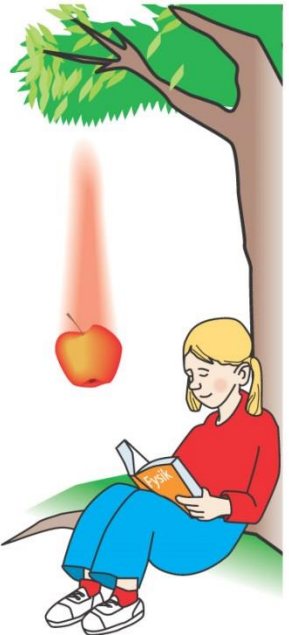




# *Introduction to Cosmology*

*Marek Demianski  
University of Warsaw*



Graviton?

Solar systems  
Galaxies

### Gravity Force

Gluons (8)

Quarks

Mesons  
Baryons

Nuclei

up quark  
u  
down quark  
d  
proton

down quark  
d  
up quark  
u  
down quark  
d  
neutron

### Strong force

### Electromagnetic force

Hydrogen atom

Water molecule

Oxygen atom

Protons and Neutrons

Electron

Photon

Atoms  
Light  
Chemistry  
Electronics

### Weak force

Bosons (W,Z)

anti-neutrino

e electron

W force carrier particle

neutron

proton

Neutron decay  
Beta decay  
Neutrino interactions  
Burning of the sun

# Basic conservation laws

Energy,

Momentum,

Angular momentum,

Electric charge,

Baryon number,

Lepton number

# Range of different forces

- Gravitational – infinite
- Electromagnetic – infinite
- Weak – very short  $\sim 10^{-16}$  m !!!
- Strong (nuclear)  $\sim 10^{-15}$  m !!!



# THE STANDARD MODEL OF FUNDAMENTAL PARTICLES AND INTERACTIONS

The Standard Model is a quantum theory that summarizes our current knowledge of the physics of fundamental particles and fundamental interactions (interactions are manifested by forces and by decay rates of unstable particles).

## FERMIONS

**matter constituents**  
spin = 1/2, 3/2, 5/2, ...

Leptons spin = 1/2			Quarks spin = 1/2		
Flavor	Mass GeV/c <sup>2</sup>	Electric charge	Flavor	Approx. Mass GeV/c <sup>2</sup>	Electric charge
$\nu_L$ lightest neutrino*	$(0-0.8)\times 10^{-9}$	0	<b>u</b> up	0.0022	2/3
<b>e</b> electron	0.000511	-1	<b>d</b> down	0.0047	-1/3
$\nu_M$ middle neutrino*	$(0.009-0.8)\times 10^{-9}$	0	<b>c</b> charm	1.27	2/3
$\mu$ muon	0.1057	-1	<b>s</b> strange	0.0934	-1/3
$\nu_H$ heaviest neutrino*	$(0.05-0.8)\times 10^{-9}$	0	<b>t</b> top	172.7	2/3
$\tau$ tau	1.777	-1	<b>b</b> bottom	4.18	-1/3

\*See the neutrino paragraph below.

**Spin** is the intrinsic angular momentum of particles. Spin is given in units of  $\hbar$ , which is the quantum unit of angular momentum where  $\hbar = h/2\pi = 6.58\times 10^{-29}$  GeV s =  $1.05\times 10^{-34}$  J s.

**Electric charges** are given in units of the proton's charge. In SI units the electric charge of the proton is  $1.60\times 10^{-19}$  coulombs.

The **energy** unit of particle physics is the electronvolt (eV), the energy gained by one electron in crossing a potential difference of one volt. **Masses** are given in GeV/c<sup>2</sup> (remember  $E = mc^2$ ) where  $1 \text{ GeV} = 10^9 \text{ eV} = 1.60\times 10^{-10}$  joule. The mass of the proton is  $0.938 \text{ GeV}/c^2 = 1.67\times 10^{-27}$  kg.

### Neutrinos

Neutrinos are produced in the sun, supernovae, reactors, accelerator collisions, and many other processes. Any produced neutrino can be described as one of three neutrino flavor states  $\nu_e$ ,  $\nu_\mu$ , or  $\nu_\tau$ , labelled by the type of charged lepton associated with its production. Each is a defined quantum mixture of the three definite-mass neutrinos  $\nu_1$ ,  $\nu_2$ , and  $\nu_3$  for which currently allowed mass ranges are shown in the table. Further exploration of the properties of neutrinos may yield powerful clues to puzzles about matter and antimatter and the evolution of stars and galaxy structures.

### Matter and Antimatter

For every particle type there is a corresponding antiparticle type, denoted by a bar over the particle symbol (unless + or - charge is shown). Particle and antiparticle have identical mass and spin but opposite charges. Some electrically neutral bosons (e.g.,  $Z^0$ ,  $\gamma$ , and  $\eta_c = c\bar{c}$  or  $\eta_b = b\bar{b}$ ) are their own antiparticles.

## Particle Processes

These diagrams are an artist's conception. Orange shaded areas represent the cloud of gluons.

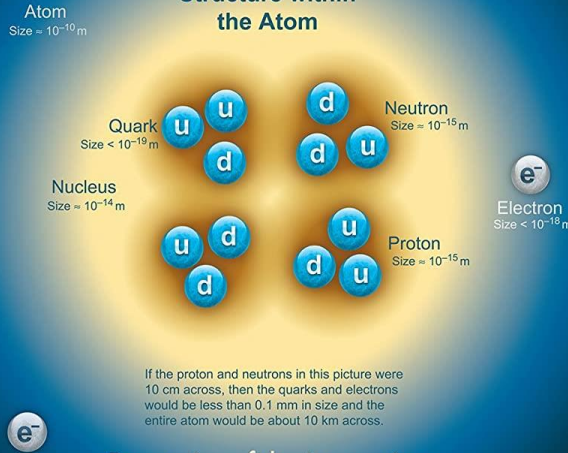
$n \rightarrow p e^- \bar{\nu}_e$

$e^+ e^- \rightarrow B^0 \bar{B}^0$

A free neutron (udd) decays to a proton (uud), an electron, and an antineutrino via a virtual (mediating) W boson. This is neutron  $\beta$  (beta) decay.

An electron and positron (antielelectron) colliding at high energy can annihilate to produce  $B^0$  and  $B^0$  mesons via a virtual Z boson or a virtual photon.

## Structure within the Atom



If the proton and neutrons in this picture were 10 cm across, then the quarks and electrons would be less than 0.1 mm in size and the entire atom would be about 10 km across.

## Properties of the Interactions

The strengths of the interactions (forces) are shown relative to the strength of the electromagnetic force for two u quarks separated by the specified distances.

Property	Gravitational Interaction	Weak Interaction (Electroweak)	Electromagnetic Interaction	Strong Interaction
Acts on:	Mass – Energy	Flavor	Electric Charge	Color Charge
Particles experiencing:	All	Quarks, Leptons	Electrically Charged	Quarks, Gluons
Particles mediating:	Graviton (not yet observed)	$W^+$ $W^-$ $Z^0$	$\gamma$	Gluons
Strength at $\begin{cases} 10^{-18} \text{ m} \\ 3\times 10^{-17} \text{ m} \end{cases}$	$10^{-41}$ $10^{-41}$	0.8 $10^{-4}$	1 1	25 60

## BOSONS

**force carriers**  
spin = 0, 1, 2, ...

Unified Electroweak spin = 1		
Name	Mass GeV/c <sup>2</sup>	Electric charge
$\gamma$ photon	0	0
$W^-$	80.38	-1
$W^+$	80.38	+1
$Z^0$ Z boson	91.188	0

Strong (color) spin = 1		
Name	Mass GeV/c <sup>2</sup>	Electric charge
<b>g</b> gluon	0	0

Higgs Boson spin = 0		
Name	Mass GeV/c <sup>2</sup>	Electric charge
<b>H</b> Higgs	125.25	0

### Higgs Boson

The Higgs boson is a critical component of the Standard Model. The associated Higgs field provides the mechanism by which fundamental particles get mass. Particles that interact more strongly with the Higgs field are more massive.

### Color Charge

Only quarks and gluons carry "strong charge" (also called "color charge") and can have strong interactions. Each quark carries three types of color charge. These charges have nothing to do with the colors of visible light. Just as electrically-charged particles interact by exchanging photons, in strong interactions, color-charged particles interact by exchanging gluons.

### Quarks Confined in Mesons and Baryons

Quarks and gluons cannot be isolated – they are confined in color-neutral particles called **hadrons**. This confinement (binding) results from multiple exchanges of gluons among the color-charged constituents. As color-charged particles (quarks and gluons) move apart, the energy in the color-force field between them increases. This energy eventually is converted into additional quark-antiquark pairs. The quarks and antiquarks then combine into hadrons; these are the particles seen to emerge.

Two types of hadrons have been observed in nature **mesons**  $q\bar{q}$  and **baryons**  $qqq$ . Among the many types of baryons observed are the proton (uud), antiproton ( $\bar{u}\bar{u}\bar{d}$ ), and neutron (udd). Quark charges add in such a way as to make the proton have charge 1 and the neutron charge 0. Among the many types of mesons are the pion  $\pi^+$  (u $\bar{d}$ ), kaon  $K^-$  (s $\bar{u}$ ), and  $B^0$  (db).

Learn more at [ParticleAdventure.org](http://ParticleAdventure.org)



## Unsolved Mysteries

Driven by new puzzles in our understanding of the physical world, particle physicists are following paths to new wonders and startling discoveries. Experiments may even find extra dimensions of space, microscopic black holes, and/or evidence of string theory.

### Why is the Universe Accelerating?



The expansion of the universe appears to be accelerating. Is this due to Einstein's Cosmological Constant? If not, will experiments reveal a new force of nature or even extra (hidden) dimensions of space?

### Why No Antimatter?



Matter and antimatter were created in the Big Bang. Why do we now see only matter except for the tiny amounts of antimatter that we make in the lab and observe in cosmic rays?

### What is Dark Matter?



Invisible forms of matter make up much of the mass observed in galaxies and clusters of galaxies. Does this dark matter consist of new types of particles that interact very weakly with ordinary matter?

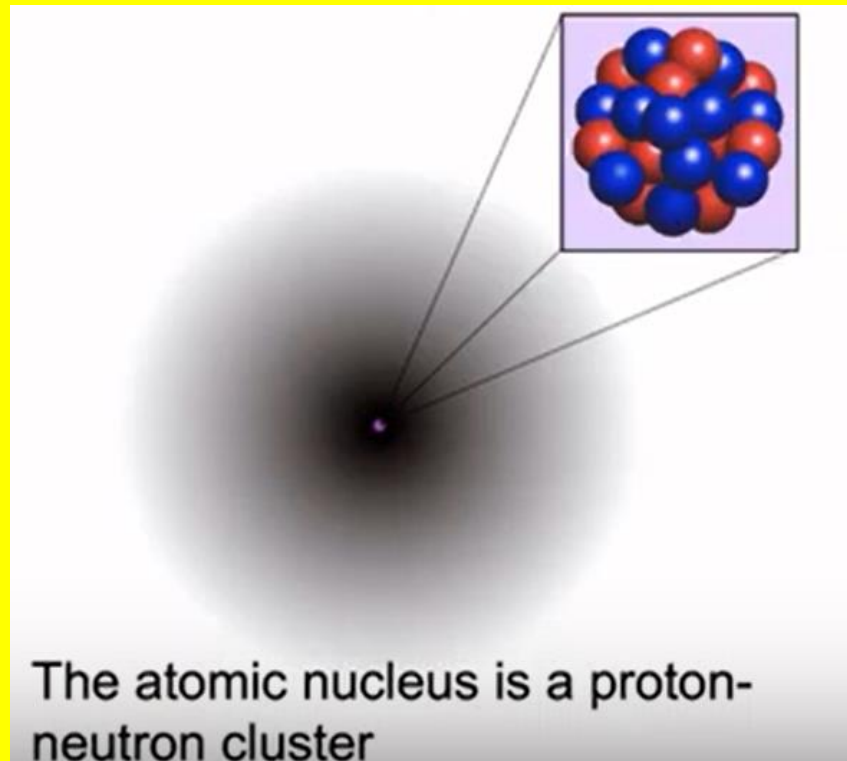
### Are there Extra Dimensions?



An indication for extra dimensions may be the extreme weakness of gravity compared with the other three fundamental forces (gravity is so weak that a small magnet can pick up a paper clip overwhelming Earth's gravity).

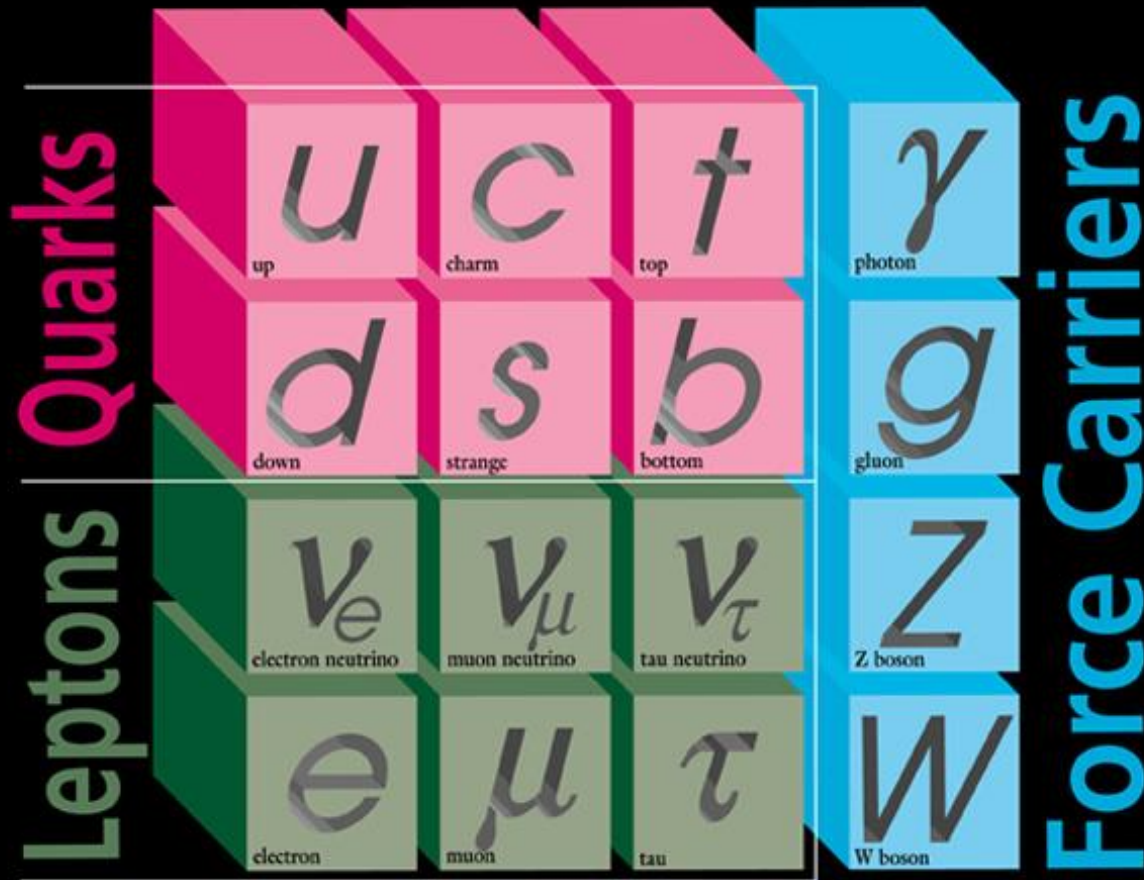
“All the matter that makes up all the living organisms and ecosystems, planets and stars, throughout every galaxy in the universe, is made of atoms, and 99.9% of the mass of all the atoms in the (visible) universe comes from the nuclei at their centers which are over 10,000 times smaller in diameter than the atoms themselves”

