External Physical Processes Influencing the Evolution of Star Clusters
Lecture VII

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Star Cluster Dynamics and Evolution

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Residual Gas Removal

Credits: Grudic 2022

Residual Gas Removal

Residual gas is gas that was not transformed into stars during the star formation process.

The following physical processes are responsible for the removal of residual gas:

- Energy injected by OB stars
- SNe explosions
- Jets from accreting massive objects

$$\eta = \frac{M_{stars}}{M_{gas} + M_{stars}} \qquad - \ SFE, star\ formation\ efficiency$$

Observation suggests that for embedded star clusters SFE is as low as 10-20% (Lada & Lada 2003). Bound star clusters have SFE around 30%

Estimated SFE for the whole Giant Molecular Clouds (GMC) are around 1-5%

Residual Gas Removal

Simple estimation of SFE

Before gas removal protostars and gas are in virial equilibrium

$$v^2 = \frac{G(M_{star} + M_{gas})}{R_0}$$

Gas removal is very fast, so the velocities of protostars are not changing. The energy of the cluster is

$$E_r = \frac{1}{2} M_{star} v^2 - \frac{G M_{star}^2}{R_0}$$

Energy after virialization has to be equal energy after gas removal

$$-\frac{G M_{star}^2}{R} = \frac{1}{2} M_{star} v^2 - \frac{G M_{star}^2}{R_0}$$

Residual Gas Removal

Using definition of v^2 we get

$$-\frac{M_{star}}{2R} = \frac{1}{2}\frac{(M_{star}+M_{gas})}{R_0} - \frac{M_{star}}{R_0}$$

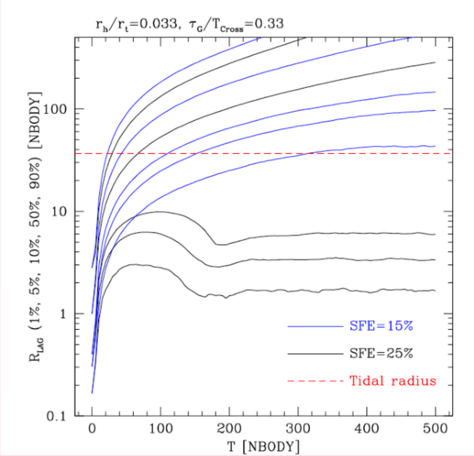
$$R = R_0 \frac{M_{star}}{M_{star}+M_{gas}} \frac{1}{2\frac{M_{star}}{M_{star}+M_{gas}} - 1}$$

It is easy see that $R > 0$ only if

$$\frac{M_{star}}{M_{star}+M_{gas}} > 0.5$$

Residual Gas Removal

Credits: Baumgardt & Kroupa 2008



Conclusions from simulations and observations

- Dependence on SFE
  - Dissolution - SFE < 30%
  - Survival - SFE > 30 %
- Dependence on gas expulsion timescale
  - $\tau/T_{cr} < 1$ - instantaneous gas removal
  - $\tau/T_{cr} > 1$ - adiabatic gas removal
- Ratio of half-mass radius to the tidal radius
  - High - cluster is easy to destroy
  - Small - cluster survive
- Dependence on metallicity
  - Metal poor clusters are more compact than metal reach clusters

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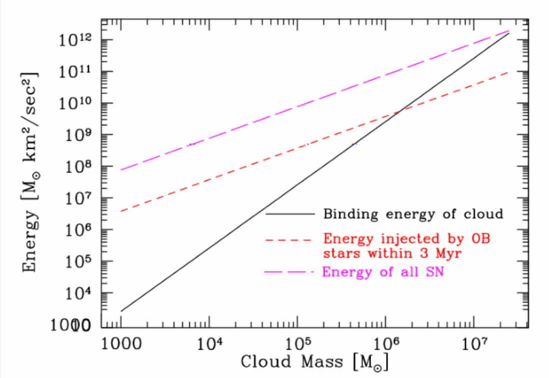
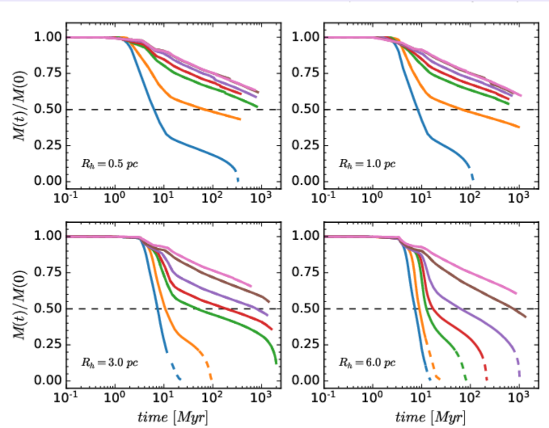
Residual Gas Removal

