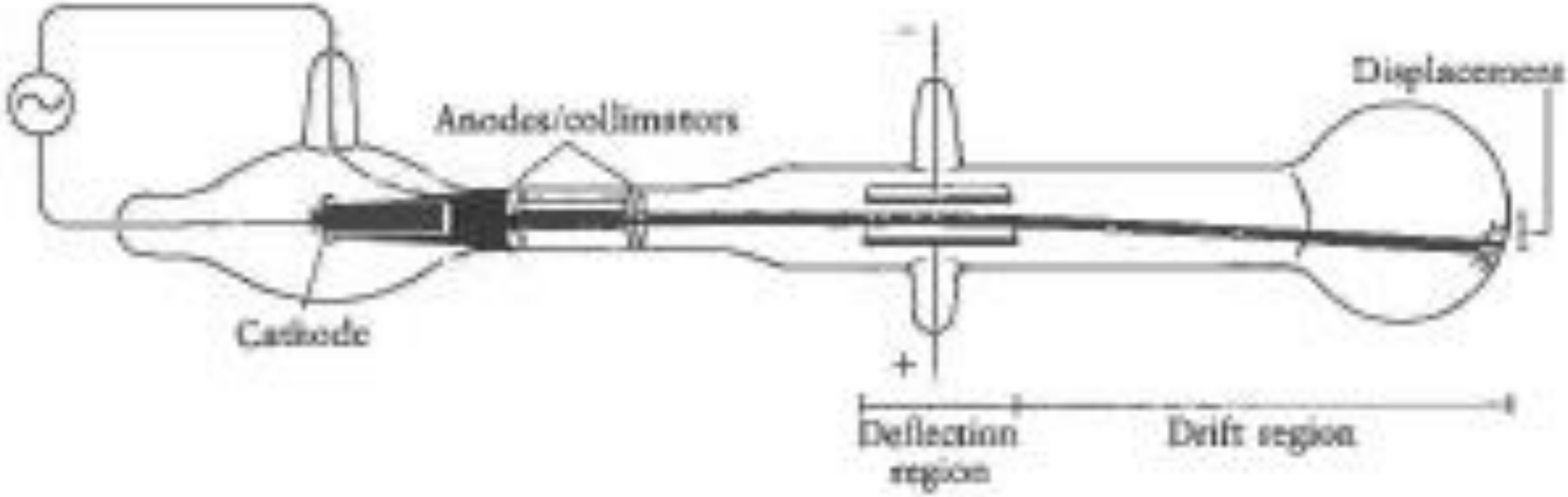
$$F_{d_1} = {}^{L}\!\Big/_{4\pi d_1^{\,2}}$$

$$F_{d_2} = {}^{L}\!\Big/_{4\pi d_2^{\,2}}$$

$$\boxed{\frac{F_{d_2}}{F_{d_1}} = \frac{d_1^{\,2}}{d_2^{\,2}}}$$

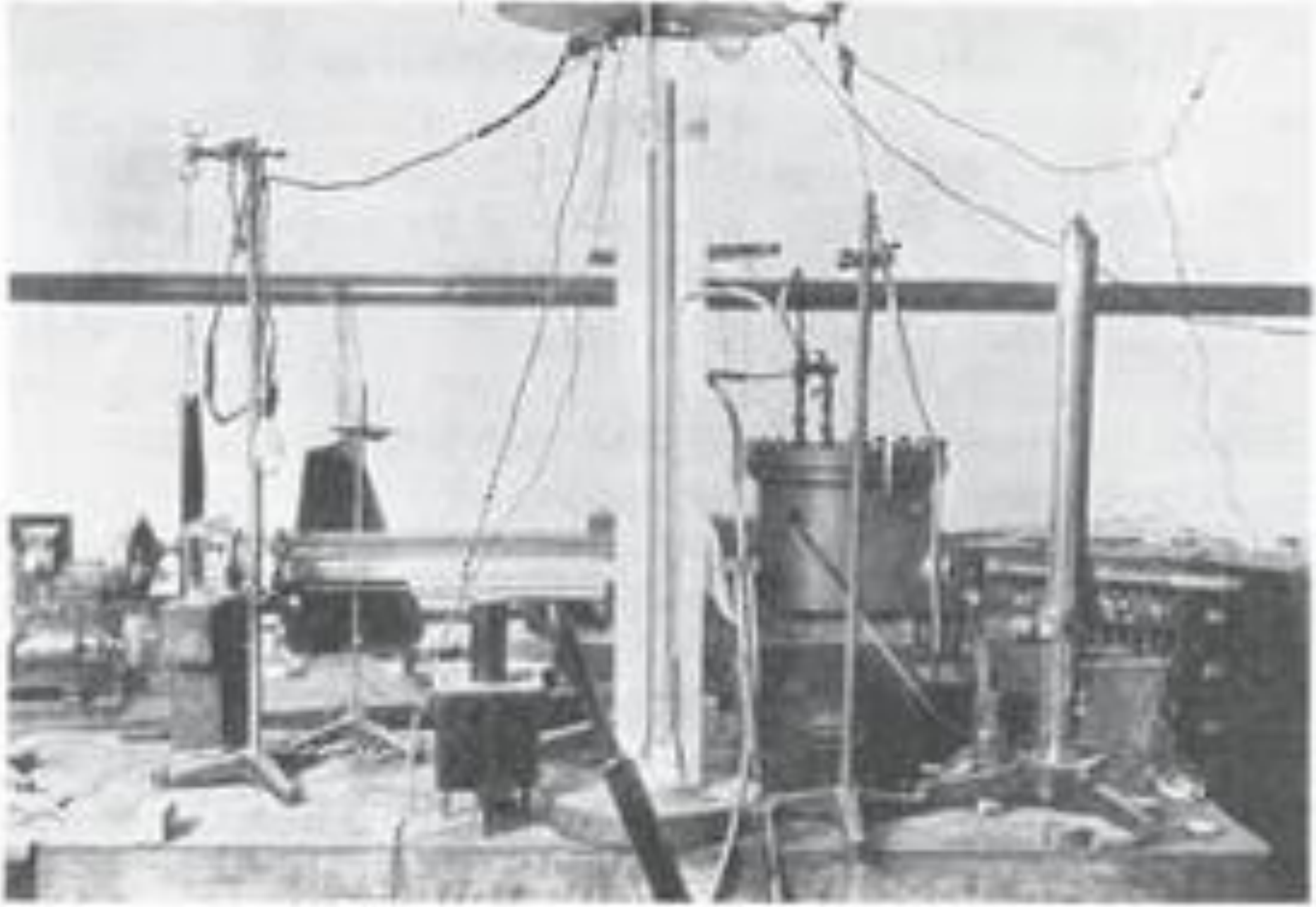
Recall: L = Luminosity = total power output from star (≠F(d))