*USNAS Decadal Study Report*





# ELEMENTARY PARTICLES



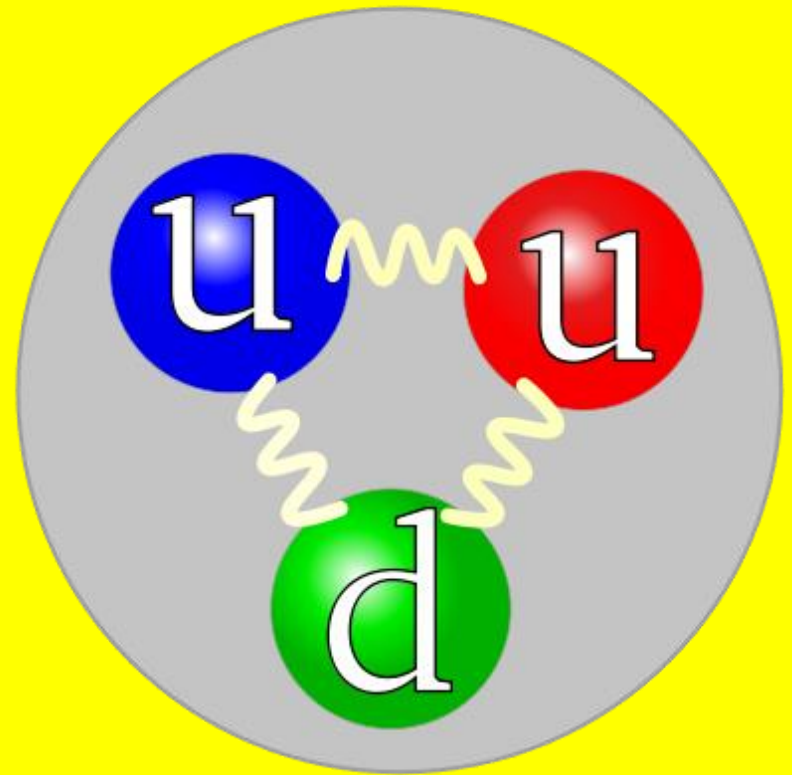
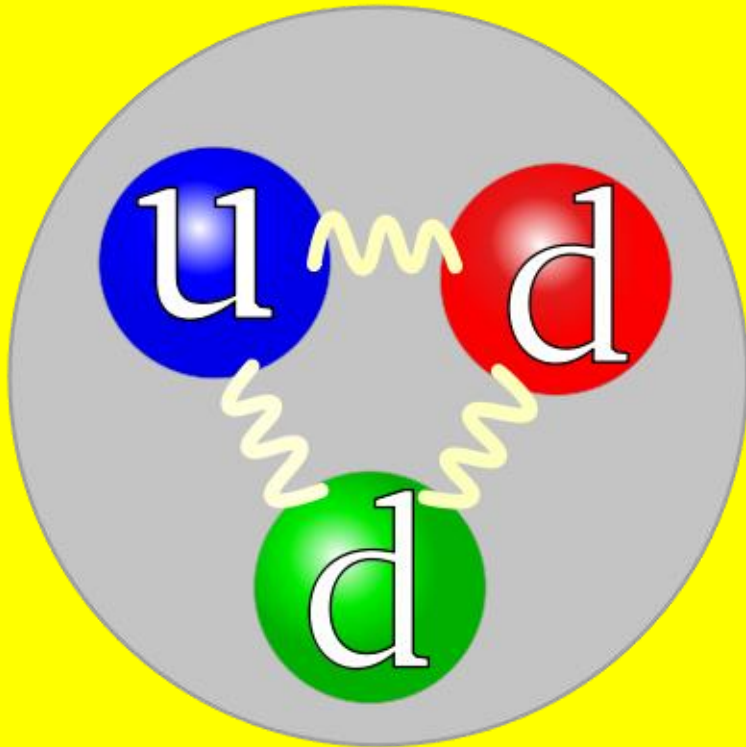
I II III  
Three Generations of Matter



# Protons and Neutrons

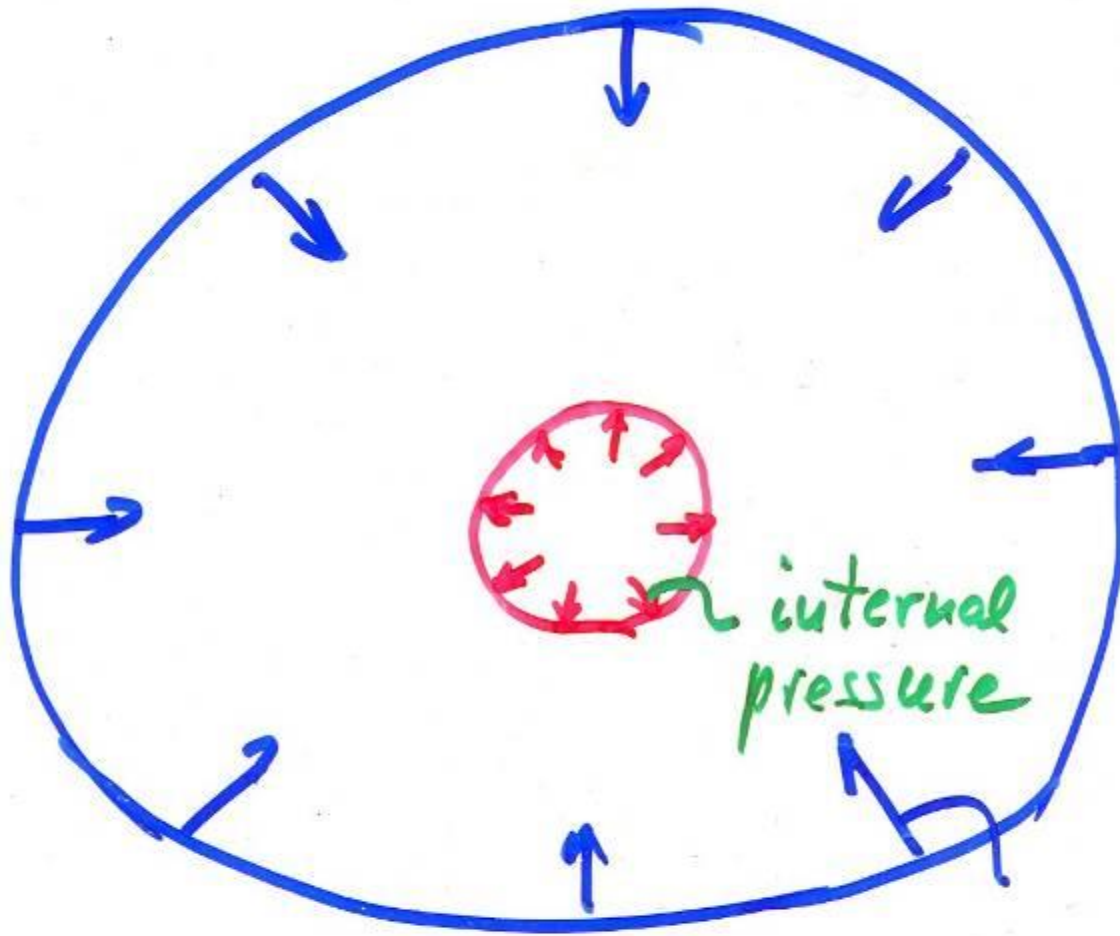
**u** charge =  $+2/3$

**d** charge =  $-1/3$

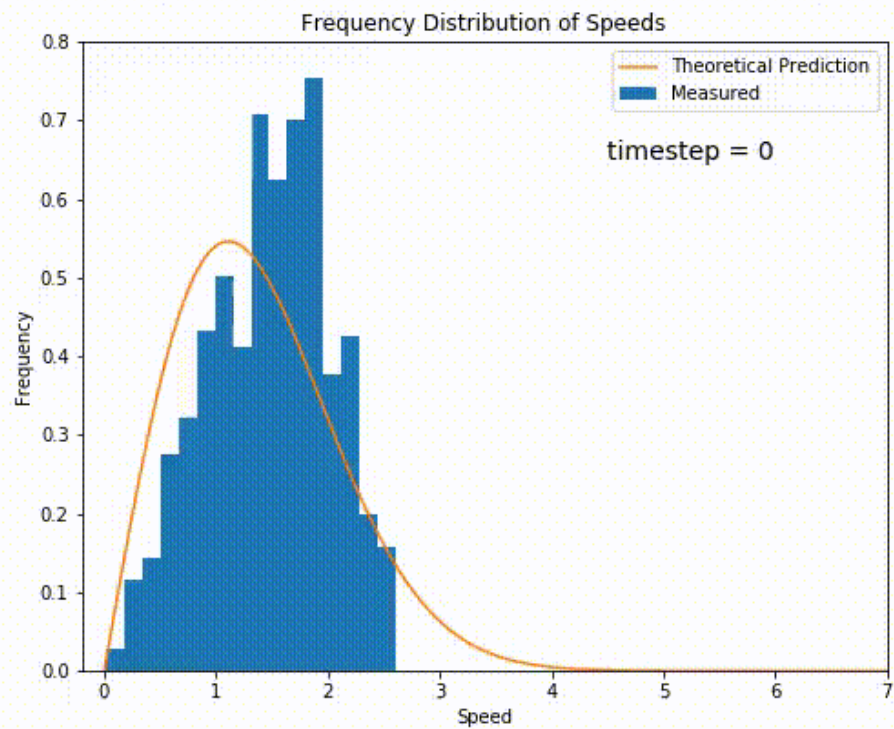
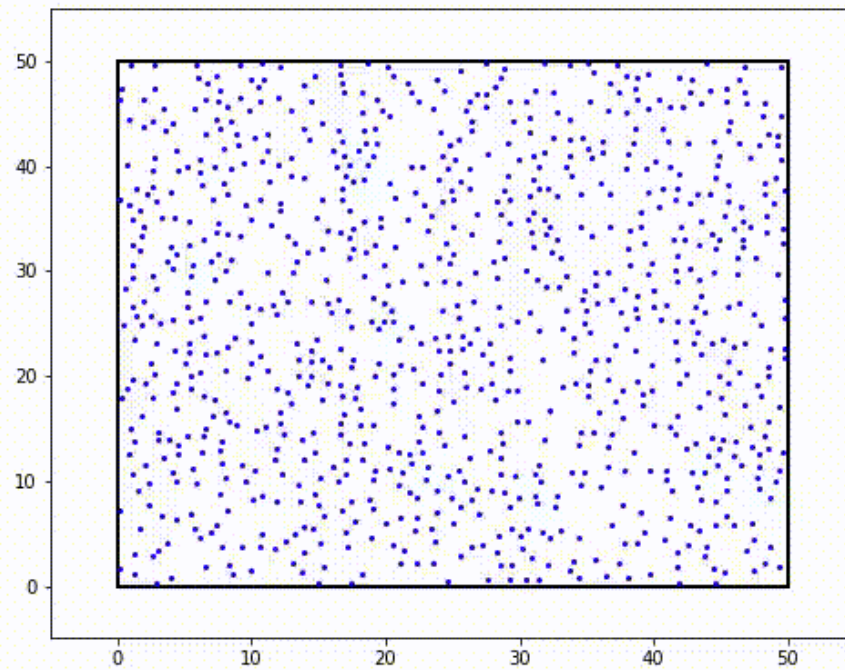


Which is a proton; which is a neutron?



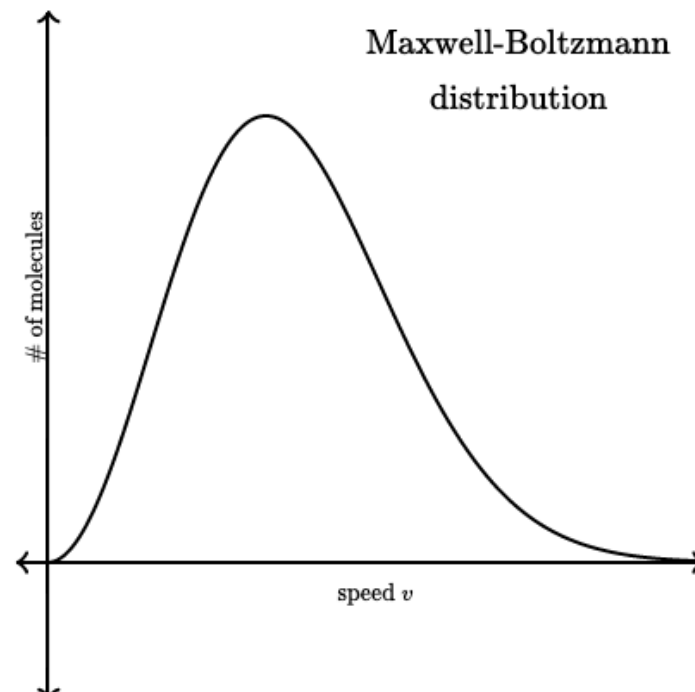


gravitational  
force



# Maxwell-Boltzmann distribution

$$f(v) = \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{m}{kT}\right)^{3/2} v^2 \exp\left[-\frac{mv^2}{2kT}\right].$$





## Maxwell-Boltzmann distribution averages

The most probable speed  $v_p$  is defined as the maximum of the M-B distribution, so it is the solution of  $\frac{df(v)}{dv} = 0$ . Using Mathematica one gets that:

$$v_p = \sqrt{\frac{2kT}{m}}.$$

The mean speed is defined as

$$\langle v \rangle = \int_0^{\infty} v f(v) dv = \sqrt{\frac{8kT}{\pi m}} = \frac{2}{\sqrt{\pi}} v_p.$$

The root mean square speed is defined as

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\int_0^{\infty} v^2 f(v) dv} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3}{2}} v_p$$

# What does *temperature* mean?

## EQUIPARTITION PRINCIPLE

All species of particles have the same average kinetic energy.