Credits: Baumgardt + 2008



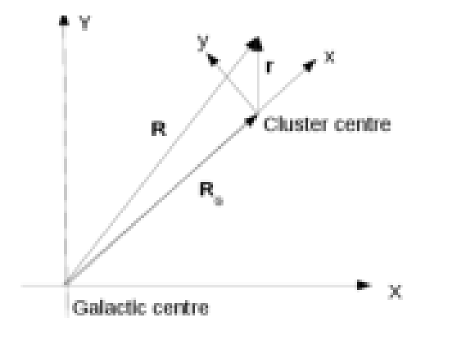
Credits: Leveque + 2023



SFE = 0.1, 0.2, 0.33, 0.4, 0.5, 0.7, 1.0

External Forces - Galactic Tides

Credits: Heggie 2017



Cluster is moving on a circular orbit of radius R_G about the Galactic Center, with angular speed $\omega(R_G)$

Let's introduce a rotating, accelerating frame of reference with origin at the center of the cluster, x-axis pointing away from the Galactic Center, y in the direction of cluster motion about the Galactic Center

The rotating reference frame is not inertial, so pseudo forces act in this frame

Credits: Heggie 2017

The equation of motion in the rotating reference frame in the case when the R_G is much larger than $\mathbf{r}$ is as follows

$$\ddot{\mathbf{r}} + 2\omega \times \mathbf{v} + \omega^2 z\mathbf{e_z} + 2R_G\omega\omega' x\mathbf{e_x} = -\nabla \varphi_c(\mathbf{r})$$

where: $\varphi_c(\mathbf{r})$ is potential due to cluster stars, $\mathbf{e_x}$ and $\mathbf{e_z}$ are unit vectors in the x and z direction respectively. ω is the modulus of the angular speed of the cluster about the Galaxy, and ω is its angular velocity vector orthogonal to the plane of motion of the cluster

$2\omega \times \mathbf{v}$ - Coriolis force

$\omega^2 z\mathbf{e_z}$ - Centrifugal force

$2R_G\omega\omega' x\mathbf{e_x}$ - Euler force

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The equation of motion can be written in the following way:

$$\ddot{\mathbf{r}} + 2\omega \times \mathbf{v} = -\bigtriangledown \varphi_{eff}(\mathbf{r})$$

where the effective potential is:

$$\varphi_{eff}(\mathbf{r}) = \varphi_c(\mathbf{r}) + R_G \omega \omega' x^2 + \frac{1}{2}\omega^2 z^2$$

The energy, so-called Jacoby energy is a conserved quantity

$$E_J = \frac{1}{2}v^2 + \varphi_{eff}(\mathbf{r})$$

$\varphi_{eff}(\mathbf{r}) \leq E$ - a star may be confined by equipotentials of φ_{eff}

External Forces - Galactic Tides

Credits: Baumgardt 2015

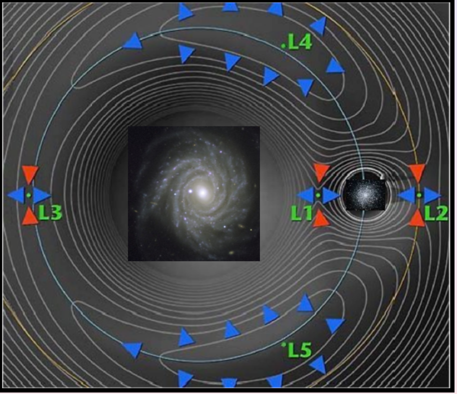
Contour-lines of constant effective potential ϕ_{eff} for two orbiting point-masses:

L1-L3:

Saddle points

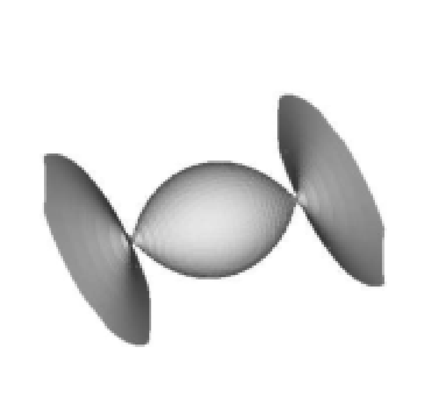
L4 and L5:

Maxima



Tidal radius for point mass Galaxy - distance to the points L_1 and L_2 is: $r_t = (M_c/3M_g)^{1/3}R$

Credits: Heggie 2017



The critical Jacoby energy needed for escape is given by: $E_{crit} = -(3GM_c/2r_t$

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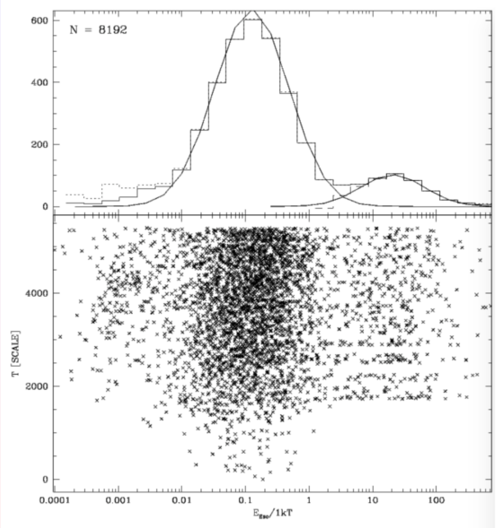
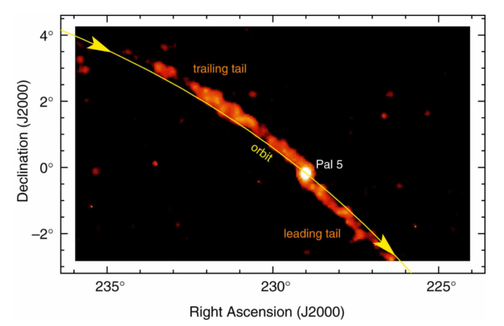
External Forces - Galactic Tides

Credits: Baumgardt 2002





Bimodal energy distribution. Slow escapers connected with relaxation. Fast escapers connected with dynamical interactions
Coriolis force deflects stars and they follow the orbit of the cluster, leading to the formation of tidal tails

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External Forces - Galactic Tides

Primordial escapers - stars with energy greater than the critical energy, $E_{crit} = -1.5GM_c/r_t$

- There are about 10% stars which are potential escapers during the evolution of the cluster
- Timescale for escape is not anymore proportional to the half-mass radius relaxation time, T_{rh}, but to $N^{-1/4}T_{rh}$, Baumgardt (2001)
- Potential escapers may look like high-speed cluster members
- Models which ignore them can't describe cluster accurately

Clusters in eccentric orbits lose stars, on average per orbit, at the same rate as clusters in circular orbits of intermediate radius - Cai + 2016

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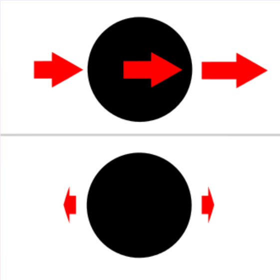
External Forces - Tidal Shocks