J J Thomson



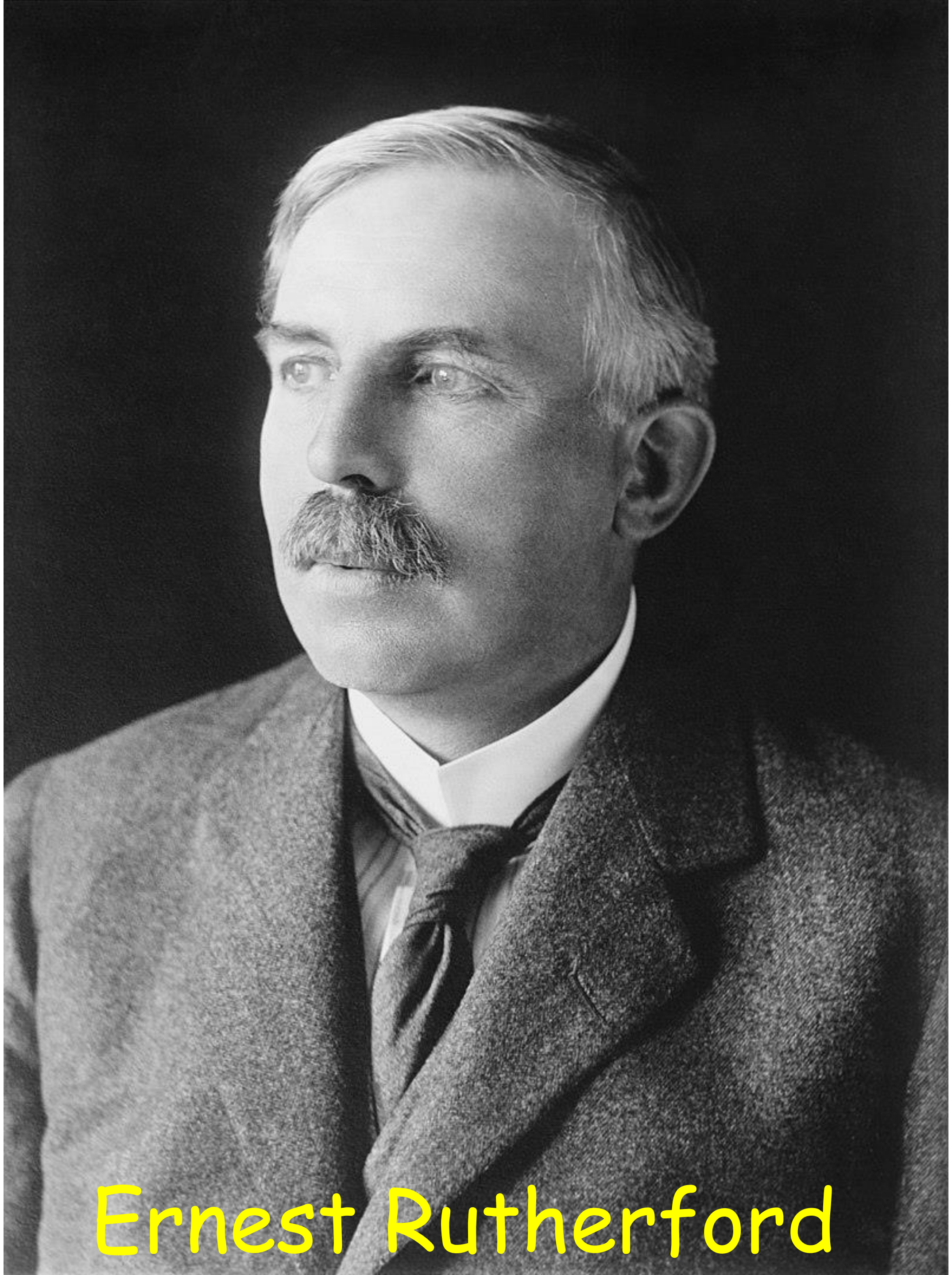
Electrons carry charge and have mass

Robert A Millikan

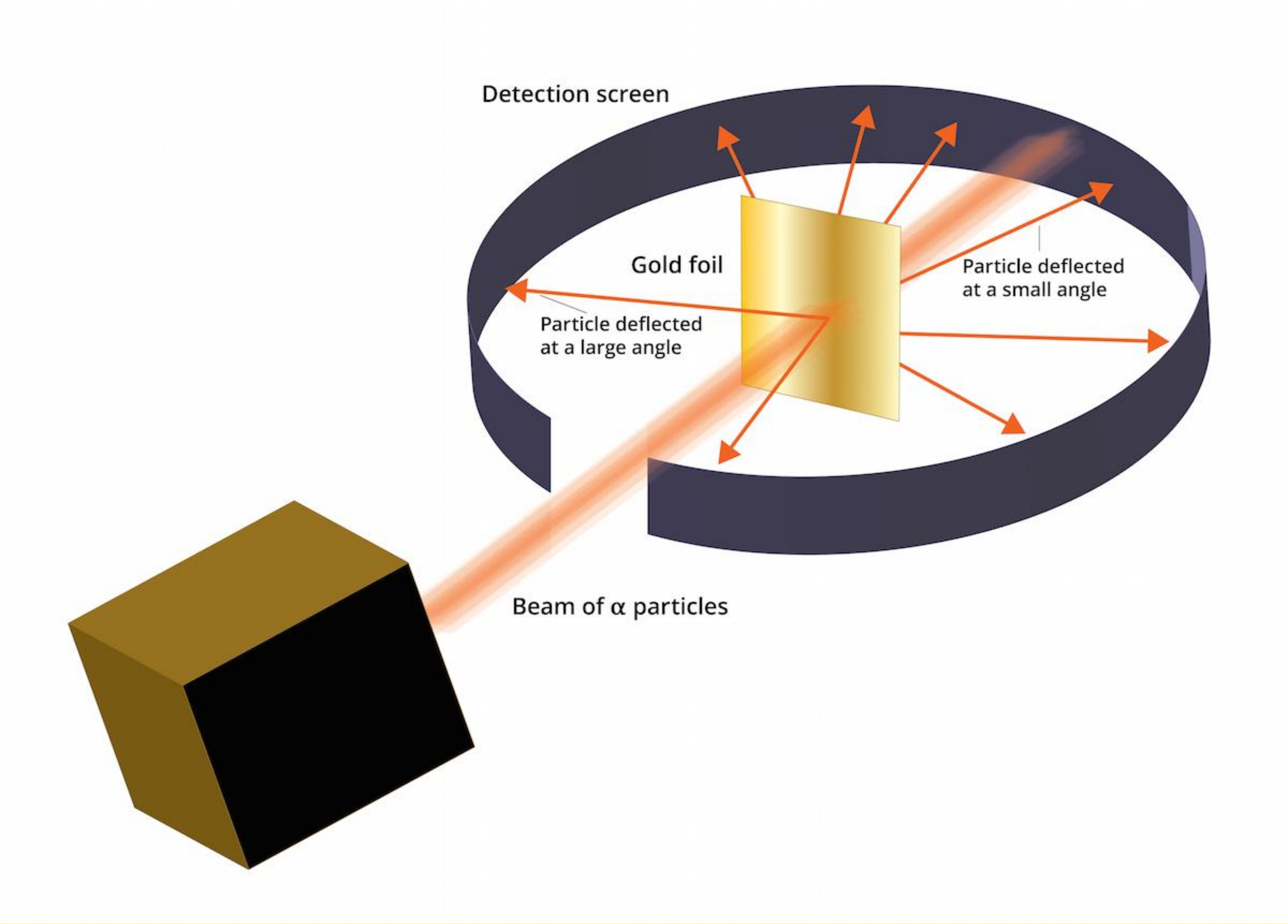


The apparatus used by Millikan in his oil-drop experiments.