According to the equipartition principle , for each kind of (monatomic) particle with mass  $m_i$  :

$$\left\langle \frac{1}{2} m_i v_i^2 \right\rangle = \frac{3kT}{2}$$

**Particles** of mass  $m_i$ , at temperature  $T$ , have **average** velocity:

$$\langle v_i \rangle = \sqrt{\frac{3kT}{m_i}}$$

## Sources of pressure

### Gas (plasma) pressure

$$p = nkT = \frac{\rho}{m} kT = \frac{\rho}{\mu m_H} kT$$

$\mu$  - mean molecular weight,  $m_H$  - mass of a hydrogen atom

### radiation pressure

$$p_{rad} = \frac{1}{3} \varepsilon_{rad} = \frac{4}{3} \frac{\sigma}{c} T^4$$

$\sigma$  - Stefan-Boltzmann constant,  $c$  - velocity of light

quantum pressure  $\leq$  Pauli exclusion principle

polytropic equation of state

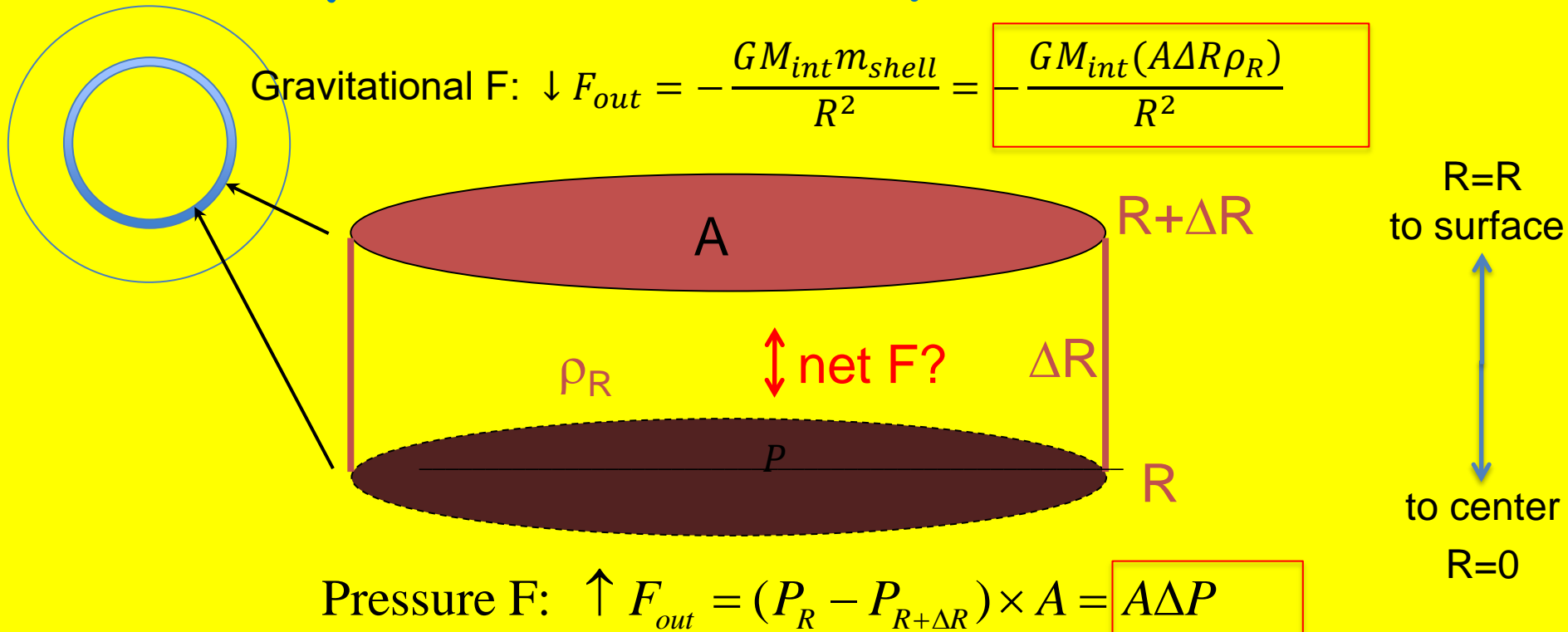
$$p = K \rho^{1 + \frac{1}{n}}$$

where  $K$  and  $n$  are real positive constants,  $n$  is called a polytropic index

Pressure does not depend on temperature !!!



# Hydrostatic Equilibrium



$$-\frac{GM_{int}(A\Delta R\rho_R)}{R^2} = A\Delta P \Rightarrow \frac{\Delta P}{\Delta R} = -\frac{GM_{int}}{R^2}\rho_R$$

$$\frac{dP}{dR}_{\text{from } R \text{ to } R+\Delta R} = -\frac{GM_{int}}{R^2}r_R \quad \frac{dP}{dR}_{\text{from } R \text{ to } R+\Delta R} = -g_R\rho_R$$

# Stellar evolution timescales

hydrodynamical

$$t_{ff} = \frac{\pi}{2} \sqrt{\frac{R^3}{2GM}} = \sqrt{\frac{3\pi}{32G \langle \rho \rangle}}$$

$$t_{ff\odot} = 1765.5s = 29.4\text{min}$$

Kelvin-Helmholtz

$$t_{K-H} = \frac{GPE}{L} = \frac{GM^2}{R L}$$

$$t_{K-H\odot} = 3.1 \cdot 10^7 \text{yr}$$

nuclear

$$t_{nuc} = \frac{0.2 \cdot 0.07Mc^2}{L}$$

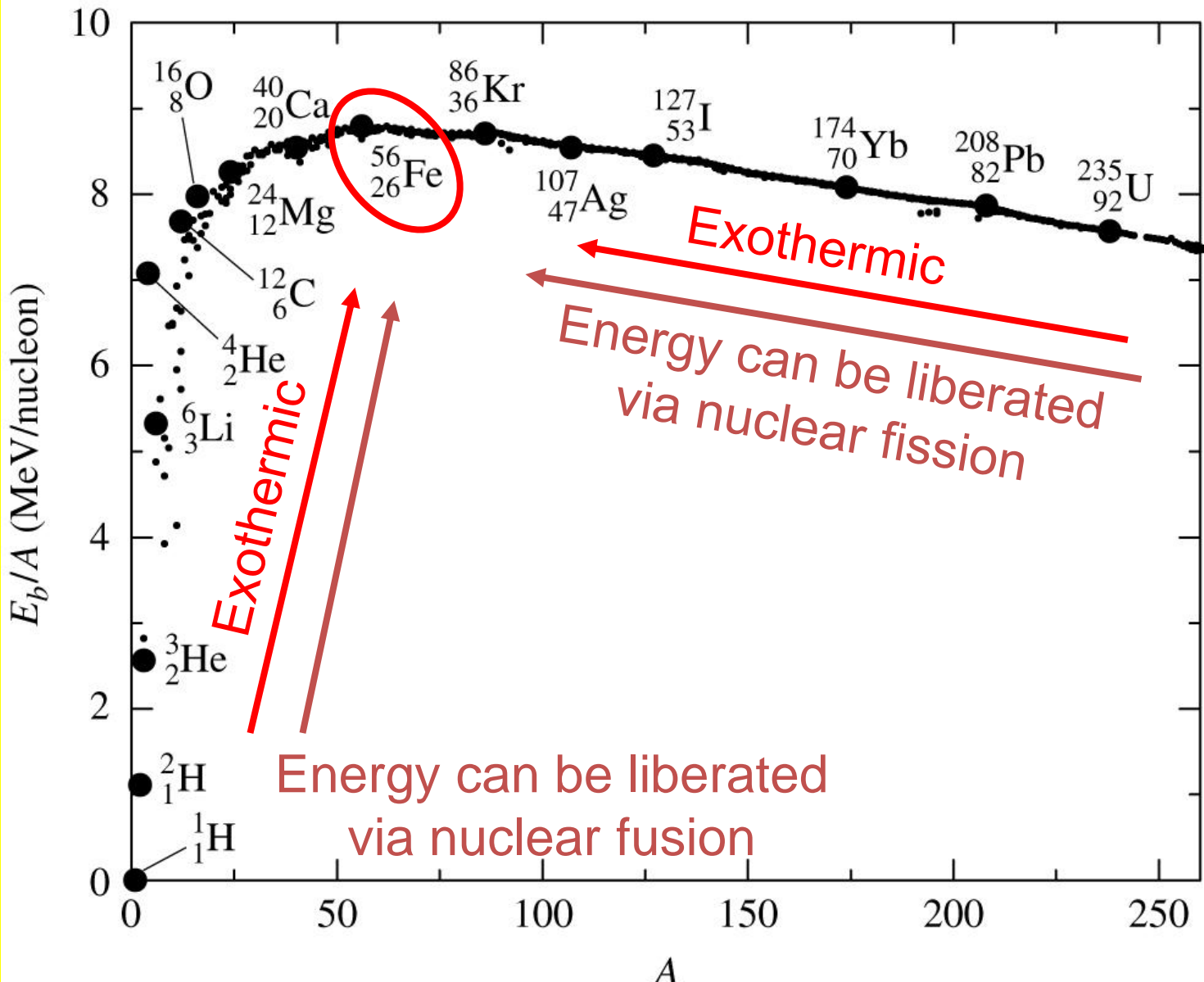
$$t_{nuc\odot} = 2 \cdot 10^{11} \text{yr}$$

Albert Einstein

$$E = mc^2$$



# Binding energy per nucleon



# Nuclear reactions

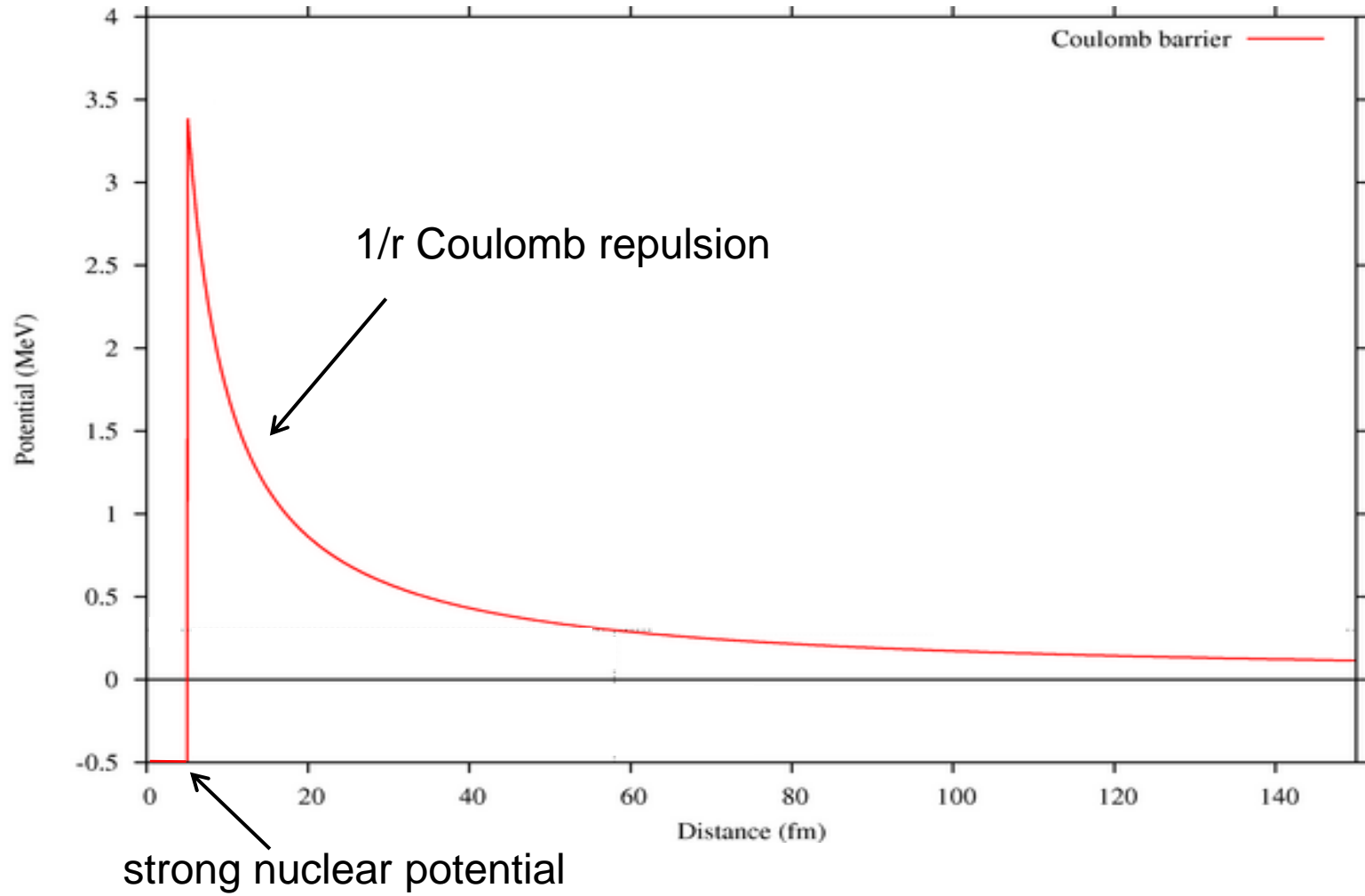
## Conservation Rules

- Conservation of charge
- Conservation of baryon number
- Conservation of lepton number

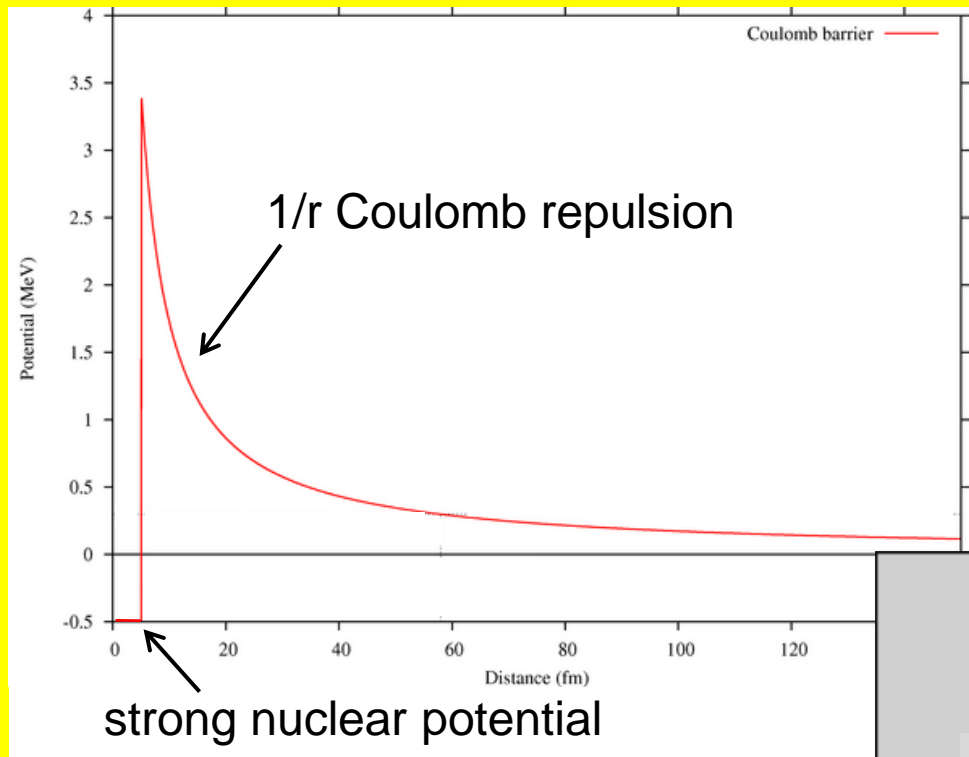
*Example: neutron decay*  $n \rightarrow p^+ + e^- + \bar{\nu}$

*Example: 1<sup>st</sup> step of pp cycle*  $p^+ + p^+ \rightarrow D + e^+ + \nu$