There are two types of tides

- Already discussed STEADY TIDES

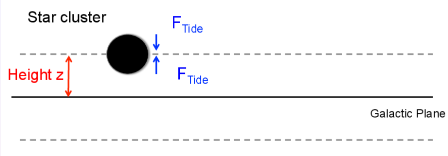


$$F_{tide} = \frac{GMm}{(R + \triangle r)^2} - \frac{GMm}{R^2}$$

$$F_{tide} = \pm \frac{2 \triangle rGMm}{R^3}$$

- Tidal shocks occur when star clusters pass near a molecular cloud, move through the disc of a spiral galaxy or move in eccentric orbits. Gravitational potential is a function of time

Disk Shocks - Spitzer 1987



Gravitational potential of the disk can be written as:

$$\frac{d^2\phi_d}{dZ^2} = 4\pi G\rho_d(Z)$$

Where Z is the distance form the Galactic plane. V is the cluster velocity, which is much larger than velocities of stars in the cluster. The gravitational acceleration is:

$$g(Z) = -\frac{d\phi_d}{dZ}$$

Let's assume that $z = Z - Z_c$, where Z_c is the distance to the cluster center, and $dz/dt = v_z$. Then the relative acceleration of a star is:

$$\left(\frac{dv_z}{dt}\right)_d = g(Z) - g(Z_c) = z\frac{dg}{dt}(Z_c)$$

dg/dZ is negative so $(dv_z/dt)_d$ is negative for positive "z" and positive for negative "z"

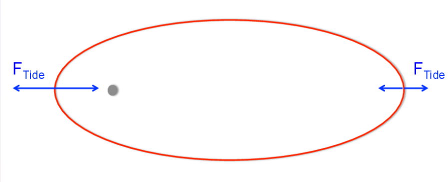
Compressive Shocks

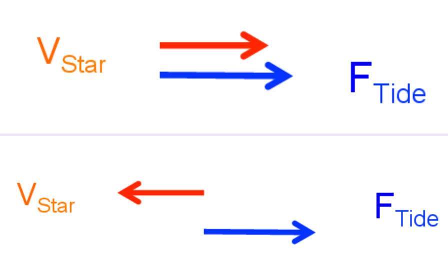
Assuming impulsive approximation (stars are motionless) and $Z_c = Vt$ we can get from the above equation $\triangle v_z = z \triangle g/V = -2zg_m/V$. So the energy change is:

$$(\triangle E)_{Av} = \frac{2z^2 g_m^2}{V}$$

Cluster is heated by Compressive Shocks

Eccentric Orbit - Credits: Baumgardt 2015





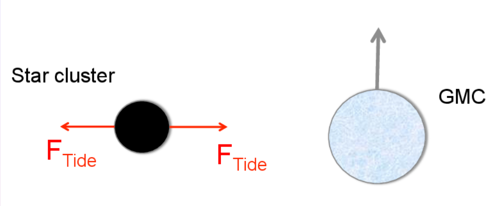
Stars moving in the direction of the tidal force will move faster by an amount of δv. Stars moving in the opposite direction will slow down by by an amount of δv

There is no analogue to the Jacoby energy

$$\triangle E = \frac{1}{2}m(v+\delta v)^2 + \frac{1}{2}m(-v+\delta v)^2 - 2\frac{1}{2}mv^2$$

$$\triangle E = m\delta v^2 > 0$$

Molecular Clouds Shocks - Spitzer 1987



Impulsive approximation- cluster velocity much larger than the velocity of stars in the cluster. Position of stars in not changed during the interaction

$$F_x = \frac{2xM_p}{R^3}, \quad F_y = -\frac{yM_p}{R^3}, \quad F_z = -\frac{zM_p}{R^3} \qquad (1)$$

where M_p is the perturber mass and R is the distance to the perturber

Straight line approximation: $R^2 = p^2 + (Vt)^2$, where p is the impact parameter
After integrating Eq.(1) over time we get:

$$\frac{\triangle v_x}{x} = -\frac{\triangle v_z}{z} = \frac{2GM_p}{p^2 V}$$

The average gain in the kinetic energy is:

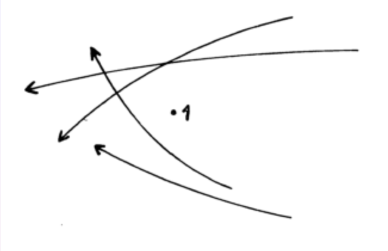
$$(\triangle E)_{Av} = \frac{(x^2+z^2)}{2}\left(\frac{2GM_p}{p^2V}\right)^2 \approx \frac{r_m^2}{3}\left(\frac{2GM_p}{p^2V}\right)^2$$

where cluster characteristic radius is equal to $r_m^2/3 = (x^2+z^2)/2$

Gain of the kinetic energy does not depend on mas of a star

External Forces - Dynamical Friction

Credits: Binney & Tremaine 1987



If v_M is small then $f(v_m) = f(0)$. We can assume for $f(v_m)$ Maxwellian distribution and $\rho(r) = n(r)m$ Let's assume that the galaxy is well approximated by isothermal sphere then

$$\rho(r) = \frac{v_c^2}{4\pi G r^2}$$

$$\frac{v_M}{dt} = F = -\frac{16\pi^2 G^2 \ln(N)}{V_M^2}(M+m)\,m\int_0^{V_M} f_m V_m^2 dV_m$$

$$F = -0.428 ln\Lambda \frac{GM^2}{r^2}$$

Force is tangential, so cluster loses angular momentum

$$\frac{dJ}{dt} = r\frac{dr}{dt} = -0.428\frac{GM}{r} ln\Lambda$$

Integration will give the DF timescale in Gyr:

$$t_{fric} = \frac{1.17 r^2 v_c}{ln\Lambda GM} = \frac{3x10^2}{ln\Lambda}\left(\frac{r}{2kpc}\right)^2\left(\frac{v_c}{230km/s}\right)\left(\frac{10^6 M_o}{M}\right)$$

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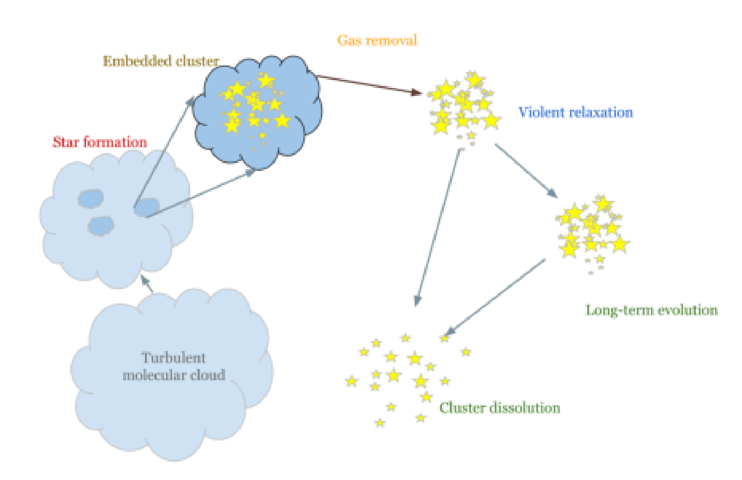
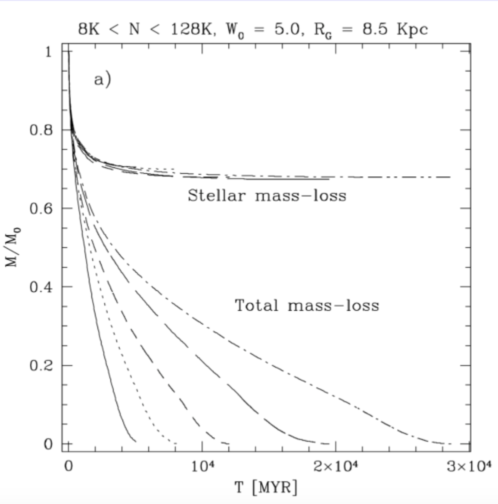
External Forces - Global Picture