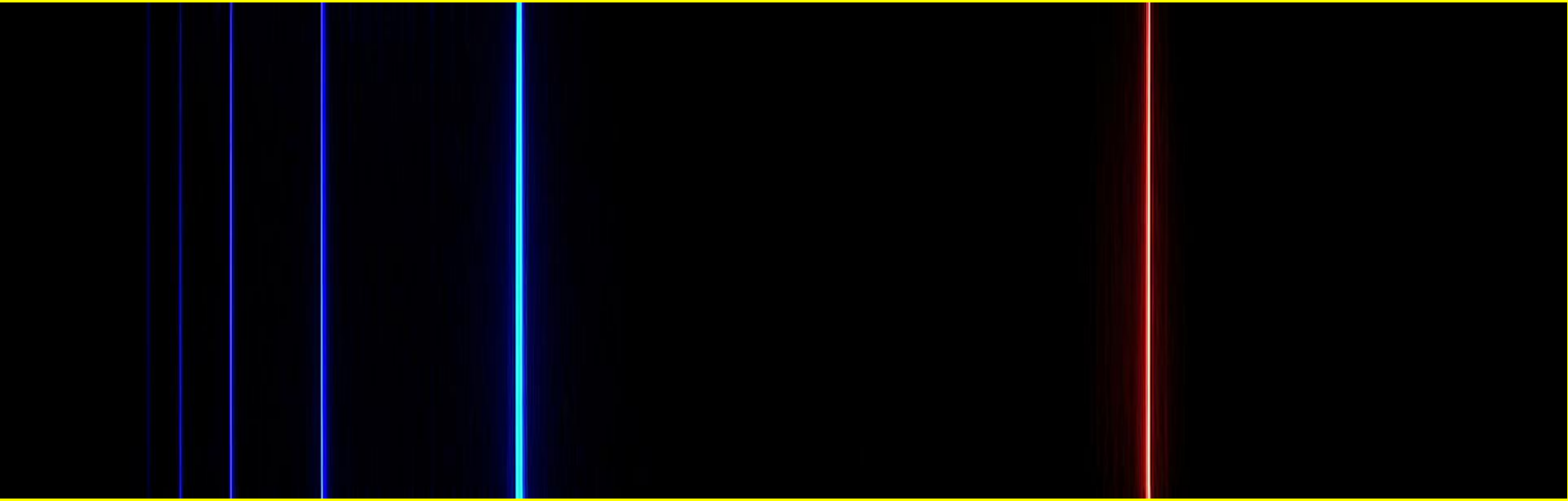
Millikan measured charge of an electron



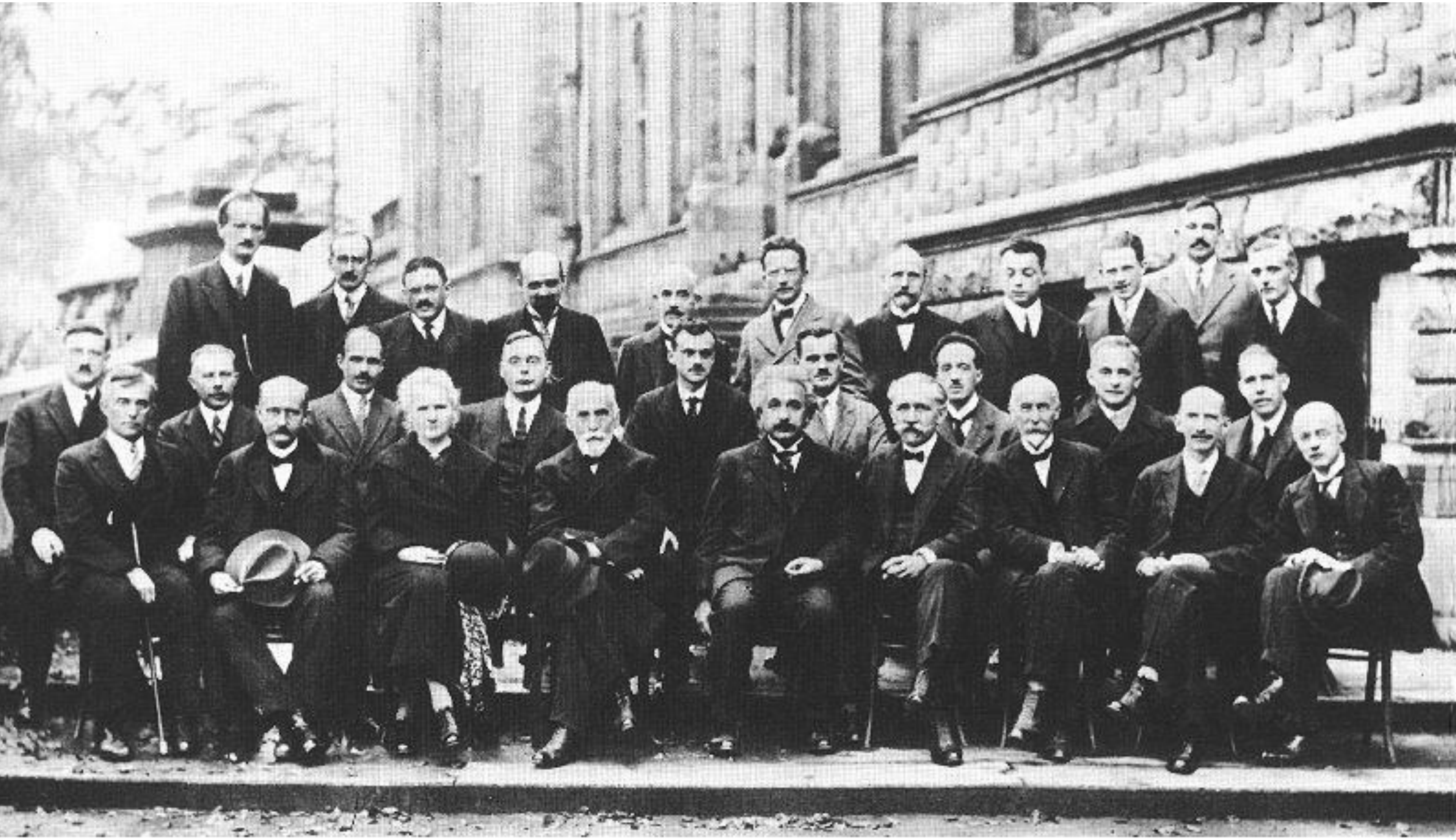
Ernest Rutherford



The Rutherford experiment (1911)



Spectrum of hydrogen gas



A. PICCARD E. HENRIOT P. EHRENFEST Ed. HERZEN Th. DE DONDER E. SCHRÖDINGER E. VERSCHAFFELT W. PAULI W. HEISENBERG R.H. FOWLER L. BRILLOUIN
P. DEBYE M. KNUDSEN W.L. BRAGG H.A. KRAMERS P.A.M. DIRAC A.H. COMPTON L. de BROGLIE M. BORN N. BOHR
I. LANGMUIR M. PLANCK Mme CURIE H.A. LORENTZ A. EINSTEIN P. LANGEVIN Ch.E. GUYE C.T.R. WILSON O.W. RICHARDSON