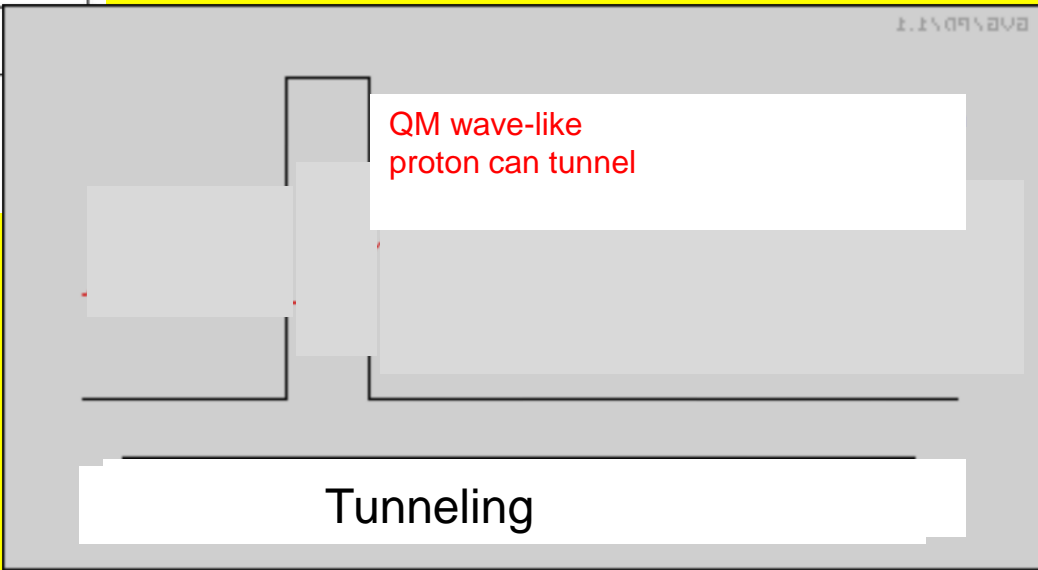
*Example: particle annihilation*  $e^- + e^+ \rightarrow \gamma + \gamma$



# Quantum-Mechanical Tunneling

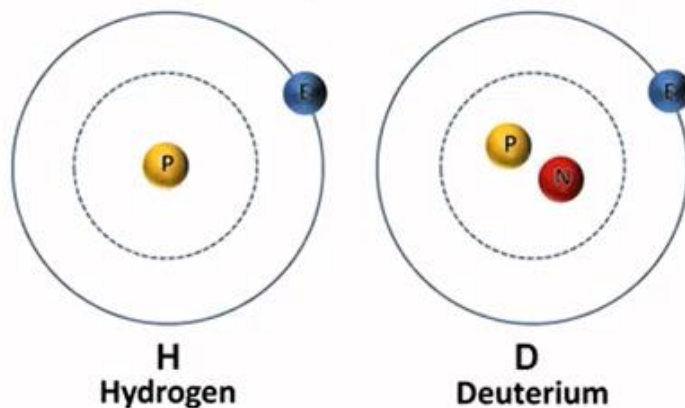


For 2 protons in the core of a star with a typical stellar interior temperature:  
**Coulomb repulsion prevents fusion**

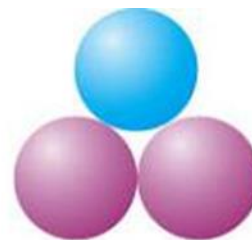
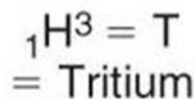
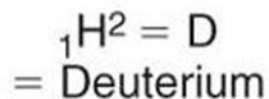


Heisenberg Unc. Pr.:  $\Delta x \Delta p \sim \hbar$   
+ small de Broglie wavelength:  $\lambda = h/p$   
means that **2 protons within  $\sim 1 \lambda$  of each other can fuse!**

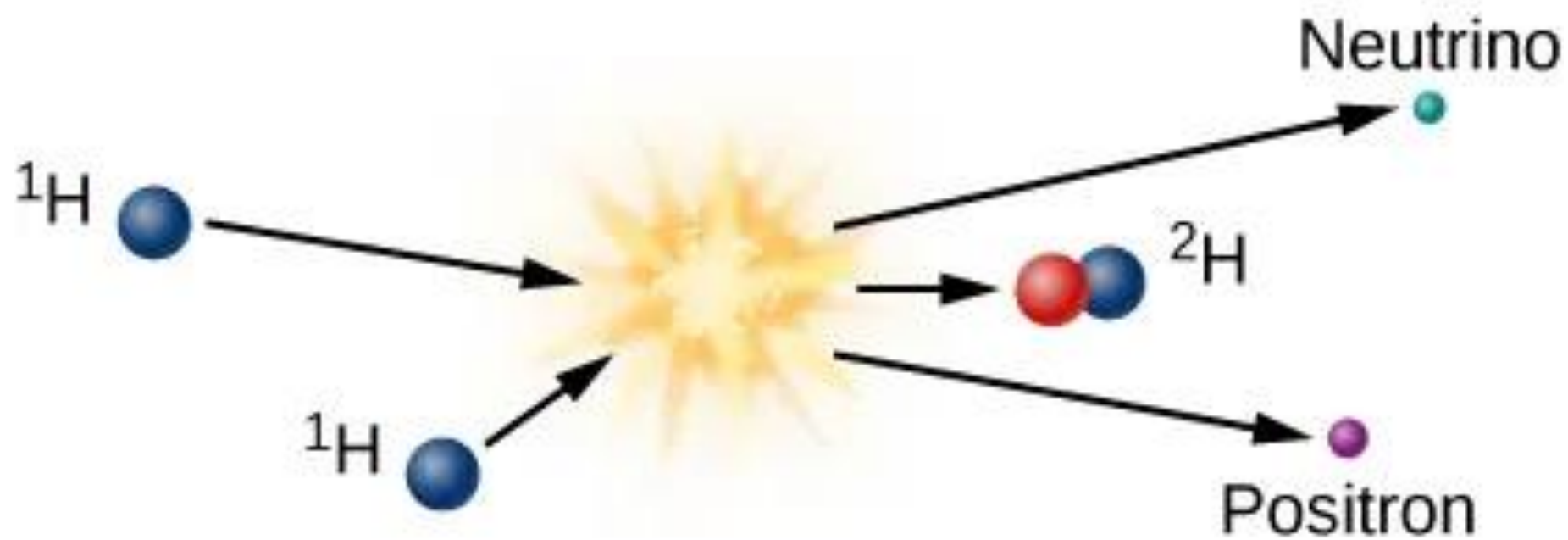
## Hydrogen Isotopes

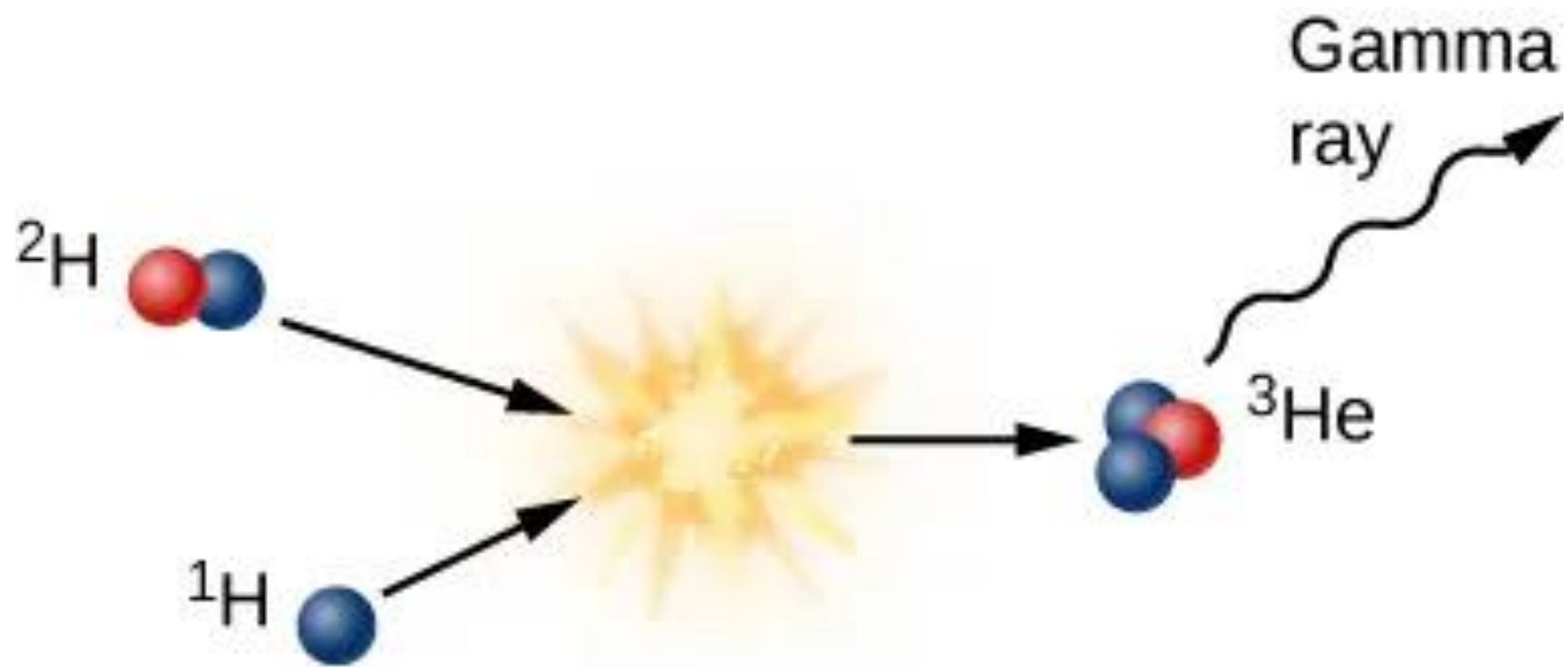


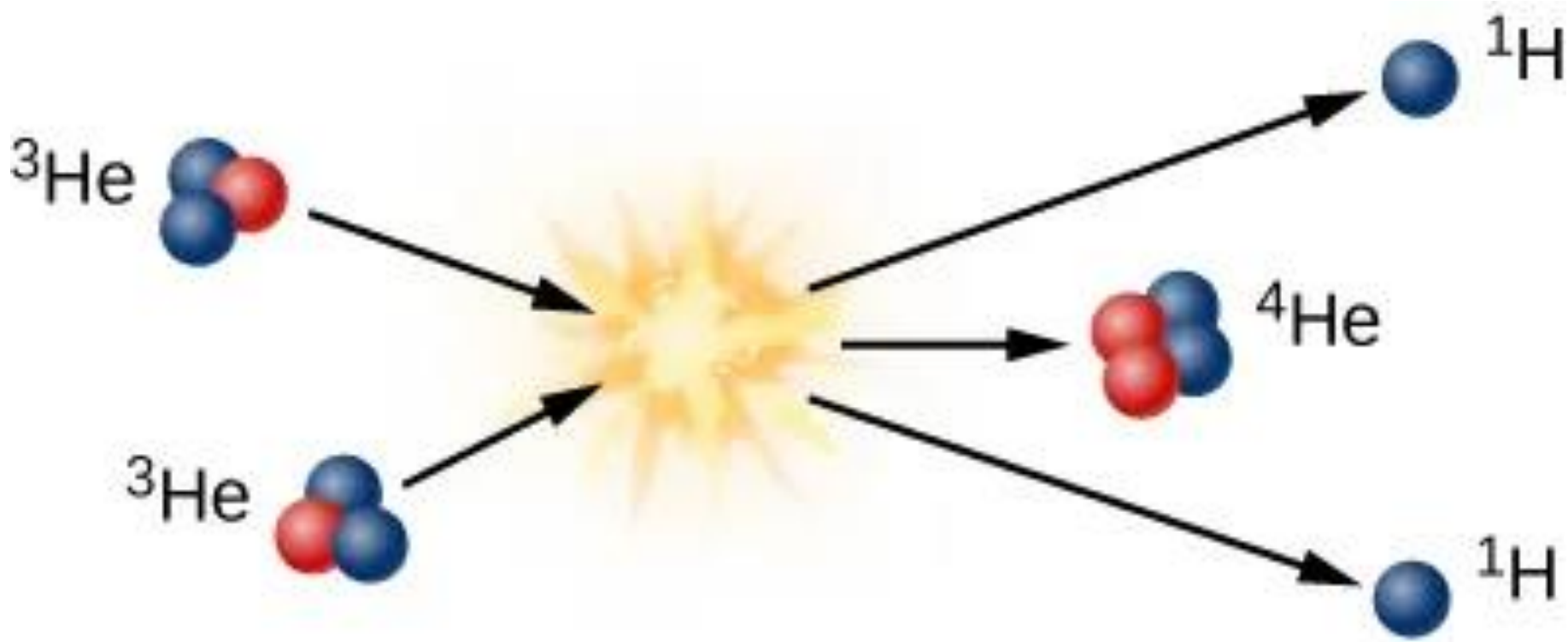
**Isotopes are essentially different versions of the elements on the periodic table, atomic particles with the same number of protons but different numbers of neutrons.**



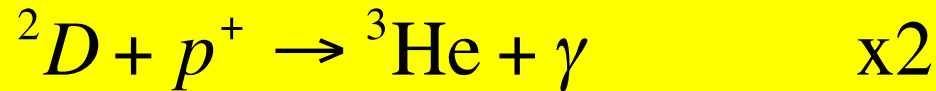
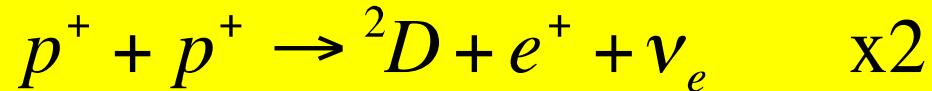








# The proton-proton cycle

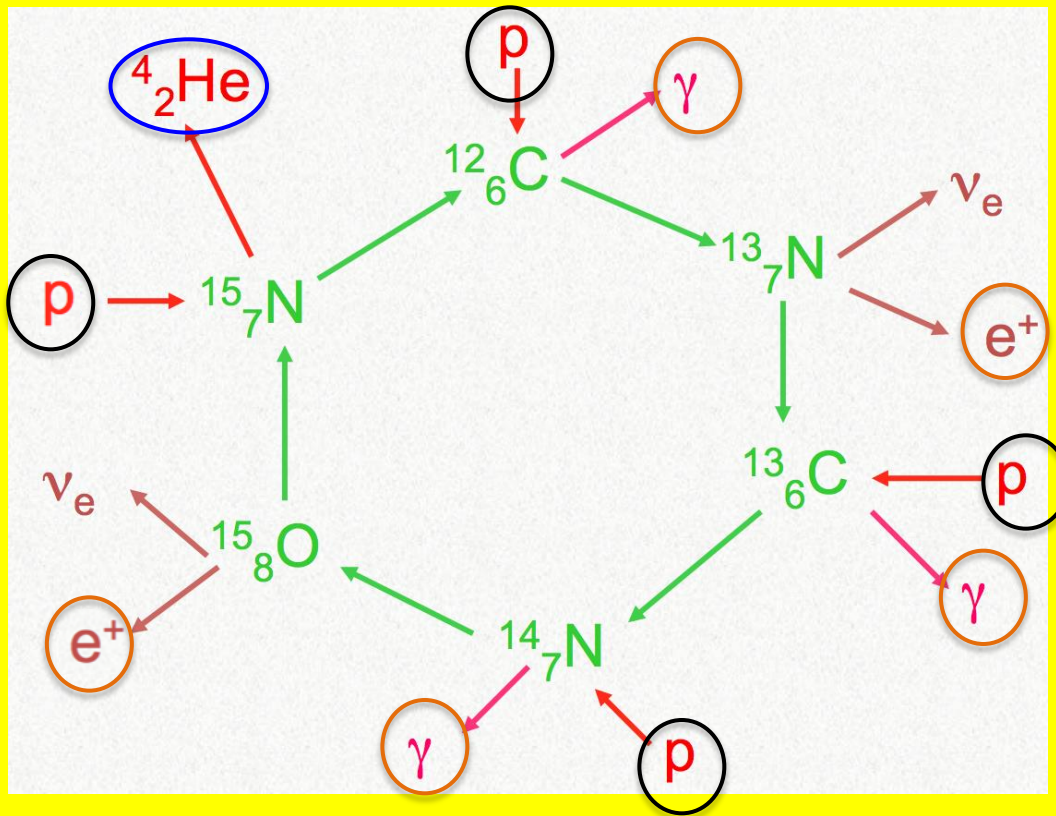


You'll calculate the details in HW problems, but for now:

- $L_{\odot} = 4 \times 10^{26} \text{ J/s}$
- 1 pp cycle yields  $\Delta m(4\text{H} - {}^4\text{He}) = (6.68 - 6.63) \times 10^{-27}$   
 $= 5 \times 10^{-29} \text{ kg} \times c^2 = 5 \times 10^{-29} \times 9 \times 10^{16} \text{ m}^2/\text{s}^2$   
 $= 4.5 \times 10^{-12} \text{ J}$

→ every second there are  $\sim 4 \times 10^{26} / 4.5 \times 10^{-12} \sim 10^{38}$  cycles

# The CNO Cycle



Net result:  $4p \rightarrow \text{He} + E$   
(neutrinos escape!)