Credits: Shukirgaliyev 2018

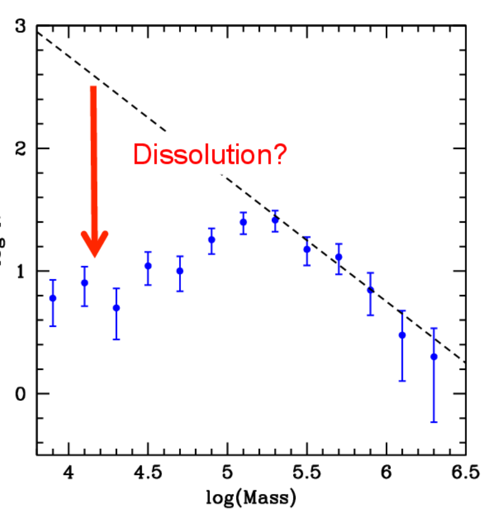




Credits: Baumgardt 2015

External Forces - Global Picture

Credits: Baumgardt 2015



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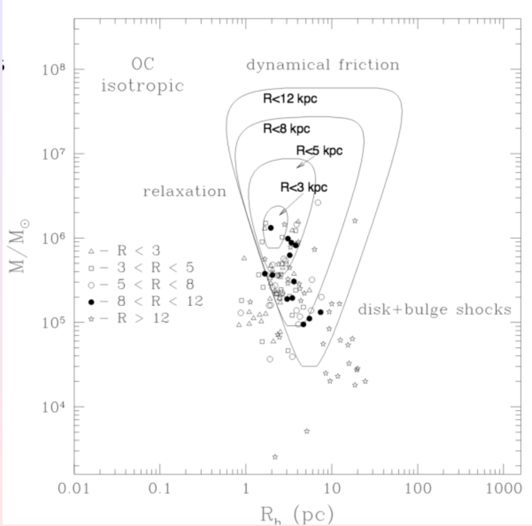
Initial Star Cluster Mass Function $N \sim M^{-2}$

Star Cluster Dissolution Processes

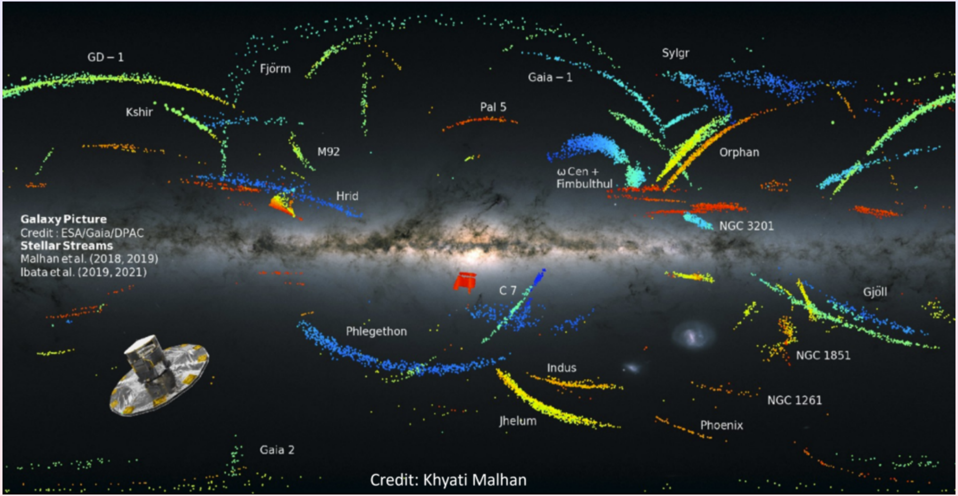
- Residual Gas Removal
- Relaxation
- Tidal Field
- Tidal Shocks
- Dynamical Friction
- Escape
- Black Hole Subsystem

External Forces - Global Picture

Credits: Gnedin & Ostriker 1997



External Forces - Global Picture



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External Forces - Short Summary

External Processes

- Steady Tides - gravitational potential constant in time, circular orbit around galaxy
- Tidal Shocks - gravitational potential is a time function
  - Eccentric orbit around galaxy
  - Interactions with Giant Molecular clouds
  - Passages through galactic plane
- Dynamical Frictions

Residual gas removal

All processes increase the rate of escape from globular clusters and may significantly shorten their lives

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