Atoms and Light

Max Planck (1900) radiation is composed of packets of energy $E = h\nu$.

Louis de Broglie (1923) particles can be treated as waves $\lambda = \frac{h}{p}$, $p = mv$ - momentum

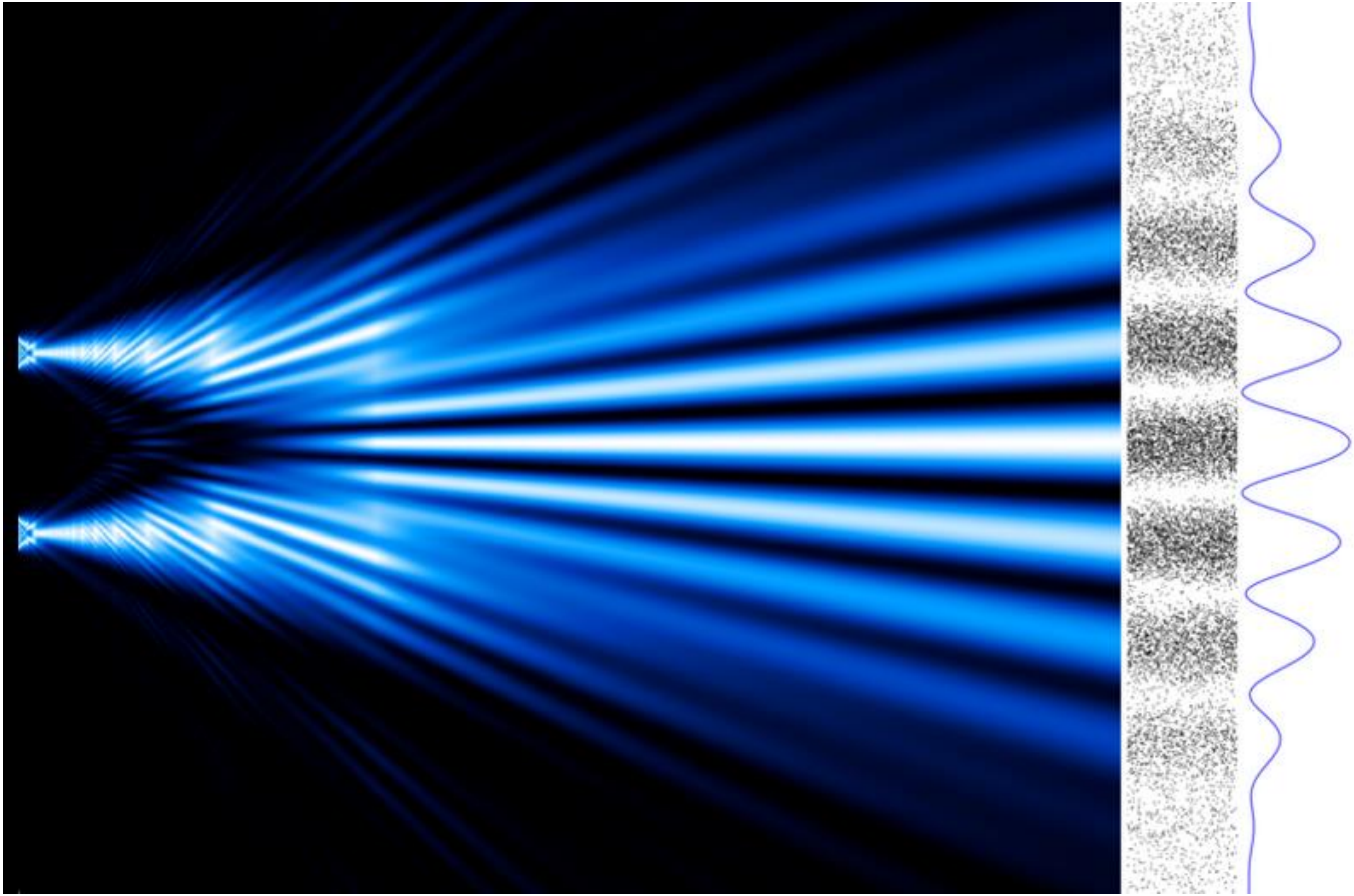
Planetary motion (Newtonian gravity)

$$\frac{m_P v^2}{r} = \frac{G m_p M_\odot}{r^2} \; .$$

Rutherford experiment (1911) positive charge concentrated in the nucleus is surrounded by a cloud of electrons.

Simple model of an hydrogen atom: central proton + electron on a circular orbit.