Why is the required temperature higher than for the pp cycle?



## Equations of stellar structure

$$\frac{dm(r)}{dr} = 4\pi\rho(r)r^2,$$

$$\frac{dP(r)}{dr} = -\frac{Gm(r)}{r^2}\rho(r),$$

$$\frac{dT(r)}{dr} = -\frac{3\rho\kappa L(r)}{16\pi a c r^2 T^3},$$

$$\frac{dL(r)}{dr} = 4\pi r^2 \varepsilon(r, T, p, \rho, CH) \cdot \rho.$$

interior boundary conditions:

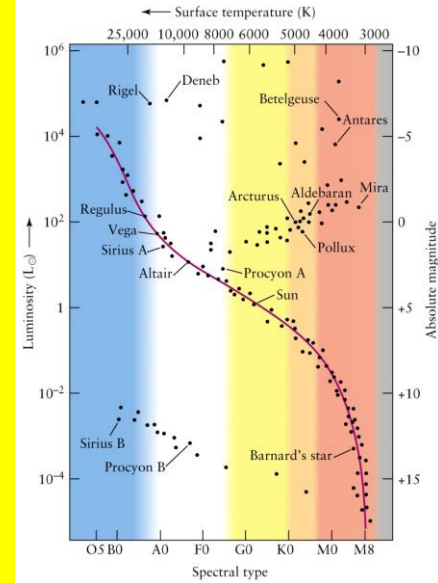
$$r = 0, \quad m(r) = 0, \quad L(r) = 0$$

surface boundary conditions:

$$\text{at } r = R, \quad P = 0, \quad T = 0$$

# Significance of the Main Sequence

- Stellar models confirm:  
L and T of MS stars are precisely what is predicted if stars fuse H into He in their cores
- Most stars are observed on the MS precisely **because** the MS phase is the longest-lasting phase in a star's life



# The virial theorem

$$GPE = - \int_0^M \frac{Gm(r)dm(r)}{r} = -\frac{3}{5} \frac{GM^2}{R}$$

$$KE = N \frac{3}{2} kT = \frac{M}{\bar{m}} \frac{3}{2} kT$$

$\bar{m}$  - mean particle mass

$$KE = -\frac{1}{2} GPE$$

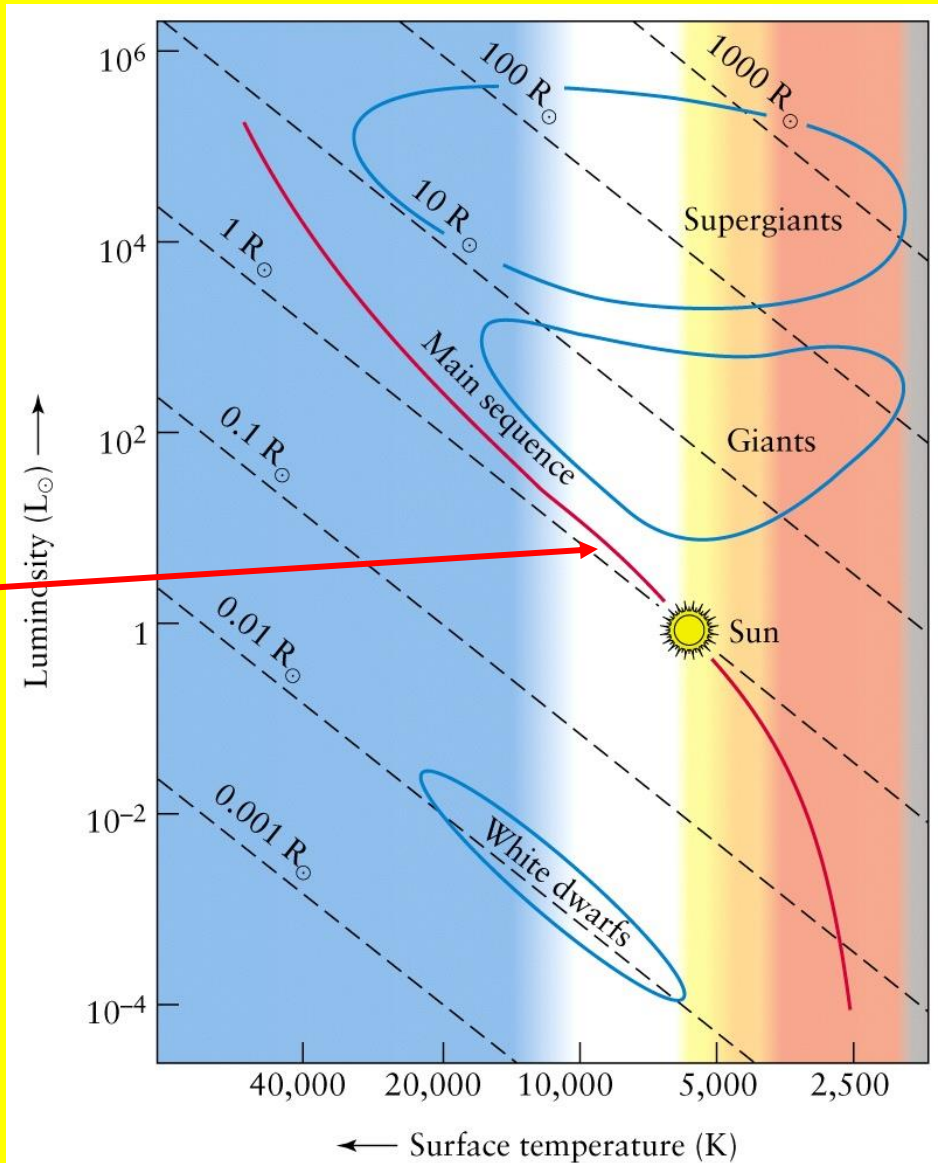
# The Main Sequence

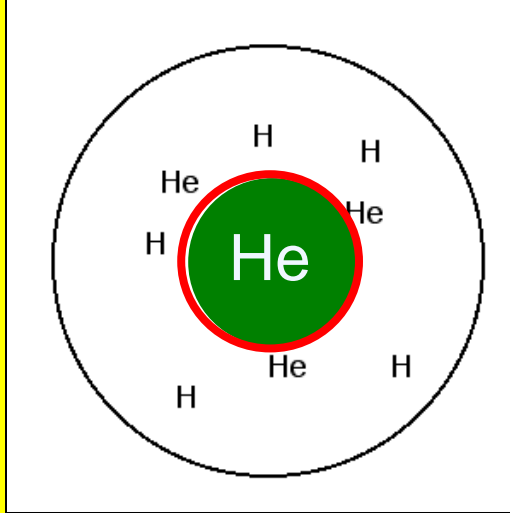
$$L \propto R^2 T^4$$

90% of stars are on  
or near MS

~10% WD

tiny fraction





After core H is depleted

Consider  $P$  vs.  $G$ :

$P$  from H core fusion now = 0, so...

Core contracts, releasing GPE.

Recall: General rule from *Virial Theorem*:

- a.  $\frac{1}{2}$  released GPE heats gas
- b.  $\frac{1}{2}$  released GPE is radiated

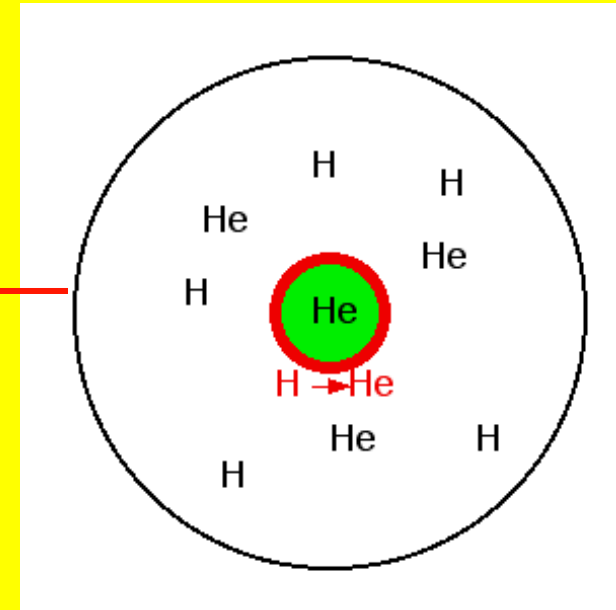
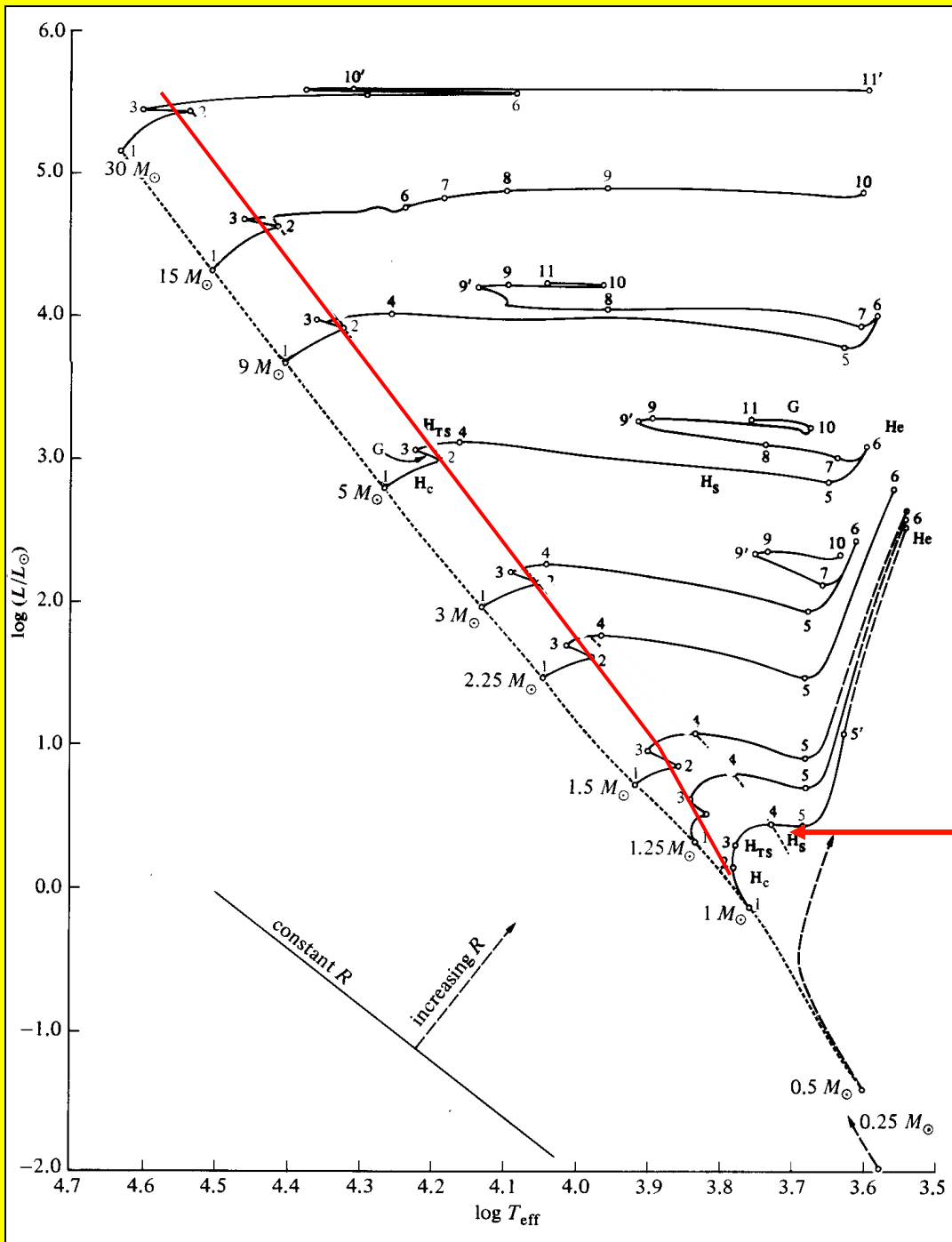
a.  $\rightarrow$  Increased  $T$  ignites H in a shell *around* core

b.  $\rightarrow$  Star's luminosity increases



# 1 $M_{\odot}$ star

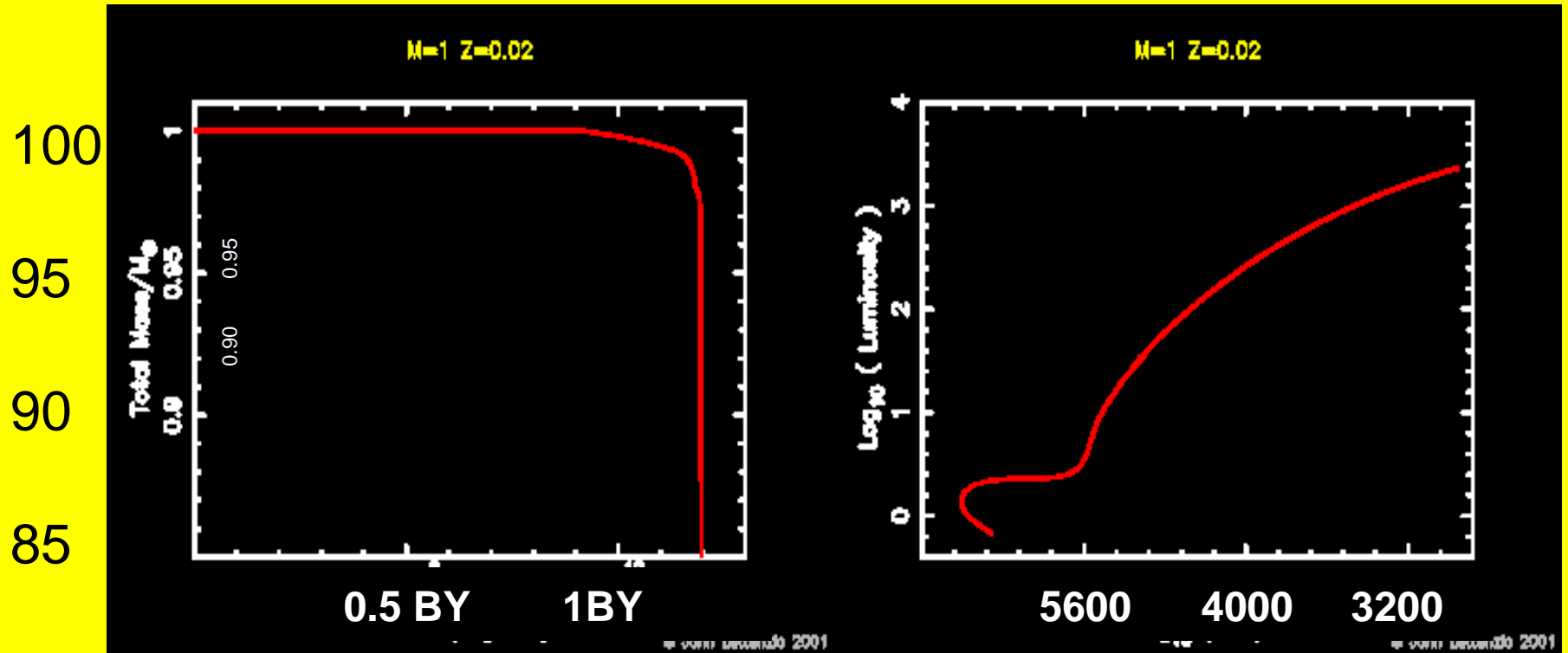
The star is now a **subgiant**, on its way to becoming a **red giant**



# Mass loss as a red giant (1 $M_{\odot}$ )

% of original mass left

HR Diagram

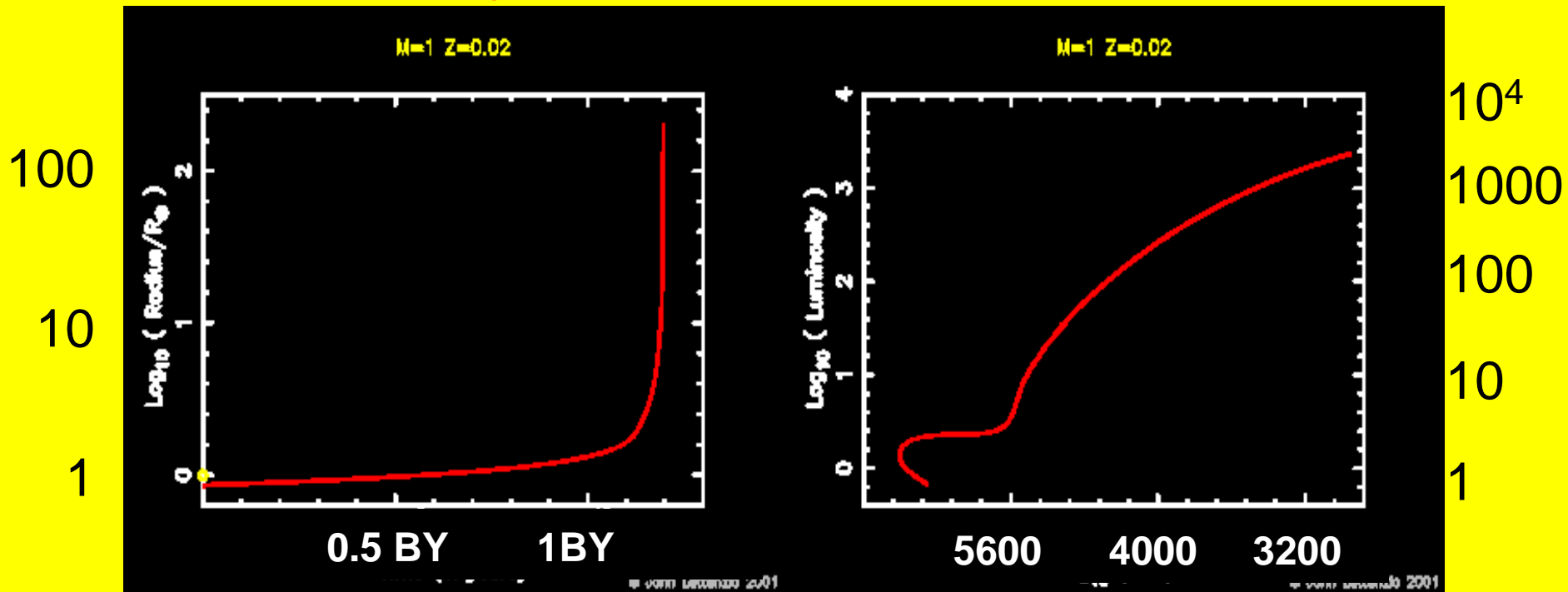


Note larger range in  $\log L$   
Compared with previous HRD

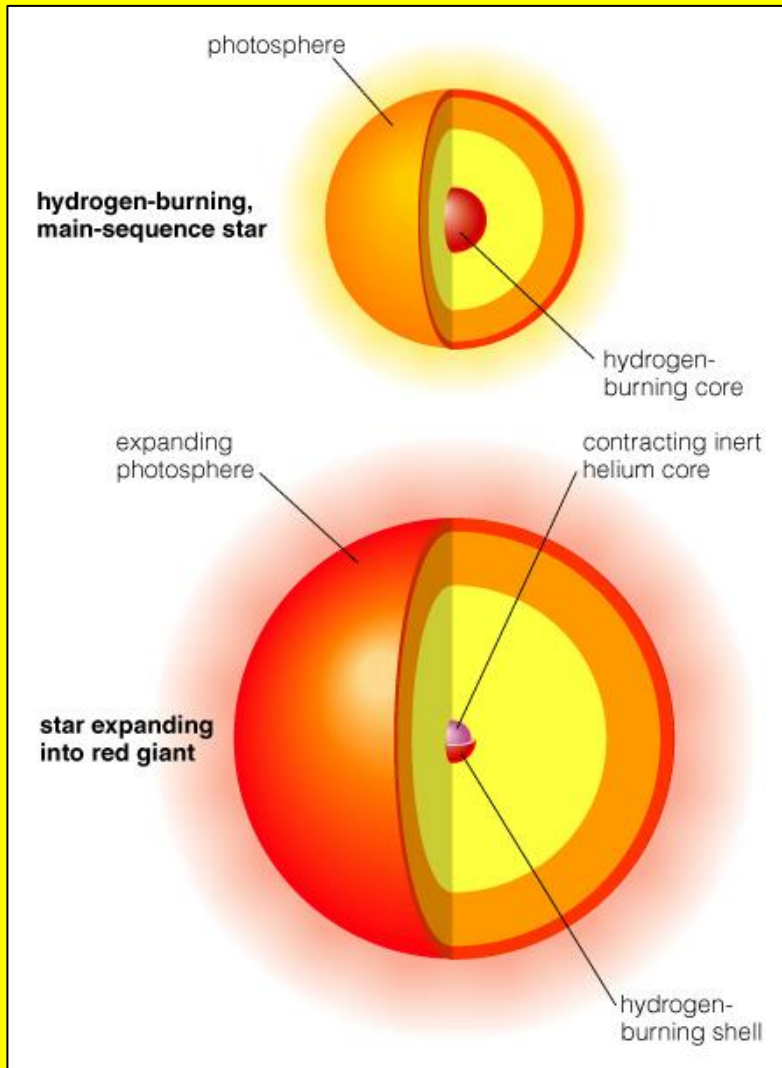
# R increases dramatically on the RGB (1 M<sub>⊙</sub>)

R/current R<sub>⊙</sub>

log (L/current L<sub>⊙</sub>)



# Broken Thermostat



- **As core contracts**,  $H \rightarrow He$  in shell around core, raising  $T_{\text{shell}}$ , accelerating fusion rate in shell
- $L_{\text{shell}}$  increases: core thermostat is **broken**: increasing shell fusion rate does **not** stop core from contracting and heating further
- Eventually  $T$  reaches He fusion temperature

# "to flash, or not to flash...."

- Lower-mass stars ( $< 2 M_{\odot}$ ):

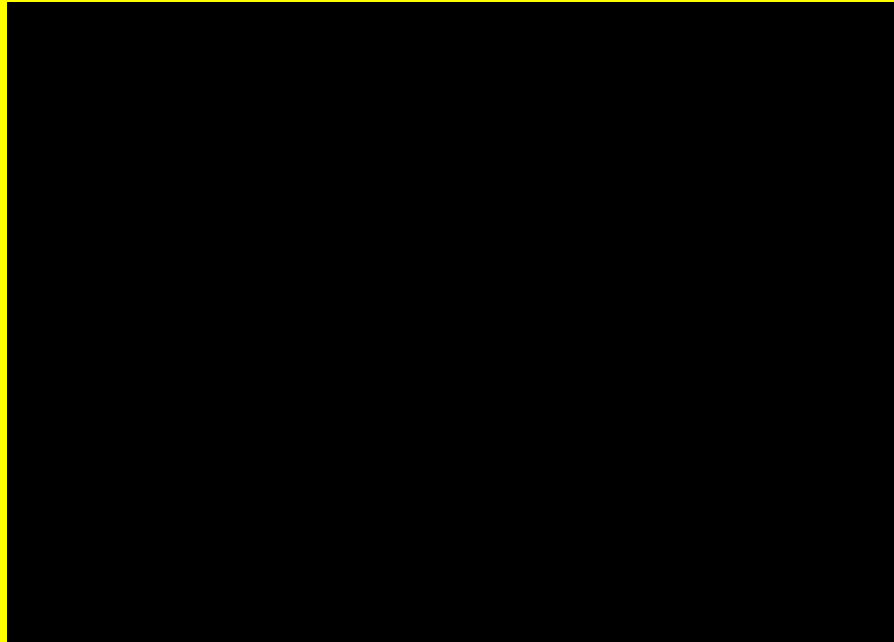
At He ignition, core density so high, electrons are **degenerate**:

- **Ideal gas:  $P \sim \rho T$**       • **Degenerate gas:  $P \sim \rho^x \Rightarrow P \neq f(T)$ !**
- Heating does not expand the gas in the core, but merely raises  $T$ , increasing reaction rates.
- Explosive reaction ensues - "Helium Flash" - core *briefly* reaches  $\sim 10^{11} L_{\odot}$  (though none reaches surface; expansion).
- Degeneracy removed and "normal" He-burning follows.

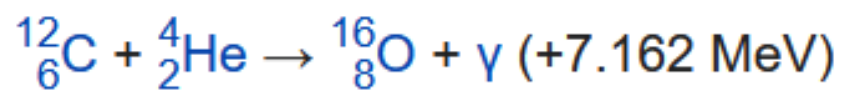
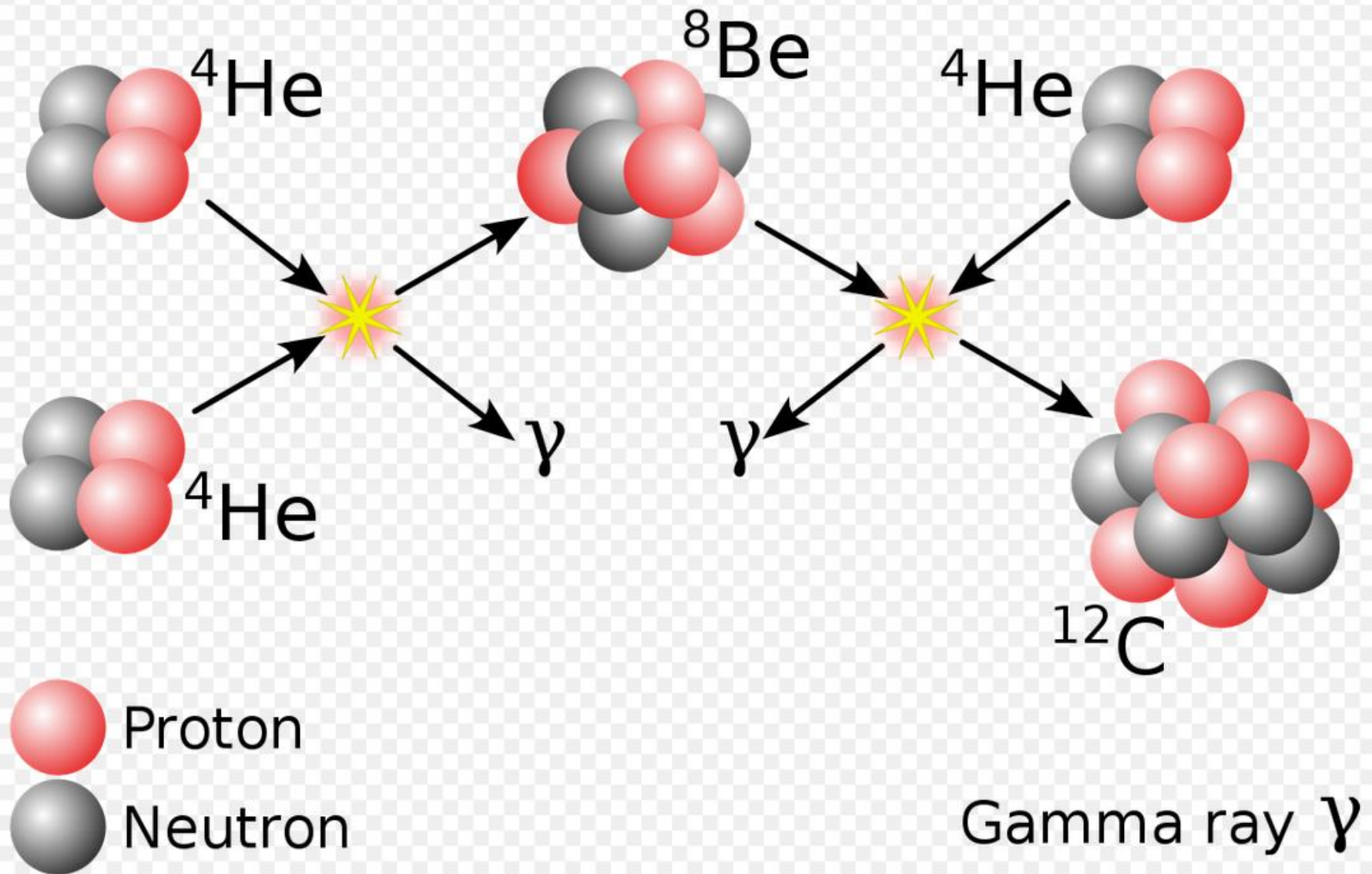
- Higher mass stars ( $> 2 M_{\odot}$ ): At He ignition, core is sufficiently low-density to be non-degenerate (radiation supplies pressure).