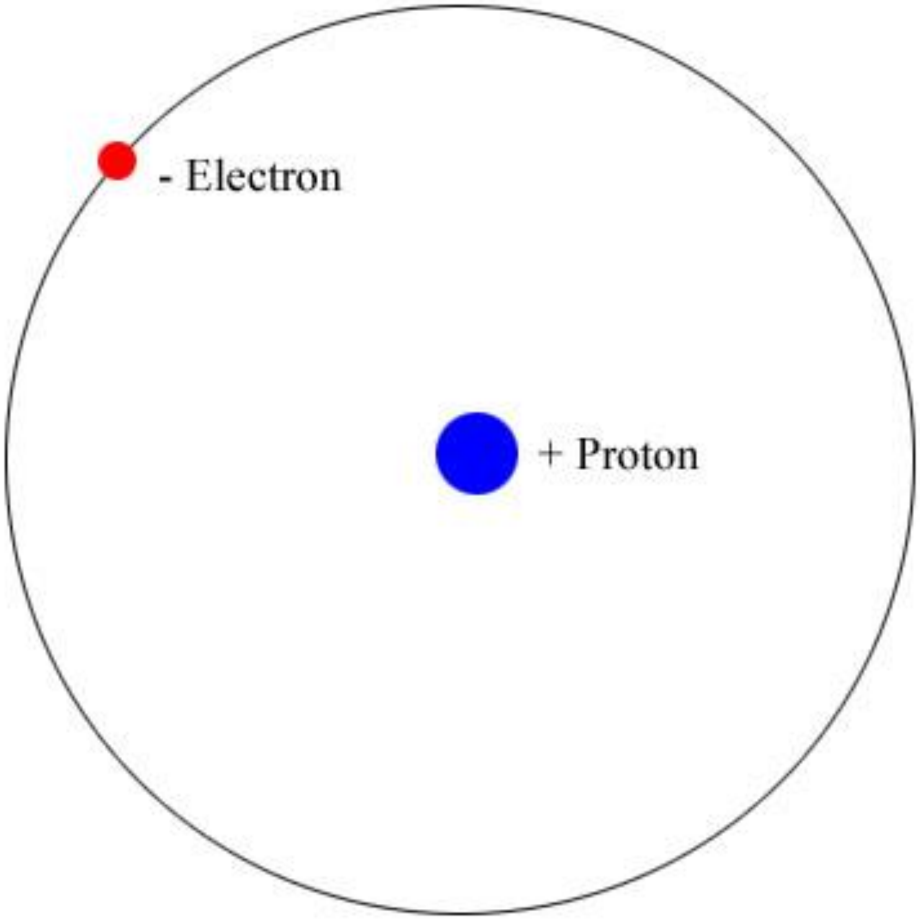
Classical electrodynamics → accelerated charge → emits radiation → inspiral collapse in 10^{-8} s !



Double slit experiment with electrons de Broglie $\lambda = \frac{h}{p}$

Bohr model of an atom

The hydrogen atom



The Bohr model of an atom

$$\alpha\frac{Ze^2}{r^2} = \frac{mv^2}{r}\,,$$

here $\alpha = 8.98 \cdot 10^9 \ N^2m^2C^{-2}$ (C - Coulomb - unit of the electric charge) is a constant related to the permittivity of empty space, Z - charge of the nucleus, e - unit of electric charge, m - mass of an electron.

Electrons can move only on orbits that satisfy quantum condition

$$J = mvr = n\hbar$$

where J - is angular momentum, n - natural number and $\hbar = \dfrac{h}{2\pi}$. It is now easy to show that:

$$r_n = \frac{\hbar^2}{\alpha m Z e^2}n^2\,.$$

Energy of an electron moving on a circular orbit around a nucleus is:

$$E = \frac{1}{2}mv^2 - \alpha\frac{Ze^2}{r} = \frac{1}{2}\alpha\frac{Ze^2}{r} - \alpha\frac{Ze^2}{r} = -\frac{1}{2}\alpha\frac{Ze^2}{r}\,.$$

Notice that $E < 0$, electron is bound to the nucleus by the Coulomb forces.

Substituting $r = r_n$, we get

$$E_n = \frac{m(\alpha Ze^2)^2}{2\hbar^2}\frac{1}{n^2}\,,$$

and

$$v_n = \frac{\alpha Ze^2}{\hbar}\frac{1}{n}\,.$$

It is time to make some calculations for the hydrogen atom but before let us introduce some more convenient units:

Energy spectrum of H atoms:

$$\Delta E = |E_n - E_m| = E_1(\frac{1}{n^2} - \frac{1}{m^2})$$

$$\frac{hc}{\lambda_{nm}} = E_1(\frac{1}{n^2} - \frac{1}{m^2})$$

$$\frac{1}{\lambda_{nm}} = \frac{E_1}{hc}(\frac{1}{n^2} - \frac{1}{m^2})$$

$$\frac{E_1}{hc} = \frac{\alpha^2 e^4 m_e}{2\hbar^2 hc} = R \text{ Rydberg constant}$$

$$R = 1.097 \cdot 10^7 \mathrm{m}^{-1} = 1.097 \cdot 10^{-3} \mathring{A}^{-1}$$

$$\frac{1}{R} = 912 \mathring{A}$$

Now let us calculate $E(1)$

$$E(1) = -\frac{1}{2}\frac{\alpha e^2}{r_1} = -\frac{1}{2}\frac{\alpha e^2}{(\frac{\hbar^2}{m_e \alpha e^2})} = -\frac{\alpha^2 e^4 m_e}{2\hbar^2} = -2.17 \times 10^{-18}\text{J} = -13.6\text{eV}$$