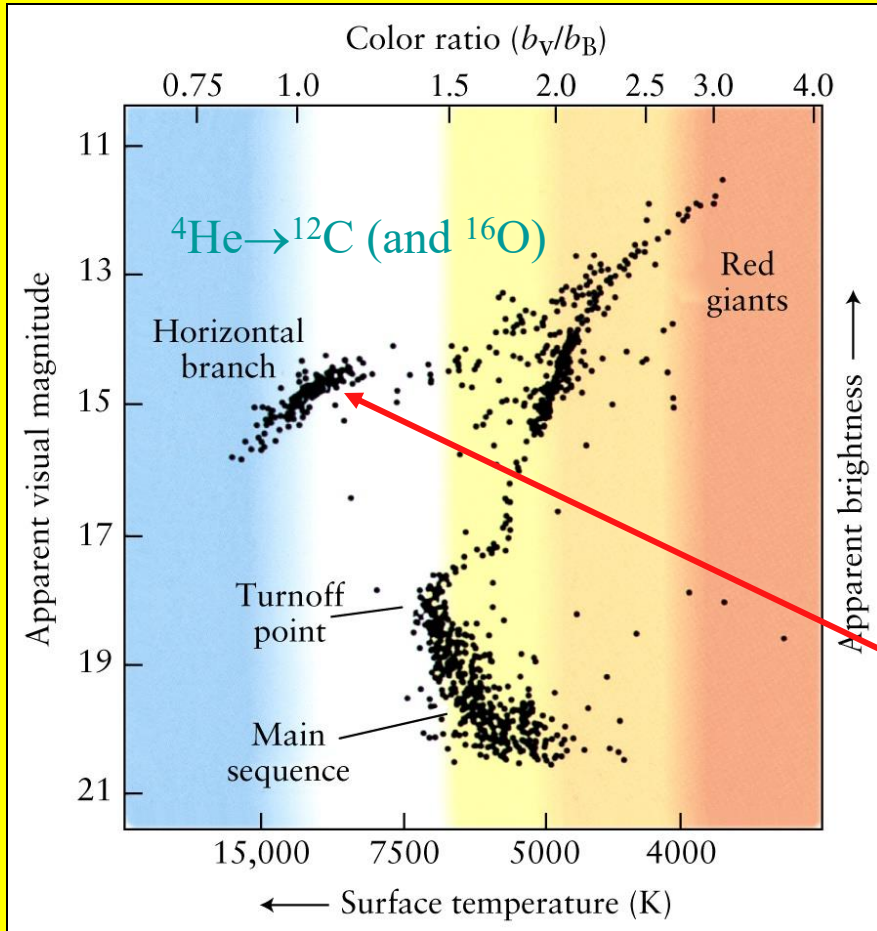
# Triple-Alpha Process



- The intermediate product  ${}^8\text{Be}$  is very unstable, and will decay if not immediately struck by another  ${}^4\text{He}$ . Thus, this is almost a 3-body interaction, as shown.
- There is a very strong temperature dependence. A 10% increase in  $T$  increases the energy generation by a factor of 50!

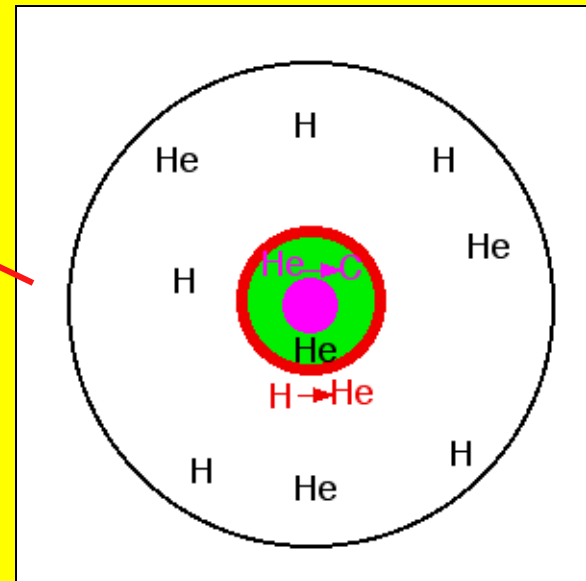


# Horizontal Branch (core He fusion)

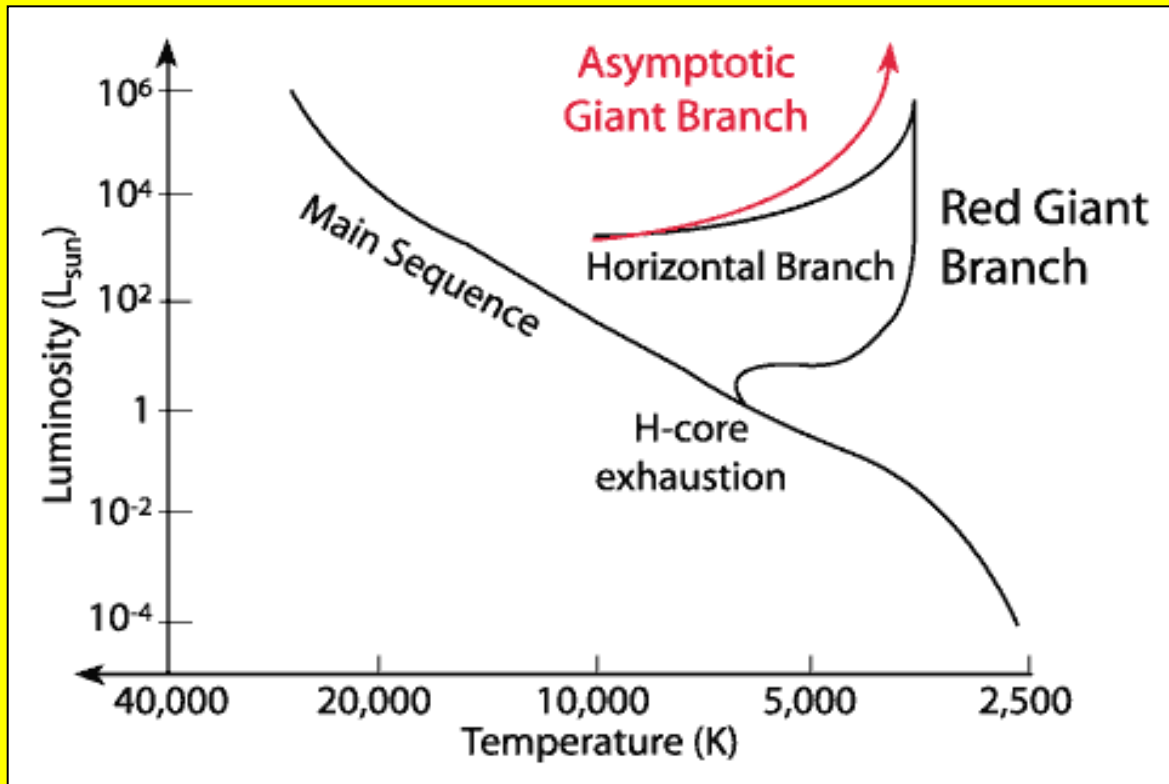


Globular cluster HRD

After the He flash, star fuses He quiescently in its core; star now on the *horizontal branch*

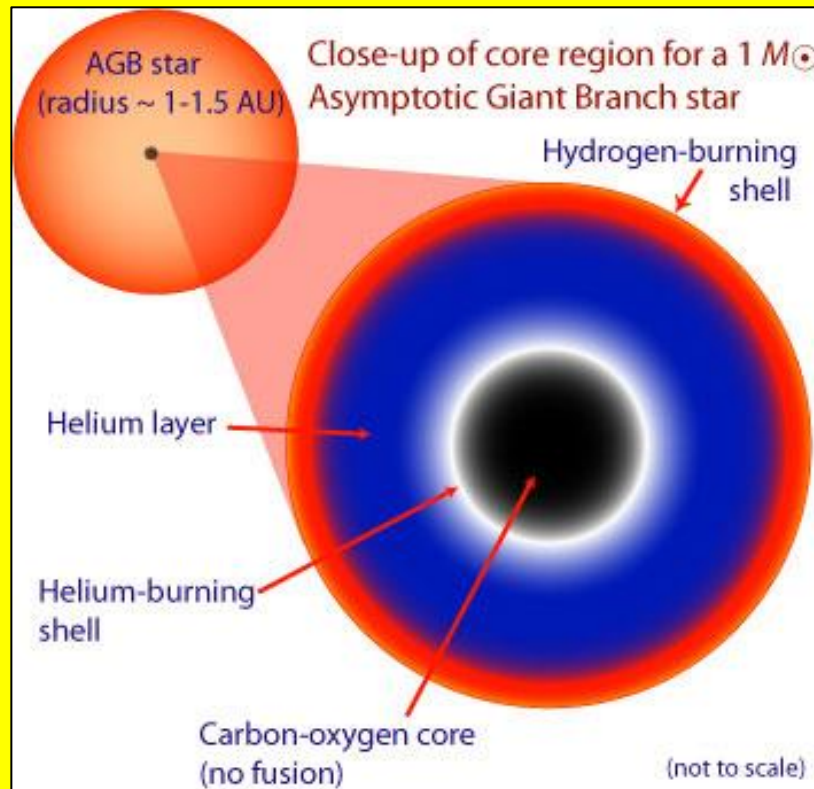


# What we mean by "branches" on the HRD



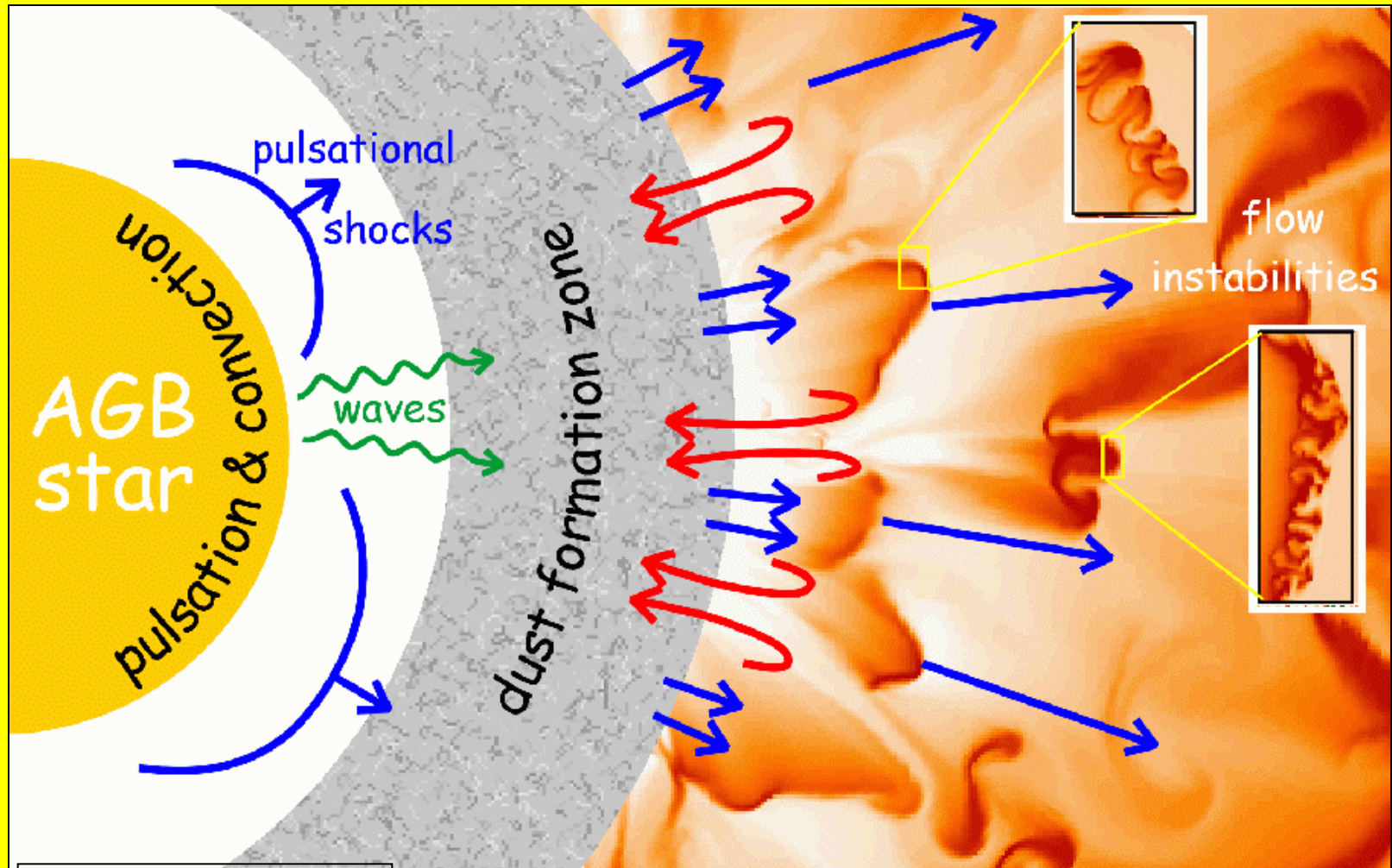
- **Red Giant Branch** stars have a contracting core and a hydrogen-burning shell.
- **Horizontal Branch** stars have a helium-burning core and a hydrogen-burning shell.
- **Asymptotic Giant Branch** stars have a contracting core, a helium-burning shell and a hydrogen-burning shell.