$$E(2) = \frac{1}{2^2}E(1) = -\frac{1}{4}13.6 = -3.4\text{eV}$$

Units

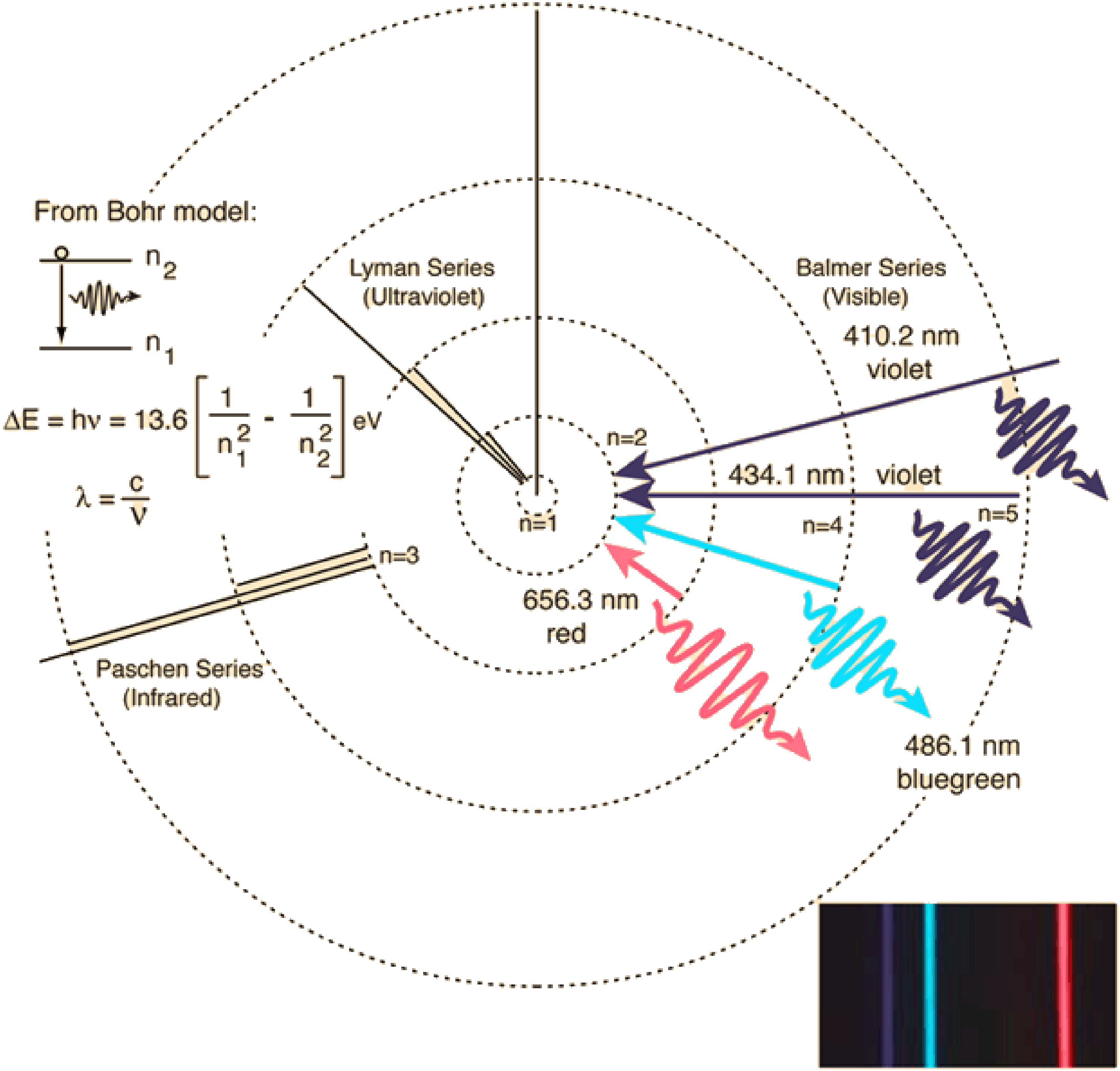
Angstrom (Å) = 10^{-8} cm or 10^{-10}m = 0.1 nm

Nanometer =10^{-9} m

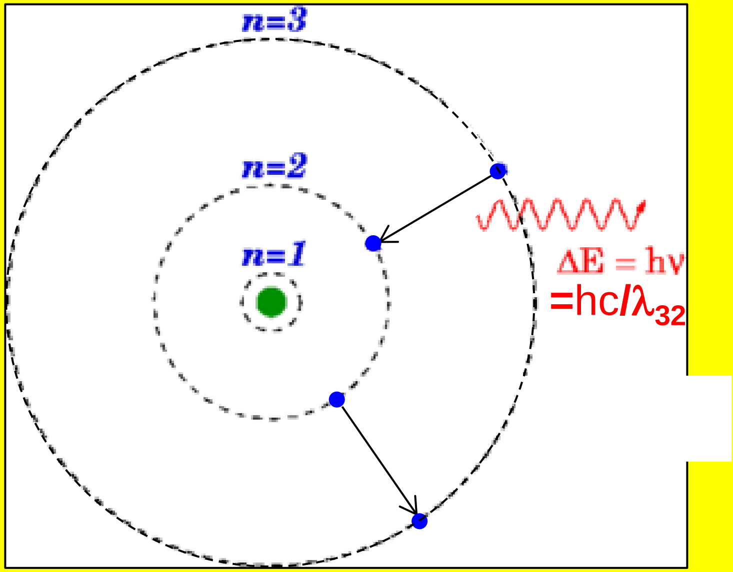
Picometer = 10^{-12} m

Femtometer = 10^{-15} m

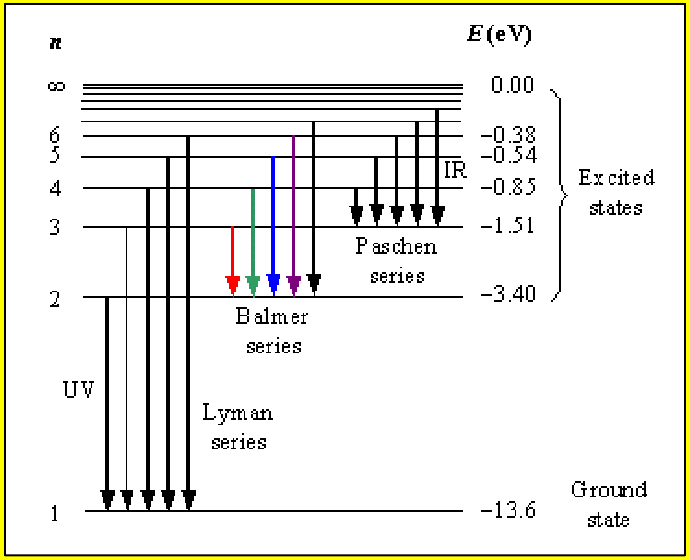
1eV = $1.6 \cdot 10^{-19}$ J


From Bohr model:
n_2
n_1
$\Delta E = h\nu = 13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]\text{eV}$
$\lambda = \frac{c}{\nu}$
Lyman Series
(Ultraviolet)
Balmer Series
(Visible)
410.2 nm
violet
n=2
434.1 nm
violet
n=1
n=4
n=5
n=3
656.3 nm
red
486.1 nm
bluegreen
Paschen Series
(Infrared)

Every atom (and ion) has a distinctive fingerprint…



$=hc/\lambda_{32}$



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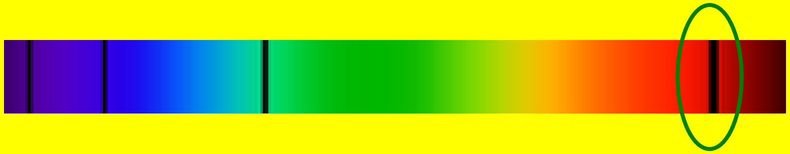
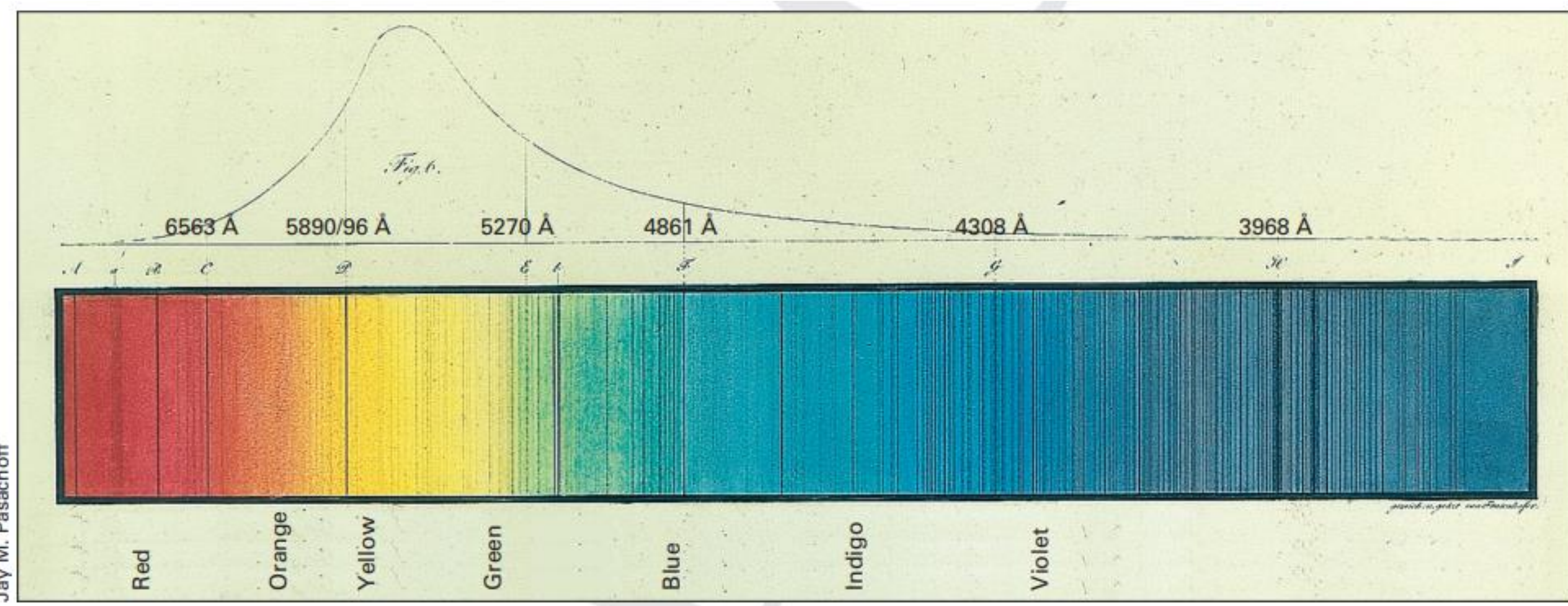
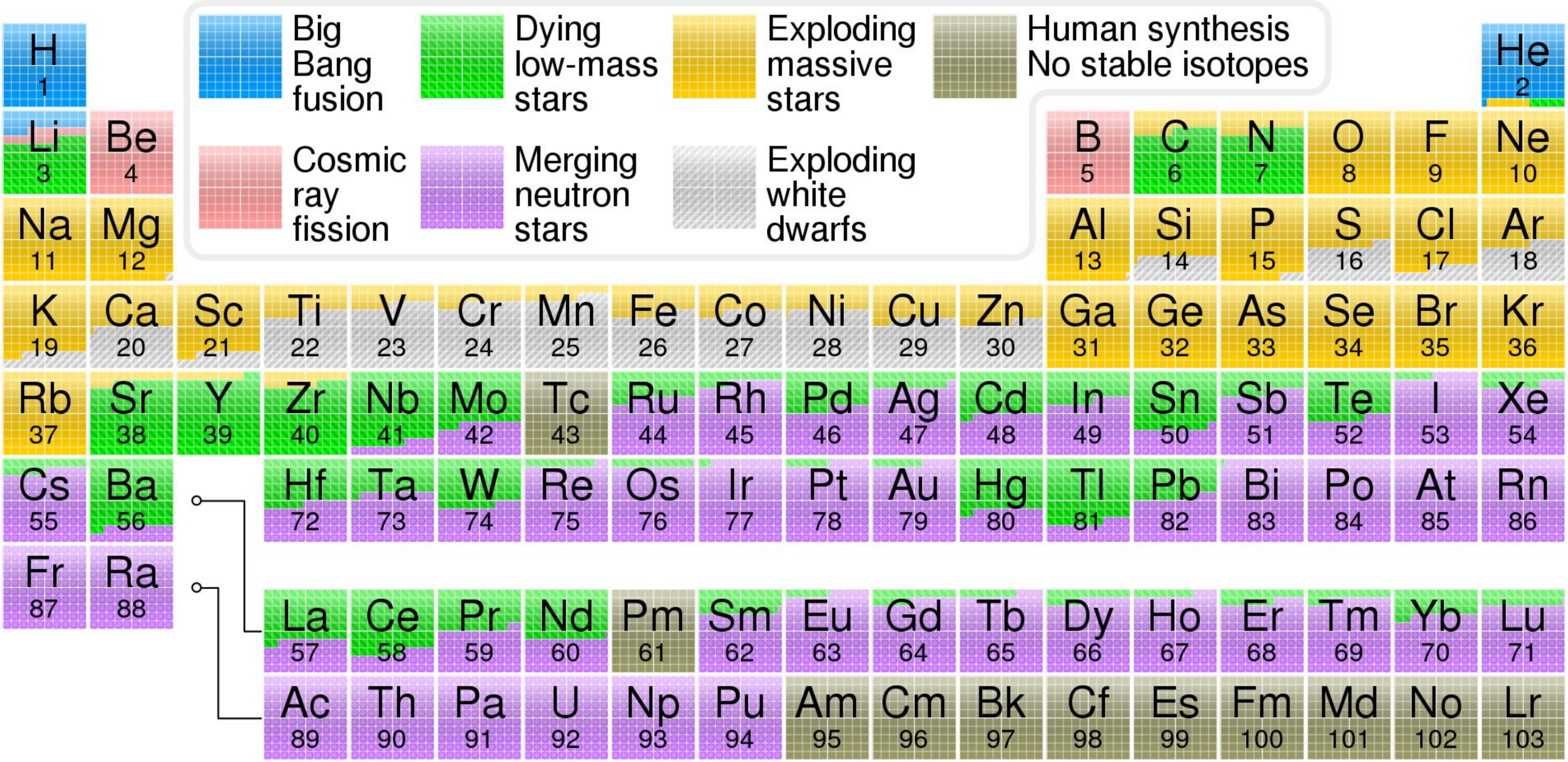