# AGB Star Core





# Region Around an AGB Star

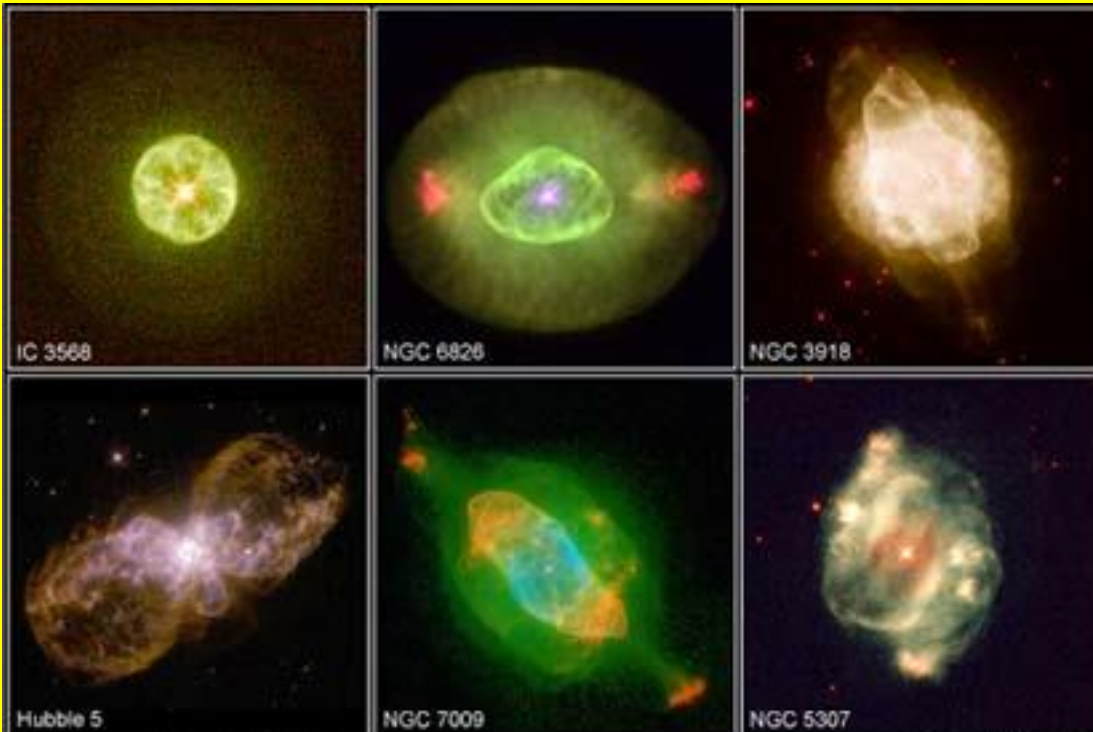


# Results of AGB Thermal Pulsing



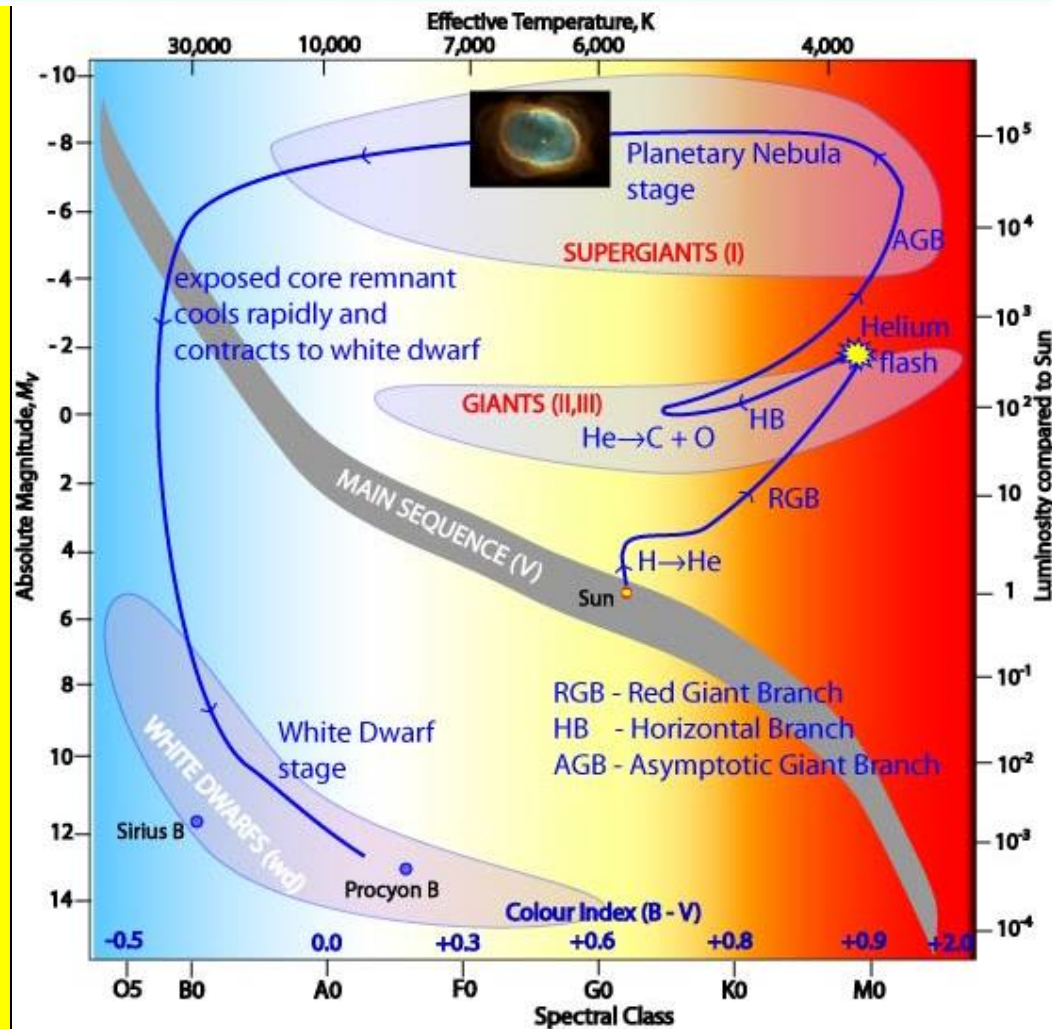
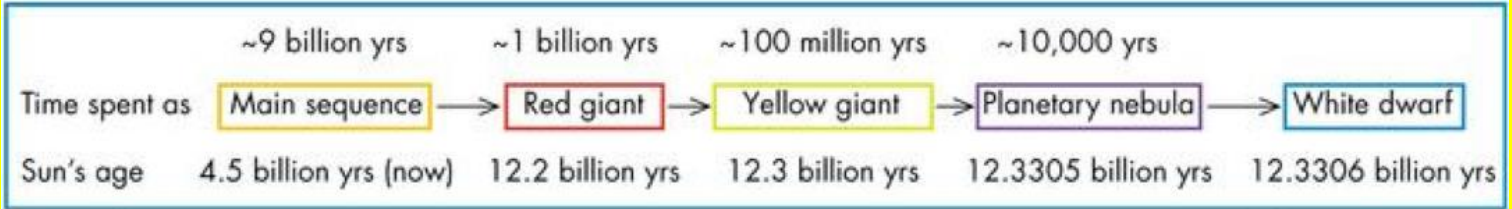
- More convective dredge-up, this time of C, s-process
- Superwind & shell ejection

[Planetary Nebula Gallery](#)





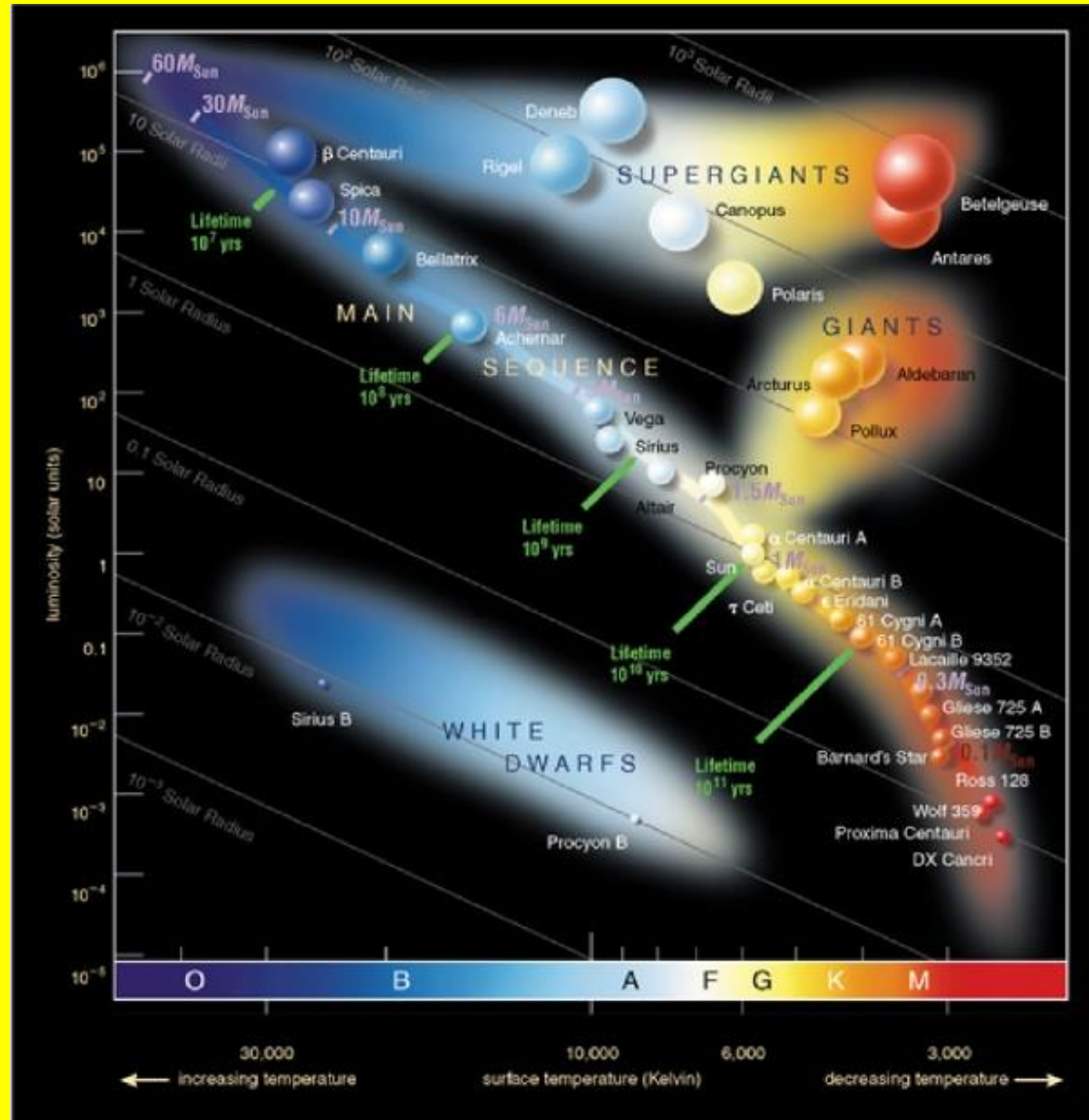
# Summary: $1 M_{\odot}$ Evolutionary Track





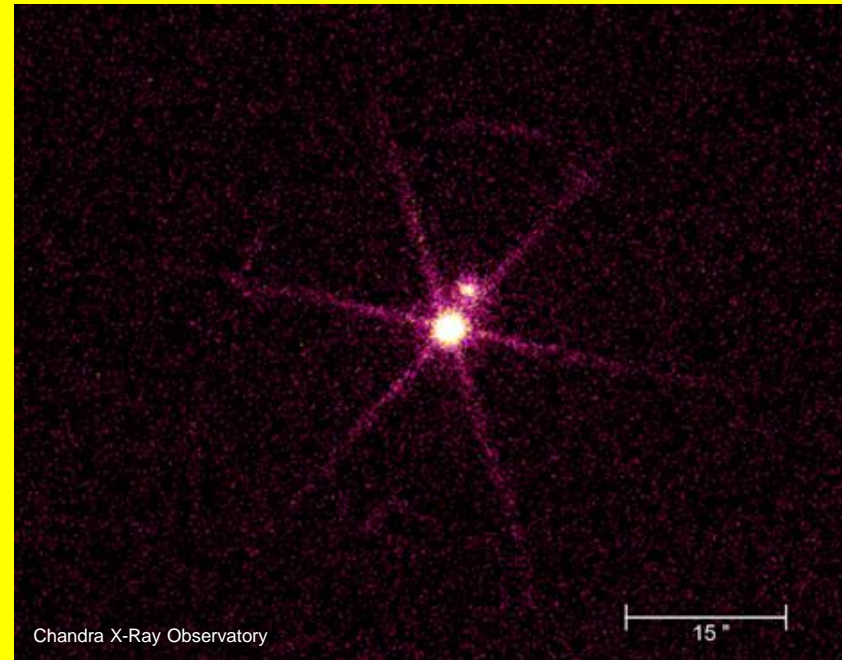
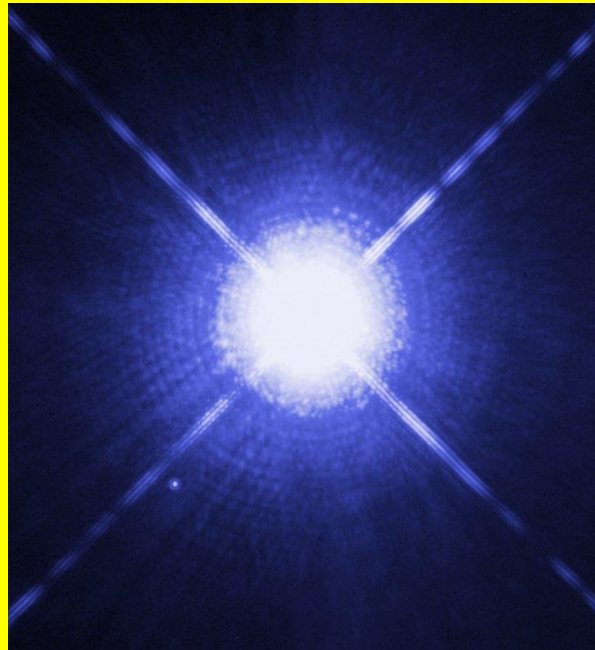
# White Dwarfs on the HR Diagram

- small
- hot
- faint





# Sirius B is the closest white dwarf (d~2.4 pc)



**TABLE 20.2 Sirius B, a Nearby White Dwarf**

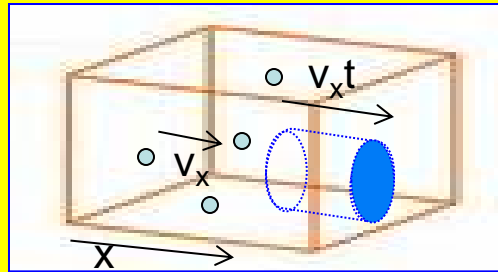
Mass	1.1 solar masses
Radius	0.0073 solar radius (5100 km)
Luminosity (total)	0.025 solar luminosity ( $9.8 \times 10^{24}$ W)
Surface temperature	27,000 K
Average density	$= 4.0 \times 10^9$ kg/m <sup>3</sup>

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$$\rho = \frac{1.1 \times 2 \times 10^{30} \text{ kg}}{\frac{4}{3} \pi \times (5.1 \times 10^6 \text{ m})^3}$$
$$= 4 \times 10^9 \frac{\text{kg}}{\text{m}^3}$$

water at STP =  $10^3$  kg/m<sup>3</sup>

# Degeneracy Produces Gas Pressure



We know all atoms are completely ionized:  $n_e = Zn_Z$

atomic number,  $Z$

number density of atom  $Z$

We also know:  $\rho = Am_p n_Z$  so solve:  $n_Z = \frac{\rho}{Am_p}$  (We are ignoring  $m_e$  and the  $p$ - $n$  mass difference.)

mass density of  $Z$       nuclear mass

now substitute for  $n_Z$ :  $\rho n_e = \frac{\rho Z}{A m_p}$  ...and we saw that  $P_{\text{deg}} \sim \frac{n_e^{5/3}}{m_e}$

$$\rho P_{\text{deg}} \sim \frac{\rho Z^{5/3}}{A} \frac{1}{m_e} \frac{\rho^{5/3}}{m_p^{5/3}}$$

i.e.,  $P_{\text{deg}} \propto \rho^{5/3}$  for a non-relativistic degenerate gas

**$P$  does not depend on  $T$  !**

# Origin of the WD Mass-Radius Relationship

From hydrostatic equilibrium:  $\frac{\Delta P}{\Delta R} = \frac{GM}{R^2} \rho$  (ignoring minus sign)

Let  $\Delta P = P$  and  $\Delta R = R$ , and  $r = \frac{M}{\frac{4}{3}\rho R^3}$  so we have:  $P = \frac{GM^2}{\frac{4}{3}\rho R^4} \longrightarrow P \propto \frac{M^2}{R^4}$

For a degenerate *non-relativistic* gas we saw  $P \propto r^{5/3}$  (back a few slides)

We also know  $r \propto \frac{M}{R^3}$ .

Equating these:  $\frac{M^2}{R^4} \propto \frac{M^{5/3}}{R^5}$ , which yields  $R \propto M^{-1/3}$

This is the Mass-Radius relation for white dwarfs...

...but it can't be true for all masses:

When electrons become relativistic, EOS changes.

For a degenerate *relativistic* gas:  $P \propto r^{4/3}$  (not 5/3)

So we get:  $\frac{M^2}{R^4} \propto \frac{M^{4/3}}{R^4}$  which has no solution for R

With the appropriate coefficients, this represents the maximum WD mass:

the **Chandrasekhar Limit ( $\sim 1.44 M_{\odot}$ )**

**This is the maximum mass that degenerate electron pressure can support against gravity.**



# White Dwarfs

- Very small ( $R \sim 6000 \text{ km}$ )
- Very hot ( $T \sim 10000 - 200000 \text{ K}$ )
- Very dense ( $\langle \rho \rangle \sim 10^9 \text{ kg/m}^3$ )
- No internal energy sources !
- $M_{\text{WD}} \sim 0.6 M_{\odot}$
- Composition He or C,O
- $M_{\text{max}} \sim 1.4 M_{\odot}$