http://socphysics.blogspot.com/2010/08/bohr-model-of-atom.html

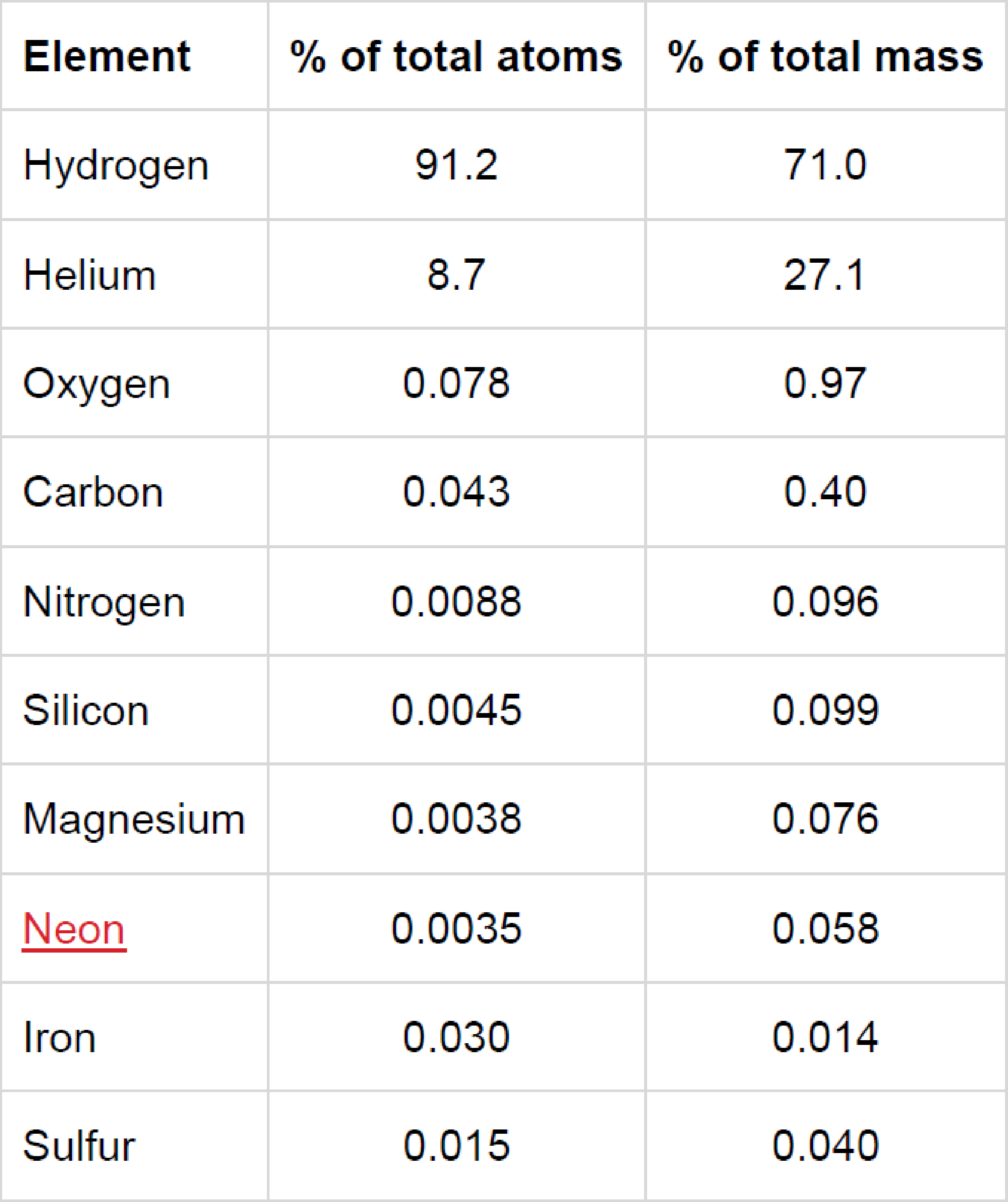
http://astronomy.nju.edu.cn/~lixd/GA/AT4/AT404/IMAGES/AACHCLS0.JPG

Fraunhofer spectral lines
Spectrum of the Sun (1814)




Big Bang fusion
Dying low-mass stars
Exploding massive stars
Human synthesis No stable isotopes
Cosmic ray fission
Merging neutron stars
Exploding white dwarfs
H 1
He 2
Li 3
Be 4
B 5
C 6
N 7
O 8
F 9
Ne 10
Na 11
Mg 12
Al 13
Si 14
P 15
S 16
Cl 17
Ar 18
K 19
Ca 20
Sc 21
Ti 22
V 23
Cr 24
Mn 25
Fe 26
Co 27
Ni 28
Cu 29
Zn 30
Ga 31
Ge 32
As 33
Se 34
Br 35
Kr 36
Rb 37
Sr 38
Y 39
Zr 40
Nb 41
Mo 42
Tc 43
Ru 44
Rh 45
Pd 46
Ag 47
Cd 48
In 49
Sn 50
Sb 51
Te 52
I 53
Xe 54
Cs 55
Ba 56
Hf 72
Ta 73
W 74
Re 75
Os 76
Ir 77
Pt 78
Au 79
Hg 80
Tl 81
Pb 82
Bi 83
Po 84
At 85
Rn 86
Fr 87
Ra 88
La 57
Ce 58
Pr 59
Nd 60
Pm 61
Sm 62
Eu 63
Gd 64
Tb 65
Dy 66
Ho 67
Er 68
Tm 69
Yb 70
Lu 71
Ac 89
Th 90
Pa 91
U 92
Np 93
Pu 94
Am 95
Cm 96
Bk 97
Cf 98
Es 99
Fm 100
Md 101
No 102
Lr 103

Chemical composition of the Sun



Element	% of total atoms	% of total mass
Hydrogen	91.2	71.0
Helium	8.7	27.1
Oxygen	0.078	0.97
Carbon	0.043	0.40
Nitrogen	0.0088	0.096
Silicon	0.0045	0.099
Magnesium	0.0038	0.076
Neon	0.0035	0.058
Iron	0.030	0.014
Sulfur	0.015	0.040